NSS Neutrosophic Sets and Systems, Vol. 77, 2025

University of New Mexico

Modeling uncertainties associated with decision-making algorithms based on similarity measures of possibility belief interval-valued fuzzy hypersoft setting

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Abstract. Hypersoft sets (HSSs) were initiated as an extension of soft sets (SSs) to address real-life scenarios involving multiple disjoint sets with different traits. One such extension is the interval-valued fuzzy hypersoft set (IVFHSS), which has proven effective in decision-making (DM). However, the IVFHSS model lacks a mechanism to incorporate the degree of acceptance of DM opinions, which is crucial for accurate decision-making. To overcome this limitation, our work aims to develop a novel hyperstructure called a possibility interval-valued fuzzy hypersoft set (PIVFHS-set). We begin by introducing essential operations and their properties, such as PIVFHS-subset, PIVFHS-null set, PIVFHS-absolute set, and complement of a PIVFHS-set. These concepts are illustrated through numerical examples to demonstrate their practical applications. Next, we delve into set-theoretic operations of PIVFHS sets, including union, intersection, AND, OR, and relevant laws. These operations are further elucidated through numerical examples, matrix representations, and graphical illustrations. Additionally, we present two algorithms based on AND and OR operations, providing step-by-step explanations and showcasing their effectiveness through illustrative examples. Furthermore, we introduce a similarity measure to facilitate pattern recognition in PIVFHS-sets, aiding users in recruitment processes. Alongside an analytical study of the advantages and disadvantages of this model, we provide suggestions for future research based on the identified limitations.

Keywords: Interval-valued fuzzy set; soft set; hypersoft set; interval-valued fuzzy hypersoft set; similarity measures; decision-making ; possibility interval-valued fuzzy hypersoft set.

1. Introduction

Dealing with situations that contain ambiguity and uncertainty posed a challenge for scholars, which prompted them to work continuously to develop mathematical tools that have the ability to efficiently deal with such situations. Scholars have already created and developed several mathematical models to address such ambiguity and uncertainty. One of these models is the fuzzy set (FS), which Zadeh [1] put forth in 1965 after deconstructing crisp set theory. The FS focuses on highlighting the true belongingness of an object entity in the initial sample space. In other words, based on FS structure, every object x in a nonempty universal set can be represented by a single membership truth function. This structure has some obstructions when dealing with some life situations (i.e., difficulty representing life problem data with a single degree). To handle this inherent difficulty, Turksen reorganises the FS structure to an interval-valued FS (IVFS) [2] by giving an interval grade to every object x in a nonempty universal set.

The theories of FS and IVFS have been widely developed and studied by means of numerous scholars, and they have employed these tools for treating uncertain information and handling realistic Multiple-criteria decision analysis (MCDA) issues. Riaz et al. [3] put forward a freethinking extension of FSs called Bipolar Picture (BPFSs) and presented some basic operations. Zulqarnain et al. [4] presented a new approach to MCDA based on the interval-valued Pythagorean fuzzy set as a generalisation of FS and IVFS. Bustince et al. [5] initiated a new class of similarity measures between two IVFSs and used these tools in stereo image matching.Ramadhani et al. [6] defined a signless Laplacian matrix on IVFS and employed it to solve multi-criteria decision making (MCDM).

Later, the researchers pointed out the need for a parametric environment that supports these tools and helps in giving more clarification of the elements of universal sets. To satisfy this void, Molodtsov [7] presented a new parameterization tool called soft set (SS). Molodtsov opened the doors for scholars around the world to develop and deeply study SSs with other fuzziness models.

Saeed et al. [8] introduced a new approach to SSs with soft members and soft elements. Azzam et al. [9] discussed the approaches of soft topological spaces like soft closure, soft interior, soft exterior, soft boundary, or soft derived set operators. Maji et al. [10] introduced the structure of fuzzy soft sets (FSSs). Jiang et al. [11] defined the distance and entropy measures between two interval-FSSs. Al-Shami et al. [12] produced very valuable researches on the hybrid structures of SSs and FSs, which they successfully applied in various areas. Khalil et al. [13] developed some algorithms based on FSS under an expert system. Al-Sharqi and other researchers employ SS to solve some real-life situations in economics [14]- [16], in medical diagnosis [17]- [18], and in computer science [19]- [20] under complex and real number settings. Peng et al. [21] investigated the IVFS-soft matrix when they disposed of a comparison issue in the IVFS-soft set. Palanikumar and Iampan [22] proposed a new strong approach to MCDM based on spherical fermatean-IVFSSs (SFIVFSSs). Qin and Ma [23] introduced a new model based on IVFSS-environment work to evaluate systems that have four bits: data collection and preprocessing, parameter reduction [24], [25], decision making [26], [27], and combination of data sets [28], [29]. More of these studies can be found through the following workers [30], [31].

Recently, Smarandache pointed out that there is a loophole in SS, which is the inability to give a clearer and accurate view of the accompanying features of parameters in the daily life situation. In response to this purpose, he presented the novel idea of "Hypersoft Set" (HSS) [32] as a novel extension of the SS. This idea attracted many researchers around the world and prompted them to make many contributions; for example, Saeed et al. [33] defined the essential operation of HSSs. Musa and Asaad [34] applied the idea of bipolarity to HSSs, following them, Al-Quran et al. [35] developed the idea of BHSSs into BFHSSs. Yolcu and Ozturk [36] proposed the notion of fuzzy HSSs (FHSSs) and their related operations. Rahman et al. [37] presented DM techniques for parameterizing a fuzzy hypersoft set (PFHSS). In order to increase the number of experts, Kamac and Saqlain [38] proposed fuzzy hypersoft expert sets (FHSESs). Bavia et al. [39] discussed certain properties of fuzzy whole hypersoft sets (FWHSSs) as a new generalization of FHSSs. Khan et al. [40] created a new hypertopic called "q-Rung Orthopair Fuzzy Hypersoft Set" and used it to analyze the cryptocurrency market. Arshad et al. [41] employed distance measures between two IVFHSSs to test the level of recovery of patients after the application of suitable medication. Arshad et al. [42] described similarity measures between IVFHSSs and tested them in pattern recognition applications. Arshad et al. [43] used the IVFHSS technique to develop an algorithm for the selection of antivirus masks during the COVID-19 pandemic. Zulqarnain et al. [44] proposed an application based on neutrosophic HSSs. Samad et al. [45] developed TOPSIS technique using NHSSs. Ahmad et al. [46] defined a novel MCDM method based on Plithogenic FNHS-sets to deal with real-life issues. Saeed and Harl [47] initiated new operations, along with properties and numerical examples of Picture FHS-set (PFHS-set). Zulqarnain et al. [48], [49] investigated the MCDM complications under intuitionistic FHS-set (IFHS-set) information which discusses the parametrization of multi-sub attributes of considered parameters. In addition, others a lot successfully studied and used many decision-making techniques [50]- [54] in solving a variety of real-world scenarios.

Moreover, the theory of possibility plays a vital role in dealing with the ambiguous nature of information, where the entity possibility degree is between 0 and 1 for any objective space. In order to take advantage of this feature, many works were presented in a fuzzy environment, which aims to promote the level of possibility for each element of the soft universe discourse.

In the following lines, some relevant previous studies are analysed to rank the research gap and the demands of the planned study. Alkhazaleh et al. [55] explored some initial properties, operations, and laws of possibility for fuzzy soft sets (PFSSs) with applications in DM problems. Khalil et al. [56] characterized the possibility of polar fuzzy soft sets (PPFSSs). Fu et al. [57] presented possibility IVFS-set (PIVFS-set) and investigated their properties. Jia-hua et al. [58] defined some related operations on possibility pythagorean fuzzy soft sets (PPFSSs). Al-Sharqi et al. [59,60] calculated similarity measures between two possibility neutrosophic soft expert sets (PNSESs), possibility interval-valued fuzzy soft sets (PIVFSSs) and applied these techniques to deal with DM problems in the clinical field. Rahman et al. [61] integrated both fuzzy parameterized possibility degree of single-valued neutrosophic hypersoft set (FPPSV-NHSS) and Sanchezs method (SM) to resolve the solid waste site selection problem (SOWSSP). Saeed et al. [62] developed a new approach to medical diagnosis based on a possible degree of NHS-set (PNHS-set) with some elementary axioms and algebraic operations of PNHS-sets. Wahab et al. [63] developed the idea of the possibility of q-rung ortho-pair FHS-set (Pq-ROFHS-set) by giving a degree of possibility for each object of q-ROFHS-set to suss out the problems associated decision-making procedure. To light up the advantages and potential applications Table 1 represents a review of the bibliometric analysis with existing optimization methods under possibility degree such as PFS-set, PIVFS-set, and PFHS-set.

Table 1. Relevant literature review with limitations.

Structures	**Authors**	**Techniques with limitations**
PFS-set	**Alkhazzleh [55]**	**1.** Each single membership value of an object in a nonempty universal set has possibility degree between [0,1]. **2.** Dealing with uncertainty in a parametric manner
PIVFS-set	**Fu et al. [57]**	**1.** Each interval membership value of an object in a nonempty universal set has possibility degree between [0,1]. **2.** Dealing with uncertainty in a parametric manner
PIVFHS-set	**Rahman et al. [64]**	**1.** Each single membership value of an object in a nonempty universal set has possibility degree between [0,1]. **2.** Dealing with uncertainty in a parametric expanded way when each parameter has multi-argument function.

Recently, using HSSs, Rahman et al. [64] proposed the glueing concept of PFHSSs with the attachment of a fuzzy possibility degree to each approximate element of FHSSs. This model deals with issues of uncertainty and ambiguity accompanying DM problems with a single membership value. But unfortunately, design makers (users) based on this structure face some difficulties in some situations when dealing with problem data with a single value. Since a single value causes some restrictions and a lack of freedom for the users when dealing with disjoint sets having sub-parametric values (HSS), To tackle this issue and increase flexibility and reliability in decision-making, in this work we will organize a novel structure, ”possibility IVFHSS (PIVFHSS),” to resolve real-world issues based on the properties of HSS and interval fuzzy form.

1.1. *Main contributions*

The following points present the major contributions of the put-forward study.

1. In order to solve the possible decision-making circumstances that include disjoint sets having sub-parametric values (HSS), the entitlement of possibility degree and consideration of IVFHSS settings are considered, and a novel mathematical model, i.e., PIVFHSS, is developed. This model is competent in providing more flexibility and freedom of a trusted DM framework.

2. The elementary properties and fundamental operations of PIVFHSSs are highlighted as well as supported by some illustrative numerical examples.

3. A novel similarity measure between PIVFHSSs is created and authenticated with a new algorithm utilized to address real-life applications for recruitment pattern recognition.

4. Two algorithms are created based on the AND-operation and OR-operation of PIVFHSSs, and a comparison table was prepared to compare their outputs.

5. The proposed model is compared with previous models under the effect of their structures, and the advantages of our proposed model are discussed.

1.2. *Paper organization*

The rest of this article is systematised in the following figure 1:

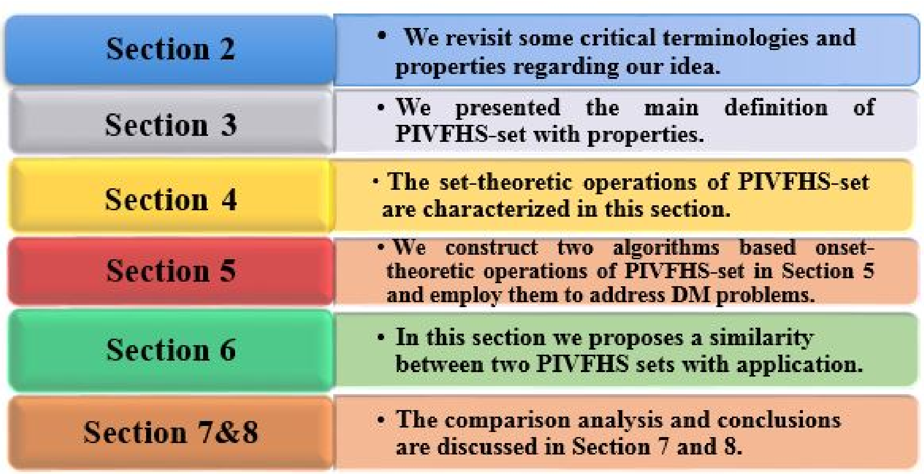

Figure 1: Represents of paper organization.

2. Preliminaries

In this part, we revisit some critical definitions and properties for our idea

Definition 2.1. [1] An FS $\ddot{\mathcal{N}}$ is characterized by $\ddot{\mathcal{N}} = \left\{\left\langle \nu, \dot{\mathcal{F}}_{\ddot{\mathcal{N}}}(\nu), \forall v \in \mathcal{V}\right\rangle\right\}$ such that $\dot{\mathcal{F}}_{\ddot{\mathcal{N}}}(\nu) : \mathcal{V} \to [0,1]$ is real-valued truth-membership.

Definition 2.2. [2] An IVFS $\ddot{\mathcal{N}}$ is characterized by $\ddot{\mathcal{N}} = \left\{\left\langle \nu, \dot{\mathcal{F}}^l{}_{\ddot{\mathcal{N}}}(\nu), \dot{\mathcal{F}}^u{}_{\ddot{\mathcal{N}}}(\nu) \forall v \in \mathcal{V}\right\rangle\right\}$ such that $\dot{\mathcal{F}}^l{}_{\ddot{\mathcal{N}}}(\nu), \dot{\mathcal{F}}^u{}_{\ddot{\mathcal{N}}}(\nu) : \mathcal{V} \to [0,1]$ are real-valued lower and upper truth-membership.

Definition 2.3. [2] **(Properties of IV-FS)**

Let $\ddot{\mathcal{N}}_1 = \left\{\langle \nu, \left[\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu)\right]\rangle \nu \in \mathcal{V}\right\}$ and $\ddot{\mathcal{N}}_2 = \left\{\langle \nu, \left[\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_2}(\nu), \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_2}(\nu)\right]\rangle \nu \in \mathcal{V}\right\}$ be two IVFSs. Then the following basic operation on IVFSs is defined as for all $\nu \in \mathcal{V}$:

(i) $\ddot{\mathcal{N}}_1 \subseteq \ddot{\mathcal{N}}_2$ if and only if $\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu) \le \dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_2}(\nu)$ and $\dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu) \le \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_2}(\nu)$.

(ii) $\ddot{\mathcal{N}}_1 = \ddot{\mathcal{N}}_2$ if and only if $\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu) = \dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_2}(\nu)$ and $\dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu) = \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_2}(\nu)$.

(iii) The complement of $\ddot{\mathcal{N}}_1$ denotes $\ddot{\mathcal{N}}_1^c$, such that $\dot{\mathcal{F}}_{\ddot{\mathcal{N}}_1^c}(\nu) = \left[1 - \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu), 1 - \dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu)\right]$.

(iv) If $\ddot{\mathcal{N}}_3 = \ddot{\mathcal{N}}_1 \cup \ddot{\mathcal{N}}_2$ then the $\dot{\mathcal{F}}_{\ddot{\mathcal{N}}_3}(\nu) = max\{\dot{\mathcal{F}}_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_2}(\nu)\} = [max[\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_2}(\nu)], max[\dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_2}(\nu)] = \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_1}(\nu) \vee \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_2}(\nu)$.

(v) $\ddot{\mathcal{N}}_3 = \ddot{\mathcal{N}}_1 \cap \ddot{\mathcal{N}}_2$ then the $\dot{\mathcal{F}}_{\ddot{\mathcal{N}}_3}(\nu) = min\{\dot{\mathcal{F}}_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_2}(\nu)\} = [min[\dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}^l_{\ddot{\mathcal{N}}_2}(\nu)], min[\dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_1}(\nu), \dot{\mathcal{F}}^u_{\ddot{\mathcal{N}}_2}(\nu)] = \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_1}(\nu) \wedge \dot{\mathcal{F}}_{\ddot{\mathcal{N}}_2}(\nu)$.

Definition 2.4. [7] A SS $\left(\tilde{\mathcal{F}}_{\mathcal{SS}}, \tilde{\mathcal{A}}\right)$ on fixed set $\tilde{\mathcal{V}}$ is stated as a mapping $\tilde{\mathcal{F}}_{\mathcal{SS}} : \tilde{\mathcal{A}} \to \mathcal{P}\left(\tilde{\mathcal{V}}\right)$ where a set of parameters $\tilde{\mathcal{A}}$ is a subset of parameters set $\tilde{\mathcal{E}}$.

Definition 2.5. [11] The structure of IVFSS is stated as:

$$\left(\ddot{\mathcal{N}}, \tilde{\mathcal{E}}\right) = \left\{\left\langle \nu_i, \left[\tilde{\mathcal{F}}^l_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu), \tilde{\mathcal{F}}^u_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu)\right]\right\rangle \middle| \tilde{a}_i \in \tilde{\mathcal{A}} \subseteq \tilde{\mathcal{E}}, \nu_i \in \tilde{\mathcal{V}}\right\}$$

where $\tilde{\mathcal{F}}^l_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu), \tilde{\mathcal{F}}^u_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu)$ $\tilde{\mathcal{F}} : \tilde{\mathcal{A}} \to \mathcal{P}^{\mathcal{IVFS}}\left(\tilde{\mathcal{V}}\right)$ denotes to lower and upper bounded grade of IVFS-set receptively

Definition 2.6. [32] The structure of HSS $\left(\tilde{\mathcal{F}}_{\mathcal{HSS}}, \ddot{\mathcal{A}}\right)$ is stated as following mapping :

$$\tilde{\mathcal{F}}_{\mathcal{HSS}} : \ddot{\mathcal{A}} \to \mathcal{P}\left(\tilde{\mathcal{V}}\right)$$

Here $\ddot{\mathcal{A}}$ written as a sequence of parameter sets like$\ddot{\mathcal{A}}_{1,k} \times \ddot{\mathcal{A}}_{2,k} \times \ddot{\mathcal{A}}_{3,k} \times ... \times \ddot{\mathcal{A}}_{n,k}$ with $\ddot{\mathcal{A}}_{n,k} \cap \ddot{\mathcal{A}}_{n,j} = \phi$ for $k \neq j$ and $\ddot{\mathcal{A}}_{n,k}, \ddot{\mathcal{A}}_{n,j}$ are sets of discrete parameters that are characteristically proportional to other characteristic parameters such that $\tilde{a}_1 \times \tilde{a}_2 \times \tilde{a}_3 \times ... \times \tilde{a}_{n,k} \in \ddot{\mathcal{A}}_{n,k}$, $\tilde{a}_1 \times \tilde{a}_2 \times \tilde{a}_3 \times ... \times \tilde{a}_{n,j} \in \ddot{\mathcal{A}}_{n,j}$ respectively.

Definition 2.7. [41] The structure of IVFHSS is stated as:

$$\left(\ddot{\mathcal{N}}, \tilde{\mathcal{E}}\right) = \left\{\left\langle \nu_i, \left[\tilde{\mathcal{F}}^l_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu), \tilde{\mathcal{F}}^u_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu)\right]\right\rangle \middle| \tilde{a}_i \in \ddot{\mathcal{A}} \subseteq \tilde{\mathcal{E}}, \nu_i \in \tilde{\mathcal{V}}\right\}$$

where $\tilde{\mathcal{F}}^l_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu), \tilde{\mathcal{F}}^u_{\ddot{\mathcal{N}}(\tilde{a}_i)}(\nu)$ $\tilde{\mathcal{F}} : \ddot{\mathcal{A}} \to \mathcal{P}^{\mathcal{IVFS}}\left(\tilde{\mathcal{V}}\right)$ denotes to lower and upper bounded grade of IVFHS-set receptively

Example 2.8. Suppose that $\tilde{\mathcal{V}} = \{\hat{v}_1, \hat{v}_2\}$ is the set of two smart-phones and $\ddot{\mathcal{A}} = \{\tilde{a}_1 = Costly, \tilde{a}_2 = Battery, \tilde{a}_3 = lightweight\} \subseteq \tilde{\mathcal{E}}$ is the set of attributes.Then the IVFSS is analyzed as follows:

$$\left(\ddot{\mathcal{N}}, \ddot{\mathcal{A}}\right) = \begin{Bmatrix} \bar{\bar{\Psi}}\left(\tilde{a}_1\right) = \left\{\left(\hat{\nu}_1, [0.24, 0.42]\right), \left(\hat{\nu}_2, [0.11, 0.74]\right)\right\} \\ \bar{\bar{\Psi}}\left(\tilde{a}_2\right) = \left\{\left(\hat{\nu}_1, [0.57, 0.74]\right), \left(\hat{\nu}_2, [0.63, 0.68]\right)\right\} \\ \bar{\bar{\Psi}}\left(\tilde{a}_3\right) = \left\{\left(\hat{\nu}_1, [0.83, 0.92]\right), \left(\hat{\nu}_2, [0.36, 0.65]\right)\right\} \end{Bmatrix}$$

3. Possibility interval-valued neutrosophic hypersoft set (PIVFHS-set)

In this part, we discuss the basic definitions of the idea of possibility interval-valued fuzzy hypersoft set (PIVFHS-set) and the fundamental operations associated with it. In addition, we support this definition with some illustrative examples.

Definition 3.1. The ordered pair $\left(\mathcal{T}_\vartheta, \tilde{\mathcal{E}}\right)$ is called the possibility interval-valued fuzzy hypersoft set (PIVFHS-set) over a nonempty hypersoft universe $\left(\mathcal{V}, \tilde{\mathcal{E}}\right)$ if

$$\mathcal{T}_\vartheta : \tilde{\mathcal{A}} \to IVF^{\mathcal{V}} \times I^{\mathcal{V}}$$

defined by

$$\mathcal{T}_\vartheta\left(\tilde{a}_i\right) = \left\{\mathcal{T}\left(\tilde{a}_i\right)\left(\nu_n\right), \vartheta\left(\tilde{a}_i\right)\left(\nu_n\right)\right\}$$

with

$$\mathcal{T}\left(\tilde{a}_i\right)\left(\nu_n\right) = \left\langle \rho^l\left(\tilde{a}_i\right)\left(\nu_n\right), \rho^u\left(\tilde{a}_i\right)\left(\nu_n\right)\right\rangle \forall \tilde{a}_i \in \tilde{\mathcal{A}} \subseteq \tilde{\mathcal{E}}, \nu_n \in \mathcal{V}.$$

Where,

(i) For $\mathcal{V} = \{\nu_1, \nu_2, \nu_3, ..., \nu_n\}$be a non-empty initial universe and $\tilde{\mathcal{A}} = \tilde{\mathcal{A}}_1 \times \tilde{\mathcal{A}}_2 \times \tilde{\mathcal{A}}_3 \times ... \times \tilde{\mathcal{A}}_n$ with $\tilde{\mathcal{A}}_i \cap \tilde{\mathcal{A}}_j \neq \phi$ such that i≠j and both of them belong to $\{1, 2, 3, ..., n\}$ are sets given as having sub-parametric values $\tilde{a}_i = 1, 2, 3...n$ respectively.

(ii) $\mathcal{T} : \tilde{\mathcal{A}} \to IVF^{\mathcal{V}}$and $\vartheta : \tilde{\mathcal{A}} \to I^{\mathcal{V}}$,$IVF^{\mathcal{V}}$ and $I^{\mathcal{V}}$ indicates the collection of all interval valued fuzzy set and fuzzy subset of non-empty initial universe $\mathcal{V}$ respectively.

(iii) $\mathcal{T}\left(\tilde{a}\right)\left(\nu_n\right)$ is the grade of interval-fuzzy membership of $\nu \in \mathcal{V}$ in $\mathcal{T}(\tilde{a})$, i.e$\left(\rho^l\left(\tilde{a}\right)\left(\nu_n\right), \rho^u\left(\tilde{a}\right)\left(\nu_n\right)\right)$ denotes to lower and upper bounded grade of interval-fuzzy memberships receptively.

(iv) $\vartheta\left(\tilde{a}\right)\left(\nu_n\right)$ is a grade of fuzzy possibility membership of $\nu \in \mathcal{V}$ in $\mathcal{T}(\tilde{a})$.

Now, from the above definition we write $\mathcal{T}(\tilde{a}_i)$ as bellow:

$$\left\{\left(\frac{\nu_1}{\mathcal{T}(\tilde{a}_1)(\nu_1)}, \vartheta\left(\tilde{a}_1\right)\left(\nu_1\right)\right), \left(\frac{\nu_2}{\mathcal{T}(\tilde{a}_2)(\nu_2)}, \vartheta\left(\tilde{a}_2\right)\left(\nu_2\right)\right), \left(\frac{\nu_3}{\mathcal{T}(\tilde{a}_3)(\nu_3)}, \vartheta\left(\tilde{a}_3\right)\left(\nu_3\right)\right), ..., \left(\frac{\nu_n}{\mathcal{T}(\tilde{a}_i)(\nu_n)}, \vartheta\left(\tilde{a}_i\right)\left(\nu_n\right)\right)\right\}$$

for $i = 1, 2, 3, ..., n$

Example 3.2. Mr. Xu works as a secondary school principal and because of his daily commute from school in the morning until back in the evening, he decided to purchase a car. There are four kinds of cars available, which create the initial universe $\mathcal{V} = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. Here, Mr.

Xu takes into account the following attributes $\tilde{\mathcal{A}}_1 = Price$, $\tilde{\mathcal{A}}_2 = Brand$ and $\tilde{\mathcal{A}}_3 = Color$, then the attribute-valued sets blending to these attributes are:

$\tilde{\mathcal{A}}_1 = \{\tilde{a}_{11} = 2000USD, \tilde{a}_{12} = 2500USD\}$

$\tilde{\mathcal{A}}_2 = \{\tilde{a}_{21} = SACA, \tilde{a}_{22} = MAZDA\}$

$\tilde{\mathcal{A}}_3 = \{\tilde{a}_{31} = White\}$

then $\tilde{\mathcal{A}} = \tilde{\mathcal{A}}_1 \times \tilde{\mathcal{A}}_2 \times \tilde{\mathcal{A}}_3 = \{\tilde{a}_{11}, \tilde{a}_{12}\} \times \{\tilde{a}_{21}, \tilde{a}_{22}\} \times \{\tilde{a}_{31}\} = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4\}$ $\forall \tilde{a}_i$ is a 3-tuple element.

Then the PIVFHSS over a nonempty hypersoft universe $\left(\mathcal{V}, \tilde{\mathcal{A}}\right)$ is given as follows:

$$\mathcal{T}_\vartheta(\tilde{a}_1) = \left\{ \left(\frac{\nu_1}{\langle[0.2,0.6]\rangle}, 0.2 \right), \left(\frac{\nu_2}{\langle[0.5,0.6]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.1,0.3]\rangle}, 0.7 \right), \left(\frac{\nu_4}{\langle[0.4,0.9]\rangle}, 0.9 \right) \right\}.$$
$$\mathcal{T}_\vartheta(\tilde{a}_2) = \left\{ \left(\frac{\nu_1}{\langle[0.5,0.9]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.3,0.4]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.3,0.7]\rangle}, 0.3 \right), \left(\frac{\nu_4}{\langle[0.2,0.6]\rangle}, 0.8 \right) \right\}.$$
$$\mathcal{T}_\vartheta(\tilde{a}_3) = \left\{ \left(\frac{\nu_1}{\langle[0.6,0.8]\rangle}, 0.7 \right), \left(\frac{\nu_2}{\langle[0.1,0.2]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.2,0.8]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.4,0.5]\rangle}, 0.3 \right) \right\}.$$
$$\mathcal{T}_\vartheta(\tilde{a}_4) = \left\{ \left(\frac{\nu_1}{\langle[0.2,0.4]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.2,0.3]\rangle}, 0.8 \right), \left(\frac{\nu_3}{\langle[0.1,0.2]\rangle}, 0.5 \right), \left(\frac{\nu_4}{\langle[0.2,0.5]\rangle}, 0.3 \right) \right\}.$$

Here also, we can present our model PIVFHSS in matrix representation:

$$\mathcal{T}_\vartheta = \begin{pmatrix} (\langle[0.2,0.6]\rangle, 0.2) & (\langle[0.5,0.6]\rangle, 0.4) & (\langle[0.1,0.3]\rangle, 0.2) & (\langle[0.4,0.9]\rangle, 0.9) \\ (\langle[0.5,0.9]\rangle, 0.6) & (\langle[0.3,0.4]\rangle, 0.4) & (\langle[0.3,0.7]\rangle, 0.3) & (\langle[0.2,0.6]\rangle, 0.8) \\ (\langle[0.6,0.8]\rangle, 0.7) & (\langle[0.1,0.2]\rangle, 0.4) & (\langle[0.2,0.8]\rangle, 0.8) & (\langle[0.4,0.5]\rangle, 0.3) \\ (\langle[0.2,0.4]\rangle, 0.6) & (\langle[0.2,0.3]\rangle, 0.8) & (\langle[0.1,0.2]\rangle, 0.5) & (\langle[0.2,0.5]\rangle, 0.3) \end{pmatrix}$$

Definition 3.3. (**PIVFHS-subset**) Let $\mathcal{T}_\vartheta, \mathcal{G}_\eta$ be two PIVFHS-sets, then $\mathcal{T}_\vartheta$ is specified to be a possibility interval-valued fuzzy hypersoft subset (PIVFHS-subset) of $\mathcal{G}_\eta$ and represented by $\mathcal{T}_\vartheta \overleftarrow{\subseteq} \mathcal{G}_\eta$, if the conditions below are met:

(i) $\vartheta(\tilde{a})$ is a fuzzy subset of $\eta(\tilde{a})$, $\forall\ \tilde{a} \in \tilde{\mathcal{A}}$.

(ii) $\mathcal{T}(\tilde{a})$ is an interval-valued fuzzy subset of $\mathcal{G}(\tilde{a})$, $\forall\ \tilde{a} \in \tilde{\mathcal{A}}$.

Example 3.4. Take $\mathcal{T}_\vartheta$ given in Example 3.2 and let

$$\mathcal{G}_\eta(\tilde{a}_1) = \left\{ \left(\frac{\nu_1}{\langle[0.3,0.7]\rangle}, 0.4 \right), \left(\frac{\nu_2}{\langle[0.6,0.6]\rangle}, 0.6 \right), \left(\frac{\nu_3}{\langle[0.2,0.4]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.5,0.9]\rangle}, 1 \right) \right\}.$$
$$\mathcal{G}_\eta(\tilde{a}_2) = \left\{ \left(\frac{\nu_1}{\langle[0.6,0.9]\rangle}, 0.7 \right), \left(\frac{\nu_2}{\langle[0.4,0.5]\rangle}, 0.5 \right), \left(\frac{\nu_3}{\langle[0.5,0.8]\rangle}, 0.4 \right), \left(\frac{\nu_4}{\langle[0.3,0.8]\rangle}, 0.9 \right) \right\}.$$
$$\mathcal{G}_{eta}(\tilde{a}_3) = \left\{ \left(\frac{\nu_1}{\langle[0.7,0.9]\rangle}, 0.8 \right), \left(\frac{\nu_2}{\langle[0.2,0.7]\rangle}, 0.8 \right), \left(\frac{\nu_3}{\langle[0.4,0.8]\rangle}, 0.9 \right), \left(\frac{\nu_4}{\langle[0.5,0.7]\rangle}, 0.5 \right) \right\}.$$
$$\mathcal{G}_\eta(\tilde{a}_4) = \left\{ \left(\frac{\nu_1}{\langle[0.7,0.9]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.3,0.5]\rangle}, 0.9 \right), \left(\frac{\nu_3}{\langle[0.2,0.2]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.3,0.6]\rangle}, 0.6 \right) \right\}.$$

then we can say $\mathcal{T}_\vartheta \overleftarrow{\subseteq} \mathcal{G}_\eta$.

Definition 3.5. (**Equality of PIVFHS-set**) Let $\mathcal{T}_\vartheta, \mathcal{G}_\eta$ be two PIVFHS-sets, then $\mathcal{T}_\vartheta$ is specified to be equal to $\mathcal{G}_\eta$ and represented by $\mathcal{T}_\vartheta = \mathcal{G}_\eta$, if $\mathcal{T}_\vartheta \overleftarrow{\subseteq} \mathcal{G}_\eta$ and $\mathcal{G}_\eta \overleftarrow{\subseteq} \mathcal{T}_\vartheta$

Example 3.6. Take $\mathcal{T}_\vartheta$ given in Example 3.2 and let

$$\mathcal{G}_{\eta} = \begin{pmatrix} (\langle[0.2,0.6]\rangle,0.2) & (\langle[0.5,0.6]\rangle,0.4) & (\langle[0.1,0.3]\rangle,0.2) & (\langle[0.4,0.9]\rangle,0.9) \\ (\langle[0.5,0.9]\rangle,0.6) & (\langle[0.3,0.4]\rangle,0.4) & (\langle[0.3,0.7]\rangle,0.3) & (\langle[0.2,0.6]\rangle,0.8) \\ (\langle[0.6,0.8]\rangle,0.7) & (\langle[0.1,0.2]\rangle,0.4) & (\langle[0.2,0.8]\rangle,0.8) & (\langle[0.4,0.5]\rangle,0.3) \\ (\langle[0.2,0.4]\rangle,0.6) & (\langle[0.2,0.3]\rangle,0.8) & (\langle[0.1,0.2]\rangle,0.5) & (\langle[0.2,0.5]\rangle,0.3) \end{pmatrix}$$

then we can say $\mathcal{T}_{\vartheta} = \mathcal{G}_{\eta}$.

Definition 3.7. (Possibility relative null IVFHS-set) A PIVFHSS $\mathcal{T}_{\vartheta}$ is called to possibility relative null IVFHS-set and denoted by $\widehat{\Phi}_{0,0}$ if $\mathcal{T}_{\vartheta}(\tilde{a}) = \langle[0,0]\rangle$ and the possibility grade $\vartheta(\tilde{a}) = 0$, $\forall\ \tilde{a} \in \tilde{\mathcal{A}}$.

Example 3.8. Assuming matrix of $\mathcal{T}_{\vartheta}$ presented in Example 3.2, we have

$$\mathcal{G}_{\widehat{\Phi}_{0,0}} = \begin{pmatrix} (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) \\ (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) \\ (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) \\ (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) & (\langle[0.0,0.0]\rangle,0.0) \end{pmatrix}$$

Definition 3.9. (Possibility relative absolute IVFHS-set) A PIVFHSS $\mathcal{T}_{\vartheta}$ is called to possibility relative absolute IVFHS-set and denoted by $\widehat{\Phi}_{1,1}$ if $\mathcal{T}_{\vartheta}(\tilde{a}) = \langle[1,1]\rangle$ and the possibility grade $\vartheta(\tilde{a}) = 1$, $\forall\ \tilde{a} \in \tilde{\mathcal{A}}$.

Example 3.10. Assuming matrix of $\mathcal{T}_{\vartheta}$ presented in Example 3.2, we have

$$\mathcal{G}_{\widehat{\Phi}_{1,1}} = \begin{pmatrix} (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) \\ (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) \\ (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) \\ (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) & (\langle[1,1]\rangle\,1) \end{pmatrix}$$

Definition 3.11. (**Complement of a PIVFHS-set**) The complement of a PIVFHS-set $\mathcal{T}_{\vartheta}$, denoted by T^c_{ϑ} and defined by $\mathcal{T}^c_{\vartheta} = \mathcal{Q}_{\eta}$ such that $\eta(\tilde{a}) = \vartheta^c(\tilde{a})$ and $\mathcal{T}^c(\tilde{a}) = \mathcal{Q}(\tilde{a})$,$\forall\ \tilde{a} \in \tilde{\mathcal{A}}$, where c is a interval-valued fuzzy complement.

Example 3.12. Assuming matrix of $\mathcal{T}_{\vartheta}$ presented in Example 3.2, we have

$$\mathcal{T}^c_{\vartheta} = \mathcal{Q}_{\eta} = \begin{pmatrix} (\langle[0.4,0.8]\rangle,0.8) & (\langle[0.4,0.5]\rangle,0.6) & (\langle[0.7,0.9]\rangle,0.8) & (\langle[0.1,0.6]\rangle,0.1) \\ (\langle[0.1,0.5]\rangle,0.4) & (\langle[0.6,0.7]\rangle,0.6) & (\langle[0.3,0.7]\rangle,0.7) & (\langle[0.4,0.8]\rangle,0.2) \\ (\langle[0.2,0.4]\rangle,0.3) & (\langle[0.8,0.9]\rangle,0.6) & (\langle[0.2,0.8]\rangle,0.2) & (\langle[0.5,0.6]\rangle,0.7) \\ (\langle[0.6,0.8]\rangle,0.4) & (\langle[0.7,0.8]\rangle,0.2) & (\langle[0.8,0.9]\rangle,0.5) & (\langle[0.5,0.8]\rangle,0.7) \end{pmatrix}$$

4. Fundamental set-theoretic operations of PIVFHS-sets

In this part, we developed the basic definitions of set-theoretic operations on PIVFHS-set and explained them with suitable numerical examples. In addition, based on these operations, we give some properties with some illustrative examples. These operations and their properties are considered an extension of the previous operations for previous models like FSs, IVFSs, SS, and HSSs.

Definition 4.1. (**Union and Intersection of PIVFHS-sets**) Let $\mathcal{T}_\vartheta$,$\mathcal{G}_\eta$ be two PIVFHSSs over a nonempty hypersoft universe $(\mathcal{V}, \mathcal{Z})$ then,
(i) $\mathcal{T}_\vartheta \cup \mathcal{G}_\eta$ is dubbed their union which is a PIVFHSS $\mathcal{C}_\pi$ such that $C(\tilde{a}) = \underline{\coprod}\{T(\tilde{a}), G(\tilde{a})\}$ and $\pi = max\{\vartheta(\tilde{a}), \eta(\tilde{a})\}$ where $\underline{\coprod}$ denotes interval-valued fuzzy union.
(ii) $\mathcal{T}_\vartheta \cap \mathcal{G}_\eta$ is dubbed their intersection which is a PIVFHSS $\mathcal{C}_\pi$ such that $C_\pi(\tilde{a}) = \bar{\prod}\{T_\vartheta(\tilde{a}), G_\eta(\tilde{a})\}$,and $\pi = min\{\vartheta(\tilde{a}), \eta(\tilde{a})\}$ where $\bar{\prod}$ denotes interval-valued fuzzy intersection.

Example 4.2. Assuming matrices of $\mathcal{T}_\vartheta$,$\mathcal{G}_\eta$ as following:

$$\mathcal{T}_\vartheta = \begin{pmatrix} (\langle[0.2, 0.6]\rangle, 0.2) & (\langle[0.5, 0.6]\rangle, 0.4) & (\langle[0.4, 0.9]\rangle, 0.9) \\ (\langle[0.5, 0.9]\rangle, 0.6) & (\langle[0.3, 0.7]\rangle, 0.3) & (\langle[0.2, 0.6]\rangle, 0.8) \\ (\langle[0.6, 0.8]\rangle, 0.7) & (\langle[0.1, 0.2]\rangle, 0.4) & (\langle[0.4, 0.5]\rangle, 0.3) \end{pmatrix}$$

and

$$\mathcal{G}_\eta = \begin{pmatrix} (\langle[0.3, 0.8]\rangle, 0.3) & (\langle[0.6, 0.9]\rangle, 0.5) & (\langle[0.5, 1]\rangle, 1) \\ (\langle[0.6, 0.8]\rangle, 0.7) & (\langle[0.7, 0.8]\rangle, 0.5) & (\langle[0.3, 0.9]\rangle, 0.9) \\ (\langle[0.7, 0.7]\rangle, 0.9) & (\langle[0.4, 0.8]\rangle, 0.8) & (\langle[0.5, 0.6]\rangle, 0.5) \end{pmatrix}$$

then we have

$$\mathcal{T}_\vartheta \bar{\cup} \mathcal{G}_\eta = \mathcal{C}_\pi = \begin{pmatrix} (\langle[0.3, 0.8]\rangle, 0.3) & (\langle[0.6, 0.9]\rangle, 0.5) & (\langle[0.5, 1]\rangle, 1) \\ (\langle[0.6, 0.9]\rangle, 0.7) & (\langle[0.7, 0.8]\rangle, 0.5) & (\langle[0.3, 0.9]\rangle, 0.9) \\ (\langle[0.7, 0.8]\rangle, 0.9) & (\langle[0.4, 0.8]\rangle, 0.8) & (\langle[0.5, 0.6]\rangle, 0.5) \end{pmatrix}$$

$$\mathcal{T}_\vartheta \bar{\cap} \mathcal{G}_\eta = \mathcal{C}_\pi = \begin{pmatrix} (\langle[0.2, 0.6]\rangle, 0.2) & (\langle[0.5, 0.6]\rangle, 0.4) & (\langle[0.4, 0.9]\rangle, 0.9) \\ (\langle[0.5, 0.8]\rangle, 0.6) & (\langle[0.3, 0.7]\rangle, 0.3) & (\langle[0.2, 0.6]\rangle, 0.8) \\ (\langle[0.6, 0.7]\rangle, 0.7) & (\langle[0.1, 0.2]\rangle, 0.4) & (\langle[0.4, 0.5]\rangle, 0.3) \end{pmatrix}$$

Proposition 4.3. *Let $\mathcal{T}_\vartheta, \mathcal{G}_\eta, \mathcal{C}_\pi$ be three PIVFHS-sets over a nonempty hypersoft universe $(\mathcal{V}, \mathcal{Z})$. Then the next properties come true:*
i. $\mathcal{T}_\vartheta \cup \mathcal{G}_\eta = \mathcal{G}_\eta \cup \mathcal{T}_\vartheta$.
ii. $\mathcal{T}_\vartheta \cap \mathcal{G}_\eta = \mathcal{G}_\eta \cap \mathcal{T}_\vartheta$.

iii. $\mathcal{T}_\vartheta \tilde{\cup} \left(\mathcal{G}_\eta \tilde{\cup} \mathcal{C}_\pi\right) = \left(\mathcal{T}_\vartheta \tilde{\cup} \mathcal{G}_\eta\right) \tilde{\cup} \mathcal{C}_\pi$

iv. $\mathcal{T}_\vartheta \tilde{\cap} \left(\mathcal{G}_\eta \tilde{\cap} \mathcal{C}_\pi\right) = \left(\mathcal{T}_\vartheta \tilde{\cap} \mathcal{G}_\eta\right) \tilde{\cap} \mathcal{C}_\pi$

Proposition 4.4. *Let $\mathcal{T}_\vartheta, \mathcal{G}_\eta, \mathcal{C}_\pi$ be three PIVFHS-sets over a nonempty hypersoft universe $(\mathcal{V}, \mathcal{Z})$. Then the next properties come true:*

i. $\mathcal{T}_\vartheta \tilde{\cup} \left(\mathcal{G}_\eta \tilde{\cap} \mathcal{C}_\pi\right) = \left(\mathcal{T}_\vartheta \tilde{\cup} \mathcal{G}_\eta\right) \tilde{\cap} \left(\mathcal{T}_\vartheta \tilde{\cup} \mathcal{C}_\pi\right)$

ii. $\mathcal{T}_\vartheta \tilde{\cap} \left(\mathcal{G}_\eta \tilde{\cup} \mathcal{C}_\pi\right) = \left(\mathcal{T}_\vartheta \tilde{\cap} \mathcal{G}_\eta\right) \tilde{\cup} \left(\mathcal{T}_\vartheta \tilde{\cap} \mathcal{C}_\pi\right)$

Proof. (i) $\forall\ \tilde{a} \in \tilde{\mathcal{A}}$, and based on the Definition 4.1. Then,

Let $\Gamma_{T_{(\tilde{a})} \tilde{\cup} \left(G_{(\tilde{a})} \tilde{\cap} C_{(\tilde{a})}\right)} (a) = \tilde{\cup} \left\{ \Gamma_{T_{(\tilde{a})}} (a), \Gamma_{\left(G_{(\tilde{a})} \tilde{\cap} C_{(\tilde{a})}\right)} (a) \right\}$

$= \tilde{\cup} \left\{ \Gamma_{T_{(\tilde{a})}} (a), \tilde{\cap} \left(\Gamma_{G_{(\tilde{a})}} (a), \Gamma_{C_{(\tilde{a})}} (a) \right) \right\}$

$= \left\{ \left\langle \tilde{a}, \max \left(\left[\Gamma_{T_{(\tilde{a})}} (a) \right], \min \left(\Gamma_{G_{(\tilde{a})}} (a), \Gamma_{C_{(\tilde{a})}} (a) \right) \right) \right\rangle \right\}$

$= \left\{ \left\langle \tilde{a}, \tilde{\cap} \left(\tilde{\cup} \left(\left[\Gamma_{T_{(\tilde{a})}} (a), \Gamma_{G_{(\tilde{a})}} (a) \right], \left[\Gamma_{T_{(\tilde{a})}} (a), \Gamma_{C_{(\tilde{a})}} (a) \right] \right) \right) \right\rangle \right\}$

$= \tilde{\cap} \left(\tilde{\cup} \left(\Gamma_{T_{(\tilde{a})} \tilde{\cup} G_{(\tilde{a})}} (a), \Gamma_{T_{(\tilde{a})} \tilde{\cup} C_{(\tilde{a})}} (a) \right) \right)$

$= \Gamma_{\left(T_{(\tilde{a})} \tilde{\cup} G_{(\tilde{a})}\right) \tilde{\cap} \left(T_{(\tilde{a})} \tilde{\cup} C_{(\tilde{a})}\right)} (a).$

and for fuzzy possibility grade

$\partial_{\vartheta_{(\tilde{a})} \tilde{\cup} \left(\eta_{(\tilde{a})} \tilde{\cap} \pi_{(\tilde{a})}\right)(\tilde{a})}$

$= \max \left\{ \partial_{\vartheta_{(\tilde{a})}} (\tilde{a}), \partial_{\left(\eta_{(\tilde{a})} \tilde{\cap} \pi_{(\tilde{a})}\right)} (\tilde{a}) \right\}$

$= \max \left\{ \partial_{\vartheta_{(\tilde{a})}} (\tilde{a}), \min \left(\partial_{\eta_{(\tilde{a})}} (\tilde{a}), \partial_{\pi_{(\tilde{a})}} (\tilde{a}) \right) \right\}$

$= \min \left\{ \max \left(\partial_{\vartheta_{(\tilde{a})}} (\tilde{a}), \partial_{\eta_{(\tilde{a})}} (\tilde{a}) \right), \max \left(\partial_{\vartheta_{(\tilde{a})}} (\tilde{a}), \partial_{\pi_{(\tilde{a})}} (\tilde{a}) \right) \right\}$

$= \min \left\{ \partial_{\vartheta_{(\tilde{a})} \tilde{\cup} \eta_{(\tilde{a})}} (\tilde{a}), \partial_{\vartheta_{(\tilde{a})} \tilde{\cup} \eta_{(\tilde{a})}} (\tilde{a}) \right\}$

$= \partial_{\left(\vartheta_{(\tilde{a})} \tilde{\cup} \eta_{(\tilde{a})}\right) \tilde{\cap} \left(\vartheta_{(\tilde{a})} \tilde{\cup} \pi_{(\tilde{a})}\right)}.$

(ii) can be verified in a similar way as in (i). □

Definition 4.5. (*AND* & *OR* operations of PIVFHS-sets) Let $(\mathcal{T}_\vartheta, \mathcal{M})$,$(\mathcal{G}_\eta, \mathcal{N})$ be PIVFHS-sets over a nonempty hypersoft universe $(\mathcal{V}, \mathcal{Z})$.Then:

i. $(\mathcal{T}_\vartheta, \mathcal{M}) \tilde{\wedge} (\mathcal{G}_\eta, \mathcal{N})$ is called as *AND*-operations, such that PIVFHS-set $(\mathcal{H}_\psi, \mathcal{W})$ and characterized by $(\mathcal{H}_\psi, \mathcal{W}) = (\mathcal{H}_\psi, \mathcal{M} \times \mathcal{N})$, where $\mathcal{H}_\psi(\tilde{m}, \tilde{n}) = (\mathcal{H}(\tilde{m}, \tilde{n})(\nu), \psi(\tilde{m}, \tilde{n})(\nu))$ $\forall(\tilde{m}, \tilde{n}) \in \mathcal{M} \times \mathcal{N}$.Such that $\mathcal{H}(\tilde{m}, \tilde{n}) = \diamond(\mathcal{T}(\tilde{m}), \mathcal{G}(\tilde{n}))$ and $\psi(\tilde{m}, \tilde{n}) = min\left(\vartheta(\tilde{m}), \eta(\tilde{n})\right)$,$\forall(\tilde{m}, \tilde{n}) \in \mathcal{M} \times \mathcal{N}$ and $\nu \in \mathcal{Z}$. Here $\diamond$ denotes interval-valued fuzzy intersection.

ii. $(\mathcal{T}_\vartheta, \mathcal{M}) \tilde{\vee} (\mathcal{G}_\eta, \mathcal{N})$ is called as *OR*-operations, such that PIVFHS-set $(\mathcal{K}_\pi, \mathcal{R})$ and characterized by $(\mathcal{K}_\psi, \mathcal{R}) = (\mathcal{K}_\pi, \mathcal{M} \times \mathcal{N})$, where $\mathcal{K}_\pi(\tilde{m}, \tilde{n}) = (\mathcal{K}(\tilde{m}, \tilde{n})(\nu), \psi(\tilde{m}, \tilde{n})(\nu))$ $\forall(\tilde{m}, \tilde{n}) \in \mathcal{M} \times \mathcal{N}$.Such that $\mathcal{H}(\tilde{m}, \tilde{n}) = \triangleright(\mathcal{T}(\tilde{m}), \mathcal{G}(\tilde{n}))$ and $\pi(\tilde{m}, \tilde{n}) = min\left(\vartheta(\tilde{m}), \eta(\tilde{n})\right)$,$\forall(\tilde{m}, \tilde{n}) \in \mathcal{M} \times \mathcal{N}$ and $\nu \in \mathcal{Z}$. Here $\triangleright$ denotes interval-valued fuzzy union.

Example 4.6. Let $\mathcal{T}_\vartheta, \mathcal{G}_\eta$ be two PIVFHS-sets over a nonempty hypersoft universe $(\mathcal{V}, \mathcal{A})$ and are defined as matrix notations as follows

$$(\mathcal{T}_\vartheta, \mathcal{M}) = \begin{pmatrix} (\langle[0.2, 0.6]\rangle, 0.2) & (\langle[0.5, 0.6]\rangle, 0.4) & (\langle[0.4, 0.9]\rangle, 0.9) \\ (\langle[0.6, 0.8]\rangle, 0.7) & (\langle[0.1, 0.2]\rangle, 0.4) & (\langle[0.4, 0.5]\rangle, 0.3) \end{pmatrix}$$

and

$$(\mathcal{G}_\eta, \mathcal{N}) = \begin{pmatrix} (\langle[0.3, 0.8]\rangle, 0.3) & (\langle[0.6, 0.9]\rangle, 0.5) & (\langle[0.5, 1]\rangle, 1) \\ (\langle[0.7, 0.7]\rangle, 0.9) & (\langle[0.4, 0.8]\rangle, 0.8) & (\langle[0.5, 0.6]\rangle, 0.5) \end{pmatrix}$$

then we have

$$(\mathcal{H}_\psi, \mathcal{W}) = \begin{pmatrix} (\langle[0.2, 0.6]\rangle, 0.2) & (\langle[0.5, 0.6]\rangle, 0.4) & (\langle[0.4, 0.9]\rangle, 0.9) \\ (\langle[0.2, 0.6]\rangle, 0.2) & (\langle[0.4, 0.6]\rangle, 0.4) & (\langle[0.4, 0.6]\rangle, 0.5) \\ (\langle[0.3, 0.8]\rangle, 0.3) & (\langle[0.1, 0.2]\rangle, 0.4) & (\langle[0.4, 0.5]\rangle, 0.3) \\ (\langle[0.6, 0.7]\rangle, 0.7) & (\langle[0.1, 0.2]\rangle, 0.4) & (\langle[0.4, 0.5]\rangle, 0.3) \end{pmatrix}$$

and

$$(\mathcal{K}_\pi, \mathcal{R}) = \begin{pmatrix} (\langle[0.3, 0.8]\rangle, 0.3) & (\langle[0.6, 0.9]\rangle, 0.5) & (\langle[0.5, 1]\rangle, 1) \\ (\langle[0.7, 0.7]\rangle, 0.9) & (\langle[0.5, 0.8]\rangle, 0.8) & (\langle[0.5, 0.9]\rangle, 0.9) \\ (\langle[0.6, 0.8]\rangle, 0.7) & (\langle[0.6, 0.9]\rangle, 0.5) & (\langle[0.5, 1]\rangle, 1) \\ (\langle[0.7, 0.8]\rangle, 0.9) & (\langle[0.4, 0.8]\rangle, 0.8) & (\langle[0.5, 0.6]\rangle, 0.5) \end{pmatrix}$$

5. Application of DM based on *AND*-operation and *OR*-operation of PIVFHS-sets

In this section, we built two robust algorithms that employ two operations on PIVFHS set, namely AND-operation and OR-operation, in order to help the user in capturing the right decision regarding the best selection. In addition, these algorithms are clarified with illustrated examples.

Example 5.1. Assume one student wants to purchase a laptop from the electronic device exhibition. There are four kinds of laptops $\nu_1, \nu_2, \nu_3, \nu_4$ that are present in the universe $\mathcal{Z} = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. Here, the student takes into account the following attributes $\tilde{y}_1 = Price$,$\tilde{y}_2 = Brand$ and $\tilde{y}_3 = Hard\ drive\ Size$, then the attribute-valued sets blending to these attributes are: $\tilde{\mathcal{A}}_1 = \{\tilde{a}_{11} = 1000USD, \tilde{a}_{12} = 1500USD\}$
$\tilde{\mathcal{A}}_2 = \{\tilde{a}_{21} = Lenovo, \tilde{a}_{22} = Apple, \tilde{a}_{23} = HP\}$, $\tilde{\mathcal{A}}_3 = \{\tilde{a}_{31} = 512GB\}$
then $\tilde{\mathcal{A}} = \tilde{\mathcal{A}}_1 \times \tilde{\mathcal{A}}_2 \times \tilde{\mathcal{A}}_3 = \{\tilde{a}_{11}, \tilde{a}_{12}\} \times \{\tilde{a}_{21}, \tilde{a}_{22}, \tilde{a}_{23}\} \times \{\tilde{a}_{31}\} = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6\}$ $\forall \tilde{a}_i$ is a 3-tuple element of $\tilde{\mathcal{A}}$.

Now, we prepare two separate algorithms depending on each definition AND-operation and an OR-operation of PIVFHS-sets to select the right laptop.

Algorithm 1. Using AND-operation of PIVFHS-sets

Step 1. Put up PIVFHS-set based on Experts.

Step 2. Compute AND-operation $\mathcal{H}_{\psi}$ of PIVFHS-sets formed in step 1.

Step 3. Compute the value $\mathcal{W}_j = \frac{\sum_{i=1}^{n} \rho_i^l(\tilde{a})(v)+\rho_i^u(\tilde{a})(v)}{n} * \frac{\sum_{i=1}^{n} \eta_i(\tilde{a})(v)}{n}$.

Step 4. Decision: choose the greatest value from the values $\mathcal{W}_j$.

Step 5. End Algorithm 1.

Below is **Figure 2**, a representation of algorithm 1 in an abbreviated way.

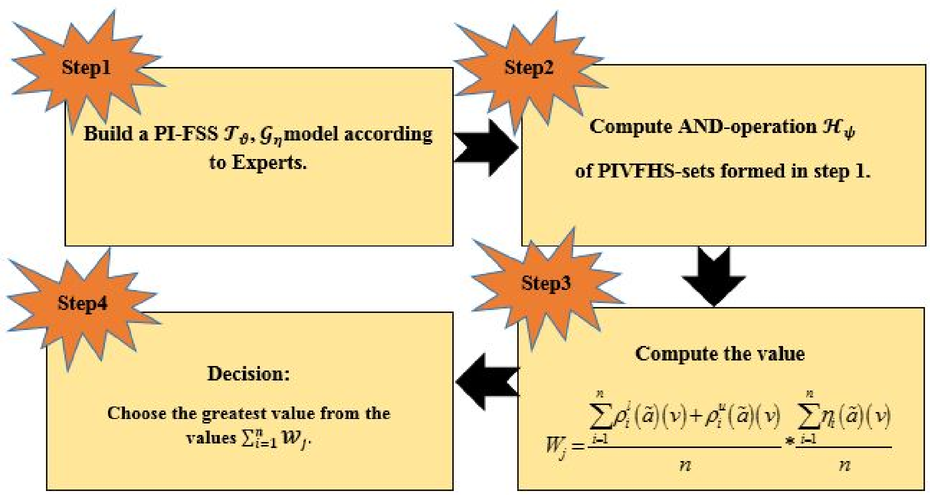

Figure 2: Representation of algorithm 1.

Step 1. Assume that there are two experts i.e $\mathcal{T}_{\vartheta}, \mathcal{G}_{\eta}$ their opinions are as follow

$$\mathcal{T}_{\vartheta}(\tilde{a}_1) = \left\{ \left(\frac{\nu_1}{\langle[0.2,0.6]\rangle}, 0.2 \right), \left(\frac{\nu_2}{\langle[0.5,0.6]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.1,0.3]\rangle}, 0.7 \right), \left(\frac{\nu_4}{\langle[0.4,0.9]\rangle}, 0.9 \right) \right\}.$$

$$\mathcal{T}_{\vartheta}(\tilde{a}_2) = \left\{ \left(\frac{\nu_1}{\langle[0.5,0.9]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.3,0.4]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.3,0.7]\rangle}, 0.3 \right), \left(\frac{\nu_4}{\langle[0.2,0.6]\rangle}, 0.8 \right) \right\}.$$

$$\mathcal{T}_{\vartheta}(\tilde{a}_3) = \left\{ \left(\frac{\nu_1}{\langle[0.6,0.8]\rangle}, 0.7 \right), \left(\frac{\nu_2}{\langle[0.1,0.2]\rangle}, 0.4 \right), \left(\frac{\nu_3}{\langle[0.2,0.8]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.4,0.5]\rangle}, 0.3 \right) \right\}.$$

$$\mathcal{T}_{\vartheta}(\tilde{a}_4) = \left\{ \left(\frac{\nu_1}{\langle[0.2,0.4]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.2,0.3]\rangle}, 0.8 \right), \left(\frac{\nu_3}{\langle[0.1,0.2]\rangle}, 0.5 \right), \left(\frac{\nu_4}{\langle[0.2,0.5]\rangle}, 0.3 \right) \right\}.$$

$$\mathcal{T}_{\vartheta}(\tilde{a}_5) = \left\{ \left(\frac{\nu_1}{\langle[0.1,0.6]\rangle}, 0.9 \right), \left(\frac{\nu_2}{\langle[0.8,0.8]\rangle}, 0.5 \right), \left(\frac{\nu_3}{\langle[0.6,0.7]\rangle}, 0.7 \right), \left(\frac{\nu_4}{\langle[0.2,0.4]\rangle}, 0.4 \right) \right\}.$$

$$\mathcal{T}_{\vartheta}(\tilde{a}_6) = \left\{ \left(\frac{\nu_1}{\langle[0.3,0.7]\rangle}, 0.8 \right), \left(\frac{\nu_2}{\langle[0.6,0.6]\rangle}, 0.5 \right), \left(\frac{\nu_3}{\langle[0.2,0.5]\rangle}, 0.2 \right), \left(\frac{\nu_4}{\langle[0.3,0.8]\rangle}, 0.4 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_1) = \left\{ \left(\frac{\nu_1}{\langle[0.3,0.7]\rangle}, 0.4 \right), \left(\frac{\nu_2}{\langle[0.6,0.6]\rangle}, 0.6 \right), \left(\frac{\nu_3}{\langle[0.2,0.4]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.5,0.9]\rangle}, 1 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_2) = \left\{ \left(\frac{\nu_1}{\langle[0.6,0.9]\rangle}, 0.7 \right), \left(\frac{\nu_2}{\langle[0.4,0.5]\rangle}, 0.5 \right), \left(\frac{\nu_3}{\langle[0.5,0.8]\rangle}, 0.4 \right), \left(\frac{\nu_4}{\langle[0.3,0.8]\rangle}, 0.9 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_3) = \left\{ \left(\frac{\nu_1}{\langle[0.7,0.9]\rangle}, 0.8 \right), \left(\frac{\nu_2}{\langle[0.2,0.7]\rangle}, 0.8 \right), \left(\frac{\nu_3}{\langle[0.4,0.8]\rangle}, 0.9 \right), \left(\frac{\nu_4}{\langle[0.5,0.7]\rangle}, 0.5 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_4) = \left\{ \left(\frac{\nu_1}{\langle[0.7,0.9]\rangle}, 0.4 \right), \left(\frac{\nu_2}{\langle[0.3,0.5]\rangle}, 0.6 \right), \left(\frac{\nu_3}{\langle[0.2,0.2]\rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle[0.3,0.6]\rangle}, 0.6 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_5) = \left\{ \left(\frac{\nu_1}{\langle[0.7,0.9]\rangle}, 0.8 \right), \left(\frac{\nu_2}{\langle[0.2,0.7]\rangle}, 0.8 \right), \left(\frac{\nu_3}{\langle[0.4,0.8]\rangle}, 0.9 \right), \left(\frac{\nu_4}{\langle[0.5,0.7]\rangle}, 0.5 \right) \right\}.$$

$$\mathcal{G}_{\eta}(\tilde{a}_6) = \left\{ \left(\frac{\nu_1}{\langle[0.5,0.7]\rangle}, 0.6 \right), \left(\frac{\nu_2}{\langle[0.1,0.3]\rangle}, 0.3 \right), \left(\frac{\nu_3}{\langle[0.1,0.5]\rangle}, 0.5 \right), \left(\frac{\nu_4}{\langle[0.4,0.7]\rangle}, 0.4 \right) \right\}.$$

Step 2. Compute AND-operation $\mathcal{H}_\psi$ of PIVFHS-sets formed in step 1.

$(\mathcal{H}_\psi, \mathcal{M}) =$

$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.5,0.6]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.1,0.3]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.1,0.3]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.9\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.5,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.1,0.3]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.4,0.7]\rangle},0.5\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.2]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.3,0.6]\rangle},0.6\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.5,0.6]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.1,0.3]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_1,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.6]\rangle},0.2\right),\left(\frac{\nu_2}{\langle[0.5,0.6]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.1,0.3]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.3,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.4]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.8\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.3,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.8\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.5\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.3,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.2]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.6\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.5\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_2,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.1,0.3]\rangle},0.3\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.3\right),\left(\frac{\nu_4}{\langle[0.2,0.6]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.4]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.3,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.4,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_3,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.1,0.2]\rangle},0.3\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.4,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_4,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.2,0.4]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.1,0.3]\rangle},0.3\right),\left(\frac{\nu_3}{\langle[0.1,0.2]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.5]\rangle},0.3\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.4]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.5,0.7]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.4,0.7]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.2]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_\psi(\tilde{a}_5,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.1,0.6]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.1,0.3]\rangle},0.3\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.2,0.4]\rangle},0.4\right)\right\}.$$

$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.4]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.7]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.7]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.2]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.6]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.7]\rangle},0.4\right)\right\}.$$
$$\mathcal{H}_{\psi}(\tilde{a}_6,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.1,0.3]\rangle},0.3\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.2\right),\left(\frac{\nu_4}{\langle[0.3,0.7]\rangle},0.4\right)\right\}.$$

TABLE 2. Values of $\mathcal{W}_i$ for *AND* Operation

$\mathcal{H}_\psi$	$\mathcal{W}_i$	**Value degree of $\mathcal{W}_i$**	$\mathcal{H}_\psi$	$\mathcal{W}_i$	**Value degree of $\mathcal{W}_i$**
$(\tilde{a}_1,\tilde{a}_1)$	$\mathcal{W}_1$	0.127	$(\tilde{a}_4,\tilde{a}_1)$	$\mathcal{W}_1$	0.118
$(\tilde{a}_1,\tilde{a}_2)$	$\mathcal{W}_2$	0.190	$(\tilde{a}_4,\tilde{a}_2)$	$\mathcal{W}_2$	0.124
$(\tilde{a}_1,\tilde{a}_3)$	$\mathcal{W}_3$	0.349	$(\tilde{a}_4,\tilde{a}_3)$	$\mathcal{W}_3$	0.144
$(\tilde{a}_1,\tilde{a}_4)$	$\mathcal{W}_4$	0.171	$(\tilde{a}_4,\tilde{a}_4)$	$\mathcal{W}_4$	0.118
$(\tilde{a}_1,\tilde{a}_5)$	$\mathcal{W}_5$	0.143	$(\tilde{a}_4,\tilde{a}_5)$	$\mathcal{W}_5$	0.144
$(\tilde{a}_1,\tilde{a}_6)$	$\mathcal{W}_6$	0.127	$(\tilde{a}_4,\tilde{a}_6)$	$\mathcal{W}_6$	0.106
$(\tilde{a}_2,\tilde{a}_1)$	$\mathcal{W}_1$	0.184	$(\tilde{a}_5,\tilde{a}_1)$	$\mathcal{W}_1$	0.131
$(\tilde{a}_2,\tilde{a}_2)$	$\mathcal{W}_2$	0.255	$(\tilde{a}_5,\tilde{a}_2)$	$\mathcal{W}_2$	0.212
$(\tilde{a}_2,\tilde{a}_3)$	$\mathcal{W}_3$	0.213	$(\tilde{a}_5,\tilde{a}_3)$	$\mathcal{W}_3$	0.165
$(\tilde{a}_2,\tilde{a}_4)$	$\mathcal{W}_4$	0.175	$(\tilde{a}_5,\tilde{a}_4)$	$\mathcal{W}_4$	0.156
$(\tilde{a}_2,\tilde{a}_5)$	$\mathcal{W}_5$	0.208	$(\tilde{a}_5,\tilde{a}_5)$	$\mathcal{W}_5$	0.286
$(\tilde{a}_2,\tilde{a}_6)$	$\mathcal{W}_6$	0.150	$(\tilde{a}_5,\tilde{a}_6)$	$\mathcal{W}_6$	0.129
$(\tilde{a}_3,\tilde{a}_1)$	$\mathcal{W}_1$	0.166	$(\tilde{a}_6,\tilde{a}_1)$	$\mathcal{W}_1$	0.183
$(\tilde{a}_3,\tilde{a}_2)$	$\mathcal{W}_2$	0.196	$(\tilde{a}_6,\tilde{a}_2)$	$\mathcal{W}_2$	0.203
$(\tilde{a}_3,\tilde{a}_3)$	$\mathcal{W}_3$	0.202	$(\tilde{a}_6,\tilde{a}_3)$	$\mathcal{W}_3$	0.209
$(\tilde{a}_3,\tilde{a}_4)$	$\mathcal{W}_4$	0.213	$(\tilde{a}_6,\tilde{a}_4)$	$\mathcal{W}_4$	0.145
$(\tilde{a}_3,\tilde{a}_5)$	$\mathcal{W}_5$	0.247	$(\tilde{a}_6,\tilde{a}_5)$	$\mathcal{W}_5$	0.208
$(\tilde{a}_3,\tilde{a}_6)$	$\mathcal{W}_6$	0.159	$(\tilde{a}_6,\tilde{a}_6)$	$\mathcal{W}_6$	0.140
Total Values	$\mathcal{W}_1$= 0.979	$\mathcal{W}_2$= 1.186			
	$\mathcal{W}_3$= 1.282	$\mathcal{W}_4$= 0.978			
	$\mathcal{W}_5$=1.048	$\mathcal{W}_6$= 0.811			
Final Decision	$\mathcal{W}_1$= ×	$\mathcal{W}_2$=×			
	$\mathcal{W}_3$= √	$\mathcal{W}_4$=×			
	$\mathcal{W}_5$=×	$\mathcal{W}_6$= ×			

The value of $\mathcal{W}_i$ presents in **Table 2** and when we take a look at **Table 2**, we noted that the value $\mathcal{W}_3$ is maximum, so it is chosen. In addition, in **Figure 3** we present a statistical chart to compare the values $\mathcal{W}_i$ obtained from **Algorithm 1**.

Algorithm 2.Using *OR*-operation of PIVFHS-sets

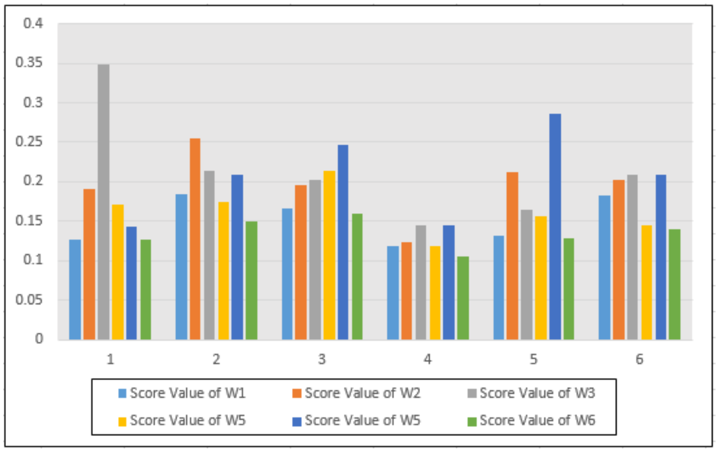

Figure 3: Statistical chart showing the variance $\mathcal{W}_i$ degree of values of $(\tilde{a}_i, \tilde{a}_j)$ under algorithm 1

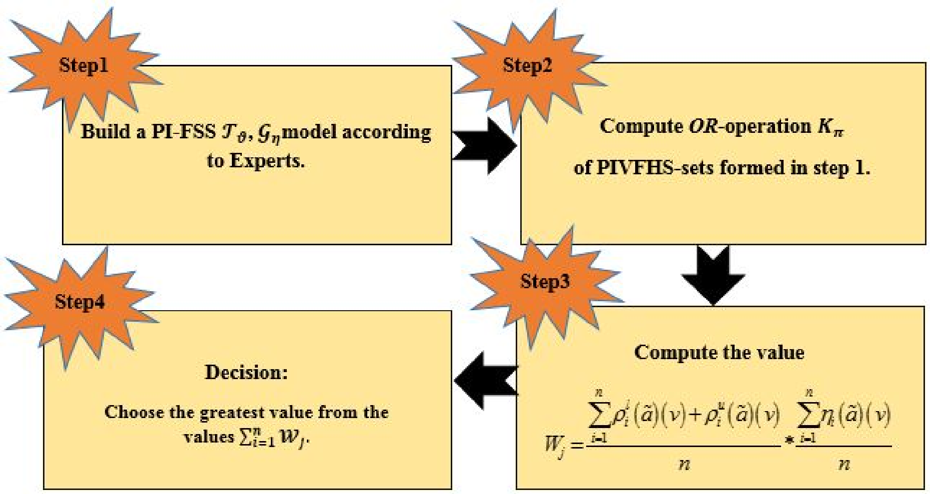

Figure 4: Representation of algorithm 2.

Step 1. Put up PIVFHS-set based on Experts.

Step 2. Compute OR-operation $\mathcal{K}_\pi$ of PIVFHS-sets formed in step 1.

Step 3. Compute the value $\mathcal{W}_i = \frac{\sum_{i=1} \rho_i^l(\tilde{a})(v) + \rho_i^u(\tilde{a})(v)}{n} * \frac{\sum_{i=1} \eta_i(\tilde{a})(v)}{n}$.

Step 4. Decision: choose the greatest value from the values $\mathcal{W}_i$.

Step 5. End Algorithm 2.

Below is **Figure 4**, a representation of algorithm 2 in an abbreviated way.

Step 2. Compute OR-operation $\mathcal{K}_\pi$ of PIVFHS-sets formed in step 1.

$(\mathcal{K}_\pi, \mathcal{M}) =$

$\mathcal{K}_\pi(\tilde{a}_1, \tilde{a}_1) = \left\{ \left(\frac{\nu_1}{\langle [0.3,0.7] \rangle}, 0.4 \right), \left(\frac{\nu_2}{\langle [0.6,0.6] \rangle}, 0.6 \right), \left(\frac{\nu_3}{\langle [0.2,0.4] \rangle}, 0.8 \right), \left(\frac{\nu_4}{\langle [0.5,0.9] \rangle}, 1 \right) \right\}.$

$\mathcal{K}_\pi(\tilde{a}_1, \tilde{a}_2) = \left\{ \left(\frac{\nu_1}{\langle [0.6,0.9] \rangle}, 0.7 \right), \left(\frac{\nu_2}{\langle [0.5,0.6] \rangle}, 0.5 \right), \left(\frac{\nu_3}{\langle [0.5,0.8] \rangle}, 0.7 \right), \left(\frac{\nu_4}{\langle [0.4,0.9] \rangle}, 0.9 \right) \right\}.$

$\mathcal{K}_\pi(\tilde{a}_1,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.5,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_1,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.4\right),\left(\frac{\nu_2}{\langle[0.5,0.6]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.2,0.3]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.9]\rangle},0.6\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_1,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.5,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.4,0.9]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_1,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.5,0.6]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.4,0.9]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},1\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.9]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.5,0.8]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.8\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.3,0.6]\rangle},0.8\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.3,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.8\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_2,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.3,0.4]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.3,0.7]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.4,0.7]\rangle},0.8\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},1\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.9]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.6]\rangle},0.6\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_3,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.1,0.3]\rangle},0.4\right),\left(\frac{\nu_3}{\langle[0.2,0.8]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.4,0.7]\rangle},0.4\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.2,0.4]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},1\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.9]\rangle},0.7\right),\left(\frac{\nu_2}{\langle[0.4,0.5]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.5,0.8]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.3,0.5]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.2,0.2]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.3,0.6]\rangle},0.6\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.2,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_4,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.2,0.3]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.1,0.5]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.4,0.7]\rangle},0.4\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.3,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.6,0.7]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},1\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.9]\rangle},0.9\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.6,0.8]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.6,0.7]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.9\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.6,0.7]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.3,0.6]\rangle},0.6\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.9\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.6,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.7]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_5,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.9\right),\left(\frac{\nu_2}{\langle[0.8,0.8]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.6,0.7]\rangle},0.7\right),\left(\frac{\nu_4}{\langle[0.4,0.7]\rangle},0.4\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_1)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.6\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.5,0.9]\rangle},1\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_2)=\left\{\left(\frac{\nu_1}{\langle[0.6,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.5,0.8]\rangle},0.4\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.9\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_3)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.8]\rangle},0.5\right)\right\}.$

$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_4)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.6\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.8\right),\left(\frac{\nu_4}{\langle[0.3,0.8]\rangle},0.6\right)\right\}.$
$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_5)=\left\{\left(\frac{\nu_1}{\langle[0.7,0.9]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.6,0.7]\rangle},0.8\right),\left(\frac{\nu_3}{\langle[0.4,0.8]\rangle},0.9\right),\left(\frac{\nu_4}{\langle[0.5,0.8]\rangle},0.5\right)\right\}.$
$\mathcal{K}_\pi(\tilde{a}_6,\tilde{a}_6)=\left\{\left(\frac{\nu_1}{\langle[0.5,0.7]\rangle},0.8\right),\left(\frac{\nu_2}{\langle[0.6,0.6]\rangle},0.5\right),\left(\frac{\nu_3}{\langle[0.2,0.5]\rangle},0.5\right),\left(\frac{\nu_4}{\langle[0.4,0.8]\rangle},0.4\right)\right\}.$

TABLE 3. Values of $\mathcal{W}_i$ for *OR* Operation

$\mathcal{K}_\pi$	$\mathcal{W}_i$	**Value degree of $\mathcal{W}_i$**	$\mathcal{K}_\pi$	$\mathcal{W}_i$	**Value degree of $\mathcal{W}_i$**
$(\tilde{a}_1,\tilde{a}_1)$	$\mathcal{W}_1$	0.35	$(\tilde{a}_4,\tilde{a}_1)$	$\mathcal{W}_1$	0.537
$(\tilde{a}_1,\tilde{a}_2)$	$\mathcal{W}_2$	0.455	$(\tilde{a}_4,\tilde{a}_2)$	$\mathcal{W}_2$	0.435
$(\tilde{a}_1,\tilde{a}_3)$	$\mathcal{W}_3$	0.506	$(\tilde{a}_4,\tilde{a}_3)$	$\mathcal{W}_3$	0.459
$(\tilde{a}_1,\tilde{a}_4)$	$\mathcal{W}_4$	0.337	$(\tilde{a}_4,\tilde{a}_4)$	$\mathcal{W}_4$	0.323
$(\tilde{a}_1,\tilde{a}_5)$	$\mathcal{W}_5$	0.563	$(\tilde{a}_4,\tilde{a}_5)$	$\mathcal{W}_5$	0.459
$(\tilde{a}_1,\tilde{a}_6)$	$\mathcal{W}_6$	0.315	$(\tilde{a}_4,\tilde{a}_6)$	$\mathcal{W}_6$	0.244
$(\tilde{a}_2,\tilde{a}_1)$	$\mathcal{W}_1$	0.468	$(\tilde{a}_5,\tilde{a}_1)$	$\mathcal{W}_1$	0.496
$(\tilde{a}_2,\tilde{a}_2)$	$\mathcal{W}_2$	0.375	$(\tilde{a}_5,\tilde{a}_2)$	$\mathcal{W}_2$	0.525
$(\tilde{a}_2,\tilde{a}_3)$	$\mathcal{W}_3$	0.515	$(\tilde{a}_5,\tilde{a}_3)$	$\mathcal{W}_3$	0.534
$(\tilde{a}_2,\tilde{a}_4)$	$\mathcal{W}_4$	0.376	$(\tilde{a}_5,\tilde{a}_4)$	$\mathcal{W}_4$	0.489
$(\tilde{a}_2,\tilde{a}_5)$	$\mathcal{W}_5$	0.516	$(\tilde{a}_5,\tilde{a}_5)$	$\mathcal{W}_5$	0.561
$(\tilde{a}_2,\tilde{a}_6)$	$\mathcal{W}_6$	0.301	$(\tilde{a}_5,\tilde{a}_6)$	$\mathcal{W}_6$	0.406
$(\tilde{a}_3,\tilde{a}_1)$	$\mathcal{W}_1$	0.484	$(\tilde{a}_6,\tilde{a}_1)$	$\mathcal{W}_1$	0.421
$(\tilde{a}_3,\tilde{a}_2)$	$\mathcal{W}_2$	0.323	$(\tilde{a}_6,\tilde{a}_2)$	$\mathcal{W}_2$	0.414
$(\tilde{a}_3,\tilde{a}_3)$	$\mathcal{W}_3$	0.425	$(\tilde{a}_6,\tilde{a}_3)$	$\mathcal{W}_3$	0.497
$(\tilde{a}_3,\tilde{a}_4)$	$\mathcal{W}_4$	0.371	$(\tilde{a}_6,\tilde{a}_4)$	$\mathcal{W}_4$	0.406
$(\tilde{a}_3,\tilde{a}_5)$	$\mathcal{W}_5$	0.459	$(\tilde{a}_6,\tilde{a}_5)$	$\mathcal{W}_5$	0.506
$(\tilde{a}_3,\tilde{a}_6)$	$\mathcal{W}_6$	0.391	$(\tilde{a}_6,\tilde{a}_6)$	$\mathcal{W}_6$	0.295
Total Values	$\mathcal{W}_1$= 2.756	$\mathcal{W}_2$= 2.527			
	$\mathcal{W}_3$= 2.936	$\mathcal{W}_4$= 2.302			
	$\mathcal{W}_5$=3.064	$\mathcal{W}_6$= 1.952			
Final Decision	$\mathcal{W}_1= \times$	$\mathcal{W}_2=\times$			
	$\mathcal{W}_3= \times$	$\mathcal{W}_4=\times$			
	$\mathcal{W}_5=\surd$	$\mathcal{W}_6= \times$			

The value of $\mathcal{W}_i$ presents in **Table 3** and when we take a look at **Table 3**, we noted that the value $\mathcal{W}_5$ is maximum, so it is chosen. In addition, in **Figure 5** we present a statistical chart to compare the values $\mathcal{W}_i$ obtained from **Algorithm 2**. We finally show a comparison of the $\mathcal{W}_i$ values for the AND-operation and the OR-operation from Algorithms 1 and 2. This can be seen in both **Figure 6**and **Table 4**.

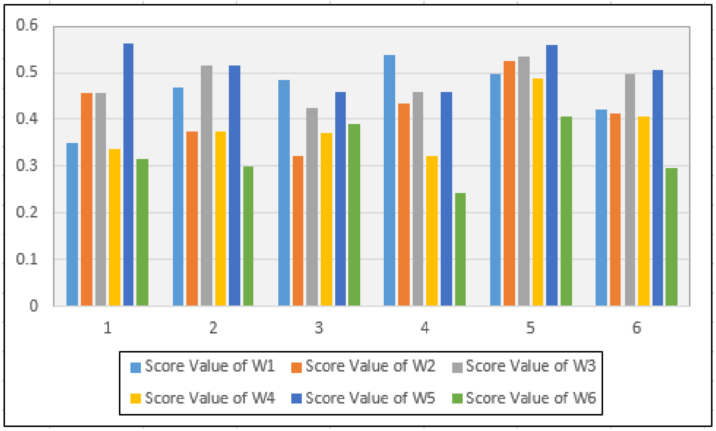

Figure 5: Statistical chart showing the variance $\mathcal{W}_i$ degree of values of $(\tilde{a}_i, \tilde{a}_j)$ under algorithm 2

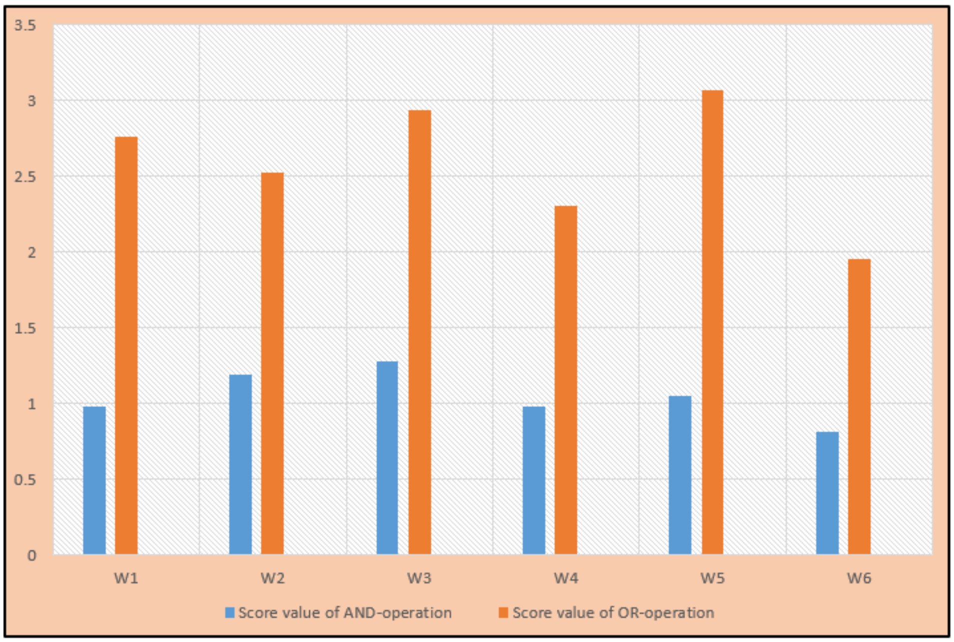

Figure 6: Statistical chart showing comparison analysis between the score values of AND and OR operations. under algorithm 1&2

TABLE 4. Comparison between the output value scores of AND and OR-operations

Aggregation Operation	$\mathcal{W}_1$	$\mathcal{W}_2$	$\mathcal{W}_3$	$\mathcal{W}_4$	$\mathcal{W}_5$	$\mathcal{W}_6$	**Ranking**
AND-Operation	0.979	1.186	1.282	0.978	1.048	0.811	$\mathcal{W}_3 \succ \mathcal{W}_2 \succ \mathcal{W}_5 \succ \mathcal{W}_1 \succ \mathcal{W}_4 \succ \mathcal{W}_6$
OR-Operation	2.756	2.527	2.936	2.302	3.064	1.952	$\mathcal{W}_5 \succ \mathcal{W}_3 \succ \mathcal{W}_1 \succ \mathcal{W}_2 \succ \mathcal{W}_4 \succ \mathcal{W}_6$

Remark 5.2. Both of the above algorithms are presented based on *AND*-operation of PIVFHS-sets and *OR*-operation of PIVFHS-sets. Therefore, the two algorithms work based on the mechanisms of both *AND*-operation and *OR*-operation. For this reason, there is a difference in the outputs of both algorithms.

We finally show a comparison of the $\mathcal{W}_i$ values for the AND-operation and the OR-operation from Algorithms 1 and 2. This can be seen in both **Figure 6** and **Table 4**.

6. Similarity Measure on PIVFHS-set

Similarity measures have been widely used by a lot researchers [65]- [69] because of their importance in calculating the percentage of similarity between two things or values, where the output of these measures is a numerical value. In FS- environment these measures are used to calculate the percentage of similarity between two FS-sets, where the output of these measures belongs to the close interval [0, 1]. Now in part, we define these measures on our proposed model IVFHS-sets in order to calculate the ratio of similarity between two PIVFHS-sets.

Definition 6.1. Let $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ be two PIVFHS-sets over $(\mathcal{V}, \mathcal{Z})$. Then the Similarity measure between $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ indicated by $\hat{S}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right)$ is defined as follows:

$$\hat{\mathbb{S}}\left(\mathcal{T}_{\tilde{a}}, \mathcal{G}_{\tilde{a}}\right) = \ddot{\mathbb{M}}\left(\mathcal{T}\left(\vartheta\right), \mathcal{G}\left(\eta\right)\right) \times \ddot{\mathbb{M}}\left(\vartheta\left(\tilde{a}\right), \eta\left(\tilde{a}\right)\right),$$

such that

$$\ddot{\mathbb{M}}\left(\mathcal{T}\left(\tilde{a}\right), \mathcal{G}\left(\tilde{a}\right)\right) = max\ddot{\mathbb{M}}_i\left(\mathcal{T}\left(\tilde{a}\right), \mathcal{G}\left(\tilde{a}\right)\right),$$
$$\ddot{\mathbb{M}}\left(\vartheta\left(\tilde{a}\right), \eta\left(\tilde{a}\right)\right) = max\ddot{\mathbb{M}}_i\left(\vartheta\left(\tilde{a}\right), \eta\left(\tilde{a}\right)\right),$$

where

$$\ddot{\mathbb{M}}_i\left(\mathcal{T}_{ij}\left(\tilde{a}\right), \mathcal{G}_{ij}\left(\tilde{a}\right)\right) = 1 - \frac{1}{\sqrt{n}}\sqrt{\sum_{j=1}^{n}\left(\hat{\phi}_{\mathcal{T}_{ij}(\tilde{a})}\left(\nu\right) - \hat{\phi}_{\mathcal{G}_{ij}(\tilde{a})}\left(\nu\right)\right)^2},$$

such that and,

$$\hat{\phi}_{\mathcal{T}_{\vartheta_{ij}}(\tilde{a})}\left(\nu\right) = \frac{\rho^l{}_{\mathcal{T}_{\vartheta_{ij}}(\tilde{a})}(\nu) + \rho^u{}_{\mathcal{T}_{\vartheta_{ij}}(\tilde{a})}(\nu)}{2}, \ \hat{\phi}_{\mathcal{G}_{\eta_{ij}}(\tilde{a})}\left(\nu\right) = \frac{\rho^l{}_{\mathcal{G}_{\eta_{ij}}(\tilde{a})}(\nu) + \rho^u{}_{\mathcal{G}_{\eta_{ij}}(\tilde{a})}(\nu)}{2}.$$

$$\ddot{\mathbb{M}}_i\left(\vartheta\left(\tilde{a}\right), \eta\left(\tilde{a}\right)\right) = 1 - \frac{\sum_{j=1}^{n}|\vartheta_{ij}(\tilde{a}) - \eta_{ij}(\tilde{a})|}{\sum_{j=1}^{n}|\vartheta_{ij}(\tilde{a}) + \eta_{ij}(\tilde{a})|}$$

Definition 6.2. Let $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ be two PIVFHSSs over $(\mathcal{V}, \mathcal{Z})$.We say that $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ are significantly similar if $\ddot{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) \geq \frac{1}{2}$.

Proposition 6.3. *Let $\mathcal{T}_\vartheta$, $\mathcal{G}_\eta$ and $\mathcal{H}_\lambda$ be three PIVFHSSs over $(\mathcal{V}, \mathcal{Z})$.Then the following results are achieved:*

(i). $\hat{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) = \hat{\mathbb{S}}\left(\mathcal{G}_\eta, \mathcal{T}_\vartheta\right)$.

(ii). $0 \leq \hat{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) \leq 1$.

(iii).If $\mathcal{T}_\vartheta = \mathcal{G}_\eta$ *then* $\hat{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) = 1$.

(iv).$\mathcal{T}_\vartheta \subseteq \mathcal{G}_\eta \subseteq \mathcal{H}_\lambda$ *then* $\hat{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) \leq \hat{\mathbb{S}}\left(\mathcal{G}_\eta, \mathcal{H}_\lambda\right)$.

(v).If $\mathcal{T}_\vartheta \cap \mathcal{G}_\eta = \Phi \Leftrightarrow \hat{\mathbb{S}}\left(\mathcal{T}_\vartheta, \mathcal{G}_\eta\right) = 0$.

Proof. The proof of these propositions is clear by Definition 6.1 and therefore omitted. □

Algorithm 3. Employing similarity measures of PIVFHS-set to recruitment pattern recognition

Step 1. Create PIVFHS-set $\mathcal{T}_{\vartheta}$ based on experts team.

Step 2. Create PIVFHS-set $\mathcal{G}_{\eta}$ based on external experts.

Step 3. Determine $\ddot{\mathbb{M}}\left(\mathcal{T}\left(\tilde{a}\right),\mathcal{G}\left(\tilde{a}\right)\right)$ and $\ddot{\mathbb{M}}\left(\vartheta\left(\tilde{a}\right),\eta\left(\tilde{a}\right)\right)$ based on Definition 6.1.

Step 4. Check similarity score based on definition 6.2.

Step 5. End Algorithm 3.

Below is Figure 7, a representation of algorithm 3 in an abbreviated way.

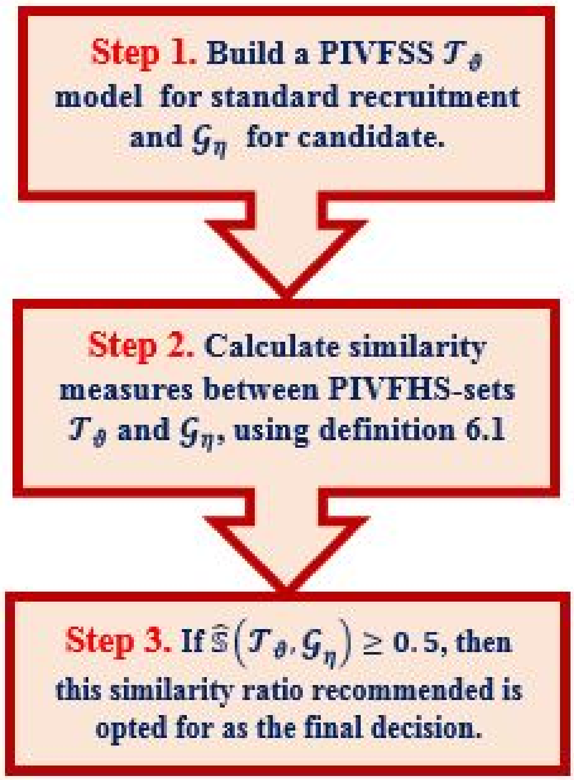

Figure 7: Representation of algorithm 3.

Example 6.4. An organization announced that there are a number of vacant posts, and for the purpose of selecting qualified people for these jobs, a committee was formed of a number of experts who belong to the human resource management department (HRMD) in this organization. In this case, there are only two option $\nu_1 = agree, \nu_2 = disagree$ that are present in the universe $\mathcal{Z} = \{\nu_1, \nu_2\}$.In this formulation, the committee members put some evaluating attributes for this recruitment which are $\tilde{y}_1 = qualification,, \tilde{y}_2 = experience$ and $\tilde{y}_3 = age$. Then the attribute-value sets corresponding to these attributes are: $\tilde{\mathcal{A}}_1 =$

$\{\tilde{a}_{11} = Master\ Degree, \tilde{a}_{12} = Diploma\}$, $\tilde{\mathcal{A}}_2 = \{\tilde{a}_{21} = 3years, \tilde{a}_{22} = 7years\}$, $\tilde{\mathcal{A}}_3 = \{\tilde{a}_{31} = 25years, \tilde{a}_{32} = 30years\}$ then $\tilde{\mathcal{X}} = \tilde{\mathcal{A}}_1 \times \tilde{\mathcal{A}}_2 \times \tilde{\mathcal{A}}_3 = \{\tilde{a}_{11}, \tilde{a}_{12}\} \times \{\tilde{a}_{21}, \tilde{a}_{22}\} \times \{\tilde{a}_{31}, \tilde{a}_{32}\} = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8\}$ $\forall \tilde{a}_i$ is a 3-tuple element of $\tilde{\mathcal{A}}$.

Now, to implement the above algorithm, let $\tilde{\mathcal{X}}_1 = \{\tilde{a}_1, \tilde{a}_3, \tilde{a}_4, \tilde{a}_7\}$ is a subset of $\tilde{\mathcal{X}}$ and we consider a model PIVFHS-set for formal recruitment is

$\mathcal{T}_\vartheta(\tilde{a}_1) = \left\{\left(\frac{\nu_1}{\langle[1,1]\rangle}, 1\right), \left(\frac{\nu_2}{\langle[1,1]\rangle}, 1\right)\right\}$.

$\mathcal{T}_\vartheta(\tilde{a}_3) = \left\{\left(\frac{\nu_1}{\langle[1,1]\rangle}, 1\right), \left(\frac{\nu_2}{\langle[1,1]\rangle}, 1\right)\right\}$.

$\mathcal{T}_\vartheta(\tilde{a}_4) = \left\{\left(\frac{\nu_1}{\langle[1,1]\rangle}, 1\right), \left(\frac{\nu_2}{\langle[1,1]\rangle}, 1\right)\right\}$.

$\mathcal{T}_\vartheta(\tilde{a}_7) = \left\{\left(\frac{\nu_1}{\langle[1,1]\rangle}, 1\right), \left(\frac{\nu_2}{\langle[1,1]\rangle}, 1\right)\right\}$.

and we organize PIVFHS-set $\mathcal{G}_\eta$ for the candidate by an expert outside the expert's team of the company.

$\mathcal{G}_\eta(\tilde{a}_1) = \left\{\left(\frac{\nu_1}{\langle[0.3,0.5]\rangle}, 0..6\right), \left(\frac{\nu_2}{\langle[0.2,0.3]\rangle}, 0.7\right)\right\}$.

$\mathcal{G}_\eta(\tilde{a}_3) = \left\{\left(\frac{\nu_1}{\langle[0.1,0.8]\rangle}, 0.5\right), \left(\frac{\nu_2}{\langle[0.6,0.8]\rangle}, 0.2\right)\right\}$.

$\mathcal{G}_\eta(\tilde{a}_4) = \left\{\left(\frac{\nu_1}{\langle[0.8,0.9]\rangle}, 0.6\right), \left(\frac{\nu_2}{\langle[0.3,0.8]\rangle}, 0.4\right)\right\}$.

$\mathcal{G}_\eta(\tilde{a}_7) = \left\{\left(\frac{\nu_1}{\langle[0.3,0.5]\rangle}, 0.7\right), \left(\frac{\nu_2}{\langle[0.2,0.4]\rangle}, 0.7\right)\right\}$.

Now we apply step 3. by computing similarity b/w $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ as stated by Definition 6.1.

$\ddot{\mathbb{M}}_1(\vartheta(\tilde{a}_1), \eta(\tilde{a}_1)) = 1 - \frac{|(1-0.6)|+|(1-0.7)|}{|(1+0.6)|+|(1+0.7)|} = 0.7879$

In the same vein:

$\ddot{\mathbb{M}}_2(\vartheta(\tilde{a}_2), \eta(\tilde{a}_2)) = 0.5193$

$\ddot{\mathbb{M}}_3(\vartheta(\tilde{a}_3), \eta(\tilde{a}_3)) = 0.6667$

$\ddot{\mathbb{M}}_4(\vartheta(\tilde{a}_4), \eta(\tilde{a}_4)) = 0.1764$

therefor

$\ddot{\mathbb{M}}(\vartheta(\tilde{a}), \eta(\tilde{a})) = max\ddot{\mathbb{M}}_i(\vartheta(\tilde{a}_i), \eta(\tilde{a}_i)) = 0.7879$

Now

$\ddot{\mathbb{M}}_1(\mathcal{T}(\tilde{a}_1), \mathcal{G}(\tilde{a}_1)) = 1 - \frac{1}{\sqrt{2}}\sqrt{(1-0.4)^2 + (1-0.25)^2} = 0.3209$

In the same vein:

$\ddot{\mathbb{M}}_2(\mathcal{T}(\tilde{a}_2), \mathcal{G}(\tilde{a}_2)) = 0.5570$

$\ddot{\mathbb{M}}_3(\mathcal{T}(\tilde{a}_3), \mathcal{G}(\tilde{a}_3)) = 0.6646$

$\ddot{\mathbb{M}}_4(\mathcal{T}(\tilde{a}_4), \mathcal{G}(\tilde{a}_4)) = 0.3480$

therefore,

$\ddot{\mathbb{M}}(\mathcal{T}(\tilde{a}_i), \mathcal{G}(\tilde{a}_i)) = max\ddot{\mathbb{M}}_i(\mathcal{T}(\tilde{a}_i), \mathcal{G}(\tilde{a}_i)) = 0.6646$

Hence, based on Definition 6.1. the degree of similarity b/w $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ is given by $\hat{\mathbb{S}}(\mathcal{T}_{\tilde{a}}, \mathcal{G}_{\tilde{a}}) = 0.6646 \times 0.7879 = 0.5236 \succ 0.5$ that means $\mathcal{T}_\vartheta$ and $\mathcal{G}_\eta$ are radically similar. Therefore, the candidate agrees to join the company.

7. Discussion and Comparison Analysis

Scholars around the world are always working to adapt their research work per the needs and requirements of different life situations. For instance, in decision-making techniques, various traits play an important role in the selection process. Therefore, researchers always resort to discovering and developing new tools that are compatible with these traits and facilitate the selection process. In literature, among different developments, Rahman et al. [57] proposed a new DM algorithmic approach known as PFHS-set. In an overview of this model, it evaluates the values of parameters, qualifications, and experience by a single value with a given possible degree between 0 and 1. In some life situations, the user needs an environment full of flexibility and reliability for apt decision-making and dealing with data that includes uncertainty. This is provided by the interval framework upon which the proposed model is built in this work. Further, the presence of an HSS collection distinguishes our concept of being able to handle sub-attributive values of parameters. In addition, in Table 5. we display a comparison analysis of our model with linked existing models based on their structural composition.

Table 5. Comparison with current models under suitable criteria.

Methods	**MD**	**DP**	**PT**	**HSAVP**	**MAAF**	**IVF**
PF-set	√	×	×	×	×	×
PFS-set	√	√	√	×	×	×
PFHS-set	√	√	√	√	√	×
PFHS-set	√	√	√	√	√	×
IVFHS-set	√	×	√	√	√	√
Our model:PIVFHS-set	√	√	√	√	√	√

Table 6. Numerical analysis of presented structure with predeveloped approaches.

Authors	**Structures**	**Ranking**	**Remark**
Yolcu and Ozturk [36]	**FHS-set**	N\A	Interval values are ignored
Rahman et al. [64]	**PFHS-set**	N\A	Interval values are ignored
Arshad et al. [41]	**IVFHS-set**	$\mathcal{W}_1 \succ \mathcal{W}_6 \succ \mathcal{W}_5 \succ \mathcal{W}_3 \succ \mathcal{W}_4 \succ \mathcal{W}_2$	Possibility degree are ignored
Proposed Study	**PIVFHS-set**	$\mathcal{W}_3 \succ \mathcal{W}_2 \succ \mathcal{W}_5 \succ \mathcal{W}_1 \succ \mathcal{W}_4 \succ \mathcal{W}_6$	The degree of probability has an intelligible effect on the ranking of values

The advantages of the proposed model can easily be distinguished by looking at Table 5, where this comparison focuses on features: MD (Membership Degree), DP (Degree of Possibility), PT (Parameter Tools), HSAVP (Handle Sub-Attributive Values of Parameters), MAAF (Multi Argument Approximate Function), and IVF (Interval Value Form) where the (√) sign indicates the presence of this feature in this model, while the (×) sign indicates that this model lacks this feature.. As for the algorithms presented in this work, they are considered more reliable in solving DM problems compared to previous algorithms that ignored both interval form, SS settings, and HSS settings. Moreover, Table 6 includes comparative experiments to illustrate the performance differences between the PIVFHS-set and other existing models that can handle the data assumed in Example 5.1 and presented in interval form when solving the same problems. As this research focused on membership in the interval value only in handling uncertainty issues in the parameter environment. Thus, this work ignores many scenarios in our daily lives that include uncertainty, vagueness, neutrality, and inconsistency. Therefore, researchers in the future can manage such scenarios by extending this model to vague settings (two memberships: True and False), intuitionistic-FS settings (two memberships: True and False), and neurosophic-SS (three memberships: True, Neutrality, and False) settings under interval form.

8. **Conclusions**

In this article, we established innovative notion of PIVFHS-set by melting both IVF-set and HS-set under possibility setting. This model is characterized by the ability to deal with uncertain issues in an apt way. Start with this model, we successfully conceptualize the basic set-theoretical operations like possibility null IVFHS-set, possibility absolute IVFHS-set, complement of PIFHS-set, union PIVFHS-sets, intersection PIVFHS-sets, AND-PIVFHS-sets, and OR-PIVFHS-sets, as well as support them with numerical examples. Furthermore, two algorithms based on PIVFHS-set operations and similarity measures between PIVFHS-sets were employed to handle real-world DM problems. In order to show the advantages of this work, the proposed model was compared with some previous models. Finally, the presented approach is based on a single organic function and therefore loses the ability to deal with issues that include both the criterion of dishonesty and a lack of neutrality, which can be explained by two concepts: IFSs and NSs. Therefore, in future work, this model can be extended to other new hybridized approaches that are more comprehensive in dealing with uncertainty issues.

Funding Statement: This article is partially funded by University of Mataram, Indonesia.
Author Contributions: All authors contributed equally to this paper. We can describe each author's individual responsibilities and contributions as follows: F. Al-Sharqi, R.H. Al-Obaidi, Z. Md. Rodzi, and M. U. Romdhini made significant contributions to the methodology, formal analysis, and write-up of the initial draft. F. Al-Sharqi and R.H. Al-Obaidi both contributed to conceptualizations, methodologies, and investigations.

Availability of Data and Materials: No data and no material were used in this study.

Conflicts of Interest: There are no conflicts of interest in this manuscript.

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Received: Aug 5, 2024. Accepted: Nov 1, 2024