Neutrosophic Sets and Systems, Vol. 77, 2025
University of New Mexico

Comprehensive Analysis Using 2-tuple Linguistic Neutrosophic MADM with Core Competencies Evaluation of Track and Field Students in Sports Colleges

Ming Tang*, Yanzhao Sun
Henan University of Economics and Law, Zhengzhou, 450000, Henan, China
*Corresponding author Email: tangming717@163.com; 20220056@huel.edu.cn

Abstract: The core competencies evaluation of track and field students in sports colleges holds significant importance. It not only helps assess students' overall abilities in track and field, such as physical fitness, athletic skills, psychological quality, and social responsibility, but also provides a scientific basis for personalized teaching and training. Through this evaluation, students' strengths and weaknesses can be identified, allowing for the optimization of training strategies to improve their athletic performance and holistic development. Additionally, this evaluation aids in cultivating students' teamwork, psychological resilience, and sense of social responsibility, ensuring that they are more competitive and adaptable in their future sports careers, thereby promoting their long-term development. The core competencies evaluation of track and field students in sports colleges is regarded as a multi-attribute decision-making (MADM) problem. In this study, we integrated the geometric Heronian mean (GHM) approach and prioritized aggregation (PA) with 2-tuple linguistic neutrosophic numbers (2TLNNs) to develop the generalized 2-tuple linguistic neutrosophic numbers prioritized GHM (G2TLNPGHM) approach. Additionally, we explored several key properties of the proposed approach. The G2TLNPGHM approach was then applied to address MADM problems under the framework of 2TLNNs. To demonstrate its practical application, the method was used as an example for evaluating the core competencies of track and field students in sports colleges. Lastly, a comprehensive comparative analysis was conducted to examine the effects of varying parameters on the results.

Keywords: MADM; 2TLNNs; G2TLNPGHM approach; core competencies evaluation

1. Introduction

In the context of athletics specialization in sports colleges, the cultivation and assessment of students' core competencies have become key research directions in recent years. Core competencies not only refer to students' athletic skills but also encompass multiple dimensions, such as psychological resilience, health awareness, teamwork abilities, and the concept of lifelong sports development. As reforms in physical education deepen, the challenge of how to scientifically evaluate the core competencies of students specializing in athletics has become crucial for improving teaching quality and talent development. First, the assessment of athletics students' core competencies should combine the improvement of sports skills with the enhancement of physical fitness, ensuring that students possess both high-competitive performance and robust physical health. This can be objectively evaluated through quantitative indicators such as technical skill tests and physical fitness assessments. Second, attention should also be given to students' mental toughness, team spirit, and ability to handle pressure during training and competitions. The development and evaluation of these psychological qualities can be conducted through observational records, behavioral analysis, and situational simulations. Additionally, students' health literacy and understanding of the value of sports should not be overlooked. Athletics demands that students adopt scientific training methods and maintain healthy lifestyle habits, while teaching should also guide students to appreciate the lifelong value of sports. In conclusion, when assessing the core competencies of athletics students in sports colleges, a multidimensional and multi-tiered evaluation system should be employed. This system should not only focus on students' athletic performance but also emphasize comprehensive assessments of their psychological, health, and social development, providing robust support and assurance for students' well-rounded growth. First, Peng, Yu and Cheng [1] studied the physical education curriculum system based on the cultivation of core competencies for vocational college students. They emphasized the need to optimize the curriculum system to enhance students' core competencies and proposed corresponding solutions to address existing problems. Then, Liu [2] explored the core competencies and capacity building of physical education teachers in vocational colleges, highlighting the crucial role of teachers in physical education and proposing the essential competencies teachers should possess. Wu [3] pointed out the lack of focus on core competencies in physical education in vocational colleges, calling for increased awareness and proposing specific improvement suggestions. In the same year, Yan, Yin and Wang [4] used literature analysis and logical reasoning to propose pathways for building a core competency system for physical education in vocational colleges, emphasizing the organic integration of core competencies, curriculum standards, and training goals. Li, Yu and Huang [5] explored the direction of physical education curriculum reform based on the needs of cultivating core competencies in vocational college students, suggesting the improvement of students' basic competencies to meet the demands of vocational education. Following this, Wu and Xu [6] conducted a preliminary study on evaluation system of physical education core competencies in vocational college students, proposing diversified and contextualized evaluation methods and emphasizing the importance of evaluating students' overall development. Liu [7] analyzed the current state of physical education core competencies in higher education institutions and identified shortcomings in students' competencies, proposing optimization strategies. Next, Ma [8] researched the construction of a physical education curriculum system for five-year vocational colleges, proposing a framework based on "one mechanism, two spaces, three goals, and three modules." Using Suzhou Agricultural Vocational College as an example, Lei [9] explored the cultivation of core competencies in minority students through physical education, offering targeted suggestions to address the weaker physical competencies of minority students. Lu, Ji and Sun [10] examined the construction of physical education curricula in medical colleges, proposing the optimization of health behavior cultivation in courses and the targeted improvement of students' physical qualities and willpower. Finally, Liu, Ning, Zhang and Sun [11] analyzed the current state of physical education curricula in vocational colleges and proposed reform measures, suggesting the optimization of policies, resource allocation, and faculty to improve the core competencies of students in physical education.

The core competencies evaluation of track and field students in sports colleges can be approached as a MADM. In this context, two decision-making approaches are particularly useful: the prioritized average (PA) approach [12], which handles the prioritization relationships among attributes, and the geometric Heronian mean (GHM) approach [12], which considers the interrelationships between input arguments. These methods are beneficial in scenarios where the evaluation process involves numerous interconnected factors that need to be weighed and prioritized carefully. Additionally, the 2TLNSs [13] is a powerful tool for handling uncertainty in evaluations. It combines the strengths of neutrosophic sets(NSs) [14-16], single-valued NSs (SVNSs) [17] and 2-tuple linguistic model [18-20], making it highly effective at portraying uncertain information. This framework has been successfully utilized in various research fields, such as evaluating bilingual teaching effectiveness in college English courses, and it offers a flexible and precise way to capture linguistic and uncertain data. To leverage the strengths of both the PA and GHM approaches, this paper integrates them with 2TLNS to propose the G2TLNPGHM approach. This new approach offers several advantages. First, it incorporates the prioritization of attributes, ensuring that more important factors have a greater influence on the final decision. Second, it accounts for the interrelationships between attributes, capturing the complex dependencies that can exist in real-world decision-making scenarios. Finally, it effectively minimizes information loss, which is crucial when evaluating core competencies that involve multifaceted and interconnected criteria, such as those of track and field students in sports colleges. The G2TLNPGHM approach is particularly well-suited for handling MADM problems within the 2TLNS framework. By allowing for both prioritization and interrelationships between attributes, it provides a comprehensive approach for decision-making under uncertain environments. This makes it an ideal tool for evaluating the core competencies of track and field students, where factors such as physical performance, technical skills, psychological resilience, and teamwork must all be considered in a balanced and integrated manner. To demonstrate practical application of G2TLNPGHM approach, the core competencies evaluation of track and field students in sports colleges is used as an example. This empirical study highlights how the G2TLNPGHM approach can be utilized to real-world decision-making scenarios, validating its effectiveness in evaluating complex, multi-attribute problems.

The main objectives and motivations behind this research can be constructed:

(1) The GHM approach and PA approach are integrated to create the G2TLNPGHM approach under the 2TLNS framework, combining the strengths of both methods.

(2) The G2TLNPGHM approach is specifically developed for handling MADM problems with 2TLNS, making it highly adaptable to situations involving uncertainty and complexity.

(3) A numerical study is constructed to apply the G2TLNPGHM approach to the core competencies evaluation of track and field students in sports colleges, demonstrating the practical utility of the approach.

(4) Comparative studies are provided to justify the effectiveness and reliability of the G2TLNPGHM approach, showcasing its advantages over other existing decision-making methods.

Overall, this study contributes a robust and flexible decision-making framework that can be applied to complex evaluations, particularly in the context of sports education and beyond. To structure this paper, the following sections are organized. In the next section, we introduce the 2TLNNSs. Section 3 presents the development of the G2TLNPGHM approach. In Section 4, we apply the G2TLNPGHM approach to an example involving the core competencies evaluation of track and field students in sports colleges, demonstrating its practical use. Finally, Section 5 provides a comprehensive conclusion, summarizing the key findings and contributions of the paper.

2. Some preliminaries

Wang et al. [13] produced the 2TLNSs.

2.1. 2TLSs

Definition 1[28-29]. Let $zs_1, zs_2, \ldots, zs_{uz}$ be linguistic variables. The ks is produced:

$$zs = \begin{Bmatrix} zs_0 = extremely\ poor, zs_1 = very\ poor, zs_2 = poor, zs_3 = medium, \\ zs_4 = good, zs_5 = very\ good, zs_6 = extremely\ good. \end{Bmatrix}$$

2.2. SVNSs

The SVNSs $z\eta$ is produced [6-7]:

$$z\eta = \left\{ \left(\theta, \phi_z(\theta), \varphi_z(\theta), \gamma_z(\theta) \right) \middle| \theta \in Q \right\} \tag{1}$$

where $\phi_z(\theta), \varphi_z(\theta), \gamma_z(\theta) \in [0,1]$ is truth-membership (TM), indeterminacy-membership (IM) and falsity-membership (FM), $0 \le \phi_z(\theta) + \varphi_z(\theta) + \gamma_z(\theta) \le 3$.

2.3. 2TLNSs

Definition 2[27]. Let $zs_1, zs_2, \ldots, zs_{uz}$ be 2TLSs. If $z\delta = \left\langle (zs_t, z\xi), (zs_i, z\psi), (zs_f, z\zeta) \right\rangle$ is produced for $zs_t, zs_i, zs_f \in zs$, $z\xi, z\psi, z\zeta \in [0, 0.5)$, where $(zs_t, z\xi), (zs_i, z\psi), (zs_f, z\zeta)$ is TM, IM and the FM through 2TLSs, the 2TLNSs is produced:

$$z\delta_j = \left\langle \left(zs_{t_j}, z\xi_j \right), \left(zs_{i_j}, z\psi_j \right), \left(zs_{f_j}, z\zeta_j \right) \right\rangle \tag{2}$$

where $0 \le \Delta^{-1}\left(zs_{t_j}, z\xi_j \right) \le uz, 0 \le \Delta^{-1}\left(zs_{i_j}, z\psi_j \right) \le uz, 0 \le \Delta^{-1}\left(zs_{f_j}, z\zeta_j \right) \le uz$,

and $0 \le \Delta^{-1}\left(zs_{t_j}, z\xi_j \right) + \Delta^{-1}\left(zs_{i_j}, z\psi_j \right) + \Delta^{-1}\left(zs_{f_j}, z\zeta_j \right) \le 3uz.$

Definition 3[27]. Let $z\delta = \left\langle (zs_t, z\xi), (zs_i, z\psi), (zs_f, z\zeta) \right\rangle$. The score function $SF(z\delta)$ and accuracy function $AF(z\delta)$ is produced:

$$ZSF\left(z\delta\right)=\frac{\left(2uz+\Delta^{-1}\left(zs_{t},z\xi\right)-\Delta^{-1}\left(zs_{i},z\psi\right)-\Delta^{-1}\left(zs_{f},z\zeta\right)\right)}{3uz},ZSF\left(z\delta\right)\in\left[0,1\right] \quad (3)$$

$$ZAF(z\delta)=\frac{uz+\Delta^{-1}\left(zs_{t},z\xi\right)-\Delta^{-1}\left(zs_{f},z\zeta\right)}{2uz},ZAF(z\delta)\in\left[0,1\right]. \quad (4)$$

Definition 4[27]. Let $z\delta_1=\left\langle\left(zs_{t_1},z\xi_1\right),\left(zs_{i_1},z\psi_1\right),\left(zs_{f_1},z\zeta_1\right)\right\rangle$ and $z\delta_2=\left\langle\left(zs_{t_2},z\xi_2\right),\left(zs_{i_2},z\psi_2\right),\left(zs_{f_2},z\zeta_2\right)\right\rangle$, then

(1) $if\ \ ZSF\left(z\delta_1\right)<ZSF\left(z\delta_2\right),\ z\delta_1<z\delta_2;$

(2) $if\ \ ZSF\left(z\delta_1\right)=ZF\left(z\delta_2\right),ZAF\left(z\delta_1\right)<ZAF\left(z\delta_2\right),\ z\delta_1<z\delta_2;$

(3) $if\ \ ZSF\left(z\delta_1\right)=ZSF\left(z\delta_2\right),ZAF\left(z\delta_1\right)=ZAF\left(z\delta_2\right),z\delta_1=z\delta_2.$

Definition 5[27]. Let $z\delta_1=\left\langle\left(zs_{t_1},z\xi_1\right),\left(zs_{i_1},z\psi_1\right),\left(zs_{f_1},z\zeta_1\right)\right\rangle$ and $z\delta_2=\left\langle\left(zs_{t_2},z\xi_2\right),\left(zs_{i_2},z\psi_2\right),\left(zs_{f_2},z\zeta_2\right)\right\rangle$, $z\delta=\left\langle\left(zs_{t},z\xi\right),\left(zs_{i},z\psi\right),\left(zs_{f},z\zeta\right)\right\rangle$ be three 2TLNNs, then

$$(1)\ \ z\delta_1\oplus z\delta_2=\left\{\begin{array}{l}\Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{t_1},z\xi_1\right)}{uz}+\frac{\Delta^{-1}\left(zs_{t_2},z\xi_2\right)}{uz}-\frac{\Delta^{-1}\left(zs_{t_1},z\xi_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{t_2},z\xi_2\right)}{uz}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{i_1},z\psi_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{i_2},z\psi_2\right)}{uz}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(z_{f_1},z\zeta_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{f_2},z\zeta_2\right)}{uz}\right)\right)\end{array}\right\};$$

$$(2)\ \ z\delta_1\otimes z\delta_2=\left\{\begin{array}{l}\Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{t_1},z\xi_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{t_2},z\xi_2\right)}{uz}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{i_1},z\psi_1\right)}{uz}+\frac{\Delta^{-1}\left(zs_{i_2},z\psi_2\right)}{uz}-\frac{\Delta^{-1}\left(zs_{i_1},z\psi_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{i_2},z\psi_2\right)}{uz}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{f_1},z\zeta_1\right)}{uz}+\frac{\Delta^{-1}\left(zs_{f_2},z\zeta_2\right)}{uz}-\frac{\Delta^{-1}\left(zs_{f_1},z\zeta_1\right)}{uz}\cdot\frac{\Delta^{-1}\left(zs_{f_2},z\zeta_2\right)}{uz}\right)\right)\end{array}\right\};$$

(3) $$\xi uz\delta=\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_t,z\xi\right)}{uz}\right)^{\xi}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_i,z\psi\right)}{uz}\right)^{\xi}\right),\Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_f,z\zeta\right)}{uz}\right)^{\xi}\right)\end{array}\right\},\xi>0;$$

(4) $$uz\delta^{\xi}=\left\{\begin{array}{l}\Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_t,z\xi\right)}{uz}\right)^{\xi}\right),\Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_i,z\psi\right)}{uz}\right)^{\xi}\right)\right),\\ \Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_f,z\zeta\right)}{uz}\right)^{\xi}\right)\right)\end{array}\right\},\xi>0.$$

2.4. GHM approach

The GHM approach [21] is produced.

Definition 7[21]. Let $\theta,\vartheta>0$, $za_i(i=1,2,\cdots,s)$ be positive values.

$$\mathrm{GHM}^{\theta,\vartheta}\left(za_1,za_2,\cdots,za_s\right)=\frac{1}{\theta+\vartheta}\left(\prod_{i=1,j=i}^{s}\left(\theta za_i+\vartheta za_j\right)\right)^{\frac{2}{s(s+1)}} \tag{5}$$

3. The G2TLNPGHM approach

3.1 The G2TLNGHM

Hu, Li and Chen [22] produced the G2TLNGHM approach.

Definition 8. Let $z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$ be 2TLNNs and let $z\theta,z\vartheta>0$. If

$$\mathrm{G2TLNGHM}^{z\theta,z\vartheta}(z\delta_1,z\delta_2,\ldots,z\delta_s)=\frac{1}{\theta+\vartheta}\left(\mathop{\otimes}_{i=1}^{s}\mathop{\otimes}_{j=i}^{s}\left(z\theta z\delta_i\oplus z\vartheta z\delta_j\right)\right)^{\frac{2}{s(s+1)}}$$

$$\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\left(\prod_{i=1,j=i}^{s}\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\cdot\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)\right)^{\frac{2}{s(s+1)}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\right),\\ \Delta\left(uz\left(1-\left(\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{k\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right)\right)^{\frac{2}{s(s+1)}}\right)^{\frac{1}{z\theta+z\vartheta}}\right),\\ \Delta\left(uz\left(1-\left(\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)\right)^{\frac{2}{s(s+1)}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\end{array}\right\} \tag{6}$$

The G2TLNGHM has some properties.

Property 1. (Idempotency) If $z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$ are same,

$$\mathrm{G2TLNGHM}^{z\theta,z\vartheta}(z\delta_1,z\delta_2,\ldots,z\delta_s)=z\delta \tag{7}$$

Property 2. (Monotonicity) Let $z\delta_{x_j}$, $z\delta_{y_j}$, $j=1,2,\ldots,s$, if $z\delta_{x_j}\le z\delta_{y_j}$, for j, then

$$\begin{aligned}&\mathrm{G2TLNGHM}^{z\theta,z\vartheta}\left(z\delta_{x_1},z\delta_{x_2},\cdots,z\delta_{x_s}\right)\\ &\le \mathrm{G2TLNGHM}^{z\theta,z\vartheta}\left(z\delta_{y_1},z\delta_{y_2},\cdots,z\delta_{y_s}\right)\end{aligned} \tag{8}$$

Property 3. (Boundedness) Let $z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$. If $z\delta^+=\left(\max_j\left(zs_{t_j},z\xi_j\right),\min_j\left(zs_{i_j},z\psi_j\right),\min_j\left(zs_{f_j},z\zeta_j\right)\right)$ and $z\delta^-=\left(\min_j\left(zs_{t_j},z\xi_j\right),\max_j\left(zs_{i_j},z\psi_j\right),\max_j\left(zs_{f_j},z\zeta_j\right)\right)$, then

$$z\delta^-\le \mathrm{G2TLNGHM}^{z\theta,z\vartheta}(z\delta_1,z\delta_2,\ldots,z\delta_s)\le z\delta^+ \tag{9}$$

3.2 The G2TLNPGHM approach

The G2TLNPGHM approach is produced on G2TLNGHM [22] and PA approach [12].

Definition 9. Let $z\delta_j = \left\langle \left(zs_{t_j}, z\xi_j\right), \left(zs_{i_j}, z\psi_j\right), \left(zs_{f_j}, z\zeta_j\right) \right\rangle$ be 2TLNNs, let $z\theta, z\vartheta > 0$. If

$$\begin{aligned} &\text{G2TLNPGHM}^{z\theta, z\vartheta}(z\delta_1, z\delta_2, \ldots, z\delta_s) \\ &= \frac{1}{z\theta + z\vartheta} \left(\mathop{\otimes}_{i=1}^{s} \mathop{\otimes}_{j=i}^{s} \left(z\theta z\delta_i \oplus z\vartheta z\delta_j \right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))} \frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}} \right) \end{aligned} \tag{10}$$

where $ZT_j = \prod_{k=1}^{j-1} ZSF\left(z\delta_j\right)\left(j = 2, \cdots, n\right)$, $ZT_1 = 1$ and $ZSF\left(k\delta_j\right)$ is score function values for $z\delta_j = \left\langle \left(zs_{t_j}, z\xi_j\right), \left(zs_{i_j}, z\psi_j\right), \left(zs_{f_j}, z\zeta_j\right) \right\rangle$.

Theorem 1. Assume $z\delta_j = \left\langle \left(zs_{t_j}, z\xi_j\right), \left(zs_{i_j}, z\psi_j\right), \left(zs_{f_j}, z\zeta_j\right) \right\rangle$ be 2TLNNs. The result by G2TLNPGHM is produced:

$$\mathrm{G2TLNPGHM}^{z\theta,z\vartheta}(z\delta_1,z\delta_2,\ldots,z\delta_s)$$
$$=\frac{1}{z\theta+z\vartheta}\left(\underset{i=1}{\overset{s}{\otimes}}\underset{j=i}{\overset{s}{\otimes}}\left(z\theta z\delta_i\oplus z\vartheta z\delta_j\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)$$
$$=\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\prod_{i=1,j=i}^{s}\left(\begin{array}{l}1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\cdot\\\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\end{array}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\right),\\
\Delta\left(uz\left(1-\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right),\\
\Delta\left(uz\left(1-\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\end{array}\right\} \tag{11}$$

where $ZT_j=\prod_{k=1}^{j-1}ZSF\left(z\delta_j\right)\left(j=2,\cdots,n\right)$, $ZT_1=1$and $ZSF\left(k\delta_j\right)$ is score function values for

$z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$.

Proof.

$$z\theta z\delta_i=\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\right),\Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\right)\end{array}\right\} \quad (12)$$

$$z\vartheta z\delta_j=\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right),\Delta\left(uz\left(1\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)\end{array}\right\} \quad (13)$$

Thus,

$$\begin{array}{l} z\theta z\delta_i\oplus z\vartheta z\delta_j \\ =\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\cdot\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right),\\ \Delta\left(uz\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)\end{array}\right\}\end{array} \quad (14)$$

Thereafter,

$$\left(z\theta z\delta_i \oplus z\vartheta z\delta_j\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}$$

$$=\left\{\begin{array}{l}\Delta\left(uz\left(1-\left(\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{k\theta}\cdot\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\right),\\ \Delta\left(uz\left(1-\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\right),\\ \Delta\left(uz\left(1-\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\right)\end{array}\right\} \tag{15}$$

Furthermore,

$$\underset{i=1}{\overset{m}{\otimes}}\underset{j=i}{\overset{m}{\otimes}}\left(z\theta z\delta_i \oplus z\vartheta z\delta_j\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}$$

$$=\left\{\begin{array}{l}\Delta\left(uz\prod_{i=1,\,j=i}^{s}\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\cdot\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right),\\ \Delta\left(uz\left(1-\prod_{i=1,\,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\right),\\ \Delta\left(uz\left(1-\prod_{i=1,\,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\right)\end{array}\right\} \tag{16}$$

Therefore,

$$
\begin{aligned}
&\text{G2TLNPGHM}^{z\theta,z\vartheta}(z\delta_1,z\delta_2,\ldots,z\delta_s)\\
&=\frac{1}{z\theta+z\vartheta}\left(\mathop{\otimes}_{i=1}^{s}\mathop{\otimes}_{j=i}^{s}\left(z\theta z\delta_i\oplus z\vartheta z\delta_j\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)\\
&=\left\{\begin{array}{l}
\Delta\left(uz\left(1-\left(1-\prod_{i=1,j=i}^{s}\left(1-\left(1-\frac{\Delta^{-1}\left(zs_{t_i},z\xi_i\right)}{uz}\right)^{z\theta}\cdot\left(1-\frac{\Delta^{-1}\left(zs_{t_j},z\xi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\right),\\
\Delta\left(uz\left(1-\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_i},z\psi_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{i_j},z\psi_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right),\\
\Delta\left(uz\left(1-\prod_{i=1,j=i}^{s}\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_i},z\zeta_i\right)}{uz}\right)^{z\theta}\cdot\left(\frac{\Delta^{-1}\left(zs_{f_j},z\zeta_j\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_i)}{\sum_{i=1}^{s}(1+ZT(z\delta_i))}\frac{1+ZT(z\delta_j)}{\sum_{j=1}^{s}(1+ZT(z\delta_j))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)
\end{array}\right\}
\end{aligned}
\tag{17}
$$

The G2TLNPGHM approach has some properties.

Property 4. (Idempotency) If $z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$ are equal, then

$$\text{G2TLNPGHM}^{z\theta,z\vartheta}\left(z\delta_1,z\delta_2,\ldots,z\delta_s\right)=z\delta \tag{18}$$

Property 5. (Monotonicity) Let $z\delta_{x_j}$, $z\delta_{y_j}$ the 2TLNNs, if $z\delta_{x_j}\le z\delta_{y_j}$, for j, then

$$
\begin{aligned}
&\text{G2TLNPGHM}^{z\theta,z\vartheta}\left(z\delta_{x_1},z\delta_{x_2},\cdots,z\delta_{x_s}\right)\\
&\le\text{G2TLNPGHM}^{z\theta,z\vartheta}\left(z\delta_{y_1},z\delta_{y_2},\cdots,z\delta_{y_s}\right)
\end{aligned}
\tag{19}
$$

Property 6. (Boundedness) Let $z\delta_j=\left\langle\left(zs_{t_j},z\xi_j\right),\left(zs_{i_j},z\psi_j\right),\left(zs_{f_j},z\zeta_j\right)\right\rangle$. If

$z\delta^{+}=\left(\max_{j}\left(zs_{t_j},z\xi_{j}\right),\min_{j}\left(zs_{i_j},z\psi_{j}\right),\min_{j}\left(zs_{f_j},z\zeta_{j}\right)\right)$ and

$z\delta^{-}=\left(\min_{j}\left(zs_{t_j},z\xi_{j}\right),\max_{j}\left(zs_{i_j},z\psi_{j}\right),\max_{j}\left(zs_{f_j},z\zeta_{j}\right)\right)$, then

$$z\delta^{-}\leq \text{G2TLNPGHM}^{z\theta,z\vartheta}(z\delta_{1},z\delta_{2},\ldots,z\delta_{s})\leq z\delta^{+} \tag{20}$$

4. Method

The MADM model is produced through G2TLNPGHM approach. Different alternatives $\{ZA_1,ZA_2,\ldots,ZA_m\}$ and different attributes $\{ZT_1,ZT_2,\ldots,ZT_n\}$. The steps of MADM model are produced through G2TLNPGHM approach (See Figure 1).

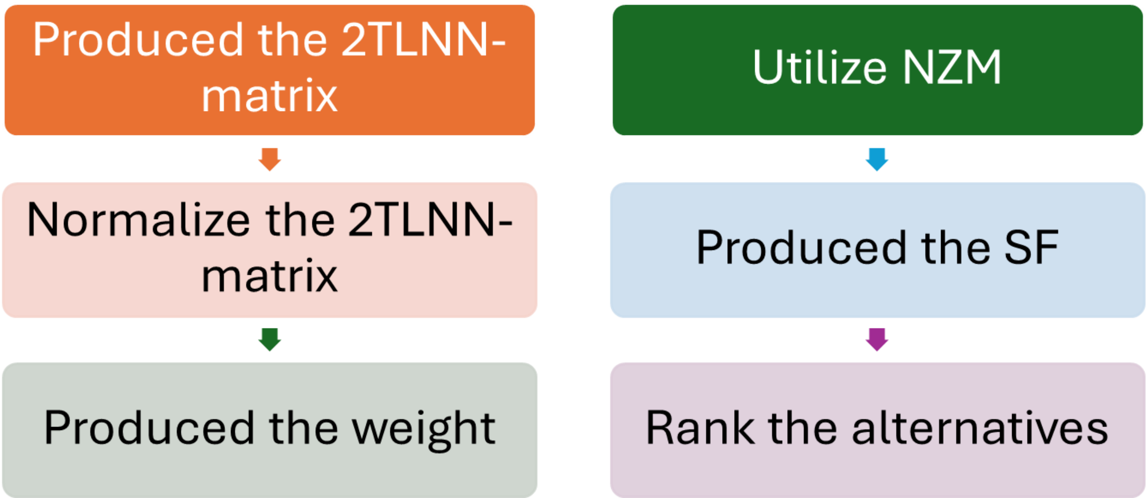

Figure 1. The steps of MADM model through G2TLNPGHM approach

Step 1. Produced the 2TLNN-matrix $ZM=\left[z\delta_{ij}\right]_{m\times n}$:

$$ZM=\left[z\delta_{ij}\right]_{m\times n}=\begin{array}{c} \\ ZA_1\\ ZA_2\\ \vdots\\ ZA_m\end{array}\begin{array}{c} \begin{array}{cccc} ZT_1 & ZT_2 & \cdots & ZT_n\end{array}\\ \begin{bmatrix} z\delta_{11} & z\delta_{12} & \cdots & z\delta_{1n}\\ z\delta_{21} & z\delta_{22} & \cdots & z\delta_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ z\delta_{m1} & z\delta_{m2} & \cdots & z\delta_{mn}\end{bmatrix}\end{array} \tag{21}$$

where $z\delta_{ij}=\left\langle\left(zs_{t_{ij}},z\xi_{ij}\right),\left(zs_{i_{ij}},z\psi_{ij}\right),\left(zs_{f_{ij}},z\zeta_{ij}\right)\right\rangle$ is the 2TLNN.

Step 2. Normalize the $ZM=\left[z\delta_{ij}\right]_{m\times n}$ to $NZM=\left[z\delta_{ij}\right]_{m\times n}$.

For benefit attributes:

$$\begin{aligned} z\delta_{ij}&=\left\langle\left(zs_{t_{ij}},z\xi_{ij}\right),\left(zs_{i_{ij}},z\psi_{ij}\right),\left(zs_{f_{ij}},z\zeta_{ij}\right)\right\rangle\\ &=\left\langle\left(zs_{t_{ij}},z\xi_{ij}\right),\left(zs_{i_{ij}},z\psi_{ij}\right),\left(zs_{f_{ij}},z\zeta_{ij}\right)\right\rangle\end{aligned} \tag{22}$$

For cost attributes:

$$z\delta_{ij} = \left\langle \left(zs_{t_{ij}}, z\xi_{ij}\right), \left(zs_{i_{ij}}, z\psi_{ij}\right), \left(zs_{f_{ij}}, z\zeta_{ij}\right) \right\rangle = \left\{ \begin{array}{l} \Delta\left(uz - \Delta^{-1}\left(zs_{t_{ij}}, z\xi_{ij}\right)\right), \Delta\left(uz - \Delta^{-1}\left(zs_{i_{ij}}, z\psi_{ij}\right)\right), \\ \Delta\left(uz - \Delta^{-1}\left(zs_{f_{ij}}, z\zeta_{ij}\right)\right) \end{array} \right\} \tag{23}$$

Step 3. Produced the weight for $ZT_{ij}\left(i = 1,2,3,4,5,\ j = 2,3,4\right)$:

$$ZT_{ij} = \prod_{\lambda=1}^{j-1} SF\left(z\delta_{i\lambda}\right)\left(i = 1,2,3,4,5,\ j = 2,3,4\right) \tag{24}$$

$$ZT_{i1} = 1,\ i = 1,2,3,4,5 \tag{25}$$

Step 4. Utilize $NZM = \left[z\delta_{ij}\right]_{m\times n}$ and G2TLNPGHM to produce the $z\delta_{i} = \left\langle \left(zs_{t_{i}}, z\xi_{i}\right), \left(zs_{i_{i}}, z\psi_{i}\right), \left(zs_{f_{i}}, z\zeta_{i}\right) \right\rangle$ of alternative ZA_i.

$$z\delta_i = \left\langle \left(zs_{t_i}, z\xi_i\right), \left(zs_{i_i}, z\psi_i\right), \left(zs_{f_i}, z\zeta_i\right) \right\rangle$$
$$= \text{G2TLNPGHM}^{z\theta, z\vartheta}\left(z\delta_{i1}, z\delta_{i2}, \ldots, z\delta_{in}\right)$$
$$= \frac{1}{z\theta + z\vartheta}\left(\underset{j=1}{\overset{n}{\otimes}} \underset{k=j}{\overset{n}{\otimes}} \left(z\theta z\delta_{ij} \oplus z\vartheta z\delta_{ik}\right)^{\frac{1+ZT(z\delta_{ij})}{\sum_{j=1}^{n}(1+ZT(z\delta_{ij}))} \frac{1+ZT(z\delta_{ik})}{\sum_{k=1}^{n}(1+ZT(z\delta_{ik}))}} \right)$$
$$= \left\{ \begin{array}{l} \Delta\left(uz\left(1-\left(1-\prod_{j=1,k=j}^{n}\left(\begin{array}{l} 1-\left(1-\frac{\Delta^{-1}\left(zs_{t_{ij}}, z\xi_{ij}\right)}{uz}\right)^{z\theta} \cdot \\ \left(1-\frac{\Delta^{-1}\left(zs_{t_{ik}}, z\xi_{ik}\right)}{uz}\right)^{z\vartheta} \end{array}\right)^{\frac{1+ZT(z\delta_{ij})}{\sum_{j=1}^{n}(1+ZT(z\delta_{ij}))} \frac{1+ZT(z\delta_{ik})}{\sum_{k=1}^{n}(1+ZT(z\delta_{ik}))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right)\right), \\ \Delta\left(uz\left(1-\prod_{j=1,k=j}^{n}\left(1-\left(\frac{\Delta^{-1}\left(zs_{i_{ij}}, z\psi_{ij}\right)}{uz}\right)^{z\theta} \cdot \left(\frac{\Delta^{-1}\left(zs_{i_{ij}}, z\psi_{ik}\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_{ij})}{\sum_{j=1}^{n}(1+ZT(z\delta_{ij}))} \frac{1+ZT(z\delta_{ik})}{\sum_{k=1}^{n}(1+ZT(z\delta_{ik}))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right), \\ \Delta\left(uz\left(1-\prod_{j=1,k=j}^{n}\left(1-\left(\frac{\Delta^{-1}\left(zs_{f_{ij}}, z\zeta_{ij}\right)}{uz}\right)^{z\theta} \cdot \left(\frac{\Delta^{-1}\left(zs_{f_{ij}}, z\zeta_{ik}\right)}{uz}\right)^{z\vartheta}\right)^{\frac{1+ZT(z\delta_{ij})}{\sum_{j=1}^{n}(1+ZT(z\delta_{ij}))} \frac{1+ZT(z\delta_{ik})}{\sum_{k=1}^{n}(1+ZT(z\delta_{ik}))}}\right)^{\frac{1}{z\theta+z\vartheta}}\right) \end{array} \right\} \quad (26)$$

Step 5. Produced the $SF\left(z\delta_i\right), AF\left(z\delta_i\right)$.

$$SF\left(z\delta_i\right) = \frac{\left(2uz + \Delta^{-1}\left(zs_{t_i}, z\xi_i\right) - \Delta^{-1}\left(zs_{i_i}, z\psi_i\right) - \Delta^{-1}\left(zs_{f_i}, z\zeta_i\right)\right)}{3uz} \quad (27)$$

$$AF(z\delta_i) = \frac{uz + \Delta^{-1}\left(zs_{t_i}, z\xi_i\right) - \Delta^{-1}\left(zs_{f_i}, z\zeta_i\right)}{2uz} \quad (28)$$

Step 6. Rank the choices $ZA_i\left(i = 1, 2, \cdots, m\right)$ and produced best one under $SF\left(z\delta_i\right), AF\left(z\delta_i\right)$.

4. Numerical example and comparisons

4.1. Numerical example

The core competencies evaluation of track and field students in sports colleges is an important tool for assessing the overall development of students, aiming to measure their comprehensive abilities in athletics. Core competencies not only refer to the students' competitive level but also encompass their physical fitness, psychological quality, athletic skills, tactical awareness, learning ability, and sense of social responsibility. This evaluation model emphasizes the all-round development of students in the field of sports, ensuring that they not only achieve excellent results on the field but also possess critical thinking, teamwork, and effective communication skills in daily life. Firstly, physical fitness and athletic skills form the foundation of the core competencies evaluation for track and field students. Students are required to have good physical conditioning and technical proficiency, including attributes such as speed, strength, endurance, and flexibility. Additionally, their athletic skills—such as starting, accelerating, jumping, and throwing—are assessed based on the precision and fluency of these technical movements. Secondly, psychological quality and tactical awareness are also important components of the evaluation. Track and field athletes need not only strong will power but also the ability to adapt quickly, think tactically, and regulate their emotions during competitions. Key aspects of the evaluation include the students' ability to handle stress, their response to unforeseen situations, and their tactical execution in various competitive scenarios. Lastly, learning ability and social responsibility reflect the students' comprehensive qualities. Track and field students must continually acquire new training methods and sports theories while also demonstrating strong self-management and teamwork capabilities. Additionally, they should exhibit a sense of social responsibility, actively engaging in community activities and serving as positive role models. Overall, the core competencies evaluation of track and field students in sports colleges is a multidimensional assessment system aimed at cultivating well-rounded sports talents. The core competencies evaluation of track and field students in sports colleges is deemed to MADM. The core competencies evaluation of track and field students in sports colleges typically includes four key indicators: physical fitness, athletic skills, psychological quality, and social responsibility. Each indicator assesses different aspects of the students' abilities and competencies, ensuring the comprehensive development of track and field athletes in both competitive performance and personal growth. Thus, five possible sports colleges $ZA_i\left(i=1,2,3,4,5\right)$ are selected under four attributes: ①ZT_1 is Physical Fitness: Physical fitness is one of the fundamental abilities for track and field students, primarily evaluating their speed, strength, endurance, flexibility, and coordination. Good physical fitness is the foundation for high-level athletic performance, affecting not only the students' competition results but also the longevity and health of their athletic careers. Fitness tests, muscle strength assessments, and endurance evaluations during training are important methods for measuring this indicator. ②ZT_2 is Athletic Skills: The evaluation of athletic skills includes the students' mastery of track and field techniques in terms of precision, proficiency, and creativity. Whether in sprints, long jumps, or throwing events, the correctness and proficiency of techniques directly impact competition outcomes. Therefore, assessing the students' technical proficiency, consistency of movements, and performance in competitions effectively reflects their level of specialized skills. ③ZT_3 is Psychological Quality: Psychological quality assesses the students' ability to cope with high-pressure environments and self-regulation. Track and field competitions require athletes to have strong psychological endurance, enabling them to remain calm, focused, and make correct tactical decisions during intense competitions. Psychological stress tests, emotional regulation during competitions, and the results of psychological resilience training are all part of this evaluation. ④ZT_4 is Social Responsibility: Social responsibility reflects the students' moral awareness and teamwork abilities. As future sports talents, students are expected not only to excel in competition but also to actively participate in social activities, demonstrating good sportsmanship and team spirit. This indicator evaluates the students' level of social engagement, teamwork awareness, and their understanding and practice of social responsibility.

The G2TLNPGHM approach is utilized to cope with core competencies evaluation of track and field students in sports colleges.

Step 1. The evaluation data is produced in Table 1.

Table 1. 2TLNNs data

	C_1	C_2	C_3	C_4
A_1	$<(zs_3, 0.12), (zs_1, 0.03), (zs_3, 0.07)>$	$<(zs_2, 0.23), (zs_3, 0.15), (zs_4, 0.08)>$	$<(zs_1, 0.17), (zs_2, 0.15), (zs_4, 0.06)>$	$<(zs_1, 0.04), (zs_3, 0.45), (zs_4, 0.34)>$
A_2	$<(zs_2, 0.32), (zs_2, 0.16), (zs_4, 0.39)>$	$<(zs_3, 0.08), (zs_1, 0.15), (zs_1, 0.04)>$	$<(zs_2, 0.04), (zs_3, 0.43), (zs_1, 0.37)>$	$<(zs_4,0.28), (zs_3,0.14), (zs_3,0.37)>$
A_3	$<(zs_4, 0.42), (zs_2, 0.12), (zs_4, 0.04)>$	$<(zs_1, 0.23), (zs_2, 0.27), (zs_2, 0.13)>$	$<(zs_4, 0.24), (zs_2, 0.16), (zs_2, 0.21)>$	$<(zs_3, 0.34), (zs_2, 0.28), (zs_4, 0.03)>$
A_4	$<(zs_5, 0.09), (zs_1, 0.18), (zs_2, 0.12)>$	$<(zs_4, 0.42), (zs_1 ,0.47), (zs_1, 0.09)>$	$<(zs_1, 0.32), (zs_4, 0.29), (zs_4, 0.16)>$	$<(zs_2, 0.06), (zs_4, 0.19), (zs_3, 0.41)>$
A_5	$<(zs_2, 0.21), (zs_2, 0.26), (zs_4, 0.29)>$	$<(zs_2, 0.05), (zs_4, 0.16), (zs_4, 0.24)>$	$<(zs_5, 0.09), (zs_1, 0.18), (zs_2, 0.05)>$	$<(zs_4, 0.06), (zs_1, 0.12), (zs_2, 0.24)>$

Step 2. Normalize the $ZM = \left[z\delta_{ij} \right]_{5\times4}$ to $NZM = \left[z\delta_{ij} \right]_{5\times4}$ (Table 2).

Table 2. $NZM = \left[z\delta_{ij} \right]_{5\times4}$

	C_1	C_2	C_3	C_4
A_1	$<(zs_3, 0.12), (zs_1, 0.03), (zs_3, 0.07)>$	$<(zs_2, 0.23), (zs_3, 0.15), (zs_4, 0.08)>$	$<(zs_1, 0.17), (zs_2, 0.15), (zs_4, 0.06)>$	$<(zs_1, 0.04), (zs_3, 0.45), (zs_4, 0.34)>$
A_2	$<(zs_2, 0.32), (zs_2, 0.16), (zs_4, 0.39)>$	$<(zs_3, 0.08), (zs_1, 0.15), (zs_1, 0.04)>$	$<(zs_2, 0.04), (zs_3, 0.43), (zs_1, 0.37)>$	$<(zs_4,0.28), (zs_3,0.14), (zs_3,0.37)>$
A_3	$<(zs_4, 0.42), (zs_2, 0.12), (zs_4, 0.04)>$	$<(zs_1, 0.23), (zs_2, 0.27), (zs_2, 0.13)>$	$<(zs_4, 0.24), (zs_2, 0.16), (zs_2, 0.21)>$	$<(zs_3, 0.34), (zs_2, 0.28), (zs_4, 0.03)>$
A_4	$<(zs_5, 0.09), (zs_1, 0.18), (zs_2, 0.12)>$	$<(zs_4, 0.42), (zs_1 ,0.47), (zs_1, 0.09)>$	$<(zs_1, 0.32), (zs_4, 0.29), (zs_4, 0.16)>$	$<(zs_2, 0.06), (zs_4, 0.19), (zs_3, 0.41)>$
A_5	$<(zs_2, 0.21), (zs_2, 0.26), (zs_4, 0.29)>$	$<(zs_2, 0.05), (zs_4, 0.16), (zs_4, 0.24)>$	$<(zs_5, 0.09), (zs_1, 0.18), (zs_2, 0.05)>$	$<(zs_4, 0.06), (zs_1, 0.12), (zs_2, 0.24)>$

Step 3. Employ Eq. (24)- Eq. (25) to produce $ZT_{ij}\ \left(i=1,\cdots,5; j=1,\cdots,4\right)$ (Table 3).

Table 3. The $ZT_{ij}\ \left(i=1,\cdots,5; j=1,\cdots,4\right)$

	C_1	C_2	C_3	C_4
A_1	1.0000	0.6010	0.4507	0.3155
A_2	1.0000	0.7796	0.5263	0.3158
A_3	1.0000	0.5198	0.3054	0.2444
A_4	1.0000	0.9096	0.7276	0.4638
A_5	1.0000	0.9908	0.5945	0.4458

Step 4. From Table 2-Table 3, the 2TLNNs is produced under G2TLNPGHM approach, $z\theta = 1, z\vartheta = 1$ (Table 4).

Table 4. Results under G2TLNPGHM approach

	G2TLNPGHM approach	G2TLNPGHM approach
A_1	$<(zs_3,0.4714), (zs_1,-0.3826) , (zs_4,-0.4487) >$	0.6142
A_2	$<(zs_1, 0.3476), (zs_5, 0.4725) , (zs_6, -0.3547) >$	0.5056
A_3	$<(zs_1,-0.3543), (zs_5,0.3714) , (zs_6,-0.1325) >$	0.3393
A_4	$<(zs_1,-0.3236), (zs_5,0.1645) , (zs_6,-0.1214) >$	0.4605
A_5	$<(zs_3, -0.4542), (zs_4,-0.4487) , (zs_5,0.1879) >$	0.5690

Step 5. From Table 4, the decision scores are constructed.

Step 6. The order is produced in Figure 2

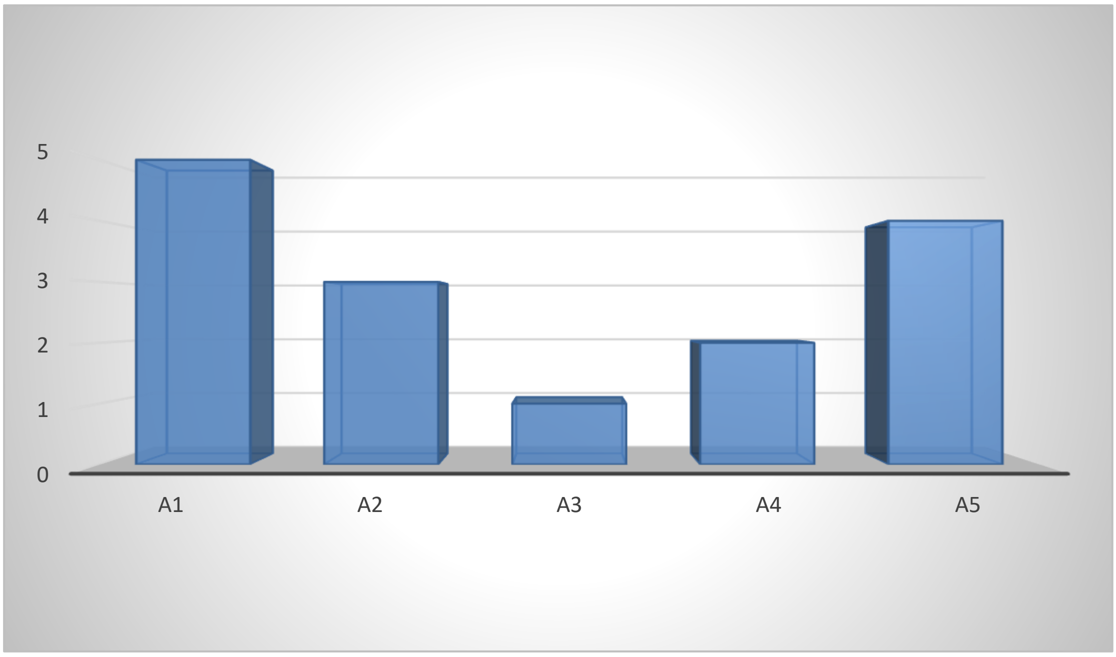

Figure 2. The rank of alternatives.

4.2. Comparative analysis

The G2TLNPGHM approach is compared with 2TLNNWA model [13], 2TLNNWG model [13], 2TLNN-CODAS model [23], 2TLNN-EDAS model [24] and 2TLNN-TODIM model [25]. The comparative results are produced in Figure 3.

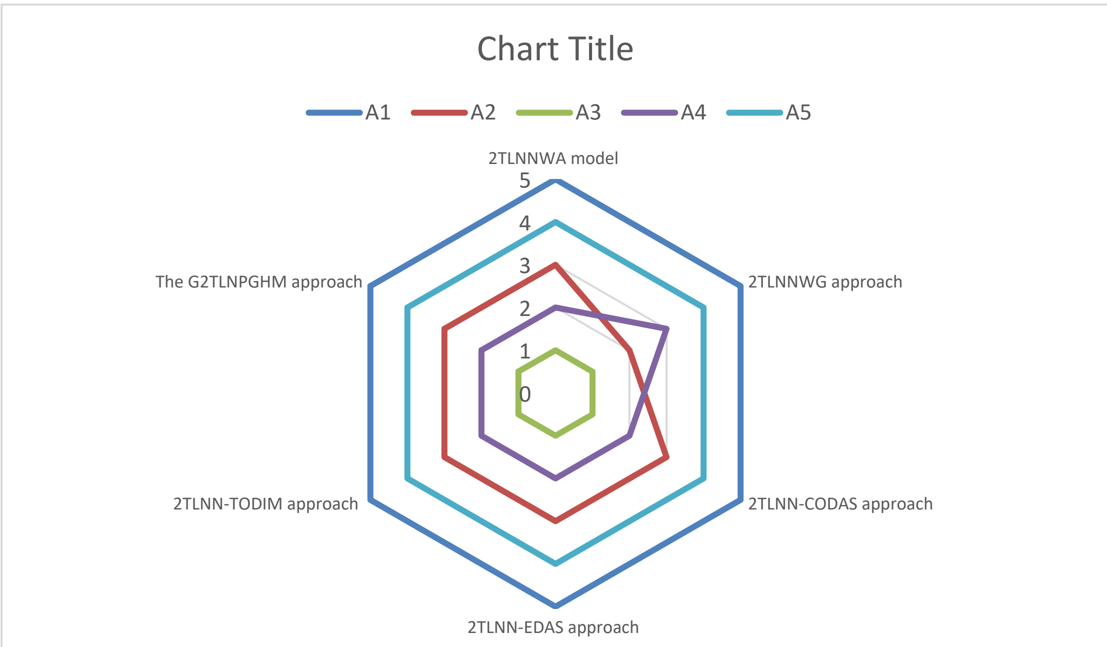

Figure 3. Comparative analysis.

Based on the WS coefficients presented in references [26, 27], the WS coefficient between the

G2TLNPGHM model and several other models, including the 2TLNNWA model, 2TLNNWG model, 2TLNN-CODAS model, 2TLNN-EDAS model and 2TLNN-TODIM model, has been calculated. The WS coefficients are produced as follows: for 2TLNNWA model, the coefficient is 1.0000; for 2TLNNWG approach, it is 0.8127; for 2TLNN-CODAS approach, it is 1.0000; for 2TLNN-EDAS approach, it is 1.0000; and for 2TLNN-TODIM approach, the coefficient is also 1.0000. The WS coefficient illustrates the degree of agreement between the ranking results of the proposed G2TLNPGHM approach and the other approaches. A WS coefficient of 1.0000 indicates that the order results of G2TLNPGHM model are identical to the order of 2TLNNWA approach, 2TLNN-CODAS approach, 2TLNN-EDAS approach, and 2TLNN-TODIM approach. However, the WS coefficient of 0.8127 for 2TLNNWG approach shows that there is a slight difference for the order compared to the G2TLNPGHM approach. Despite this minor difference, the rankings are still relatively close.

From Figure 3, it could be observed that while the order of different methods differs slightly, all methods consistently identified the same optimal and worst-performing sports colleges. This consistency across methods further validates the reasonableness and effectiveness of the proposed G2TLNPGHM approach. By producing similar results to established methods, the G2TLNPGHM approach demonstrates its reliability in evaluating and ranking sports colleges.

5. Managerial Implications

Evaluating the quality of management work in university student organizations has several managerial implications, providing insights into organizational effectiveness and leadership development such as:

A. Enhancing Organizational Structure and Leadership Skills: Leadership Development: Evaluations help identify gaps in student leadership skills, allowing university managers to implement training programs that focus on team management, decision-making, and effective communication. Role Clarity and Accountability: Assessment of management work highlights areas where responsibilities may be unclear, helping establish clear roles and accountability measures within the organization, reducing redundancy and improving efficiency.

B. Improving Student Engagement and Satisfaction: Tailoring Activities to Student Interests:

Evaluation results can reveal what students value in their organizations. Managers can then guide organizations to design activities that better align with student interests, enhancing engagement and satisfaction. Increasing Participation Rates: Insights into student preferences allow managers to suggest program adjustments that attract and retain more students, thereby increasing participation and fostering a more vibrant campus community.

C. Optimizing Resource Allocation and Utilization Effective Budget Management: Evaluating the quality of management work can highlight areas where resources are either underutilized or overspent, enabling managers to better allocate budgets and maximize the impact of available funds. Efficient Use of Facilities and Resources: Assessments help identify inefficiencies in resource usage, prompting student organizations to use university facilities, equipment, and materials more effectively.

D. Promoting Organizational Accountability and Transparency Setting Performance Standards: Quality evaluations create a basis for setting performance standards, which increases accountability within student organizations and aligns activities with broader university goals. Transparent Decision-Making Processes: By assessing decision-making processes, evaluations can encourage student leaders to adopt transparent practices that build trust and credibility within their organizations and with university stakeholders.

6. Conclusion

The purpose of evaluating the core competencies of track and field students in sports colleges is to comprehensively assess their overall abilities in training and competition, promoting their holistic development. By evaluating their physical fitness, athletic skills, psychological qualities, and sense of social responsibility, the evaluation identifies their strengths and weaknesses and provides a basis for personalized training programs. This evaluation not only aims to improve their athletic performance but also focuses on cultivating psychological resilience, teamwork, and social responsibility, helping students achieve long-term success in both their future sports careers and personal lives, becoming well-rounded sports professionals. In this study, we integrate the GHM approach and PG approach under 2TLNNs to develop G2TLNPGHM approach. This newly proposed approach is then applied to an example involving the core competencies evaluation of track and field students in sports colleges, demonstrating its practical utility.

The key contributions of this study are produced:

- ✧ The GHM approach and PA approach are employed to formulate the G2TLNPGHM approach within the framework of 2TLNSs.

- The G2TLNPGHM approach is specifically developed for MADM problems under 2TLNSs, enhancing decision-making processes that consider uncertainty and imprecision.
- An empirical study is conducted to assess the core competencies of track and field students in sports colleges, providing the concrete example study of how the constructed approach can be effectively applied.
- Several comparative studies are presented to validate the reliability of G2TLNPGHM approach, demonstrating its advantages over other existing methods.

Overall, this research contributes a novel aggregation approach tailored for decision-making scenarios involving linguistic and neutrosophic information, and its application in sports education showcases its potential in real-world evaluations.

References

[1] X. Peng, S. Yu, X. Cheng, Research on the physical education curriculum system based on the cultivation of core literacy of students in vocational colleges, Sports, (2016) 112-114.

[2] Q. Liu, Research on the core literacy and capacity building of physical education teachers in vocational colleges, Education and Teaching Forum, (2017) 25-26.

[3] X. Wu, Physical education core literacy: Realistic deficiencies and innovative interpretations in vocational colleges, Shandong Sports Science and Technology, 40 (2018) 84-88.

[4] S. Yan, X. Yin, B. Wang, Research on the construction of the core literacy system of physical education in vocational colleges, Contemporary Sports Technology, 8 (2018) 150-152.

[5] H. Li, H. Yu, X. Huang, Exploration of the physical education curriculum system based on the cultivation of core literacy of students in vocational colleges, Contemporary Sports Technology, 9 (2019) 88-90.

[6] X. Wu, C. Xu, Preliminary study on the evaluation system of physical education core literacy of students in vocational colleges, Sports Science and Technology, 41 (2020) 143-145.

[7] H. Liu, Research on the current situation and development strategies of students' core literacy in physical education in higher education institutions, Sports Vision, (2021) 81-82.

[8] S. Ma, Research on the construction of the physical education curriculum system in five-year vocational colleges under core literacy goals, Sports Vision, (2022) 68-70.

[9] Q. Lei, Research on the cultivation of physical education core literacy of ethnic minority students in vocational colleges: A case study of suzhou agricultural vocational college, Ice and Snow Sports Innovation Research, (2023) 87-90.

[10] L. Lu, Z. Ji, M. Sun, Research on the construction of physical education curriculum in medical colleges from the perspective of core literacy, Contemporary Sports Technology, 14 (2024) 55-59.

[11] F. Liu, Y. Ning, D. Zhang, H. Sun, Analysis of the current situation and construction of reform guarantee measures for physical education curriculum in vocational colleges from the perspective of core literacy, Health and Beauty, (2024) 99-101.

[12] R.R. Yager, Prioritized aggregation operators, International Journal of Approximate Reasoning, 48 (2008) 263-274.

[13] J. Wang, G.W. Wei, Y. Wei, Models for green supplier selection with some 2-tuple linguistic neutrosophic number bonferroni mean operators, Symmetry-Basel, 10 (2018) 36.

[14] F.A. Smarandache, Unifying field in logics. Neutrosophy: Neutrosophic probability, set and

logic, American Research Press, Rehoboth, 1999.
[15] F. Smarandache, Foundation of superhyperstructure & neutrosophic superhyperstructure (review paper), Neutrosophic Sets and Systems, 63 (2024) 367-381.
[16] F. Smarandache, Foundation of revolutionary topologies: An overview, examples, trend analysis, research issues, challenges, and future directions, Neutrosophic Systems with Applications, 13 (2024) 45-66.
[17] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, Multispace Multistruct, (2010) 410-413.
[18] F. Herrera, L. Martinez, A 2-tuple fuzzy linguistic representation model for computing with words, Ieee Transactions on Fuzzy Systems, 8 (2000) 746-752.
[19] F. Herrera, L. Martinez, An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making, International Journal of Uncertainty Fuzziness and Knowledge-Based Systems, 8 (2000) 539-562.
[20] F. Herrera, L. Martinez, The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency, International Journal of Uncertainty Fuzziness and Knowledge-Based Systems, 9 (2001) 33-48.
[21] D.J. Yu, Intuitionistic fuzzy geometric heronian mean aggregation operators, Applied Soft Computing, 13 (2013) 1235-1246.
[22] W. Hu, B. Li, L. Chen, An intelligent decision methodology for physical health evaluation of college students based on generalized 2-tuple linguistic neutrosophic geometric hm operators, Journal of Intelligent & Fuzzy Systems, 43 (2022) 7213-7225.
[23] P. Wang, J. Wang, G.W. Wei, J. Wu, C. Wei, Y. Wei, Codas method for multiple attribute group decision making under 2-tuple linguistic neutrosophic environment, Informatica, 31 (2020) 161-184.
[24] P. Wang, J. Wang, G.W. Wei, Edas method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment, Journal of Intelligent & Fuzzy Systems, 37 (2019) 1597-1608.
[25] J. Wang, G.W. Wei, M. Lu, Todim method for multiple attribute group decision making under 2-tuple linguistic neutrosophic environment, Symmetry-Basel, 10 (2018) 486.
[26] W. Salabun, K. Urbaniak, A new coefficient of rankings similarity in decision-making problems, in: 20th Annual International Conference on Computational Science (ICCS), Springer International Publishing Ag, Amsterdam, NETHERLANDS, 2020, pp. 632-645.
[27] W. Sałabun, J. Wątróbski, A. Shekhovtsov, Are mcda methods benchmarkable? A comparative study of topsis, vikor, copras, and promethee ii methods, Symmetry, 12 (2020) 1549.

Received: Aug 5, 2024. Accepted: Nov 4, 2024