Neutrosophic Sets and Systems, Vol. 77, 2025
University of New Mexico

ELECTRE I approach for multi-criteria group decision-making in single-valued neutrosophic N-soft environment

Fathima Perveen P A [1,*] and Sunil Jacob John[2]

[1] Department of Mathematics, Al-Ameen College, Edathala, Aluva-683 561, Kerala, India
[2] Department of Mathematics, National Institute of Technology, Calicut-673 601, Kerala, India

* Correspondence: perveenpa@gmail.com

Abstract: The single-valued neutrosophic N-soft set model is a superior tool for capturing ambiguity and incompleteness in real-life circumstances. In this paper, we extend the ELECTRE I method to the single-valued neutrosophic N-soft ELECTRE I method in multi-criteria group decision- making environment. To explain the outranking relationship among alternatives with respect to different parameters, this new technique is developed by introducing the ideas of strong, midrange, and weak single-valued neutrosophic N-soft concordance and discordance sets. Also, we propose an algorithm for the proposed method along with a flowchart. Finally, an illustrative example is solved to validate and prove the relevance of our proposed method

Keywords: Neutrosophic set, single-valued neutrosophic N-soft set, ELECTRE I method, outranking relation, concordance and discordance sets.

---

1. Introduction

Multi-criteria group decision-making (MCGDM), which enables teams of experts to assess and rank several possibilities according to a set of criteria, has become an essential methodology in several sectors. This approach delivers a structured framework for managing difficult decision-making situations, improving teamwork, and assisting professionals in various fields, including the social sciences, economics, environmental studies, and healthcare, in coming to logical, informed judgements. [1, 2, 3, 4, 5, 6, 7]. The use of MCGDM procedures has become increasingly popular in real-world settings that call for organised decision-making, like resource management, urban development, and sustainability planning [8, 9].

Numerous MCGDM models, including TOPSIS [10], PROMETHEE [11], VIKOR [12], ELECTRE [13], MABAC [14], and KEMIRA [15], have been created to successfully solve these challenges. Across a variety of fields, these techniques have been used to assist decision-makers in organising complicated data and prioritising options when competing standards are involved. For example, TOPSIS and VIKOR are frequently utilised in project management and supplier selection because of their ability to reliably evaluate options according to how close they are to perfect solutions [16, 17]. Out of the various decision-making techniques, the ELECTRE (ELimination and Choice Expressing REality) approach is one of the most successful methods which makes the pairwise comparison of alternatives by outranking relation and offering as accurate and suitable a set of actions as feasible by removing alternatives that are outranked by others, depending on several criteria. Furthermore, new developments have shown how well blended decision-making paradigms work in domains like precision livestock management, underscoring the adaptability of MCGDM approaches in challenging decision-making situations [18, 19].

Further, Roy [20] drew attention to the fluctuation of distinct objects in the reference set by inventing ELECTRE-I, an enhanced version of ELECTRE. Again, Several researchers generalized and developed other innovative methods such as ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE TRI-C, and ELECTRE IS [21, 22, 23, 24]. These models are being used more and more to solve complicated decision-making problems in domains where managing multi-dimensional criteria is crucial, such as transportation, finance, medical science, and agriculture [25, 26]. In order to increase ELECTRE's efficacy in predictive and real-time decision contexts, recent research has presented hybrid models that include ELECTRE with machine learning and optimisation techniques [27, 28].

Fuzzy set (FS) theory, first presented by Zadeh [29], has been frequently used in conjunction with MCGDM techniques to address ambiguity and uncertainty in decision-making. To deal with hesitation and imprecision in data, fuzzy extensions such as intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PyFSs), and q-rung orthopair fuzzy sets (qROFSs) have been employed [30, 31, 32, 33]. These modifications have been used in fields like risk assessment and medical diagnostics that demand high accuracy levels. Building on these ideas, Smarandache [34, 35] created neutrosophic sets (NSs), which manage data fusion and complicated ambiguities frequently found in large-scale decision-making by introducing separate truth, indeterminacy, and falsity membership functions. Neutrosophic extensions, like the single-valued neutrosophic set (SVNS) [36], have been successfully used in fields where processing nuanced information is essential, such as information technology and engineering [37, 38, 39, 40, 41].

Soft set theory, introduced by Molodtsov [42] gives MCGDM an additional degree of adaptability. Which provides binary evaluations of objects, which might be restrictive when used in real-world scenarios where non-binary evaluations are necessary. When assessments must contain more nuanced, graded evaluations, traditional mathematical models like FSs, IFSs, and NSs also fail. Non-binary ratings, which might be represented by stars, dots, grades, or other generalised multilevel values, are frequently required for ranking and evaluation in the real world. The N-soft set [43], an improved model of soft sets that better captures graded evaluations, was introduced to close this gap. Recent uses of soft and N-soft sets show their value in real-time decision-making situations and dynamic systems, like automated monitoring and disaster management [44, 45].

In this paper, we propose a novel adaptation of these methods by developing the $\mathrm{SVN_NSS}$ ELECTRE I approach, a model that combines the flexibility of single-valued neutrosophic N-soft sets ($\mathrm{SVN_NSS}$) with the robust outranking structure of ELECTRE I. By offering a more generalised model that can handle uncertain, partial, and graded data, our method aims to overcome the shortcomings of current MCGDM approaches. For group decision-making in settings where ambiguity and the requirement for multi-level evaluations are common, the $\mathrm{SVN_NSS}$ ELECTRE I technique is especially well-suited.

The following is the structure of the research paper: In Section 2, certain fundamental definitions regarding neutrosophic N-soft sets are recalled. The notion of $\mathrm{SVN_NSS}$ is proposed and some of its algebraic operations are defined. In Section 3, a mathematical explanation of the MCGDM problem, as well as some important terminologies related to the $\mathrm{SVN_NSS}$-ELECTRE I techniques, are described. Section 4 provides a flowchart and the algorithm for the proposed $\mathrm{SVN_NSS}$-ELECTRE I approach. In Section 5, a numerical example of a site selection problem is solved as an application of the proposed algorithm. Finally, Section 6 concludes the work with recommendations for future work.

2. Preliminaries

In this section, we will go through some basic concepts and some of the impacts of N-soft and neutrosophic sets, which will be useful in the coming sections.

Definition 2.1. [29] Let Σ be the universal set of discourse. A fuzzy set Ψ on Σ is an object of the form $\Psi = \{(\varsigma, \mu_\Psi(\varsigma)) | \varsigma \in \Sigma\}$, where $\mu_\Psi : \Sigma \rightarrow [0, 1]$ is the membership function of Ψ, the value $\mu_\Psi(\varsigma)$ is the grade of membership of ς in Ψ.

Definition 2.2. [34] Let Σ be a space of objects or points, and let $\varsigma \in \Sigma$ be a generic element. A neutrosophic set N over Σ is defined by a truth membership function T_N , an indeterminacy membership function I_N , and a falsity-membership function F_N , where for each $\varsigma \in \Sigma$, $T_N(\varsigma)$, $I_N(\varsigma)$, and $F_N(\varsigma)$, are real standard or non-standard subsets of $]0^-, 1^+[$. That is T_N , I_N , F_N : $\Sigma \rightarrow]0^-, 1^+[$.

Definition 2.3. [6] Let Σ be a space of objects or points, and let $\varsigma \in \Sigma$ be a generic element. A single-valued neutrosophic set S over Σ is defined by a truth membership function T_S, an indeterminacy membership function I_S, and a falsity-membership function F_S, where for each $\varsigma \in \Sigma$, $T_S(\varsigma)$, $I_S(\varsigma)$, $F_S(\varsigma) \in [0, 1]$. It can be represented as:

- If Σ is continuous, then

$$S = \int_{\Sigma} \langle T_S(\varsigma), I_S(\varsigma), F_S(\varsigma)\rangle/\varsigma, \qquad \varsigma \in \Sigma$$

- If $\Sigma = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_n\}$ is discrete, then

$$S = \sum_{1}^{n} \langle T_S(\varsigma), I_S(\varsigma), F_S(\varsigma)\rangle/\varsigma, \qquad \varsigma \in \Sigma$$

Definition 2.4. [42] Let Σ be the universal set with parameter set K. A soft set over Σ is a pair $\langle \aleph, L\rangle$, where $\aleph$ is a function from L to $P(\Sigma)$, where $P(\Sigma)$ is the power set of Σ and $L \subseteq K$.

Definition 2.5. [46] A neutrosophic soft set over a universal set Σ with parameter set K is a pair $\langle \aleph, L\rangle$, where $\aleph$ is a mapping from L to $NS(\Sigma)$, where $NS(\Sigma)$ is the set of all neutrosophic sets over Σ and $L \subseteq K$.
For each $\varpi \in L$, $\aleph(\varpi)$ is a neutrosophic set such that $\aleph(\varpi) = \{(\varsigma, T_{\aleph(\varpi)}(\varsigma), I_{\aleph(\varpi)}(\varsigma), F_{\aleph(\varpi)}(\varsigma)) \mid \varsigma \in \Sigma)\}$, where $T_{\aleph(\varpi)}(\varsigma), I_{\aleph(\varpi)}(\varsigma), F_{\aleph(\varpi)}(\varsigma) \in [0, 1]$ are the truth membership degree, indeterminacy membership degree, and falsity membership degree respectively.

Definition 2.6. [43] Let Σ be the set of objects and let K be the set of parameters with $L \subseteq K$. Let $O_G = \{0, 1, 2, \ldots, N - 1\}$ be the collection of ordered grades where $N \in \{2, 3, \ldots\}$. Then the triplet $(\aleph, L, N)$ is called an N-soft set over Σ, where for all $\varsigma \in \Sigma, \varpi \in L$, and, $\mathrm{g}_{\mathrm{L}(\varpi)} \in O_G$, $\aleph : L \rightarrow 2^{\Sigma \times O^G}$ is a map defined by $\aleph(\varpi) = (\varsigma, \mathrm{g}_{\mathrm{L}(\varpi)})$.

Definition 2.7. [47] Let Σ be the universal set with set of attributes K and $L \subseteq \Sigma$. Let $O_G = \{0, 1, 2, \ldots, N - 1\}$ be the collection of ordered grades where $N \in \{2, 3, \ldots\}$. Then a neutrosophic N-soft set over Σ, denoted by Γ_L, is a triplet $(\aleph, \Theta, N)$, where $\Theta = (\Gamma, L, N)$ is a neutrosophic soft set over Σ. That is,

$$\Gamma_L = \{(\varpi, \Pi_L(\varpi)) : \Pi_L(\varpi) = \{(\langle \varsigma, T_{L(\varpi)}(\varsigma), I_{L(\varpi)}(\varsigma), F_{L(\varpi)}(\varsigma)\rangle, g_{L(\varpi)}(\varsigma)), g_L \in O_G, \varsigma \in \Sigma, \varpi \in L\}$$

where T_L, I_L, $F_L \in [0, 1]$ are the truth membership degree, indeterminacy membership degree, and falsity membership degree respectively.

2.1. Single-Valued Neutrosophic N-Soft Sets

Definition 2.8. Let Σ be the initial universal set and K be the set of parameters under consideration, and $L \subseteq K$. Let $O_G = \{0,1,2,\ldots,N-1\}$ be the collection of ordered grades where $N \in \{2,3,\ldots\}$. We say that $(\aleph, L, \boldsymbol{N})$ is a single-valued neutrosophic N-soft set ($SVN_N SS$) on Σ if $\aleph : L \to 2^{SVNS(\Sigma)\times O_G}$, where $SVNS(\Sigma)$ is the collection of all single-valued neutrosophic sets over Σ. That is for each $\varpi \in L$,

$$\aleph(\varpi) = \{(\langle\varsigma, T_{\aleph(\varpi)}(\varsigma), I_{\aleph(\varpi)}(\varsigma), F_{\aleph(\varpi)}(\varsigma)\rangle, g_{\aleph(\varpi)}(\varsigma)) | \varsigma \in \Sigma\}$$

where $T_{\aleph(\varpi)}(\varsigma)$ is the degree of membership, $I_{\aleph(\varpi)}(\varsigma)$ is the degree of indeterminacy, and $F_{\aleph(\varpi)}(\varsigma)$ is the degree of falsity of elements in Σ, and $T_{\aleph(\varpi)}(\varsigma), I_{\aleph(\varpi)}(\varsigma), F_{\aleph(\varpi)}(\varsigma) \in [0,1]$, $g_{\aleph(\varpi)}(\varsigma) \in O_G$. For each $\varsigma \in \Sigma$ and $\varpi \in L$, the element $(\langle\varsigma, T_{\aleph(\varpi)}(\varsigma), I_{\aleph(\varpi)}(\varsigma), F_{\aleph(\varpi)}(\varsigma)\rangle, g_{\aleph(\varpi)}(\varsigma))$ is called the single-valued neutrosophic N-soft number ($SVN_N SS$).

Example 2.1. Suppose that $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3\}, K \supseteq L = \{\varpi_1, \varpi_2\}$. Define $SVN_7 SS$ as follows:

$$(\aleph, L, 7) = \{\aleph(\varpi_1) = \{(\langle\varsigma_1, 0.8, 0.5, 0.2\rangle, 3), (\langle\varsigma_2, 0.9, 0.2, 0.5\rangle, 5), (\langle\varsigma_3, 0.6, 0.8, 0.4\rangle, 6)\}, \aleph(\varpi_2) = \{(\langle\varsigma_1, 0.4, 0.3, 0.5\rangle, 6), (\langle\varsigma_2, 0.7, 0.4, 0.8\rangle, 3), (\langle\varsigma_3, 0.5, 0.7, 0.3\rangle, 4)\}\}$$

The matrix representation of the $SVN_7 SS$ $[\aleph, L, 7]$ is given as follows:

$$[\aleph, L, 7] = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{matrix} \varpi_1 \qquad\qquad \varpi_2 \\ \begin{bmatrix} (\langle 0.8,0.5,0.2\rangle, 3) & (\langle 0.4,0.3,0.5\rangle, 6) \\ (\langle 0.9,0.2,0.5\rangle, 5) & (\langle 0.7,0.4,0.8\rangle, 3) \\ (\langle 0.6,0.8,0.4\rangle, 6) & (\langle 0.5,0.7,0.3\rangle, 4) \end{bmatrix} \end{matrix}$$

Definition 2.9. Let $(\aleph_1, L_1, N)$ and $(\aleph_2, L_2, N)$ be two $SVN_N SSs$ over Σ, where $L_1, L_2 \subseteq K$ and $N \in O_G$. Then the algebraic sum of $(\aleph_1, L_1, N)$ and $(\aleph_2, L_2, N)$ is an $SVN_N SS$, denoted by $(\aleph_1, L_1, N) \boxplus (\aleph_2, L_2, N)$, is defined as follows:

$$(\aleph_1, L_1, N) \boxplus (\aleph_2, L_2, N) = (\aleph, L, N)$$

Where $L = L_1 \cup L_2$ and

$$\aleph(\varpi) = \begin{cases} \aleph_1(\varpi); \text{ if } \varpi \in L_1 - L_2 \\ \aleph_2(\varpi); \text{ if } \varpi \in L_2 - L_1 \\ \aleph_1(\varpi) \boxplus \aleph_2(\varpi); \text{ if } \varpi \in L_1 \cap L_2 \end{cases}$$

Where

$$\aleph_1(\varpi) \boxplus \aleph_2(\varpi) = \{(\langle\varsigma, T_{\aleph_1(\varpi)}(\varsigma) + T_{\aleph_2(\varpi)}(\varsigma) - T_{\aleph_1(\varpi)}(\varsigma).T_{\aleph_2(\varpi)}(\varsigma), I_{\aleph_1(\varpi)}(\varsigma).I_{\aleph_2(\varpi)}(\varsigma), (\varsigma).F_{\aleph_2(\varpi)}(\varsigma)\rangle, \max\left(g_{\aleph_1(\varpi)}(\varsigma), g_{\aleph_2(\varpi)}(\varsigma)\right) | \varsigma \in \Sigma\}$$

Example 2.2. Suppose that the universal set $\Sigma = \{\varsigma_1, \varsigma_2\}$ and the parameter set $K = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$ with $L_1 = \{\varpi_1, \varpi_2\}$ and $L_2 = \{\varpi_2, \varpi_3\}$ are two subsets of K. Consider two $SVN_5 SSs$ $(\aleph_1, L_1, 5)$ and $(\aleph_2, L_2, 5)$ as follows:

$$[\aleph_1, L_1, 5] = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 \qquad\qquad \varpi_2 \\ \begin{bmatrix} (\langle 0.5, 0.3, 0.4\rangle, 4) & (\langle 0.5, 0.2, 0.4\rangle, 2) \\ (\langle 0.7, 0.8, 0.9\rangle, 3) & (\langle 0.6, 0.1, 0.2\rangle, 3) \end{bmatrix} \end{matrix}$$

$$[\aleph_2, L_2, 5] = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \end{matrix} \begin{matrix} \varpi_1 \qquad\qquad \varpi_2 \\ \begin{bmatrix} (\langle 0.3, 0.4, 0.9\rangle, 1) & (\langle 0.5, 0.2, 0.4\rangle, 2) \\ (\langle 0.8, 0.2, 0.7\rangle, 4) & (\langle 0.2, 0.1, 0.5\rangle, 2) \end{bmatrix} \end{matrix}$$

Then the algebraic sum $(\aleph_1, L_1, 5) \boxplus (\aleph_2, L_2, 5 = (\aleph, \mathrm{L}, 5)$ is given by

$$[\aleph, L, 5] = \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (\langle 0.50, 0.30, 0.40\rangle, 4) & (\langle 0.65, 0.08, 0.36\rangle, 2) & (\langle 0.40, 0.50, 0.60\rangle, 3) \\ \varsigma_2 & (\langle 0.70, 0.80, 0.90\rangle, 3) & (\langle 0.92, 0.02, 0.14\rangle, 4) & (\langle 0.20, 0.10, 0.50\rangle, 2) \end{matrix}$$

Definition 2.10. Let $(\aleph_1, L_1, N)$ and $(\aleph_2, L_2, N)$ be two $\mathrm{SVN_NSSs}$ over Σ, where $L_1, L_2 \subseteq K$ and $N \in O_G$. Then the algebraic product of sum of $(\aleph_1, L_1, N)$ and $(\aleph_2, L_2, N)$ is an $\mathrm{SVN_NSS}$, denoted by $(\aleph_1, L_1, N) \boxdot (\aleph_2, L_2, N)$, is defined as follows:

$$(\aleph_1, L_1, N) \boxdot (\aleph_2, L_2, N) = (\aleph, L, N)$$

Where $L = L_1 \cup L_2$ and

$$\aleph(\varpi) = \begin{cases} \aleph_1(\varpi); \text{ if } \varpi \in L_1 - L_2 \\ \aleph_2(\varpi); \text{ if } \varpi \in L_2 - L_1 \\ \aleph_1(\varpi) \boxdot \aleph_2(\varpi); \text{ if } \varpi \in L_1 \cap L_2 \end{cases}$$

Where

$$\aleph_1(\varpi) \boxdot \aleph_2(\varpi) = \{(\langle \varsigma, T_{\aleph_1(\varpi)}(\varsigma).T_{\aleph_2(\varpi)}(\varsigma), I_{\aleph_1(\varpi)}(\varsigma) + I_{\aleph_2(\varpi)}(\varsigma) - I_{\aleph_1(\varpi)}(\varsigma).I_{\aleph_2(\varpi)}(\varsigma),$$

$$F_{\aleph_1(\varpi)}(\varsigma) + F_{\aleph_2(\varpi)}(\varsigma) - F_{\aleph_1(\varpi)}(\varsigma).F_{\aleph_2(\varpi)}(\varsigma)\rangle, min\left(g_{\aleph_1(\varpi)}(\varsigma), g_{\aleph_2(\varpi)}(\varsigma)\right) | \varsigma \in \Sigma\}$$

Example 2.3. Consider Example 2.2. Then the algebraic product $(\aleph_1, L_1, 5) \boxdot (\aleph_2, L_2, 5) = (\aleph, \mathrm{L}, 5)$ is given by

$$[\aleph, L, 5] = \begin{matrix} & \varpi_1 & \varpi_2 & \varpi_3 \\ \varsigma_1 & (\langle 0.50, 0.30, 0.40\rangle, 4) & (\langle 0.15, 0.52, 0.94\rangle, 1) & (\langle 0.40, 0.50, 0.60\rangle, 3) \\ \varsigma_2 & (\langle 0.70, 0.80, 0.90\rangle, 3) & (\langle 0.48, 0.28, 0.76\rangle, 3) & (\langle 0.20, 0.10, 0.50\rangle, 2) \end{matrix}$$

Definition 2.11. Let $(\aleph, L, N)$ be a $\mathrm{SVN_NSS}$ over Σ. Then the scalar multiplication operation on $(\aleph, L, N)$ with the scalar $\lambda\ (\lambda > 0)$, denoted by $\lambda(\aleph, L, N)$, is defined as

$$\lambda(\aleph, L, N) = (\aleph_\lambda, \mathrm{L}, N) = \left\{\left(\left\langle \varsigma, 1 - \left(1 - T_{\aleph(\varpi)}(\varsigma)\right)^\lambda, I_{\aleph(\varpi)}(\varsigma)^\lambda, F_{\aleph(\varpi)}(\varsigma)^\lambda \right\rangle, \mathrm{g}_{\aleph_\lambda(\varpi)}(\varsigma)\right) \middle| \varsigma \in \Sigma\right\}$$

Where,

$$\mathrm{g}_{\aleph^\lambda(\varpi)}(\varsigma) = \begin{cases} \left[\mathrm{g}_{\aleph(\varpi)}(\varsigma)^\lambda\right]; \text{if } \left[\mathrm{g}_{\aleph(\varpi)}(\varsigma)^\lambda\right] \leq \mathrm{N} - 1 \\ \mathrm{N} - 1; \text{ otherwise} \end{cases}$$

Where, $\left[\mathrm{g}_{\aleph(\varpi)}(\varsigma)^\lambda\right]$ is the greatest integer less than or equal to $\mathrm{g}_{\aleph(\varpi)}(\varsigma)^\lambda$.

Example 2.4. Consider the $\mathrm{SVN_7SS}$ in Example 2.1. Suppose that $\lambda = 0.5$, then

1. The scalar multiplication operation 0.5(ℵ, L, 7) is given by,

$$[\aleph_{0.5}, \mathrm{L}, 7] = \begin{matrix} & \varpi_1 & \varpi_2 \\ \varsigma_1 & (\langle 0.55, 0.71, 0.45\rangle, 1) & (\langle 0.23, 0.55, 0.71\rangle, 3) \\ \varsigma_2 & (\langle 0.68, 0.45, 0.71\rangle, 2) & (\langle 0.45, 0.63, 0.89\rangle, 0) \\ \varsigma_3 & (\langle 0.37, 0.89, 0.63\rangle, 3) & (\langle 0.29, 0.84, 0.55\rangle, 2) \end{matrix}$$

2. The exponentiation operation (ℵ, L, 7)0.5 is given by,

$$[\aleph^{0.5}, \mathrm{L}, 7] = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{cc} \varpi_1 & \varpi_2 \end{array} \\ \begin{bmatrix} (\langle 0.89, 0.29, 0.11\rangle, 1) & (\langle 0.63, 0.16, 0.29\rangle, 2) \\ (\langle 0.95, 0.11, 0.29\rangle, 2) & (\langle 0.84, 0.23, 0.55\rangle, 1) \\ (\langle 0.77, 0.55, 0.23\rangle, 3) & (\langle 0.71, 0.45, 0.16\rangle, 2) \end{bmatrix} \end{array}$$

3. The Single-Valued Neutrosophic N-Soft ELECTRE I Method

In this section, we develop a novel approach to group decision-making problems, called single-valued neutrosophic N-soft ELECTRE I, by combining the notion of $\mathrm{SVN_NSS}$ and ELECTRE method. This innovative method is very effective to solve the MCGDM problems with single- valued neutrosophic N-soft information.

3.1. The MCGDM Problem

As part of operations research (OR), MCGDM can rank the alternatives by considering many relevant criteria or parameters. For an MCGDM problem involving single-valued neutrosophic N-soft information, let $\Sigma = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_n\}$ be the set of alternatives with parameter set $K = \{\varpi_1, \varpi_2, \ldots, \varpi_o\}$, and $D = \{d_1, d_2, \ldots, d_p\}$ be the set of decision-makers. The decision-maker d_q $(q = 1, 2, \ldots, p)$ provides his evaluation data by combining the feasible alternative ς_i, (i = 1, 2, ..., n) with the parameter ϖ_j, $(j = 1,2, \ldots, n)$ under consideration, which is described by single-valued neutrosophic N-soft decision matrix $M^{(q)} = [m_{ij}^{(q)}]_{n\times o} = [\aleph_q, K, N]_{n\times o}$, where,

$$m_{ij}^{q} = \left\{ \left(\left\langle \varsigma, T_{\aleph_q(\varpi_j)}(\varsigma_i), I_{\aleph_q(\varpi_j)}(\varsigma_i), F_{\aleph_q(\varpi_j)}(\varsigma_i) \right\rangle, g_{\aleph_q(\varpi_j)}(\varsigma_i) \right) \middle| \varsigma \in \Sigma \right\}$$

That is, the $\mathrm{SVN_NSDM}$

$$M^{(q)} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_n \end{array} \begin{array}{c} \begin{array}{cccc} \varpi_1 & \varpi_2 & \cdots & \varpi_o \end{array} \\ \begin{bmatrix} m_{11}^{(q)} & m_{12}^{(q)} & \cdots & m_{1o}^{(q)} \\ m_{21}^{(q)} & m_{22}^{(q)} & \cdots & m_{2o}^{(q)} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n1}^{(q)} & m_{n2}^{(q)} & \cdots & m_{no}^{(q)} \end{bmatrix} \end{array}$$

Example 3.1. Suppose that $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, $K = \{\varpi_1, \varpi_2, \varpi_3\}$, and $D = \{d_1\}$. Take $N = 5$. Then

$$M^{(1)} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \\ \begin{bmatrix} (\langle 0.8,0.2,0.3\rangle, 4) & (\langle 0.7,0.5,0.2\rangle, 2) & (\langle 0.9,0.4,0.2\rangle, 4) \\ (\langle 0.7,0.3,0.4\rangle, 3) & (\langle 0.8,0.2,0.1\rangle, 3) & (\langle 0.3,0.2,0.4\rangle, 1) \\ (\langle 0.4,0.3,0.4\rangle, 2) & (\langle 0.6,0.4,0.3\rangle, 1) & (\langle 0.4,0.6,0.1\rangle, 3) \end{bmatrix} \end{array}$$

3.2. The Single-Valued Neutrosophic N-soft Concordance and Discordance Sets

The concordance and discordance sets are defined in the ELECTRE method as measures of satisfaction and dissatisfaction for the decision-maker when preferring an alternative over another. The alternative having greater the membership degree, lower the indeterminacy degree, lower the falsity degree, and greater the grade, is supposed to be superior to other available alternatives. The $\mathrm{SVN}_N\mathrm{S}$ concordance and discordance sets are classified as strong, midrange, weak concordance and discordance sets.

Suppose that $(\aleph_q, K, N)$ be an $\mathrm{SVN}_N\mathrm{SS}$ over the universe Σ with parameter set K, and let ς_μ and ς_ν $(\mu, \nu = 1,2,\ldots,n, \mu \neq \nu)$ be any two alternatives in Σ. Then the $\mathrm{SVN_NS}$ concordance set is the set of subscripts of those parameters about which ς_μ preferred to ς_ν and are classified as follows:

1) The $\mathrm{SVN_NS}$ strong concordance set $S_{C_{\mu\nu}}$:

$$S_{C_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) \geq T_{\aleph(\varpi_j)}(\varsigma_\nu), I_{\aleph(\varpi_j)}(\varsigma_\mu) \leq I_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) \leq F_{\aleph(\varpi_j)}(\varsigma_\nu), \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\mu) \geq \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (1)$$

2) The $\mathrm{SVN_NS}$ midrange concordance set $M_{C_{\mu\nu}}$:

$$M_{C_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) \geq T_{\aleph(\varpi_j)}(\varsigma_\nu), I_{\aleph(\varpi_j)}(\varsigma_\mu) \geq I_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) \leq F_{\aleph(\varpi_j)}(\varsigma_\nu), \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\mu) \geq \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (2)$$

3) The $\mathrm{SVN}_N\mathrm{S}$ weak concordance set $W_{C_{\mu\nu}}$:

$$W_{C_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) \geq T_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) \geq F_{\aleph(\varpi_j)}(\varsigma_\nu), \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\mu) \geq \mathrm{g}_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (3)$$

Where $\{j = 1, 2, \ldots, o\}$ is the collection of subscripts of all parameters under consideration. Each of these $\mathrm{SVN_NS}$ concordance sets signify the various extents of preference of ς_μ andς_ν. In the case of $\mathrm{SVN_NS}$ midrange concordance set, the degree of indeterminacy of alternative ς_μ is higher than the alternative ς_ν, corresponding to the parameter ϖ_j. Therefore, when comparing $S_{C_{\mu\nu}}$ and $M_{C_{\mu\nu}}$, $S_{C_{\mu\nu}}$ is more concordant. Also, the degree of falsity of ς_μ is greater than ς_ν corresponding to the parameter ϖ_j. This implies that the $\mathrm{SVN_NS}$ midrange concordance set $M_{C_{\mu\nu}}$ is more concordant than the $\mathrm{SVN_NS}$ weak concordance set $W_{C_{\mu\nu}}$.

Example 3.2. Consider Example 3.1, The $\mathrm{SVN_5S}$ concordance sets are then computed and presented in matrix form as follows:

1) The $\mathrm{SVN_5S}$ strong concordance set $S_{C_{\mu\nu}}$:

$$S_C = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{matrix} \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \left[\begin{matrix} - & \{1\} & \{1\} \\ \{2\} & - & \{1,2\} \\ \phi & \phi & - \end{matrix}\right] \end{matrix}$$

2) The $\mathrm{SVN_NS}$ midrange concordance set $M_{C_{\mu\nu}}$:

$$M_C = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{matrix} \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \left[\begin{matrix} - & \{3\} & \{2\} \\ \phi & - & \{1\} \\ \phi & \{3\} & - \end{matrix}\right] \end{matrix}$$

3) The $\mathrm{SVN_NS}$ weak concordance set $W_{C_{\mu\nu}}$:

$$W_C = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{matrix} \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \left[\begin{matrix} - & \phi & \{3\} \\ \phi & - & \{1\} \\ \phi & \phi & - \end{matrix}\right] \end{matrix}$$

Similarly, the $\mathrm{SVN_NS}$ discordant set is the collection of subscripts of those parameters about which ς_μ is not preferable than ς_ν, are classified as follows:

1) The $\mathrm{SVN_NS}$ strong discordance set $S_{D_{\mu\nu}}$:

$$S_{D_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) < T_{\aleph(\varpi_j)}(\varsigma_\nu), I_{\aleph(\varpi_j)}(\varsigma_\mu) > I_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) > F_{\aleph(\varpi_j)}(\varsigma_\nu), g_{\aleph(\varpi_j)}(\varsigma_\mu) < g_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (4)$$

2) The $\mathrm{SVN_NS}$ midrange discordance set $M_{D_{\mu\nu}}$:

$$M_{D_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) < T_{\aleph(\varpi_j)}(\varsigma_\nu), I_{\aleph(\varpi_j)}(\varsigma_\mu) < I_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) > F_{\aleph(\varpi_j)}(\varsigma_\nu), g_{\aleph(\varpi_j)}(\varsigma_\mu) < g_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (5)$$

3) The $\mathrm{SVN_NS}$ weak concordance set $W_{D_{\mu\nu}}$:

$$W_{D_{\mu\nu}} = \{j | T_{\aleph(\varpi_j)}(\varsigma_\mu) < T_{\aleph(\varpi_j)}(\varsigma_\nu), F_{\aleph(\varpi_j)}(\varsigma_\mu) < F_{\aleph(\varpi_j)}(\varsigma_\nu), g_{\aleph(\varpi_j)}(\varsigma_\mu) < g_{\aleph(\varpi_j)}(\varsigma_\nu)\} \quad (6)$$

The $\mathrm{SVN_NS}$ discordance set can be considered as the complementary subset of $\mathrm{SVN_NS}$ concor- dance set. The alternative having lower the membership degree, higher the indeterminacy degree, higher the falsity degree, and lower the grade, is supposed to be greater discordant than the other alternatives. By considering the $\mathrm{SVN_NS}$ midrange discordance set, the smaller the degree of in- determinacy of ς_μ than ς_ν implies that $S_{D_{\mu\nu}}$ is more discordant than $M_{D_{\mu\nu}}$. Moreover, since the degree of falsity of alternative ς_μ is smaller than the alternative ς_ν in $\mathrm{SVN_NS}$ weak discordance set, $M_{D_{\mu\nu}}$ is more discordant than $W_{D_{\mu\nu}}$.

Example 3.3. Consider Example 3.1, The $\mathrm{SVN_5S}$ discordance sets are then computed and presented in matrix form as follows:

1) The $\mathrm{SVN_5S}$ strong concordance set $S_{D_{\mu\nu}}$:

$$S_D = \begin{array}{c} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \overset{\begin{array}{ccc} \varsigma_1 & \varsigma_2 & \varsigma_3 \end{array}}{\begin{bmatrix} - & \{2\} & \phi \\ \{1\} & - & \phi \\ \{1\} & \{2\} & - \end{bmatrix}}$$

2) The $\mathrm{SVN_NS}$ midrange concordance set $M_{D_{\mu\nu}}$:

$$M_D = \begin{array}{c} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \overset{\begin{array}{ccc} \varsigma_1 & \varsigma_2 & \varsigma_3 \end{array}}{\begin{bmatrix} - & \phi & \phi \\ \{3\} & - & \{3\} \\ \{2\} & \phi & - \end{bmatrix}}$$

3) The $\mathrm{SVN_NS}$ weak concordance set $W_{D_{\mu\nu}}$:

$$W_D = \begin{array}{c} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \overset{\begin{array}{ccc} \varsigma_1 & \varsigma_2 & \varsigma_3 \end{array}}{\begin{bmatrix} - & \phi & \phi \\ \phi & - & \phi \\ \{3\} & \phi & - \end{bmatrix}}$$

3.3. The Single-Valued Neutrosophic N-soft Concordance and Discordance Index

3.3.1. The Single-Valued Concordance Index and Matrix

The means of weights given to the parameter involved in the appropriate concordance set deter- mine the probable concordance index. Suppose that $\omega_j = \left(\langle T_\omega(\omega_j), I_\omega(\omega_j), F_\omega(\omega_j)\rangle, g_\omega(\omega_j)\right)$ is the SVN_NS weights associated with the parameter ω_j, then the concordance index $CI_{\mu\nu} = (\langle T^{(j)}_{CI_{\mu\nu}}, I^{(j)}_{CI_{\mu\nu}}, F^{(j)}_{CI_{\mu\nu}}\rangle, g^{(j)}_{CI_{\mu\nu}}), (\mu, \nu = 1,2,\dots,n, \mu \neq \nu)$ of the alternatives ς_μ and ς_ν defined as follows:

$$CI_{\mu\nu} = \omega_{S_C} \sum_{j\in S_{C_{\mu\nu}}} \omega_j \boxplus \omega_{M_C} \sum_{j\in M_{C_{\mu\nu}}} \omega_j \boxplus \sum_{j\in W_{C_{\mu\nu}}} \omega_j \tag{7}$$

where $\sum$ denote the algebraic summation. ω_{S_C}, ω_{M_C} and ω_{W_C} are the weights given by the decision-makers, to the SVN_NS strong, midrange, and weak concordance sets respectively. The SVN_NS concordance index illustrate the relative dominance of the alternative ς_μ over the alter- native ς_ν according to the Equation 7. Also a higher or lower value of $CI_{\mu\nu}$ suggests that ς_μ has higher or lower dominance and concordance with ς_ν . Then the SVN_NS concordance matrix $C_M = [CI, K, \boldsymbol{N}]_{n\times n} = [CI_{\mu\nu}]_{n\times n}$ containing the concordance indices $CI_{\mu\nu}$ as entries can be assembled as follows:

$$C_M = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_{n-1} \\ \varsigma_n \end{array} \begin{array}{c} \begin{array}{ccccc} \varsigma_1 & \varsigma_2 & \cdots & \varsigma_{(n-1)} & \varsigma_n \end{array} \\ \begin{bmatrix} - & CI_{12} & \cdots & CI_{1(n-1)} & CI_{1n} \\ CI_{\mu\nu} & - & \cdots & CI_{2(n-1)} & CI_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CI_{(n-1)1} & CI_{(n-1)2} & \cdots & - & CI_{(n-1)n} \\ CI_{n1} & CI_{n2} & \cdots & CI_{n(n-1)} & - \end{bmatrix} \end{array}$$

3.3.2. The Single-Valued Discordance Index and Matrix

The SVN_NS discordance indices illustrate how the ratings of alternative ς_μ are significantly weaker than the evaluation of ς_ν. The index $DI_{\mu\nu}$ is obtained for every element of the discordance set using the elements of aggregated weighted SVN_NS decision matrix $M = [m_{ij}]_{n\times n} = [\aleph, K, N]_{n\times n}$, and is defined as follows:

$$DI_{\mu\nu} = \left(d_{\mu\nu}, g_{\mu\nu}\right) \tag{8}$$

Where,

$$DI_{\mu\nu} = \frac{\max\limits_{(j\in S_{D_{\mu\nu}}\cup M_{D_{\mu\nu}}\cup W_{D_{\mu\nu}})} \{\omega_{S_D}.d(m_{\mu j}, m_{\nu j}),\ \omega_{M_D}.d(m_{\mu j}, m_{\nu j}),\ \omega_{W_D}.d(m_{\mu j}, m_{\nu j})\}}{\max\limits_{j\in\{1,2,\dots,o\}} d(m_{\mu j}, m_{\nu j})} \tag{9}$$

$$g_{\mu\nu} = \max_{(j\in S_{D_{\mu\nu}}\cup M_{D_{\mu\nu}}\cup W_{D_{\mu\nu}})} \left|g_{\aleph(\varpi_j)}(\varsigma_\mu) - g_{\aleph(\varpi_j)}(\varsigma_\nu)\right| \tag{10}$$

$(0 \leq d_{\mu\nu} \leq 1), (\mu, \nu = 1,2,\dots,n, \mu \neq \nu)$, ω_{S_D} , ω_{M_D} and ω_{W_D} are the weights of SVN_NS strong, midrange, and weak discordance sets respectively, given by the decision-makers. And

$$d(m_{\mu j}, m_{\nu j}) = \sqrt{\frac{1}{3}\left[\mathrm{T}_{\aleph(\varpi_j)}(\varsigma_\mu) - \mathrm{T}_{\aleph(\varpi_j)}(\varsigma_\nu)\right]^2 + \left[\mathrm{I}_{\aleph(\varpi_j)}(\varsigma_\mu) - \mathrm{I}_{\aleph(\varpi_j)}(\varsigma_\nu)\right]^2 + \left[\mathrm{F}_{\aleph(\varpi_j)}(\varsigma_\mu) - \mathrm{F}_{\aleph(\varpi_j)}(\varsigma_\nu)\right]^2} \quad (11)$$

The $\mathrm{SVN_N S}$ discordance index $DI_{\mu\nu}$, as defined by its formula, shows the relative difference in the performance of competing alternatives ς_μ and ς_ν in terms of discordance parameter. Also, the stronger objection to the assertion "ς_μ is at least as good as ς_ν". Then the $\mathrm{SVN_N S}$ discordance matrix $D_M = [DI_{\mu\nu}]_{n\times n}$ can be constructed as follows:

$$D_M = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_{n-1} \\ \varsigma_n \end{matrix} \begin{matrix} \varsigma_1 & \varsigma_2 & \cdots & \varsigma_{(n-1)} & \varsigma_n \\ - & DI_{12} & \cdots & DI_{1(n-1)} & DI_{1n} \\ DI_{\mu\nu} & - & \cdots & DI_{2(n-1)} & DI_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ DI_{(n-1)1} & DI_{(n-1)2} & \cdots & - & DI_{(n-1)n} \\ DI_{n1} & DI_{n2} & \cdots & DI_{n(n-1)} & - \end{matrix}$$

Example 3.5. Consider Example 3.1, Then the discordance matrix D_M is computed as follows:

$$D_M = \begin{matrix} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{matrix} \begin{matrix} \varsigma_1 & \varsigma_2 & \varsigma_3 \\ - & (0.501,1) & \phi \\ (1,2) & - & (0.334,2) \\ (0.259,2) & (0.227,2) & - \end{matrix}$$

3.4 The Outranking Relations

To model the priorities among the pairs of alternatives, outranking relations are used. If there is significant evidence to believe that ς_μ is preferable to ς_ν or at least one is as excellent as the other, an alternative ς_μ outranks the other alternative ς_ν. The most flexible idea of $\mathrm{SVN_N S}$ outranking relation can be used to represent the objective of preference ranking of one alternative over the others in the procedure of multi-criteria decision-making.

3.4.1. The Single-Valued Neutrosophic N-Soft Effective Concordance Matrix

To effectively identify the preeminence among a given pair of alternatives in a concordant point of view, a comparison can be made between the indices in the $\mathrm{SVN}_N\mathrm{S}$ concordance matrix C_M and the threshold value named concordance level, denoted as $\check{C}L = \left(\left\langle T_{\check{C}L_{\mu\nu}}, I_{\check{C}L_{\mu\nu}}, F_{\check{C}L_{\mu\nu}}\right\rangle, \mathrm{g}_{\check{C}L_{\mu\nu}}\right)$, and evaluated from the $\mathrm{SVN}_N\mathrm{S}$ concordance indices as:

$$\check{C}L = \left(\frac{1}{n(n-1)}\left\langle \sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} T_{CI_{\mu\nu}}^{(j)}, \sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} I_{CI_{\mu\nu}}^{(j)}, \sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} F_{CI_{\mu\nu}}^{(j)}\right\rangle, \left[\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} \mathrm{g}_{CI_{\mu\nu}}^{(j)}\right]\right) \quad (12)$$

where $(\mu, \nu = 1,2,\dots,n, \mu \neq \nu)$ and $\left[\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} \mathrm{g}_{CI_{\mu\nu}}^{(j)}\right]$ is the greatest integer less than or equal to $\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} \mathrm{g}_{CI_{\mu\nu}}^{(j)}$.

The probability of dominance of alternative ς_μ over ς_ν increases, when $CI_{\mu\nu}$ outperforms this concordant level $\check{C}L$. Each and every entries in the SVN_NS concordance matrix C_M are compared with the concordance level $\check{C}L$, on the basis of this, the concordant Boolean matrix $\Lambda = [\Lambda_{\mu\nu}]_{n\times n}$ can be calculated as

$$\Lambda = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_{n-1} \\ \varsigma_n \end{array} \begin{array}{c} \begin{array}{ccccc} \varsigma_1 & \varsigma_2 & \cdots & \varsigma_{(n-1)} & \varsigma_n \end{array} \\ \begin{bmatrix} - & \Lambda_{12} & \cdots & \Lambda_{1(n-1)} & \Lambda_{1n} \\ \Lambda_{\mu\nu} & - & \cdots & \Lambda_{2(n-1)} & \Lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Lambda_{(n-1)1} & \Lambda_{(n-1)2} & \cdots & - & \Lambda_{(n-1)n} \\ \Lambda_{n1} & \Lambda_{n2} & \cdots & \Lambda_{n(n-1)} & - \end{bmatrix} \end{array}$$

Where,

$$\Lambda_{\mu\nu} = \begin{cases} 1; if\ CI_{\mu\nu} \geq \check{C}L\ (T_{CI_{\mu\nu}}^{(j)} \geq T_{\check{C}L_{\mu\nu}}, I_{CI_{\mu\nu}}^{(j)} \leq I_{\check{C}L_{\mu\nu}}, F_{CI_{\mu\nu}}^{(j)} \geq F_{\check{C}L_{\mu\nu}}, and\ g_{CI_{\mu\nu}}^{(j)} \geq g_{\check{C}L_{\mu\nu}}) \\ 0; otherwise \end{cases} \quad (13)$$

Example 3.6. Consider Example 3.1, Then the concordance level $\check{C}L$ and the concordant Boolean matrix Λ are calculated as follows:

$$\check{C}L = (\langle 0.684, 0.177, 0.121\rangle, 3)$$

$$\Lambda = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varsigma_1 & \varsigma_2 & \varsigma_3 \end{array} \\ \begin{bmatrix} - & 1 & 1 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix} \end{array}$$

3.4.2. The Single-Valued Neutrosophic N-Soft Effective Discordance Matrix

Another method involves evaluating elements in the discordance matrix D_M on the basis of discordance level in order to obtain the dominant data of alternatives from a discordant perspective. That is, to what extent is one alternative unsatisfactory to another. The discordance level $\breve{D}L = (\breve{d}_{\mu\nu}, \breve{g}_{\mu\nu})$ can be calculated as follows:

$$\breve{D}L = \left(\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} d_{\mu\nu}, \left[\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} g_{\mu\nu}\right]\right) \quad (14)$$

where $\left[\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} g_{\mu\nu}\right]$ is the greatest integer less than or equal to $\frac{1}{n(n-1)}\sum_{\mu,\mu\neq\nu}^{n}\sum_{\nu,\mu\neq\nu}^{n} g_{\mu\nu}$ and $(\mu, \nu = 1,2,\ldots,n, \mu \neq \nu)$.

Since $DI_{\mu\nu}$ reflects the degree to which an alternative ς_μ is inferior to ς_ν, and when it exceeds the discordance level $\breve{D}L$, it confirms the conclusion that ς_μ is absolutely not preferable than ς_ν. On the basis of discordance indices, the discordant Boolean matrix $\Omega = [\Omega_{\mu\nu}]_{n\times n}$ can be computed as

$$\varsigma_1 \quad \varsigma_2 \quad \cdots \quad \varsigma_{(n-1)} \quad \varsigma_n$$

$$\Omega = \begin{matrix} \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_{n-1} \\ \varsigma_n \end{matrix} \begin{bmatrix} - & \Omega_{12} & \cdots & \Omega_{1(n-1)} & \Omega_{1n} \\ \Omega_{\mu\nu} & - & \cdots & \Omega_{2(n-1)} & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Omega_{(n-1)1} & \Omega_{(n-1)2} & \cdots & - & \Omega_{(n-1)n} \\ \Omega_{n1} & \Omega_{n2} & \cdots & \Omega_{n(n-1)} & - \end{bmatrix}$$

Where,

$$\Omega_{\mu\nu} = \begin{cases} 0;\ \text{if } DI_{\mu\nu} \geq \breve{D}L(d_{\mu\nu} > \breve{d}_{\mu\nu} \text{ and } g_{\mu\nu} > \breve{g}_{\mu\nu}) \\ 1;\ \text{otherwise} \end{cases} \quad (15)$$

Example 3.7. Consider Example 3.1, Then the discordance level $\breve{D}L$ and the concordant Boolean matrix Ω are calculated as follows:

$$\breve{D}L = (0.387,1)$$

$$\Omega = \begin{matrix} & \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \varsigma_1 & - & 1 & 0 \\ \varsigma_2 & 1 & - & 0 \\ \varsigma_3 & 0 & 0 & - \end{matrix}$$

3.4.3. Aggregated Outranking Boolean Matrix

By combining the effective concordance and discordance information of the outranking relationship among the competing alternatives in terms of different parameters, the aggregated outranking matrix can be obtained. In a corresponding manner, the aggregated Boolean outranking matrix $\Delta = [\Delta_{\mu\nu}]_{n \times n}$ can be defined as follows:

$$\Delta = \begin{matrix} & \varsigma_1 & \varsigma_2 & \cdots & \varsigma_{(n-1)} & \varsigma_n \\ \varsigma_1 & - & \Delta_{12} & \cdots & \Delta_{1(n-1)} & \Delta_{1n} \\ \varsigma_2 & \Delta_{\mu\nu} & - & \cdots & \Delta_{2(n-1)} & \Delta_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \varsigma_{n-1} & \Delta_{(n-1)1} & \Delta_{(n-1)2} & \cdots & - & \Delta \\ \varsigma_n & \Delta_{n1} & \Delta_{n2} & \cdots & \Delta_{n(n-1)} & - \end{matrix}$$

Where,

$$\Delta_{\mu\nu} = \Lambda_{\mu\nu} * \Omega_{\mu\nu} \quad (16)$$

A non-zero value of $\Delta_{\mu\nu}$ (That is 1) completely implies that the alternative ς_μ has strict dominance over the alternative ς_ν.

Example 3.8. Consider Example 3.1, Then the Boolean outranking matrix Δ is computed as follows:

$$\Delta = \begin{matrix} & \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \varsigma_1 & - & 1 & 0 \\ \varsigma_2 & 0 & - & 0 \\ \varsigma_3 & 0 & 0 & - \end{matrix}$$

3.5 Discription of Outranking Graph

A graph is a structure consisting of a collection of elements, some of which are "connected" in some way. The final phase of the ELECTRE 1 approach is to use the outranking matrix Δ to extract as much of a compact set of alternatives as feasible in order to find the best solution to the MCDM problem. Accordingly, a simple directed graph $G = (V, M)$ is drawn, where V denote the set of vertices representing the alternatives and M denote the set of edges. Then the following cases can be occur, for any pair of alternatives ς_{μ} and ς_{ν}.

Case 1: $\Delta_{\mu\nu}= 1$; then draw an arc from ς_{μ} to ς_{ν} indicating that ς_{μ} is performing better than ς_{ν}. (That is, $\varsigma_{\mu} \succ \varsigma_{\nu}$)

Case 2: $\Delta_{\mu\nu}= 1$ and $\Delta_{\nu\mu}= 1$; then draw a two-way arrowed arc to represent the indifferent relationship between the alternatives ς_{μ} and ς_{ν} (That is $\varsigma_{\mu} \approx \varsigma_{\nu}$).

Case 3: $\Delta_{\mu\nu}= 0$; then there will be no arc connecting the two vertices, and the corresponding alternatives ς_{μ} and ς_{ν} are incomparable (That is $\varsigma_{\mu}\ ?\, \varsigma_{\nu}$).

4. Algorithm for Single-Valued Neutrosophic N-soft ELECTRE I Method

Step 1: The neutrosophic numbers are used to indicate the significance of decision-makers using linguistic terms. Let $N_q = (T_q, I_q, F_q)$ be the neutrosophic number corresponding to the $\mathrm{SVN_NS}$ number, for rating of decision-maker d_q. Then the weight of d_q can be computed as

$$\xi_q = \frac{1-\frac{1}{2(N-1)}\sqrt{(N-1)^2\left(\left(1-T_q\right)^2+I_q^2+F_q^2\right)+\left(N-1-\mathrm{g}_q\right)^2}}{\sum_{q=1}^{p}\left(1-\frac{1}{2(N-1)}\sqrt{(N-1)^2\left(\left(1-T_q\right)^2+I_q^2+F_q^2\right)+\left(N-1-\mathrm{g}_q\right)^2}\right)} \tag{17}$$

where $\sum_{q=1}^{p} \xi_q = 1$.

Step 2: Corresponding to each decision-maker d_q with weight $\xi_q (q = 1,2,\dots,p)$, $M^q = [m^{(q)}]_{n\times o}$ is the $\mathrm{SVN}_N\mathrm{S}$ decision matrix. To generate an aggregated $\mathrm{SVN}_N\mathrm{S}$ decision matrix $\widetilde{M} = [\widetilde{m}_{ij}]_{n\times o} = [\,\widetilde{\aleph}, K, N]_{n\times o}$ in a group decision-making technique, all the individual opinions of decision-makers must be united into a collective opinion. The aggregated $\mathrm{SVN}_N\mathrm{S}$ decision matrix $\widetilde{M}$ can be obtained as follows:

$$\widetilde{m}_{ij} = \left(\left\langle 1-\prod_{q=1}^{p}\left(1-T_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}, \prod_{q=1}^{p}\left(I_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}, \prod_{q=1}^{p}\left(F_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}\right\rangle, \left[\prod_{q=1}^{p}\left(\mathrm{g}_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}\right]\right) \tag{18}$$

where $\left[\prod_{q=1}^{p}\left(\mathrm{g}_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}\right]$ is the greatest integer less than or equal to $\prod_{q=1}^{p}\left(\mathrm{g}_{\aleph_q(\varpi_j)}(\varsigma_i)\right)^{\xi_q}$. Here $\widetilde{m}_{ij} = \left(\left\langle T_{\widetilde{\aleph}(\varpi_j)}(\varsigma_i), I_{\widetilde{\aleph}(\varpi_j)}(\varsigma_i), F_{\widetilde{\aleph}(\varpi_j)}(\varsigma_i)\right\rangle, \mathrm{g}_{\widetilde{\aleph}(\varpi_j)}(\varsigma_i)\right)$, $(i = 1,2,\dots,n \;\&\; j = 1,2,\dots,o)$. Thus the aggregated $\mathrm{SVN_NS}$ decision matrix $\widetilde{M} = [\widetilde{m}_{ij}]_{n\times o} = [\,\widetilde{\aleph}, K, N]_{n\times o}$ can be constructed as follows:

$$\widetilde{M} = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_n \end{array} \begin{array}{c} \begin{array}{cccc} \varpi_1 & \varpi_2 & \dots & \varpi_o \end{array} \\ \begin{bmatrix} \widetilde{m}_{11} & \widetilde{m}_{12} & \cdots & \widetilde{m}_{1o} \\ \widetilde{m}_{21} & \widetilde{m}_{22} & \cdots & \widetilde{m}_{2o} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{m}_{n1} & \widetilde{m}_{n2} & \dots & \widetilde{m}_{no} \end{bmatrix} \end{array}$$

Step 3: Each parameter has its own importance, which may not be equal. By combining all the opinions of decision-makers, a matrix of important degrees of the appropriate parameter is constructed for the ranking of each parameter. Suppose that $\omega_j^{(q)} = \left(\left\langle T_j^{(q)}, I_j^{(q)}, F_j^{(q)}\right\rangle, g_j^{(q)}\right)$ is the SVN_NS number assigned to the parameter ω_j by the decision-maker d_q. To find the weight ω_j of each parameter, the evaluation of decision makers on parameters are aggregated by using the following equation.

$$\omega_j = \left(\left\langle 1-\prod_{q=1}^{p}\left(1-T_j^{(q)}\right)^{\xi_q}, \prod_{q=1}^{p}\left(I_j^{(q)}\right)^{\xi_q}, \prod_{q=1}^{p}\left(F_j^{(q)}\right)^{\xi_q}\right\rangle, \left[\prod_{q=1}^{p}\left(g_j^{(q)}\right)^{\xi_q}\right]\right) \quad (19)$$

where $\left[\prod_{q=1}^{p}\left(g_j^{(q)}\right)^{\xi_q}\right]$ is the greatest integer less than or equal to $\prod_{q=1}^{p}\left(g_j^{(q)}\right)^{\xi_q}$, and $W = [\omega_1, \omega_2, \dots, \omega_o]$ where $\omega_j = \left(\langle T_W(\varpi_j), I_W(\varpi_j), F_W(\varpi_j)\rangle, g_W(\varpi_j)\right)$, $j = 1,2,\dots,o$).

Step 4: The aggregated weighted SVN_NS decision matrix $M = [m_{ij}]_{n\times o} = [\aleph, K, N]_{n\times o}$ can be calculated using multiplicative operator

$$\begin{aligned} m_{ij} &= \tilde{m}_{ij} \boxtimes \omega_j \\ &= \left(\left\langle T_{\tilde{\aleph}(\varpi_j)}(\varsigma_i).T_W(\varpi_j), I_{\tilde{\aleph}(\varpi_j)}(\varsigma_i) + I_W(\varpi_j) - I_{\tilde{\aleph}(\varpi_j)}(\varsigma_i).I_W(\varpi_j), F_{\tilde{\aleph}(\varpi_j)}(\varsigma_i) + \right.\right. \\ &\quad \left.\left. F_W(\varpi_j) - F_{\tilde{\aleph}(\varpi_j)}(\varsigma_i).F_W(\varpi_j)\right\rangle, \min\left\{g_{\tilde{\aleph}(\varpi_j)}(\varsigma_i), g_W(\varpi_j)\right\}\right) \end{aligned} \quad (20)$$

The aggregated weighted SVN_NS decision matrix M can be created as follows:

$$M = \begin{matrix} & \varpi_1 & \varpi_2 & \dots & \varpi_o \\ \varsigma_1 & m_{11} & m_{12} & \cdots & m_{1o} \\ \varsigma_2 & m_{21} & m_{22} & \cdots & m_{2o} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \varsigma_n & m_{n1} & m_{n2} & \dots & m_{no} \end{matrix}$$

Step 5: Based on the membership degree, indeterminacy degree, falsity degree, and grade of alternative over different parameters, the SVN_NS strong, midrange, weak concordance and discordance sets $S_{C_{\mu\nu}}, M_{C_{\mu\nu}}, W_{C_{\mu\nu}}, S_{D_{\mu\nu}}, M_{D_{\mu\nu}}$, and $W_{D_{\mu\nu}}$ can be computed by using equations 1-6 respectively.

Step 6: Compute the concordance index $CI_{\mu\nu} = (\langle T_{CI_{\mu\nu}}^{(j)}, I_{CI_{\mu\nu}}^{(j)}, F_{CI_{\mu\nu}}^{(j)}\rangle, g_{CI_{\mu\nu}}^{(j)})$, $(\mu, \nu = 1,2,\dots,n, \mu \neq \nu)$ by utilizing Equation 7 and create the concordant matrix C_M.

Step 7: Compute the discordance index $DI_{\mu\nu} = (d_{\mu\nu}, g_{\mu\nu})$ by using Equation 8 and construct the discordance matrix D_M.

Step 8: Using Equation 12, calculate the concordance level $\check{C}L = \left(\left\langle T_{\check{C}L_{\mu\nu}}, I_{\check{C}L_{\mu\nu}}, F_{\check{C}L_{\mu\nu}}\right\rangle, g_{\check{C}L_{\mu\nu}}\right)$ to analyze the relative superiority of pairs of alternatives, and then construct the effective concordance Boolean matrix $\Lambda = [\Lambda_{\mu\nu}]_{n\times n}$ as per Equation 13.

Step 9: Using Equation 14, evaluate the discordance level $\breve{D}L = (\check{d}_{\mu\nu}, \breve{g}_{\mu\nu})$ to investigate the relative non-dominance of two alternatives and then construct the effective discordance matrix $\Omega = [\Omega_{\mu\nu}]_{n\times n}$ as per Equation 15.

Step 10: Compute the aggregated outranking Boolean matrix $\Delta = [\Delta_{\mu\nu}]_{n\times n}$ by using Equation 16.

Step 11: To eliminate the less desirable alternatives and determine the most acceptable collection of alternatives, draw the directed outranking graph $G = (V, M)$ as described in Section 3.5.

The flowchart of the proposed single-valued neutrosophic N-soft ELECTRE I method is presented in Figure 1.

5. Numerical Example

To demonstrate the applicability of our proposed $\mathrm{SVN}_N S$-ELECTRE I method, we address a site selection problem for launching a manufacturing plant in a group decision-making environment in this section.

5.1 Site Selection for a Manufacturing plant

Finding the appropriate location, in the right region, and a suitable plot of land is becoming highly relevant for most businesses. As the global economic environment in which a firm must make a location selection becomes more uncertain by the day, it is critical to make a solid and future-proof decision on where to locate a new manufacturing plant. Companies may begin the location selection process for a new manufacturing plant for a variety of reasons, including cost reduction, capacity expansion to facilitate business growth, entry into new markets, tapping into new labor pools, and rationalization following a merger or acquisition. The construction of a new manufacturing unit necessitates major investments and internal modifications. Because the investments are made over a lengthy period of time, the new location's impact should similarly be good for a long period of time. That is, if a location decision is made wrongly, the negative consequences will last for a long time.

For the selection of a suitable location or site for a manufacturing unit, the following factors can be considered.

- Costs: Real estate, land, logistical costs, taxes, labor, and other pertinent aspects that can be transformed into money.
- The business environment quality: Factors that cannot be easily translated into currency but have a direct impact on the performance of the new activities. Consider labor availability, access, supplier availability, customs rules, and the simplicity with which you may conduct business.
- Risk: All external business disruption risk elements that a company cannot control but that could have a significant impact on future operations, such as inflation, transparency risk, currency exchange rate risk, natural disaster risk, and so on.

In this work, we find the best suitable site for a company to start their manufacturing unit. We have to choose the most convincing site from a set of six sites $\{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \varsigma_6\}$, which are selected by some primary evaluations. The set $\Sigma = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \varsigma_6\}$ is considered as the set of alternatives. The company mainly considers five important parameters of criteria $K = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5\}$ for the selection process. They are given as follows:

- ϖ_1: Suitable size (The size of the site must exceed the minimum plant layout requirement. Unless future plant expansions are planned, sites should not be much larger than the required size, as this has an impact on affordability.)

- ϖ_2: Availability (The location should be for sale and free of encumbrances such as contam ination or legal wrangling. It can take a long time to fix these issues.)
- ϖ_3: Accessibility (The location should be easily accessible for delivering process equipment to the site, as well as for feed material and product transportation. Access should ideally not entail the construction of new roads, rail spurs, or bridges.)
- ϖ_4: Affordability (The demanding price for the site should be reasonable and based on an evaluation. The selecting committee should establish a maximum price that they are ready to pay based on the project's profitability.)
- ϖ_5: Political stability (If a development in a foreign country is taken into account, this may be applicable. It might refer to the government's continuous backing for a project or the facility's physical security.)

The following steps constitute the process for selecting an appropriate site for starting the manufacturing plant using the proposed single-valued neutrosophic N-soft ELECTRE I method, and here we are taking $N = 7$. In other words, the single-valued neutrosophic 7 soft ELECTRE I method for the site selection process containing the following steps:

Step 1: Computation of the weights of the decision-makers.

Suppose that there are four decision-makers $D = \{d_1, d_2, d_3, d_4\}$. Table 1 lists the linguistic terms utilized in the relative importance rating of experts, as well as the parameters that were examined. Using Equation 17, the weight of the decision-makers are calculated and listed in Table 2.

Table 1: Linguistic terms for the weights of experts and parameters

Linguistic terms	**SVN$_7$S Number**
Highly Important (HI)	$(\langle 0.9, 0.1, 0.2\rangle, 6)$
Important (I)	$(\langle 0.8, 0.3, 0.1\rangle, 5)$
Average (A)	$(\langle 0.6, 0.3, 0.4\rangle, 3)$
Not Important (NI)	$(\langle 0.3, 0.7, 0.6\rangle, 2)$
Highly Not Important (HN)	$(\langle 0.1, 0.8, 0.9\rangle, 1)$

Table 2: The significance of decision-makers and their weights

$\boldsymbol{D}$	**Linguistic terms**	**SVN$_7$S Number**	**Weights (ξ_q)**
$\boldsymbol{d_1}$	A	$(\langle \mathbf{0.6}, \mathbf{0.3}, \mathbf{0.4}\rangle, \mathbf{3})$	0.2274
$\boldsymbol{d_2}$	HI	$(\langle \mathbf{0.9}, \mathbf{0.1}, \mathbf{0.2}\rangle, \mathbf{6})$	0.3540
$\boldsymbol{d_3}$	I	$(\langle \mathbf{0.8}, \mathbf{0.3}, \mathbf{0.1}\rangle, \mathbf{5})$	0.3166
$\boldsymbol{d_4}$	NI	$(\langle \mathbf{0.3}, \mathbf{0.7}, \mathbf{0.6}\rangle, \mathbf{2})$	0.1020

Step 2: Construction of single-valued neutrosophic 7 soft aggregated matrix.

Linguistic terms for the weights of the parameters are given in Table 3. The evaluation information of the decision-makers with respect to the parameters under considerations are shown in Table 4. Then the corresponding SVN$_7$S decision matrices $M^{(1)}$, $M^{(2)}$, $M^{(3)}$, and $M^{(4)}$ are given as follows:

Table 3: Linguistic terms for the weights of alternatives

Linguistic terms	**SVN_7S Number**
Highly Preferable (HP)	$(\langle 0.9, 0.1, 0.1\rangle, 6)$
Most Preferable (MP)	$(\langle 0.8, 0.2, 0.1\rangle, 5)$
Preferable (P)	$(\langle 0.7, 0.3, 0.4\rangle, 4)$
Average Preferable (AP)	$(\langle 0.6, 0.4, 0.5\rangle, 3)$
Un-Preferable (UP)	$(\langle 0.4, 0.6, 0.8\rangle, 2)$
Most Un-Preferable (MU)	$(\langle 0.2, 0.8, 0.7\rangle, 1)$
Highly Un-Preferable (HU)	$(\langle 0.1, 0.9, 0.9\rangle, 1)$

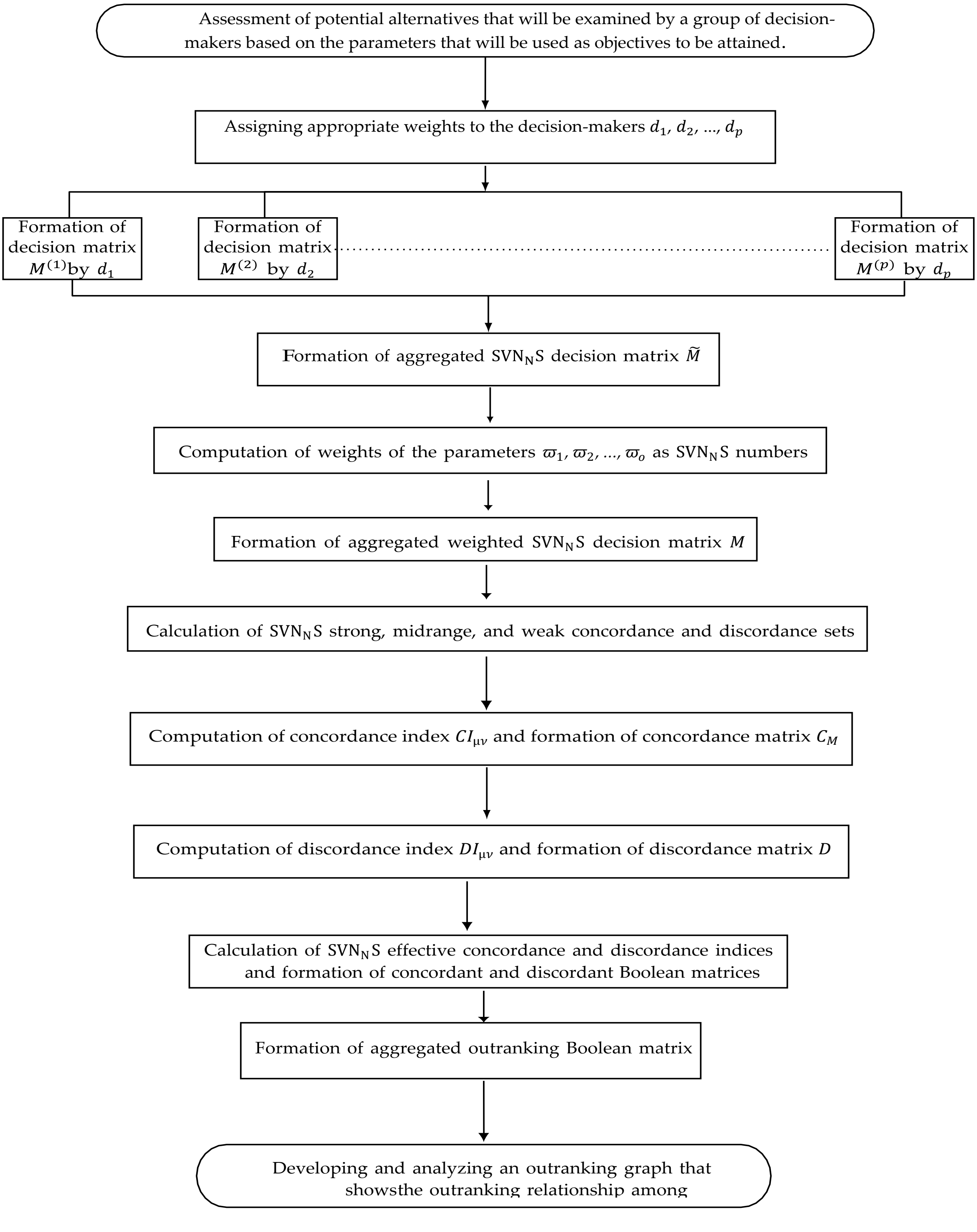

Figure 1: Flowchart of SVN_NS -ELECTRE I method

Table 4: The significance of decision-makers and their weights

D	**Σ**	$\boldsymbol{\varpi_1}$	$\boldsymbol{\varpi_2}$	$\boldsymbol{\varpi_3}$	$\boldsymbol{\varpi_4}$	$\boldsymbol{\varpi_5}$
d_1	ς_1	HP	P	P	AP	UP
	ς_2	UP	MP	HP	AP	HP
	ς_3	HP	HP	P	AP	HP
	ς_4	HP	P	MU	P	UP
	ς_5	P	MP	AP	HP	P
	ς_6	P	HP	AP	P	P
d_2	ς_1	AP	P	HP	MP	MP
	ς_2	P	AP	HP	P	P
	ς_3	MP	P	HP	HP	AP
	ς_4	UP	AP	MP	P	AP
	ς_5	P	AP	AP	P	P
	ς_6	P	HP	MP	AP	P
d_3	ς_1	HP	P	MU	AP	MP
	ς_2	MP	P	HP	MP	P
	ς_3	HP	AP	HP	MP	MP
	ς_4	HU	P	AP	P	MP
	ς_5	MP	HP	P	P	UP
	ς_6	P	AP	AP	HP	P
d_4	ς_1	P	MP	AP	MP	HP
	ς_2	HP	MU	P	P	MP
	ς_3	HP	P	MP	MP	P
	ς_4	AP	HU	MU	P	MP
	ς_5	P	P	AP	AP	MP
	ς_6	AP	P	HP	MP	HU

$$M^{(1)}) = [\aleph_1, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array}
\begin{array}{c} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \end{array} \\
\begin{bmatrix}
(\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.4, 0.6, 0.8\rangle, 2) \\
(\langle 0.4, 0.6, 0.8\rangle, 2) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) \\
(\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) \\
(\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.2, 0.8, 0.7\rangle, 1) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.4, 0.6, 0.8\rangle, 2) \\
(\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) \\
(\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4)
\end{bmatrix} \end{array}$$

$$M^{(2)}) = [\aleph_2, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array}
\begin{array}{c} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \end{array} \\
\begin{bmatrix}
(\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\
(\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4) \\
(\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.6, 0.4, 0.5\rangle, 3) \\
(\langle 0.4, 0.6, 0.8\rangle, 2) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) \\
(\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) \\
(\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4)
\end{bmatrix} \end{array}$$

$$M^{(3)}) = [\aleph_3, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.2, 0.8, 0.7\rangle, 1) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.7, 0.3, 0.4\rangle, 4) \\ (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.1, 0.9, 0.9\rangle, 1) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.4, 0.6, 0.8\rangle, 2) \\ (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) \end{array}$$

$$M^{(4)}) = [\aleph_4, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.9, 0.1, 0.1\rangle, 6) \\ (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.2, 0.8, 0.7\rangle, 1) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.7, 0.3, 0.4\rangle, 4) \\ (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.1, 0.9, 0.9\rangle, 1) & (\langle 0.2, 0.8, 0.7\rangle, 1) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.8, 0.2, 0.1\rangle, 5) \\ (\langle 0.6, 0.4, 0.5\rangle, 3) & (\langle 0.7, 0.3, 0.4\rangle, 4) & (\langle 0.9, 0.1, 0.1\rangle, 6) & (\langle 0.8, 0.2, 0.1\rangle, 5) & (\langle 0.1, 0.9, 0.9\rangle, 1) \end{array}$$

Now, the aggregated SVN_7S decision matrix $\widetilde{M} = [\widetilde{\aleph}, K, 7]$ is constructed by using Equation 18 and is given below.

$$\widetilde{M} = [\widetilde{\aleph}, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \\ (\langle 0.817, 0.183, 0.204\rangle, 4) & (\langle 0.712, 0.288, 0.347\rangle, 5) & (\langle 0.714, 0.286, 0.299\rangle, 2) & (\langle 0.708, 0.292, 0.240\rangle, 3) & (\langle 0.761, 0.239, 0.160\rangle, 4) \\ (\langle 0.724, 0.276, 0.262\rangle, 3) & (\langle 0.665, 0.335, 0.334\rangle, 3) & (\langle 0.888, 0.112, 0.115\rangle, 5) & (\langle 0.718, 0.282, 0.271\rangle, 4) & (\langle 0.776, 0.224, 0.253\rangle, 4) \\ (\langle 0.872, 0.128, 0.100\rangle, 5) & (\langle 0.744, 0.256, 0.313\rangle, 4) & (\langle 0.862, 0.138, 0.137\rangle, 5) & (\langle 0.816, 0.183, 0.144\rangle, 4) & (\langle 0.772, 0.228, 0.204\rangle, 4) \\ (\langle 0.564, 0.436, 0.493\rangle, 2) & (\langle 0.628, 0.372, 0.470\rangle, 3) & (\langle 0.607, 0.393, 0.316\rangle, 2) & (\langle 0.700, 0.300, 0.400\rangle, 4) & (\langle 0.672, 0.328, 0.284\rangle, 3) \\ (\langle 0.736, 0.264, 0.258\rangle, 4) & (\langle 0.786, 0.214, 0.204\rangle, 4) & (\langle 0.635, 0.365, 0.466\rangle, 3) & (\langle 0.759, 0.244, 0.299\rangle, 4) & (\langle 0.757\ 0.243, 0.265\rangle, 3) \\ (\langle 0.691, 0.306, 0.409\rangle, 3) & (\langle 0.827, 0.173, 0.192\rangle, 4) & (\langle 0.728, 0.272, 0.240\rangle, 3) & (\langle 0.775, 0.225, 0.242\rangle, 4) & (\langle 0.664, 0.336, 0.434\rangle, 3) \end{array}$$

Step 3: Calculation of the weights of parameters

The four decision-makers gave the weights of parameters under consideration in the form of linguistic terms, and are given in Table 5. The corresponding SVN7S numbers are provided in table 6. Now, the weights of the parameters are calculated by aggregating the evaluation of decision-makers by using Equation 19. That is,

$$W = \begin{bmatrix} \omega_1 = (\langle 0.805, 0.203, 0.198\rangle, 4) \\ \omega_2 = (\langle 0.825, 0.189, 0.183\rangle, 4) \\ \omega_3 = (\langle 0.866, 0.158, 0.150\rangle, 5) \\ \omega_4 = (\langle 0.768, 0.268, 0.166\rangle, 4) \\ \omega_5 = (\langle 0.837, 0.148, 0.256\rangle, 4) \end{bmatrix}^T$$

Table 5: Weights of the parameters in linguistic terms

	d_1	d_2	d_3	d_4
ϖ_1	I	HI	A	I
ϖ_2	A	I	HI	HI
ϖ_3	HI	HI	I	I
ϖ_4	I	I	A	HI
ϖ_5	HI	A	HI	HI

Table 6: Weights of the parameters by decision-makers

	d_1	d_2	d_3	d_4
ϖ_1	$(\langle 0.8, 0.3, 0.1\rangle, 5)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.6, 0.3, 0.4\rangle, 3)$	$(\langle 0.8, 0.3, 0.1\rangle, 5)$
ϖ_2	$(\langle 0.6, 0.3, 0.4\rangle, 3)$	$(\langle 0.8, 0.3, 0.1\rangle, 5)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$
ϖ_3	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.8, 0.3, 0.1\rangle, 5)$	$(\langle 0.8, 0.3, 0.1\rangle, 5)$
ϖ_4	$(\langle 0.8, 0.3, 0.1\rangle, 5)$	$(\langle 0.8, 0.3, 0.1\rangle, 5)$	$(\langle 0.6, 0.3, 0.4\rangle, 3)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$
ϖ_5	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.6, 0.3, 0.4\rangle, 3)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$	$(\langle 0.9, 0.1, 0.2\rangle, 6)$

Step 4: Formation of aggregated weighted SVN_7S decision matrix

The aggregated weighted SVN_7S decision matrix is calculated by using Equation 20, and is given as follows:

$$M = [\aleph, K, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{ccccc} \varpi_1 & \varpi_2 & \varpi_3 & \varpi_4 & \varpi_5 \end{array} \\ \begin{bmatrix} (\langle 0.658, 0.349, 0.362\rangle, 4) & (\langle 0.587, 0.423, 0.466\rangle, 4) & (\langle 0.618, 0.399, 0.404\rangle, 2) & (\langle 0.544, 0.484, 0.366\rangle, 3) & (\langle 0.637, 0.352, 0.375\rangle, 4) \\ (\langle 0.583, 0.423, 0.408\rangle, 3) & (\langle 0.549, 0.461, 0.456\rangle, 3) & (\langle 0.769, 0.252, 0.248\rangle, 5) & (\langle 0.551, 0.474, 0.392\rangle, 4) & (\langle 0.650, 0.339, 0.444\rangle, 4) \\ (\langle 0702, 0.305, 0.278\rangle, 4) & (\langle 0.614, 0.397, 0.439\rangle, 4) & (\langle 0.746, 0.274, 0.267\rangle, 5) & (\langle 0.627, 0.402, 0.286\rangle, 4) & (\langle 0.646, 0.342, 0.408\rangle, 4) \\ (\langle 0.454, 0.550, 0.593\rangle, 2) & (\langle 0.518, 0.491, 0.567\rangle, 3) & (\langle 0.526, 0.489, 0.419\rangle, 2) & (\langle 0.538, 0.488, 0.499\rangle, 4) & (\langle 0.562, 0.427, 0.467\rangle, 3) \\ (\langle 0.592, 0.413, 0.405\rangle, 4) & (\langle 0.648, 0.363, 0.350\rangle, 4) & (\langle 0.550, 0.465, 0.546\rangle, 3) & (\langle 0.583, 0.444, 0.415\rangle, 4) & (\langle 0.634\ 0.355, 0.453\rangle, 3) \\ (\langle 0.556, 0.449, 0.526\rangle, 3) & (\langle 0.682, 0.329, 0.340\rangle, 4) & (\langle 0.630, 0.387, 0.354\rangle, 3) & (\langle 0.595, 0.433, 0.368\rangle, 4) & (\langle 0.556, 0.434, 0.579\rangle, 3) \end{bmatrix} \end{array}$$

Step 5: Calculation of SVN_7S concordance and discordance sets.

The SVN_7S concordance and discordance sets are evaluated by using equations 1-6, and are given as follows:

1. The SVN_7S strong concordance set $S_{C_{\mu\nu}}$:

$$S_C = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \{1\} & \phi & \{1,2,3,5\} & \{1,5\} & \{1,5\} \\ \{3\} & - & \{3\} & \{1,2,3,4,5\} & \{3,5\} & \{1,3,5\} \\ \{1,2,3,4\} & \{1,2,4\} & - & \{1,2,3,4,5\} & \{1,3,4,5\} & \{1,3,4,5\} \\ \phi & \phi & \phi & - & \phi & \{5\} \\ \{2\} & \{1,2\} & \{2\} & \{1,2,4,5\} & - & \{1,5\} \\ \{2,3\} & \{2,4\} & \{2\} & \{1,2,3,4\} & \{2,3,4\} & - \end{bmatrix} \end{array}$$

2. The SVN_7S midrange concordance set $M_{C_{\mu\nu}}$:

$$M_C = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \phi & \phi & \phi & \phi & \phi \\ \phi & - & \phi & \phi & \phi & \phi \\ \phi & \phi & - & \phi & \phi & \phi \\ \phi & \phi & \phi & - & \phi & \phi \\ \phi & \phi & \phi & \phi & - & \phi \\ \phi & \phi & \phi & \phi & \phi & - \end{bmatrix} \end{array}$$

3. The SVN_7S week concordance set $W_{C_{\mu\nu}}$:

$$W_C = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \{2\} & \phi & \phi & \phi & \phi \\ \{4,5\} & - & \{5\} & \phi & \phi & \phi \\ \phi & \phi & - & \phi & \phi & \phi \\ \phi & \phi & \phi & - & \phi & \phi \\ \{4\} & \{4\} & \phi & \{3\} & - & \phi \\ \{4\} & \phi & \phi & \phi & \phi & - \end{bmatrix} \end{array}$$

4. The SVN_7S strong discordance set $S_{D_{\mu\nu}}$:

$$S_D = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \{3\} & \{3,4\} & \phi & \phi & \{3\} \\ \{1\} & - & \{1,2\} & \phi & \{1,2\} & \{2\} \\ \phi & \phi & - & \phi & \phi & \phi \\ \{1,2,5\} & \{1,3,5\} & \{1,2,3,5\} & - & \{1,2\} & \{1,2,3\} \\ \{5\} & \{3,5\} & \{3,5\} & \phi & - & \phi \\ \{1,5\} & \{3,5\} & \{1,3,5\} & \phi & \{1\} & - \end{bmatrix} \end{array}$$

5. The SVN_7S midrange discordance set $M_{D_{\mu\nu}}$:

$$M_D = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \phi & \phi & \phi & \phi & \phi \\ \phi & - & \phi & \phi & \phi & \phi \\ \phi & \phi & - & \phi & \phi & \phi \\ \phi & \phi & \phi & - & \phi & \phi \\ \phi & \phi & \phi & \phi & - & \phi \\ \phi & \phi & \phi & \phi & \phi & - \end{bmatrix} \end{array}$$

6. The SVN_7S week discordance set $W_{D_{\mu\nu}}$:

$$W_D = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & \{4\} & \{5\} & \phi & \{4\} & \{4\} \\ \{2\} & - & \phi & \phi & \phi & \phi \\ \phi & \phi & - & \phi & \phi & \phi \\ \phi & \phi & \phi & - & \{3\} & \phi \\ \phi & \phi & \phi & \phi & - & \phi \\ \phi & \phi & \phi & \phi & \phi & - \end{bmatrix} \end{array}$$

Step 6: Computation of concordance index and formation of concordance matrix.

Let $(\omega_{S_C}, \omega_{M_C}, \omega_{W_C}) = (1, \frac{2}{3}, \frac{1}{3})$ be the weights of the SVN_7S strong, midrange, and weak concordance sets, respectively, given by the decision-makers. The concordance indices $CI_{\mu\nu}$ $(\mu, \nu = 1, 2, \ldots, n, \mu \neq \nu)$ is calculated using Equation 7, and constructed SVN_7S concordance matrix C_M, where entries are the concordance indices $CI_{\mu\nu}$ $(\mu, \nu = 1, 2, \ldots, n, \mu \neq \nu)$.

$$C_M = [CI, \varpi, 7] =$$

$$\begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & (\langle 0.891, 0.117, 0.112\rangle, 4) & \phi & (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.968, 0.030, 0.051\rangle, 4) & (\langle 0.968, 0.030, 0.051\rangle, 4) \\ (\langle 0.955, 0.054, 0.052\rangle, 5) & - & (\langle 0.927, 0.084, 0.095\rangle, 5) & (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.978, 0.023, 0.038\rangle, 5) & (\langle 0.996, 0.005, 0.001\rangle, 5) \\ (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.992, 0.010, 0.002\rangle, 4) & (\langle 0.927, 0.084, 0.095\rangle, 5) & (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.999, 0.001, 0.001\rangle, 5) \\ \phi & \phi & - & \phi & \phi & (\langle 0.837, 0.148, 0.256\rangle, 5) \\ (\langle 0.893, 0.122, 0.102\rangle, 4) & (\langle 0.983, 0.025, 0.020\rangle, 4) & (\langle 0.825, 0.189, 0.183\rangle, 4) & (\langle 0.999, 0.001, 0.001\rangle, 5) & - & (\langle 0.968, 0.030, 0.051\rangle, 4) \\ (\langle 0.986, 0.019, 0.015\rangle, 4) & (\langle 0.959, 0.051, 0.008\rangle, 4) & (\langle 0.825, 0.189, 0.183\rangle, 4) & (\langle 0.999, 0.001, 0.001\rangle, 5) & (\langle 0.995, 0.008, 0.001\rangle, 5) & - \end{bmatrix} \end{array}$$

Step 7: Computation of discordance index and formation of discordance matrix.

Let $(\omega_{S_D}, \omega_{M_D}, \omega_{W_D}) = (1, \frac{2}{3}, \frac{1}{3})$ be the weights of the SVN_7S strong, midrange, and weak discordance sets, respectively, given by the decision-makers. The discordance indices $DI_{\mu\nu}$ $(\mu, \nu = 1, 2, \ldots, n, \mu \neq \nu)$ is calculated using Equation 8, and constructed SVN_7S discordance matrix D_M, where entries are the concordance indices $DI_{\mu\nu}$ $(\mu, \nu = 1, 2, \ldots, n, \mu \neq \nu)$

$$D_M = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & (1,3) & (1,3) & \phi & (0.1569,1) & (0.3016,1) \\ (0.4386,1) & - & (1,1) & \phi & (4103,1) & (0.9976,1) \\ \phi & \phi & - & \phi & \phi & \phi \\ (1,2) & (1,3) & (1,3) & - & (1,2) & (1,1) \\ (0.4574,1) & (1,2) & (1,2) & \phi & - & \phi \\ (1,1) & (1,2) & (1,2) & \phi & (0.5908,1) & - \end{bmatrix} \end{array}$$

Step 8: Calculation of single-valued neutrosophic 7 soft effective concordance matrix.
Using Equation 12 and the single-valued neutrosophic 7 soft concordance indices, the concordance level $\breve{C}L$ can be computed as:

$$\breve{C}L = (\langle 0.798, 0.038, 0.041 \rangle, 3)$$

Now, the concordant Boolean matrix Λ can be calculated by using Equation 13, and given as follows:

$$\Lambda = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & 0 & 0 & 1 & 0 & 0 \\ 0 & - & 0 & 1 & 1 & 1 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 1 & 0 & 1 & - & 0 \\ 1 & 0 & 0 & 1 & 1 & - \end{bmatrix} \end{array}$$

Step 9: Calculation of single-valued neutrosophic 7 soft effective discordance matrix.
Using Equation 14 and the single-valued neutrosophic 7 soft discordance indices, the discordance level $\breve{D}L$ can be computed as:

$$\breve{D}L = (0.5451, 1)$$

Now, the discordant Boolean matrix Ω can be calculated by using Equation 15, and given as follows:

$$\Omega = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & 0 & 0 & 1 & 1 & 1 \\ 1 & - & 0 & 1 & 1 & 0 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 1 & 0 & 0 & 1 & - & 0 \\ 0 & 0 & 0 & 1 & 0 & - \end{bmatrix} \end{array}$$

Step 10: Formation of aggregated outranking Boolean matrix.
The effective aggregated outranking Boolean matrix Δ is constructed using Equation 16, and given as:

$$\Delta = \begin{array}{c} \\ \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \\ \varsigma_5 \\ \varsigma_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 & \varsigma_5 & \varsigma_6 \end{array} \\ \begin{bmatrix} - & 0 & 0 & 1 & 0 & 0 \\ 0 & - & 0 & 1 & 1 & 0 \\ 1 & 1 & - & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 1 & - & 0 \\ 0 & 0 & 0 & 1 & 0 & - \end{bmatrix} \end{array}$$

Step 11: Construction of the directed outranking graph.
Based on the aggregated outranking Boolean matrix Δ, the directed outranking graph $G = (V, M)$ can be constructed, and shown in Figure 2.

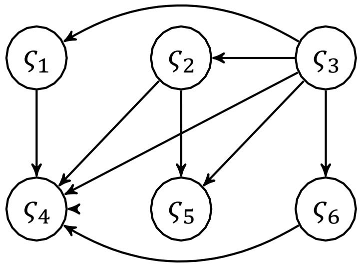

Figure 2: Outranking relationships between sites under consideration

The ranking between the alternatives (sites) given in Table 7 by exploring the decision graph. Thus it can be observed that, the site ς_3 is more preferable than the other sites ς_1, ς_2, ς_4, ς_5, and ς_6 for starting the manufacturing plant.

Table 7: Exploration of the directed outranking graph

Sites	Incomparable sites	Submissive sites	SVN_7S-ELECTRE I ranking
ς_1	$\varsigma_2, \varsigma_5, \varsigma_6$	ς_4	4
ς_2	ς_1, ς_6	$\varsigma_1, \varsigma_2, \varsigma_5$	2
ς_3	-	ς_4, ς_5	1
ς_4	-	$\varsigma_1, \varsigma_2, \varsigma_4, \varsigma_5, \varsigma_6$	5
ς_5	ς_1, ς_6	ς_4	3
ς_6	$\varsigma_1, \varsigma_2, \varsigma_5$	ς_4	4

5. Conclusion

ELECTRE is a traditional outranking-based MCDM methodology for obtaining the most preferable collection of alternatives, more optimal for the corresponding decision-making problem. In this paper, we proposed the SVN_NS -ELECTRE I method to deal with MCGDM problems. An algorithm based on the proposed method is presented along with a flowchart. The novel ELECTRE I technique for SVN_NS MCGDM demonstrates a new strategy to efficiently deal with problems in business management. Also, this established approach is projected to be beneficial in any group decision-making context, including industrial engineering, environmental management, and medical sciences. We hope to extend our work to SVN_NS-ELECTRE II method, and to other ELECTRE generalizations in group decision-making in the future due to the flexibility of the SVN_NS model.

5. Acknowledgements

During the development of the manuscript, the authors gratefully acknowledges the financial assistance provided by the University Grants Commission (UGC) of India. The authors also express their heartfelt gratitude to Al-Ameen College, Edathala, Aluva, Kerala, India, for providing a conducive research environment and the necessary infrastructure to carry out this work.

Conflict of Interest

The authors declare that they have no conflict of interest.

References

1. Du, Yuan-Wei,; Jiao-Jiao Zhong, Dynamic multicriteria group decision-making method with automatic reliability and weight calculation. *Information Sciences*, **2023**, 634, 400-422.
2. Gao, Hengxia, ; Yanbing Ju,; Xiao-Jun Zeng,; Wenkai Zhang, Satisfaction-driven consensus model for social network MCGDM with incomplete information under probabilistic linguistic trust. *Computers & Industrial Engineering*, **2021**, 154, 107099.
3. Li, Shengli,; Cuiping Wei, A large scale group decision making approach in healthcare service based on sub-group weighting model and hesitant fuzzy linguistic information. *Computers & Industrial Engineering*, **2020**, 144, 106444.
4. Salam, Amira,; Mai Mohamed,; Rui Yong,; Jun Ye, A Robust Decision-Making Model for Medical Supplies via Selecting Appropriate Unmanned Aerial Vehicle. *Neutrosophic Systems with Applications*, **2024**, 18, 14-30.
5. Dursun, Mehtap,; E. Ertugrul Karsak ; Melis Almula Karadayi, A fuzzy multicriteria group decision making framework for evaluating health-care waste disposal alternatives. *Expert Systems with Applications*, **2011**, 38(9), 11453-11462.
6. Peng, Juan-juan,; Jian-qiang Wang,; Hong-yu Zhang,; Xiao-hong Chen, An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. *Applied Soft Computing*, **2014**, 25, 336-346.
7. Zhan, J.,; Masood Malik, H.,; Akram, M, Novel decision-making algorithms based on intuitionistic fuzzy rough environment. *International Journal of Machine Learning and Cybernetics*, **2019**, 10, 1459-1485.
8. Marimuthu, D., Meidute-Kavaliauskiene, I., Mahapatra, G. S., Cincikaite, R., Roy, P., Vasilis Vasiliauskas, A, Sustainable urban conveyance selection through MCGDM using a new ranking on generalized interval type-2 trapezoidal fuzzy number. *Mathematics*, **2022**, 10(23), 4534.
9. Zhong, S., Zhang, J., He, X., and Liu, S, Sustainable supply chain partner selection and order allocation: A hybrid fuzzy PL-TODIM based MCGDM approach. *Plos one*, **2022**, 17(9), e0271194.
10. Hwang, C. L., Yoon, K., Hwang, C. L., Yoon, K, Methods for multiple attribute decision making. *Multiple attribute decision making: methods and applications a state-of-the-art survey*, **1981**, 58-191.
11. Brans, J. P., Vincke, P., and Mareschal, B, How to select and how to rank projects: The PROMETHEE method. *European journal of operational research*, **1986**, 24(2), 228-238.
12. Opricovic, S., Tzeng, G. H., Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European journal of operational research*, **2004**, 156(2), 445-455.
13. Benayoun, R., Roy, B., Sussmann, B, ELECTRE: Une m´ethode pour quider le choix en presence de points de vue multiples, Note de travail 49, *SEMA-METRA International,Direction Scientifique*, **1966**
14. Pamucar, D., Cirovic, G, The selection of transport and handling resources in ´ logistics centers using Multi-Attributive Border Approximation area Comparison (MABAC). *Expert systems with applications,* **2015**, 42(6), 3016-3028.
15. Krylovas, A., Zavadskas, E. K., Kosareva, N., Dadelo, S, New KEMIRA method for determining criteria priority and weights in solving MCDM problem. *International journal of information technology & decision making*, **2014**, 13(06), 1119-1133.
16. Goswami, S. S., Behera, D. K, An analysis for selecting best smartphone model by AHP-TOPSIS decision-making methodology. *International Journal of Service Science, Management, Engineering, and Technology (IJSSMET)*, **2021**, 12(3), 116-137.
17. Palanikumar, M. P., Arulmozhi, K., MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators. *Neutrosophic Sets and Systems*, **2022**, 51(1), 35.
18. Mohamed, M., Metwaly, A. A., Ibrahim, M., Smarandache, F., Voskoglou, M, Clarivate Optimal Livestock via Enigmatic Nature of Blended Decision-Making Paradigm: Practicing Comparative Methodologies. *Precision Livestock*, **2024**, 1, 75-88.

19. Elsayed, A., Mohamed, M., Smarandache, F., Voskoglou, M, Automated Livestock Practices: Incorporation Emerging Contemporary Technologies Toward Sustainable Livestock in Era of Smart Cities. *Infinite Study*, **2024**
20. [20] Roy, B, Classement et choix en pr´esence de points de vue multiples. *Revue fran¸caise d'informatique et de recherche op´erationnelle*, **1968**, 2(8), 57-75.
21. Figueira, J. R., Mousseau, V., Roy, B, ELECTRE methods. Multiple criteria decision analysis: *State of the art surveys*, **2016**, 155-185.
22. de Almeida, A. T., Multicriteria decision model for outsourcing contracts selection based on utility function and ELECTRE method. *Computers & operations research*, **2007**, 34(12), 3569-3574.
23. Roy, B, Partial preference analysis and decision-aid: The fuzzy outranking relation concept. *Conflicting objectives in Decisions*, **1977**, 40-75.
24. Corner, J. L., Kirkwood, C. W, Decision analysis applications in the operations research literature, 1970–1989. *Operations Research*, **1991**, 39(2), 206-219.
25. Haurant, P., Oberti, P., Muselli, M, Multicriteria selection aiding related to photovoltaic plants on farming fields on Corsica island: A real case study using the ELECTRE outranking framework. *Energy policy*, **2011**, 39(2), 676-688.
26. Cavallaro, F. A comparative assessment of thin-film photovoltaic production processes using the ELECTRE III method. *Energy policy*, **2010**, 38(1), 463-474.
27. Montenegro de Barros, G. M., Pereira, V., Roboredo, M. C, ELECTRE tree: a machine learning approach to infer ELECTRE Tri-B parameters. *Data Technologies and Applications*, **2021**, 55(4), 586-608.
28. Singh, T., Pattnaik, P., Pruncu, C. I., Tiwari, A., Fekete, G, Selection of natural fibers based brake friction composites using hybrid ELECTRE-entropy optimization technique. *Polymer Testing*, **2020**, 89, 106614.
29. Zadeh, L.A, Fuzzy set, *Information and Control*, **1965**, 8(3), 338–353.
30. Atanassov K.T, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **1986**, 20, 87-96.
31. Yager, R. R, Pythagorean fuzzy subsets, *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS) , IEEE*, **2013**, 57-61.
32. Yager, R. R, Generalized orthopair fuzzy sets, *IEEE Transactions on Fuzzy Systems*, **2016**, 25(5), 1222-1230.
33. Yager, R. R,; Alajlan, N, Approximate reasoning with generalized orthopair fuzzy sets, *Information Fusion*, **2017**, 38, 65-73.
34. Smarandache, F, Neutrosophic set-a generalization of the intuitionistic fuzzy set, *International journal of pure and applied mathematics*, **2005**, 24(3), 287.
35. Smarandache, F, Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited), *Journal of New Theory*, **2019**, 29, 1-31.
36. Wang, H., Smarandache, F., Zhang, Y., Sunderraman, Single valued neutrosophic sets. *Multispace Multistructure*, **2010**, 4, 410–413.
37. Singh, S., Sharma, S, Divergence Measures and Aggregation Operators for SingleValued Neutrosophic Sets with Applications in Decision-Making Problems. *Neutrosophic Systems with Applications*, **2024**, 20, 27-44.
38. Mohamed, Z., Ismail, M. M., Abd El-Gawad, A, Sustainable supplier selection using neutrosophic multi-criteria decision making methodology. *Sustainable Machine Intelligence Journal*, **2023**, 3, 2-1.
39. El-Douh, A. A., Lu, S., Abdelhafeez, A., Aziz, A. S, Assessment the Health Sustainability using Neutrosophic MCDM Methodology: Case Study COVID-19. *Sustainable Machine Intelligence Journal*, **2023**, 3, 1-1.
40. Mohamed, M., Elsayed, A., Arain, B., Ye, J, An Efficient Neutrosophic Approach for Evaluating Possible Industry 5.0 Enablers in Consumer Electronics: A Case Study. *Neutrosophic Systems with Applications*, **2024**, 20, 10-26.
41. Mohamed, M., Salam, A., Ye, J., Yong, R, Single-valued neutrosophic MCDM approaches integrated with MEREC and RAM for the selection of UAVs in forest fire detection and management. *Neutrosophic systems with applications*, **2024**, 19, 1-14.
42. Molodtsov, D, Soft set theory—first results. *Computers & mathematics with applications*, **1999**, 37(4-5), 19-31.
43. Fatimah, F., Rosadi, D., Hakim, R. F., Alcantud, J. C. R, N-soft sets and their decision making algorithms. *Soft Computing*, **2018**, 22, 3829-3842.

44. Akram, M., Farooq, A., Shabir, M., Al-Shamiri, M. M. A., Khalaf, M. M, Group decision-making analysis with complex spherical fuzzy N-soft sets. *Math. Biosci. Eng*, **2022**, 19(5), 4991-5030.
45. Fatimah, F., Alcantud, J. C. R, The multi-fuzzy N-soft set and its applications to decision-making. *Neural Computing and Applications*, **2021**, 33, 11437-11446.
46. Maji, P. K, A neutrosophic soft set approach to a decision making problem. *Annals of Fuzzy mathematics and Informatics*, **2012**, 3(2), 313-319.
47. Riaz, M., Naeem, K., Zareef, I., Afzal, Neutrosophic N-soft sets with TOPSIS method for multiple attribute decision making. *Neutrosophic sets and systems*, **2020**, 32, 145.

Received: Aug 11, 2024. Accepted: Nov 7, 2024