An Overview of Neutrosophic Graphs

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Abstract

A new area of study called "neutrosophic graph theory" uses neutrosophic logic to expand on traditional graph theory to include ambiguous, indeterminate, and uncertain information. This paper gives a thorough introduction to neutrosophic graphs, including their definition, components, uses, and difficulties. Neutrosophic graphs let you show and analyze relationships that aren't clear-cut because they give vertices and edges different levels of truth, indeterminacy, and falsity. Image processing, social network analysis, pattern recognition, and decision-making in the face of uncertainty all use neutrosophic graphs. However, we still need to resolve issues like interpretability and computational complexity. All things considered, neutrosophic graphs provide a viable framework for representing and evaluating ambiguous information in graph structures, with potential applications in a variety of fields.

Keywords

Decision-making problems; neutrosophic graph; uncertainty

1. Introduction

Modern mathematics and applied sciences have shown significant interest in the study of graphs. The reason for this is that graphs possess a broad spectrum of applications across diverse disciplines such as engineering, biology, computer science, and social sciences. Conventional graph theory sometimes faces limitations when addressing the uncertainty and imprecision that is present in many real-world problems. Several proposals have suggested expanding and altering graph theory to address these limitations. Neutrosophic graphs have emerged as a very resilient and adaptable architecture.

Cantor originally introduced traditional set theory, the foundation of classical graph theory, to represent accurate and predictable relationships. Because graphs are insufficient for modeling uncertain optimization issues due to the inherent uncertainty and complexity of Real-Life Events, To Address This Disparity, Zadeh [1] Introduced The concept of Fuzzy Sets (FSs) which expand upon traditional sets by permitting a range of membership degrees ranging from 0 to 1. FSs offer a method for representing incomplete truths and managing uncertainty to a limited degree. Although they are useful, numerous real-world situations involve data that is not just imprecise but also uncertain and contradictory. In many fields such as decision-making, the information at hand may exhibit gaps, ambiguities, or contradicting evidence. Neutrosophic sets provide a more inclusive method for representing and analyzing situations by explicitly considering the various elements of uncertainty involved. Neutrosophic sets can better capture the intricacies of uncertain knowledge by incorporating three distinct elements: truth, indeterminacy, and falsity. Mathematicians Florentin Smarandache [2] proposed the use of Neutrosophic Set (NS) to enhance the ability to model and analyze data with more complexity and ambiguity than fuzzy sets alone can handle. This development facilitates more precise and dependable decision-making and problem-solving in domains where uncertainty plays a crucial role.

The concept of Single-Valued NS (SVNS) was introduced by Wang et al. [3]. These sets are an extension of intuitionistic FSs [4]. These entities are distinguished by the inclusion of three separate membership functions: truth, indeterminacy, and falsehood. Each membership function in this set has values that range from [0, 1]. This technology's development makes it possible to depict uncertainty in a more sophisticated manner. Overview of SVNS was presented by Pramanik [5]. Broumi et al. [6] came up with the concept of single-valued neutrosophic graphs, which they presented. In order to simulate complicated interactions in data that contain not only fuzziness but also indeterminacy and inconsistency, these graphs extend the principles of S-VNS to graph theory. This makes it possible to model these relationships. By utilizing this framework, one is able to more effectively examine and express the complexities of situations that happen in the real world.

Majumdar and Samanta [7] looked into the similarity and entropy of SVNSs to learn more about their structure and how to measure uncertainty. Yang et al. [8] advanced the field even further by discussing single-valued neutrosophic linkages and investigating their interconnections and comparisons. Ye [9] used these theoretical ideas in Multi Attribute Decision-Making (MADM) problems and showed correlation coefficients for SVNSs, building on these foundations to show that they are useful in real life. Ye [10] later proposed a) method using aggregation operators to streamline the application of SVNSs in complex MADM situations. Biswas et al. [11, 12, 13] presented the TOPSIS strategy in the Single Valued Neutrosophic Number (SVNN) environment. Pramanik et al. [14] presented the cross entropy-based Multi Attribute Group Decision Making (MAGDM) under SVNN environment. Pramanik et al. [15], presented the Best-Worst Method (BWM) TOPSIS strategy, under the SVNN environment. These approaches simplified and expedited the evaluation of several criteria in the presence of ambiguity, making it more suitable for real-world use. In sum, these developments demonstrate how neutrosophic set theory is constantly developing and how useful it is for dealing with complicated, ambiguous, indeterminate, and inconsistent problems in the real world. These scholars' combined efforts have greatly enhanced the discipline, producing strong methodologies and tools for better decision-making across different domains.

Broumi et al. [16] presented the Neutrosophic Graph (NG) in 2016. Akram and Shahzadi [17] developed the SVNS based graphs. Akram, and Sitara [18] presented the SVNS set based graph structures and made a big step forward in the field by applying the ideas of SVNSs to graph theory. This innovation facilitated more precise modelling of uncertainty in graphs. Subsequent research has explored several new concepts related to neutrosophic graphs and their applications, expanding the utility and applicability of these graphs in various domains. In addition, the studies [19, 20] discussed newly formulated concepts about NGs and hypergraphs. Subsequently, Raut et al. determined the shortest concise route within a network by utilizing the neutrosophic number [21, 22, 23,24].

The most significant contributions for this work are as follows:

1) This article covers different types of operations of NGs, such as Cartesian product, lexicographic product, union, composition and join, as well as their properties.

2) This study investigates the importance of a novel class of graphs and their application in MADM scenarios.

The remaining parts of the paper organize their contents in the following manner: This paper's Section-2 focuses on elucidating the purpose behind the research and outlining its significant contributions. Section -3 provides a concise overview of several definitions. Section -4 presents an overview of the many categories of neutrosophic graphs. Section-5 elucidates the various functions and activities performed by neutrosophic graphs. Section-6 presents a specific numerical example that demonstrates the use of SVNNs in the process of decision-making. Section-7 summarizes the article's conclusions.

2. Motivation

NGs simulate real-world scenarios in which the attributes of vertices or relationships between them may be ambiguous, imprecise, or unpredictable. Neutrosophic graphs provide a more versatile and all-encompassing approach to representing and analysing complex systems and networks by introducing the indeterminacy component. The relationships between them may be ambiguous, imprecise, or unpredictable. Neutrosophic graphs provide a more versatile and all-encompassing approach to representing and analyzing complex systems and networks by introducing the indeterminacy component.

Some important motivations and applications for neutrosophic graphs are as follows:

1. NGs can model uncertain or incomplete information, providing a more realistic representation of real-world scenarios.
2. NGs improve decision-making in uncertain situations, allowing for more informed and robust decisions.
3. NGs can represent uncertain or indeterminate relationships between individuals.
4. NGs are useful for analysing computer networks and communication systems with uncertain or indeterminate links or nodes.
5. You can use NGs for image processing and pattern recognition with imprecise or indeterminate pixel values or feature representations.

3. Preliminary:

3.1 Neutrosophic set (N-Set)

A NS is an extension of the intuitionistic fuzzy sets, which includes the concept of indeterminacy. The system is characterized by three separate membership functions i.e. truth, indeterminacy, and falsity. Consider U as the set of all possible elements under consideration. A NS A in the universal set U is defined as:

$$= \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

where:

The function $T_A(x): U \rightarrow [0,1]$ represents the degree of truth to which x belongs to the set A.

The function $I_A\,(x): U \rightarrow [0,1]$ represents the degree of indeterminacy of x belonging to A.
The function $F_A(x): U \rightarrow [0,1]$ represents the degree of falsity of x in A.

3.2 Interval valued NSs [25]

An Inter valued NS A on a universe of discourse X is a structure having the form as $A = \{\langle x, T_A\,(x), I_A\,(x), F_A(x)\rangle |\, x \in X\}$ where
$T_A\,(x) = [T_A^{-}\,(x), T_A^{+}\,(x)]$, $I_A\,(x) = [I_A^{-}\,(x), I_A^{+}\,(x)]$, and
$F_A\,(x) = [F_A^{-}\,(x), F_A^{+}\,(x)]$ indicate the truth, indeterminacy and falsity membership degree.
$0 \le (T_A\,(x))^3 + (F_A(x))^3 \le 1$ and $0 \le (I_A\,(x))^3 \le 1$

$$0 \le (T_A\,(x))^3 + \left(F_A(x)\right)^3 + \left(I_A\,(x)\right)^3 \le 2\,, \forall\, x \in X$$

Means

$$0 \le (T_A^{+}\,(x))^3 + \left(F_A^{+}(x)\right)^3 + \left(I_A^{+}\,(x)\right)^3 \le 2\,, \forall\, x \in X$$

3.3 NGs

In NG theory, a NG is defined as follows:

Consider a set V that consists of vertices (nodes) and a set E that consists of edges (arcs) connecting these vertices. Both V and E are non-empty. A NG G = (V, E) is represented by each of these three mappings:

1. A membership function that represents truth: T: V → [0, 1]
2. A membership function that represents indeterminacy: I: V → [0, 1]
3. A membership function that represents falsity: F: V → [0, 1]

such that 0 ≤ T(v) + I(v) + F(v) ≤ 3 for all v ∈ V.

The functions T, I, and F are used to assign membership values to each vertex v ∈ V, representing the truth, indeterminacy, and falsity degrees, respectively, of the proposition "v is a member of the graph G."

Similarly, the membership values of an edge (u, v) ∈ E are defined by the following three mappings:

1. A membership function that represents truth: T: E → [0, 1]
2. A membership function that represents indeterminacy: I: E → [0, 1]
3. A membership function that represents falsity: F: E → [0, 1]

such that 0 ≤ T(u, v) + I(u, v) + F(u, v) ≤ 3 for all (u, v) ∈ E.

These membership functions represent the truth, indeterminacy, and falsity degrees, respectively, of the proposition "the edge (u, v) is a member of the graph G."

4. Operations on NGs

In NG theory, various operations can be performed on NGs to obtain new graphs or analyze the properties of existing ones. Here are some common operations on NGs and illustrative examples.

4.1 Cartesian Product of NGs

The Cartesian product of two NGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a new NG $G = G_1 \times G_2$, where the vertex set is the Cartesian product of the vertex sets of G_1 and G_2, and the edge set is defined by specific rules involving the edges of G_1 and G_2.i.e $G_1 \times G_2 = (V_1 \times V_2\,, E_1 \times E_2\,)$

$$\hat{T}_{V_1 \times V_2}(v_1, v_2) = min\{\hat{T}_{V_1}(v_1), \hat{T}_{V_2}(v_2)\}$$
$$\hat{I}_{V_1 \times V_2}(v_1, v_2) = max\{\hat{I}_{V_1}(v_1), \hat{I}_{V_2}(v_2)\}$$

$$\hat{F}_{V_1 \times V_2}(v_1, v_2) = max\{\hat{F}_{V_1}(v_1), \hat{F}_{V_2}(v_2)\}$$
$$\forall (v_1, v_2) \in (V_1, V_2)$$

The membership value of the edges in $G_1 \times G_2$ can be computed as

$$\hat{T}_{E_1 \times E_2}((v, e_1), (v, e_2)) = min\{\hat{T}_{V_1}(v), \hat{T}_{E_2}(e_1, e_2)\}$$
$$\hat{I}_{E_1 \times E_2}((v, e_1), (v, e_2)) = max\{\hat{I}_{V_1}(v), \hat{I}_{E_2}(e_1, e_2)\}$$
$$\hat{F}_{E_1 \times E_2}((v, e_1), (v, e_2)) = max\{\hat{F}_{V_1}(v), \hat{F}_{E_2}(e_1, e_2)\}$$
$$\forall v \in V_1, (e_1, e_2) \in E_2$$
$$\hat{T}_{E_1 \times E_2}((e_1', \alpha), (e_2', \alpha)) = min\{\hat{T}_{E_1}(e_1', e_2'), \hat{T}_{V_2}(\alpha)\}$$
$$\hat{I}_{E_1 \times E_2}((e_1', \alpha), (e_2', \alpha)) = max\{\hat{I}_{E_1}(e_1', e_2'), \hat{I}_{V_2}(\alpha)\}$$
$$\hat{F}_{E_1 \times E_2}((e_1', \alpha), (e_2', \alpha)) = max\{\hat{F}_{E_1}(e_1', e_2'), \hat{F}_{V_2}(\alpha)\}$$

$$\forall \alpha \in V_2, (e_1', e_2') \in E_1$$

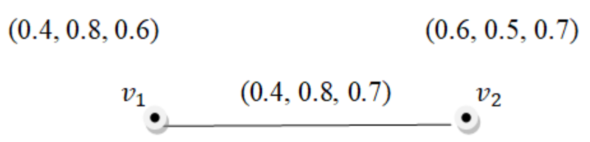
4.2 Composition of Neutrosophic Graphs

- The composition of two NGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a new NG $G = G_1 \circ G_2$,defined as follows: $G_1 \circ G_2 = (V_1 \circ V_2, E_1 \circ E_2)$ where

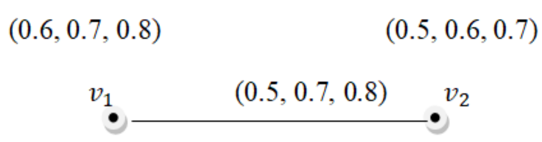
$$\hat{T}_{V_1 \circ V_2}(v_1, v_2) = min\{\hat{T}_{V_1}(v_1), \hat{T}_{V_2}(v_2)\}$$
$$\hat{I}_{V_1 \circ V_2}(v_1, v_2) = max\{\hat{I}_{V_1}(v_1), \hat{I}_{V_2}(v_2)\}$$
$$\hat{F}_{V_1 \circ V_2}(v_1, v_2) = min\{\hat{F}_{V_1}(v_1), \hat{F}_{V_2}(v_2)\}$$
$$\forall (v_1, v_2) \in (V_1, V_2)$$
$$\hat{T}_{E_1 \circ E_2}((\alpha, e_1), (\alpha, e_2)) = min\{\hat{T}_{V_1}(\alpha), \hat{T}_{E_2}(e_1, e_2)\}$$
$$\hat{I}_{E_1 \circ E_2}((\alpha, e_1), (\alpha, e_2)) = max\{\hat{I}_{V_1}(\alpha), \hat{I}_{E_2}(e_1, e_2)\}$$
$$\hat{F}_{E_1 \circ E_2}((\alpha, e_1), (\alpha, e_2)) = max\{\hat{F}_{V_1}(\alpha), \hat{F}_{E_2}(e_1, e_2)\}$$
$$\forall \alpha \in V_1, (e_1, e_2) \in E_2$$
$$\hat{T}_{R_1 \circ R_2}((u_1, \gamma), (v_1, \gamma)) = min\{\hat{T}_{R_1}(u_1, v_1), \hat{T}_{P_2}(\gamma)\}$$
$$\hat{I}_{R_1 \circ R_2}((u_1, \gamma), (v_1, \gamma)) = min\{\hat{I}_{R_1}(u_1, v_1), \hat{I}_{P_2}(\gamma)\}$$
$$\hat{F}_{R_1 \circ R_2}((u_1, \gamma), (v_1, \gamma)) = min\{\hat{F}_{R_1}(u_1, v_1), \hat{F}_{P_2}(\gamma)\}$$
$$\forall \gamma \in V_2, (u_1, v_1) \in E_1$$
$$\hat{T}_{R_1 \circ R_2}((u_1, u_2), (v_1, v_2)) = min\{\hat{T}_{P_2}(u_2), \hat{T}_{P_2}(v_2), \hat{T}_{R_1}(u_1, v_1)\}$$
$$\hat{I}_{R_1 \circ R_2}((u_1, u_2), (v_1, v_2)) = max\{\hat{I}_{P_2}(u_2), \hat{I}_{P_2}(v_2), \hat{I}_{R_1}(u_1, v_1)\}$$
$$\hat{F}_{R_1 \circ R_2}((u_1, u_2), (v_1, v_2)) = max\{\hat{F}_{P_2}(u_2), \hat{F}_{P_2}(v_2), \hat{F}_{R_1}(u_1, v_1)\}$$
$$\forall (u_1, u_2), (v_1, v_2) \in E_1$$

Example 4.2.1

Consider two NGs G_1 & G_2, as presented below.



G_1



G_2

Then the composition of $G_1 \circ G_2$, is shown graphically in Figure 1.

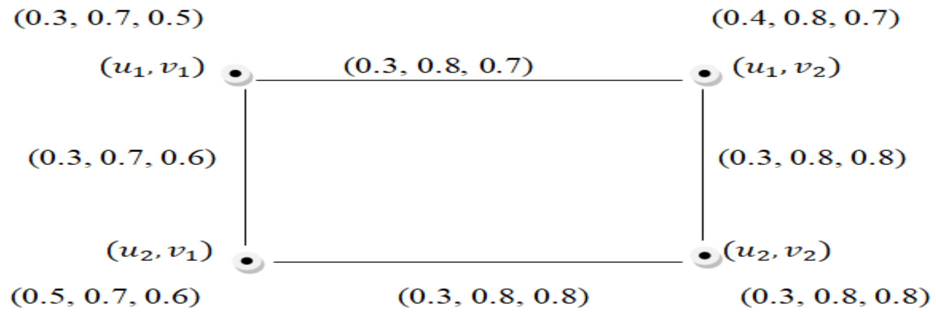


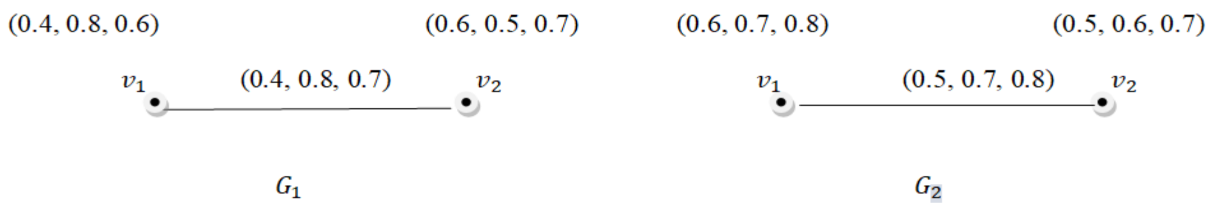
Figure 1. Composition of two NGs

4.3 The lexicographic product

The lexicographic product of two NGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a new NG$G = G_1 \cdot G_2$,defined as follows: $G_1 \cdot G_2 = (V_1 \cdot V_2, E_1 \cdot E_2)$ where

$$\hat{T}_{V_1 \cdot V_2}(v_1, v_2) = min\{\hat{T}_{V_1}(v_1), \hat{T}_{V_2}(v_2)\}$$
$$\hat{I}_{V_1 \cdot V_2}(v_1, v_2) = max\{\hat{I}_{V_1}(v_1), \hat{I}_{V_2}(v_2)\}$$
$$\hat{F}_{V_1 \cdot V_2}(v_1, v_2) = max\{\hat{F}_{V_1}(v_1), \hat{F}_{V_2}(v_2)\}$$
$$\forall (v_1, v_2) \in (V_1, V_2)$$
$$\hat{T}_{E_1 \cdot E_2}((\beta, e_1), (\beta, e_2)) = min\{\hat{T}_{V_1}(\beta), \hat{T}_{E_2}(e_1, e_2)\}$$
$$\hat{I}_{E_1 \cdot E_2}((\beta, e_1), (\beta, e_2)) = max\{\hat{I}_{V_1}(\beta), \hat{I}_{E_2}(e_1, e_2)\}$$
$$\hat{F}_{E_1 \cdot E_2}((\beta, e_1), (\beta, e_2)) = max\{\hat{F}_{V_1}(\beta), \hat{F}_{E_2}(e_1, e_2)\}$$
$$\forall \beta \in V_1, (e_1, e_2) \in E_2$$
$$\hat{T}_{E_1 \cdot E_2}((v_1, v_2), (e_1, e_2)) = min\{\hat{T}_{V_2}(v_2), \hat{T}_{V_2}(e_2), \hat{T}_{E_1}(v_1, e_1)\}$$
$$\hat{I}_{E_1 \cdot E_2}((v_1, v_2), (e_1, e_2)) = max\{\hat{I}_{V_2}(u_2), \hat{I}_{V_2}(v_2), \hat{I}_{E_1}(v_1, e_1)\}$$
$$\hat{F}_{E_1 \cdot E_2}((v_1, v_2), (e_1, e_2)) = max\{\hat{F}_{V_2}(u_2), \hat{F}_{V_2}(v_2), \hat{F}_{E_1}(v_1, e_1)\}$$
$$\forall (v_1, v_2) \in E_1, (e_1, e_2) \in E_1$$

Example 4.3.1
Consider two NGs G_1 & G_2, as presented below.



Then the lexicographic product of G_1 & G_2 *is denoted by* $G_1 \cdot G_2$, is shown graphically in Figure 2.

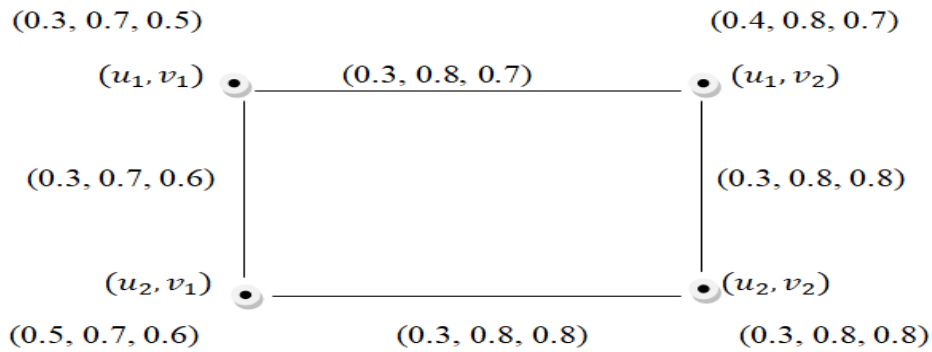


Figure 2. Lexicographic product of two NGs

4.4 Union of NGs

The union of two NGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a new NG $G = G_1 \cup G_2$, where the vertex set is the union of the vertex sets of G_1 and G_2, and the edge set is the union of the edge sets of G_1 and G_2., is defined as follows: $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

where

$$\hat{T}_{V_1 \cup V_2}(v) = \begin{cases} \hat{T}_{V_1}(v)\ if\ v \in V_1 - V_2 \\ \hat{T}_{V_2}(v)\ if\ v \in V_2 - V_1 \\ max\{\hat{T}_{V_1}(v_1), \hat{T}_{V_2}(v_2)\}\ if\ v \in V_1 \cup V_2 \end{cases}$$

$$\hat{I}_{V_1 \cup V_2}(v) = \begin{cases} \hat{I}_{V_1}(v)\ if\ v \in V_1 - V_2 \\ \hat{I}_{V_2}(v)\ if\ v \in V_2 - V_1 \\ min\{\hat{I}_{V_1}(v_1), \hat{I}_{V_2}(v_2)\}\ if\ v \in V_1 \cup V_2 \end{cases}$$

$$\hat{F}_{V_1 \cup V_2}(v) = \begin{cases} \hat{F}_{V_1}(v)\ if\ v \in V_1 - V_2 \\ \hat{F}_{V_2}(v)\ if\ v \in V_2 - V_1 \\ min\{\hat{F}_{V_1}(v_1), \hat{F}_{V_2}(v_2)\}\ if\ v \in V_1 \cup V_2 \end{cases}$$

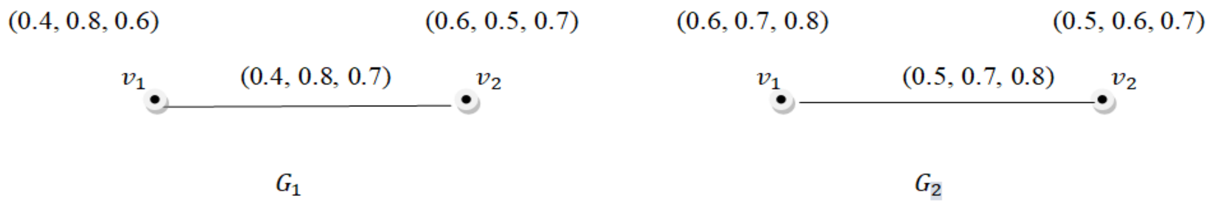
$$\hat{T}_{V_1 \cup V_2}(v_1, v_2) = \begin{cases} \hat{T}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{T}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ max\{\hat{T}_{E_1}(v_1, v_2), \hat{T}_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

$$\hat{I}_{V_1 \cup V_2}(v_1, v_2) = \begin{cases} \hat{I}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{I}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ min\{\hat{I}_{E_1}(v_1, v_2), \hat{I}_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

$$\hat{F}_{V_1 \cup V_2}(v_1, v_2) = \begin{cases} \hat{F}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{F}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ min\{\hat{F}_{E_1}(v_1, v_2), F_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

Example 4.4.1

Consider two NGs G_1 and G_2, as presented below.



Then the union of G_1 *and* G_2, is denoted by $G_1 \cup G_2$ and is shown graphically in Figure 3. below. Neutrosophic graphs

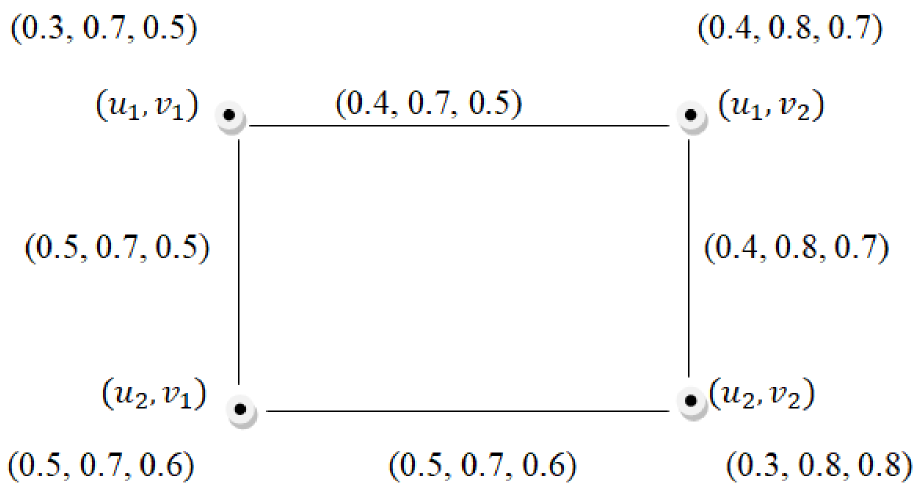


Figure 3. Union of two NGs

4.5 Intersection of NGs

The intersection of two NGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a new NG $G = G_1 \cap G_2$, where the vertex set is the intersection of the vertex sets of G_1 and G_2, and the edge set is the intersection of the edge sets of G_1 and G_2., is defined as follows: $G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2)$

where

$$\hat{T}_{V_1 \cap V_2}(v) = \begin{cases} \hat{T}_{V_1}(v) \text{ } if \text{ } v \in V_1 - V_2 \\ \hat{T}_{V_2}(v) \text{ } if \text{ } v \in V_2 - V_1 \\ min\{\hat{T}_{V_1}(v_1), \hat{T}_{V_2}(v_2)\} \text{ } if \text{ } v \in V_1 \cup V_2 \end{cases}$$

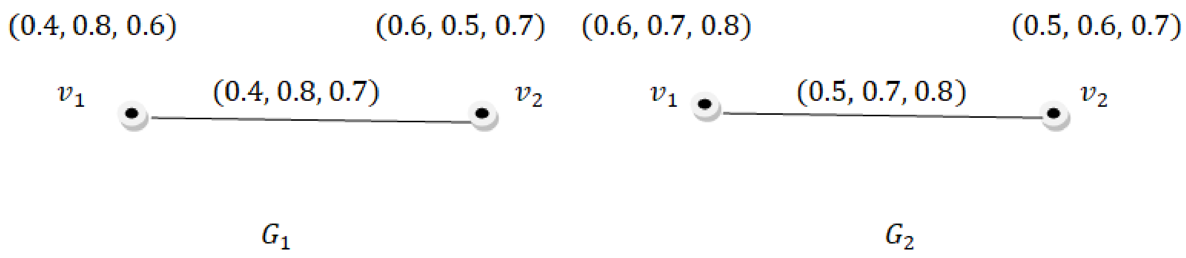
$$\hat{I}_{V_1 \cap V_2}(v) = \begin{cases} \hat{I}_{V_1}(v) \text{ } if \text{ } v \in V_1 - V_2 \\ \hat{I}_{V_2}(v) \text{ } if \text{ } v \in V_2 - V_1 \\ max\{\hat{I}_{V_1}(v_1), \hat{I}_{V_2}(v_2)\} \text{ } if \text{ } v \in V_1 \cup V_2 \end{cases}$$

$$\hat{F}_{V_1 \cap V_2}(v) = \begin{cases} \hat{F}_{V_1}(v) \text{ } if \text{ } v \in V_1 - V_2 \\ \hat{F}_{V_2}(v) \text{ } if \text{ } v \in V_2 - V_1 \\ max\{\hat{F}_{V_1}(v_1), \hat{F}_{V_2}(v_2)\} \text{ } if \text{ } v \in V_1 \cup V_2 \end{cases}$$

$$\hat{T}_{V_1 \cap V_2}(v_1, v_2) = \begin{cases} \hat{T}_{E_1}(v_1, v_2) \text{ } if \text{ } (v_1, v_2) \in E_1 - E_2 \\ \hat{T}_{E_2}(v_1, v_2) \text{ } if \text{ } (v_1, v_2) \in E_2 - E_1 \\ min\{\hat{T}_{E_1}(v_1, v_2), \hat{T}_{E_2}(v_1, v_2)\} \text{ } if \text{ } (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

$$\hat{I}_{V_1\cap V_2}(v_1,v_2)=\begin{cases}\hat{I}_{E_1}(v_1,v_2)\ if\ (v_1,v_2)\ \in\ E_1-E_2\\ \hat{I}_{E_2}(v_1,v_2)\ if\ (v_1,v_2)\ \in\ E_2-E_1\\ max\{\hat{I}_{E_1}(v_1,v_2),\hat{I}_{E_2}(v_1,v_2)\}\ if\ (v_1,v_2)\ \in\ E_1\cup E_2\end{cases}$$
$$\hat{F}_{V_1\cap V_2}(v_1,v_2)=\begin{cases}\hat{F}_{E_1}(v_1,v_2)\ if\ (v_1,v_2)\ \in\ E_1-E_2\\ \hat{F}_{E_2}(v_1,v_2)\ if\ (v_1,v_2)\ \in\ E_2-E_1\\ max\{\hat{F}_{E_1}(v_1,v_2),F_{E_2}(v_1,v_2)\}\ if\ (v_1,v_2)\ \in\ E_1\cup E_2\end{cases}$$

Example 4.5.1
Consider two NGsG_1 and G_2, as presented below.



Then the intersection of two graphs mis denioted by G_1 $\cap$ G_2, and is e shown graphically in Figuren4.

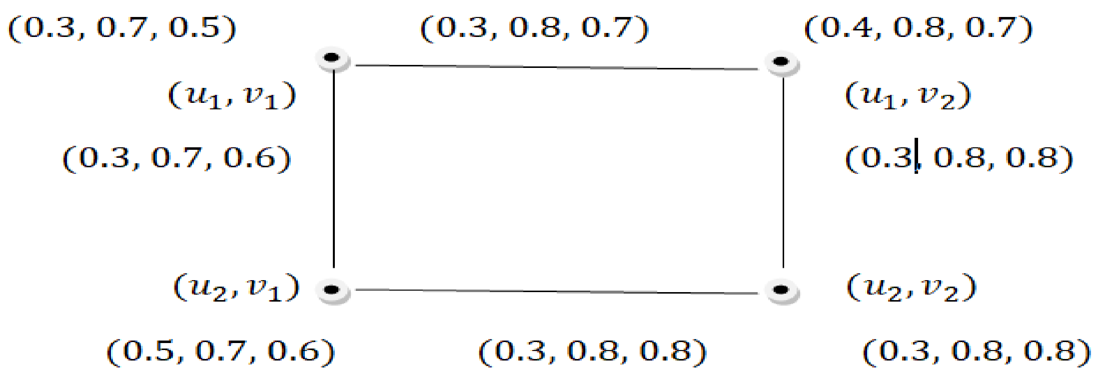


Figure 4. Intersection of two NGs

4.6 Join of NGs

The join of two NGs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a new NG $G=G_1+G_2$, where the vertex set and the edge set is G_1 and G_2 , denoted by G_1 $+G_2$, is defined as follows: $G_1+G_2=(V_1+\ V_2\ ,E_1+E_2\ )$ where

$$\hat{T}_{V_1+V_2}(v)=\begin{cases}\hat{T}_{V_1}(v)\ if\ v\ \in\ V_1-V_2\\ \hat{T}_{V_2}(v)\ if\ v\ \in\ V_1-V_2\\ min\{\hat{T}_{V_1}(v),\hat{T}_{V_2}(v)\}\ if\ v\in\ V_1\cup V_2\end{cases}$$
$$\hat{I}_{V_1+V_2}(v)=\begin{cases}\hat{I}_{V_1}(v)\ if\ v\ \in\ V_1-V_2\\ \hat{I}_{V_2}(v)\ if\ v\ \in\ V_1-V_2\\ max\{\hat{I}_{V_1}(v),\hat{I}_{V_2}(v)\}\ if\ v\in\ V_1\cup V_2\end{cases}$$

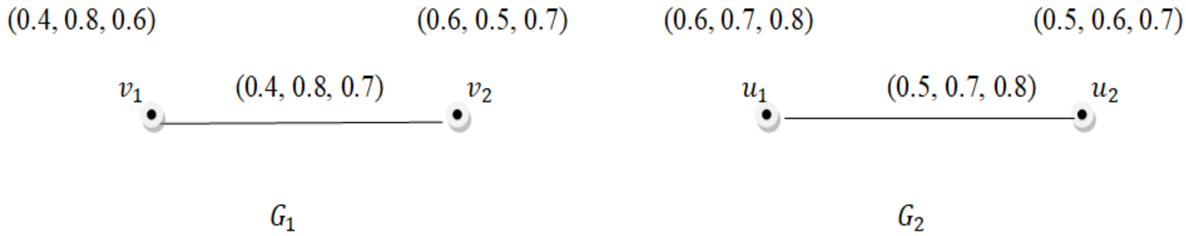
$$\hat{F}_{V_1+V_2}(v) = \begin{cases} \hat{F}_{V_1}(v)\ if\ v \in V_1 - V_2 \\ \hat{F}_{V_2}(v)\ if\ v \in V_1 - V_2 \\ max\{\hat{F}_{V_1}(v), \hat{F}_{V_2}(v)\}\ if\ v \in V_1 \cup V_2 \end{cases}$$

$$\hat{T}_{V_1+V_2}(v_1, v_2) = \begin{cases} \hat{T}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{T}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ min\{\hat{T}_{E_1}(v_1, v_2), \hat{T}_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

$$\hat{I}_{V_1+V_2}(v_1, v_2) = \begin{cases} \hat{I}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{I}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ max\{\hat{I}_{E_1}(v_1, v_2), \hat{I}_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

$$\hat{F}_{V_1+V_2}(v_1, v_2) = \begin{cases} \hat{F}_{E_1}(v_1, v_2)\ if\ (v_1, v_2) \in E_1 - E_2 \\ \hat{F}_{E_2}(v_1, v_2)\ if\ (v_1, v_2) \in E_2 - E_1 \\ max\{\hat{F}_{E_1}(v_1, v_2), \hat{F}_{E_2}(v_1, v_2)\}\ if\ (v_1, v_2) \in E_1 \cup E_2 \end{cases}$$

Example 4.6.1
Consider two NGs G_1 *and* G_2, as presented below.



Then the join of G_1 *and* G_2 *is deoted by* $G_1 + G_2$, and is shown graphically in Figure 5.

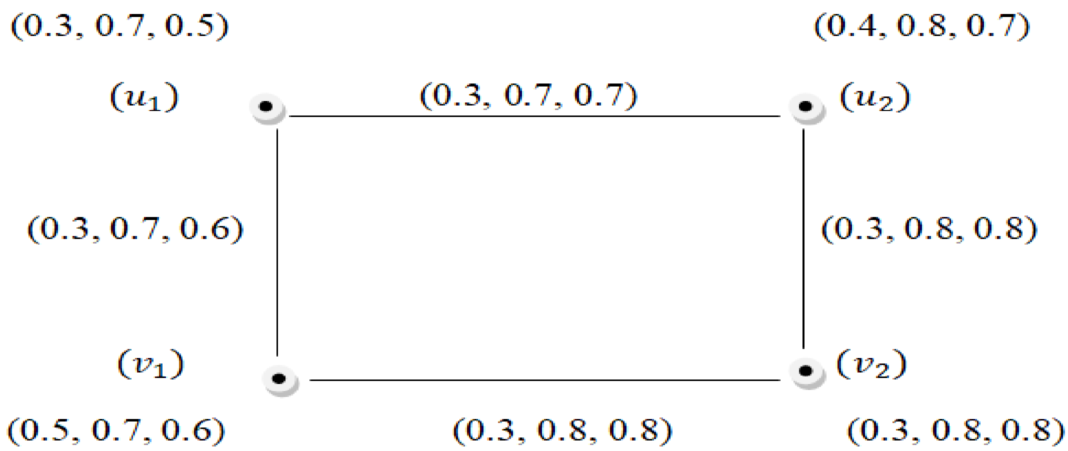


Figure 5. Join of two NGs

5. Example illustrating the use of NGs in decision-making.

Below is a numerical illustration that demonstrates the use of NGs in the decision-making process:

Suppose an organization is contemplating three potential sites ($A, B, and\ C$) for establishing a new branch. The decision-makers have established five criteria ($C_1, C_2, C_3, C_4, and\ C_5$) that will serve as guidelines for their decision-making process. These factors include market potential, transportation infrastructure, labor availability, operating expenses, and environmental restrictions.

We can model this decision-making problem using a neutrosophic graph, where we represent the locations as vertices and the criteria as edges connecting the vertices. The membership values associated with each edge will represent the level of truth, indeterminacy, and falsehood in relation to each location's eligibility for the applicable criterion.

Suppose that the edges have the following membership values:

$$T(A, C_1) = 0.7, I(A, C_1) = 0.2, F(A, C_1) = 0.1$$
$$T(A, C_2) = 0.6, I(A, C_2) = 0.3, F(A, C_2) = 0.1$$
$$T(A, C_3) = 0.8, I(A, C_3) = 0.1, F(A, C_3) = 0.1$$
$$T(A, C_4) = 0.7, I(A, C_4) = 0.2, F(A, C_4) = 0.1$$
$$T(A, C_5) = 0.6, I(A, C_5) = 0.3, F(A, C_5) = 0.1$$

$$T(B, C_1) = 0.5, I(B, C_1) = 0.4, F(B, C_1) = 0.1$$
$$T(B, C_2) = 0.7, I(B, C_2) = 0.2, F(B, C_2) = 0.1$$
$$T(B, C_3) = 0.6, I(B, C_3) = 0.3, F(B, C_3) = 0.1$$
$$T(B, C_4) = 0.8, I(B, C_4) = 0.1, F(B, C_4) = 0.1$$
$$T(B, C_5) = 0.7, I(B, C_5) = 0.2, F(B, C_5) = 0.1$$

$$T(C, C_1) = 0.6, I(C, C_1) = 0.3, F(C, C_1) = 0.1$$
$$T(C, C_2) = 0.5, I(C, C_2) = 0.4, F(C, C_2) = 0.1$$
$$T(C, C_3) = 0.7, I(C, C_3) = 0.2, F(C, C_3) = 0.1$$
$$T(C, C_4) = 0.6, I(C, C_4) = 0.3, F(C, C_4) = 0.1$$
$$T(C, C_5) = 0.8, I(C, C_5) = 0.1, F(C, C_5) = 0.1$$

To make a decision, decision-makers can examine membership values and use various decision-making strategies, such as aggregating membership values or ranking each criterion based on importance.

For example, if all criteria are equally relevant, decision-makers could calculate the total score for each site by adding the truth membership values and subtracting the indeterminacy and falsity membership values.

$$Score(A) = 0.7 + 0.6 + 0.8 + 0.7 + 0.6 - (0.2 + 0.3 + 0.1 + 0.2 + 0.3) - (0.1 + 0.1 + 0.1 + 0.1 + 0.1) = 2.4$$
$$Score(B) = 0.5 + 0.7 + 0.6 + 0.8 + 0.7 - (0.4 + 0.2 + 0.3 + 0.1 + 0.2) - (0.1 + 0.1 + 0.1 + 0.1 + 0.1) = 2.2$$
$$Score(C) = 0.6 + 0.5 + 0.7 + 0.6 + 0.8 - (0.3 + 0.4 + 0.2 + 0.3 + 0.1) - (0.1 + 0.1 + 0.1 + 0.1 + 0.1) = 2.0$$

Based on the overall scores, Location A gets the highest score of 2.4, suggesting it may be the best option based on all criteria and membership values.

However, depending on their preferences and the decision-making situation, decision-makers may also consider other aspects, such as the relative relevance of each criterion or other aggregation methods.

This example demonstrates neutrosophic graphs' ability to simulate decision-making scenarios involving multiple criteria and ambiguity or uncertainty in the available data. Edge membership values enable a more sophisticated decision-making process by representing each alternative's suitability to the criterion.

6. Conclusion:

In conclusion, NGs offer a powerful and versatile framework for representing and analyzing complex systems and networks, particularly those involving uncertainty and indeterminacy. Neutrosophic graph theory could make big contributions to many fields if more research is done, theory is improved, and people from different fields work together. This would allow for more accurate and complete modeling of real-life situations and help people make better decisions. Moreover, real-world case studies and applications of neutrosophic graphs in various domains are essential to validate their effectiveness and demonstrate their practical value. These practical applications can also provide insights and feedback to refine and extend the theoretical foundations of neutrosophic graph theory in extended version.

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