Neutrosophic Crisp Set Theory

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Abstract. The purpose of this paper is to introduce new types of neutrosophic crisp sets with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp point and neutrosophic crisp relations. Possible applications to database are touched upon.

Keywords: Neutrosophic Set, Neutrosophic Crisp Sets; Neutrosophic Crisp Relations; Generalized Neutrosophic Sets; Intuitionistic Neutrosophic Sets.

1 Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [16, 17, 18] and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 19,20, 21], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 4, 23] such as a neutrosophic set theory. In this paper we introduce new types of neutrosophic crisp set. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp points and relation between two new neutrosophic crisp notions. Finally, we introduce and study the notion of neutrosophic crisp relations.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [7, 11, 12, 20]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^-0, 1^+\right[$ is nonstandard unit interval.

Definition 2.1 [7]

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$,

Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

ϕ_N may be defined as four types:

i) Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii) Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv) Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

1) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

iii) Type3: $X_N = \langle X, X, \phi \rangle$,

iv) Type4: $X_N = \langle X, X, X \rangle$,

Definition 2.2 [6, 7]

Let $A = \langle A_1, A_2, A_3 \rangle$ a NCS on X, then the complement of the set A (A^c, for short) may be defined as three kinds

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

Definition 2.3 [6, 7]

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$)

($A \subseteq B$) may be defined as two types:

1) Type1: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or
2) Type2: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$.

Definition 2.5 [6, 7]

Let X be a non-empty set, and NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ are NCSS Then

1) $A \bigcap B$ may be defined as two types:
   i. Type1: $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or
   ii. Type2: $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$
2) $A \bigcup B$ may be defined as two types:
   i) Type1: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or
   ii) Type2: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$.

3 Some Types of Neutrosophic Crisp Sets

We shall now consider some possible definitions for some types of neutrosophic crisp sets

Definition 3.1

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) **(Neutrosophic Crisp Set with Type 1)** If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$. (NCS-Type1 for short).
2) **(Neutrosophic Crisp Set with Type 2)** If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type2 for short).
3) **(Neutrosophic Crisp Set with Type 3)** If satisfying $A_1 \cap A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type3 for short).

Definition 3.3

1) **(Neutrosophic Set [9, 16, 17]):** Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^+$ and $0^- \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+$.

2) **(Generalized Neutrosophic Set [8]):** Let X be a non-empty fixed set. A generalized neutrosophic (GNS for short) set A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^+$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \leq 0.5$ and $0^- \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+$.

3) **(Intuitionistic Neutrosophic Set [22]).** Let X be a non-empty fixed set. An intuitionistic neutrosophic set A (INS for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0.5 \leq \mu_A(x), \sigma_A(x), \nu_A(x)$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \leq 0.5$, $\mu_A(x) \wedge \nu_A(x) \leq 0.5$, $\sigma_A(x) \wedge \nu_A(x) \leq 0.5$, and ${}^-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 2^+$. A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.

Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously on NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS ⟶ Generalized NS ⟶ NS

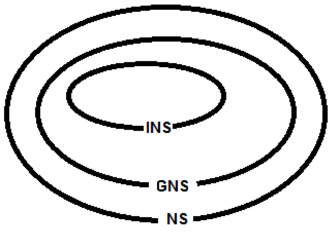

Fig. 1. Represents the relation between types of NS

Corollary 3.1

Let X non-empty fixed set and $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be INS on X Then:

1) Type1- A^c of INS be a GNS.
2) Type2- A^c of INS be a INS.
3) Type3- A^c of INS be a GNS.

Proof

Since A INS then $\mu_A(x), \sigma_A(x), \nu_A(x)$, and

$\mu_A(x) \wedge \sigma_A(x) \le 0.5, \nu_A(x) \wedge \mu_A(x) \le 0.5$

$\nu_A(x) \wedge \sigma_A(x) \le 0.5$ Implies

$\mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x) \le 0.5$ then is not to be Type1- A^c INS. On other hand the Type 2- A^c,

$A^c = \langle \nu_A(x), \sigma_A(x), \mu_A(x) \rangle$ be INS and Type3- A^c,

$A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ and $\sigma^c{}_A(x) \le 0.5$ implies to

$A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ GNS and not to be INS

Example 3.1

Let $X = \{a, b, c\}$, and A, B, C are neutrosophic sets on X, $A = \langle (0.7,0.9,0.8) \setminus a, (0.6,0.7,0.6) \setminus b, (0.9,0.7,0.8 \setminus c \rangle$,

$B = \langle (0.7,0.9,0.5) \setminus a, (0.6,0.4,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$

$C = \langle (0.7,0.9,0.5) \setminus a, (0.6,0.8,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$ By the Definition 3.3 no.3 $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \ge 0.5$, A be not GNS and INS,

$B = \langle (0.7,0.9,0.5) \setminus a, (0.6,0.4,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$ not INS, where $\sigma_A(b) = 0.4 < 0.5$. Since

$\mu_B(x) \wedge \sigma_B(x) \wedge \nu_B(x) \le 0.5$ then B is a GNS but not INS.

$A^c = \langle (0.3,0.1,0.2) \setminus a, (0.4,0.3,0.4) \setminus b, (0.1,0.3,0.2 \setminus c \rangle$

Be a GNS, but not INS.

$B^c = \langle (0.3,0.1,0.5) \setminus a, (0.4,0.6,0.5) \setminus b, (0.1,0.5,0.2 \setminus c \rangle$

Be a GNS, but not INS, C be INS and GNS,

$C^c = \langle (0.3,0.1,0.5) \setminus a, (0.4,0.2,0.5) \setminus b, (0.1,0.5,0.2 \setminus c \rangle$

Be a GNS but not INS.

Definition 3.2

A NCS-Type1 ϕ_{N_1}, X_{N_1} in X as follows:

1) ϕ_{N1} may be defined as three types:
   i) Type1: $\phi_{N_1} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N_1} = \langle \phi, X, \phi \rangle$, or
   iii) Type3: $\phi_N = \langle \phi, \phi, \phi \rangle$.
2) X_{N1} may be defined as one type
   Type1: $X_{N_1} = \langle X, \phi, \phi \rangle$.

Definition 3.3

A NCS-Type2, ϕ_{N_2}, X_{N2} in X as follows:

1) ϕ_{N2} may be defined as two types:
   i) Type1: $\phi_{N_2} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N_2} = \langle \phi, X, \phi \rangle$
2) X_{N2} may be defined as one type
   Type1: $X_{N_2} = \langle X, \phi, \phi \rangle$

Definition 3.4

A NCS-Type 3, ϕ_{N3}, X_{N3} in X as follows:

1) ϕ_{N3} may be defined as three types:
   i) Type1: $\phi_{N3} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N3} = \langle \phi, X, \phi \rangle$, or
   iii) Type3: $\phi_{N3} = \langle \phi, X, X \rangle$.
2) X_{N3} may be defined as three types
   i) Type1: $X_{N3} = \langle X, \phi, \phi \rangle$,
   ii) Type2: $X_{N3} = \langle X, X, \phi \rangle$,
   iii) Type3: $X_{N3} = \langle X, \phi, X \rangle$,

Corollary 3.2

In general

1- Every NCS-Type 1, 2, 3 are NCS.
2- Every NCS-Type 1 not to be NCS-Type2, 3.
3- Every NCS-Type 2 not to be NCS-Type1, 3.
4- Every NCS-Type 3 not to be NCS-Type2, 1, 2.
5- Every crisp set be NCS.

The following Venn diagram represents the relation between NCSs

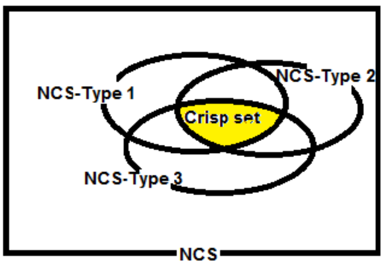

Fig 1. Venn diagram represents the relation between NCSs

Example 3.2

Let $X = \{a, b, c, d, e, f\}$, $A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$, $D = \langle \{a, b\}, \{e, c\}, \{f, d\} \rangle$ be a NCS-Type 2,

$B=\langle\{a,b,c\},\{d\},\{e\}\rangle$ be a NCT-Type1 but not NCS-Type 2, 3. $C=\langle\{a,b\},\{c,d\},\{e,f,a\}\rangle$ be a NCS-Type 3.but not NCS-Type1, 2.

Definition 3.5

Let X be a non-empty set, $A=\langle A_1,A_2,A_3\rangle$

1)If A be a NCS-Type1 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement Type1: $A^c=\langle A_3,A_2,A_1\rangle$.

2) If A be a NCS-Type 2 on X, then the complement of the set A (A^c, for short) may be defined as one kind of complement $A^c=\langle A_3,A_2,A_1\rangle$.

3) If A be NCS-Type3 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement defined as three kinds of complements

(C_1) Type1: $A^c=\langle A^c{}_1,A^c{}_2,A^c{}_3\rangle$,

(C_2) Type2: $A^c=\langle A_3,A_2,A_1\rangle$

(C_3) Type3: $A^c=\langle A_3,A^c{}_2,A_1\rangle$

Example 3.3

Let $X=\{a,b,c,d,e,f\}$, $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$ be a NCS-Type 2, $B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$ be a NCS-Type1., $C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ NCS-Type 3, then the complement $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$,

$A^c=\langle\{f\},\{e\},\{a,b,c,d\}\rangle$ NCS-Type 2, the complement of $B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$, $B^c=\langle\{d,e\},\{\phi\},\{a,b,c\}\rangle$ NCS-Type1. The complement of $C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ may be defined as three types:

Type 1: $C^c=\langle\{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$.

Type 2: $C^c=\langle\{e,f\},\{a,b,e,f\},\{a,b\}\rangle$,

Type 3: $C^c=\langle\{e,f\},\{c,d\},\{a,b\}\rangle$,

Proposition 3.1

Let $\{A_j : j\in J\}$ be arbitrary family of neutrosophic crisp subsets on X, then

1) $\cap A_j$ may be defined two types as :

i) Type1: $\cap A_j=\langle\cap Aj_1,\cap A_{j_2},\cup A_{j_3}\rangle$,or

ii) Type2: $\cap A_j=\langle\cap Aj_1,\cup A_{j_2},\cup A_{j_3}\rangle$.

2) $\cup A_j$ may be defined two types as :

1) Type1: $\cup A_j=\langle\cup Aj_1,\cap A_{j_2},\cap A_{j_3}\rangle$ or

2) Type2: $\cup A_j=\langle\cup Aj_1,\cup A_{j_2},\cap A_{j_3}\rangle$.

Definition 3.6

(a) If $B=\langle B_1,B_2,B_3\rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B)=\langle f^{-1}(B_1),f^{-1}(B_2),f^{-1}(B_3)\rangle$.

(b) If $A=\langle A_1,A_2,A_3\rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A)=\langle f(A_1),f(A_2),f(A_3)^c)\rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.3

Let A, $\{A_i : i\in J\}$, be a family of NCS in X, and B, $\{B_j : j\in K\}$ NCS in Y, and $f : X\to Y$ a function. Then

(a) $A_1\subseteq A_2 \Leftrightarrow f(A_1)\subseteq f(A_2)$,
$B_1\subseteq B_2 \Leftrightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$,

(b) $A\subseteq f^{-1}(f(A))$ and if f is injective, then $A=f^{-1}(f(A))$,

(c) $f^{-1}(f(B))\subseteq B$ and if f is surjective, then $f^{-1}(f(B))=B$,

(d) $f^{-1}(\cup B_i)=f^{-1}(B_i)$, $f^{-1}(\cap B_i)=\cap f^{-1}(B_i)$,

(e) $f(\cup A_{i_i})=\cup f(A_{ii})$; $f(\cap A_{i_i})\subseteq\cap f(A_{ii})$; and if f is injective, then $f(\cap A_{i_i})=\cap f(A_{ii})$;

(f) $f^{-1}(Y_N)=X_N$, $f^{-1}(\phi_N)=\phi_N$.

(g) $f(\phi_N)=\phi_N$, $f(X_N)=Y_N$, if f is subjective.

Proof

Obvious

4 Neutrosophic Crisp Points

One can easily define a nature neutrosophic crisp set in X, called "neutrosophic crisp point" in X, corresponding to an element X:

Definition 4.1

Let $A=\langle A_1,A_2,A_3\rangle$,be a neutrosophic crisp set on a set X, then $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, $p_1\neq p_2\neq p_3\in X$ is called a neutrosophic crisp point on A.

A NCP $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, is said to be belong to a neutrosophic crisp set $A=\langle A_1,A_2,A_3\rangle$, of X, denoted by $p\in A$, if may be defined by two types

Type 1: $\{p_1\}\subseteq A_1,\{p_2\}\subseteq A_2$ and $\{p_3\}\subseteq A_3$ or

Type 2: $\{p_1\}\subseteq A_1,\{p_2\}\supseteq A_2$ and $\{p_3\}\subseteq A_3$

Theorem 4.1

Let $A=\langle A_1,A_2,A_3\rangle$ and $B=\langle B_1,B_2,B_3\rangle$, be neutrosophic crisp subsets of X. Then $A\subseteq B$ iff $p\in A$ implies $p\in B$ for any neutrosophic crisp point p in X.

Proof

Let $A\subseteq B$ and $p\in A$, Type 1: $\{p_1\}\subseteq A_1,\{p_2\}\subseteq A_2$ and $\{p_3\}\subseteq A_3$ or Type 2: $\{p_1\}\subseteq A_1,\{p_2\}\supseteq A_2$ and $\{p_3\}\subseteq A_3$ Thus $p\in B$. Conversely, take any point in X. Let $p_1\in A_1$ and $p_2\in A_2$ and $p_3\in A_3$. Then p is a neutrosophic crisp point in X. and $p\in A$. By the hypothesis $p\in B$. Thus $p_1\in B_1$ or Type1: $\{p_1\}\subseteq B_1,\{p_2\}\subseteq B_2$ and $\{p_3\}\subseteq B_3$ or Type 2: $\{p_1\}\subseteq B_1,\{p_2\}\supseteq B_2$ and $\{p_3\}\subseteq B_3$. Hence $A\subseteq B$.

Theorem 4.2

Let $A=\langle A_1,A_2,A_3\rangle$, be a neutrosophic crisp subset of X. Then $A=\cup\{p: p\in A\}$.

Proof

Obvious

Proposition 4.1

Let $\{A_j: j\in J\}$ is a family of NCSs in X. Then

(a_1) $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle\in\bigcap_{j\in J}A_j$ iff $p\in A_j$ for each $j\in J$.

(a_2) $p\in\bigcup_{j\in J}A_j$ iff $\exists j\in J$ such that $p\in A_j$.

Proposition 4.2

Let $A=\langle A_1,A_2,A_3\rangle$ and $B=\langle B_1,B_2,B_3\rangle$ be two neutrosophic crisp sets in X. Then $A\subseteq B$ iff for each p we have $p\in A\Leftrightarrow p\in B$ and for each p we have $p\in A\Rightarrow p\in B$. iff $A=B$ for each p we have $p\in A\Rightarrow p\in B$ and for each p we have $p\in A\Leftrightarrow p\in B$.

Proposition 4.3

Let $A=\langle A_1,A_2,A_3\rangle$ be a neutrosophic crisp set in X. Then $A=\cup<\{p_1: p_1\in A_1\},\{p_2: p_2\in A_2\},\{p_3: p_3\in A_3\}$.

Definition 4.2

Let $f: X\to Y$ be a function and p be a neutrosophic crisp point in X. Then the image of p under f, denoted by $f(p)$, is defined by $f(p)=\langle\{q_1\},\{q_2\},\{q_3\}\rangle$, where $q_1=f(p_1), q_2=f(p_2)$ and $q_3=f(p_3)$. It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p)=q$, where $q=f(p)$, and it is exactly the same meaning of the image of a NCP under the function f.

Definition 4.3

Let X be a nonempty set and $p\in X$. Then the neutrosophic crisp point p_N defined by $p_N=\langle\{p\},\phi,\{p\}^c\rangle$ is called a neutrosophic crisp point (NCP for short) in X, where NCP is a triple ({only element in X}, empty set,{the complement of the same element in X }). Neutrosophic crisp points in X can sometimes be inconvenient when express neutrosophic crisp set in X in terms of neutrosophic crisp points. This situation will occur if $A=\langle A_1,A_2,A_3\rangle$ NCS-Type1, $p\notin A_1$. Therefore we shall define "vanishing" neutrosophic crisp points as follows:

Definition 4.4

Let X be a nonempty set and $p\in X$ a fixed element in X Then the neutrosophic crisp set $p_{N_N}=\langle\phi,\{p\},\{p\}^c\rangle$ is called vanishing" neutrosophic crisp point (VNCP for short) in X. where VNCP is a triple (empty set,{only element in X}, { the complement of the same element in X}).

Example 4.1

Let $X=\{a,b,c,d\}$ and $p=b\in X$. Then $p_N=\langle\{b\},\phi,\{a,c,d\}\rangle$, $p_{N_N}=\langle\phi,\{b\},\{a,c,d\}\rangle$, $P=\langle\{b\},\{a\},\{d\}\rangle$.

Now we shall present some types of inclusion of a neutrosophic crisp point to a neutrosophic crisp set:

Definition 4.5

Let $p_N=\langle\{p\},\phi,\{p\}^c\rangle$ is a NCP in X and $A=\langle A_1,A_2,A_3\rangle$ a neutrosophic crisp set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) p_{N_N} be VNCP in X and $A = \left\langle A_1, A_2, A_3 \right\rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_3$.

Remark 4.2

p_N and p_{N_N} are NCS-Type1

Proposition 4.4

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p_N \in \bigcap_{j \in J} A_j$ iff $p_N \in A_j$ for each $j \in J$.

(a_2) $p_{N_N} \in \bigcap_{j \in J} A_j$ iff $p_{N_N} \in A_j$ for each $j \in J$.

(b_1) $p_N \in \bigcup_{j \in J} A_j$ iff $\exists j \in J$ such that $p_N \in A_j$.

(b_2) $p_{N_N} \in \bigcap_{j \in J} A_j$ iff $\exists j \in J$ such that $p_{N_N} \in A_j$.

Proof

Straightforward.

Proposition 4.5

Let $A = \left\langle A_1, A_2, A_3 \right\rangle$ and $B = \left\langle B_1, B_2, B_3 \right\rangle$ are two neutrosophic crisp sets in X. Then $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$. $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof

Obvious

Proposition 4.6

Let $A = \left\langle A_1, A_2, A_3 \right\rangle$ be a neutrosophic crisp set in X. Then $A = \left(\cup \{p_N : p_N \in A\}\right) \cup \left(\cup \{p_{NN} : p_{NN} \in A\}\right)$.

Proof

It is sufficient to show the following equalities:

$A_1 = \left(\cup \{p\} : p_N \in A\}\right) \cup \left(\cup \{\phi : p_{NN} \in A\}\right)$, $A_3 = \phi$

and $A_3 = \left(\cap \left\{\{p\}^c : p_N \in A\right\}\right) \cap \left(\cap \left\{\{p\}^c : p_{NN} \in A\right\}\right)$

which are fairly obvious.

Definition 4.6

Let $f : X \rightarrow Y$ be a function and p_N be a nutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$ is defined by $f(p_N) = \left\langle \{q\}, \phi, \{q\}^c \right\rangle$ where $q = f(p)$.

Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \left\langle \phi, \{q\}, \{q\}^c \right\rangle$ where $q = f(p)$.

It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$ where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under he function f. $f(p_{NN})$, is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

Proposition 4.7

States that any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup\{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup\{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \left\langle A_1, A_2, A_3 \right\rangle$, then $\underset{N}{A} = \left\langle A_1, \phi, A_1^c \right\rangle$ and $\underset{NN}{A} = \left\langle \phi, A_2, A_3 \right\rangle$.

Proposition 4.8

Let $f : X \rightarrow Y$ be a function and $A = \left\langle A_1, A_2, A_3 \right\rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

Proposition 4.9

Let $A = \left\langle A_1, A_2, A_3 \right\rangle$ and $B = \left\langle B_1, B_2, B_3 \right\rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$.

b) $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof

Obvious

Proposition 4.10

Let $A = \left\langle A_1, A_2, A_3 \right\rangle$ be a neutrosophic crisp set in X. Then $A = \left(\cup \{p_N : p_N \in A\}\right) \cup \left(\cup \{p_{NN} : p_{NN} \in A\}\right)$.

Proof

It is sufficient to show the following equalities:
$A_1 = \left(\cup \{p \} : p_N \in A\}\right) \cup \left(\cup \{\phi : p_{NN} \in A\}\right)$ $A_3 = \phi$ and $A_3 = \left(\cap \{\{p \}^c : p_N \in A\}\right) \cap \left(\cap \{\{p\}^c : p_{NN} \in A\}\right)$, which are fairly obvious.

Definition 4.7

Let $f : X \to Y$ be a function.

(a) Let p_N be a neutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$, is defined by $f(p_N) = \left\langle \{q\}, \phi, \{q\}^c \right\rangle$, where $q = f(p)$.

(b) Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \left\langle \phi, \{q\}, \{q\}^c \right\rangle$, where $q = f(p)$. It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function f. $f(p_{NN})$ is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

Proposition 4.11

Any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup\{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup\{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then $\underset{N}{A} = \left\langle x, A_1, \phi, A_1^c \right\rangle$ and $\underset{NN}{A} = \langle x, \phi, A_2, A_3 \rangle$.

Proposition 4.12

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

5 Neutrosophic Crisp Set Relations

Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three crisp nonempty sets

Definition 5.1

Let X and Y are two non-empty crisp sets and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y. Then

i) The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by

$A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$ on $X \times Y$.

ii) We will call a neutrosophic crisp relation $R \subseteq A \times B$ on the direct product $X \times Y$.

The collection of all neutrosophic crisp relations on $X \times Y$ is denoted as $NCR(X \times Y)$

Definition 5.2

Let R be a neutrosophic crisp relation on $X \times Y$, then the inverse of R is donated by R^{-1} where $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$ on $Y \times X$.

Example 5.1

Let $X = \{a,b,c,d\}$, $A = \langle \{a,b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d,b\} \rangle$ then the product of two neutrosophic crisp sets given by

$A \times B = \langle \{(a,a),(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle$ and

$B \times A = \langle \{(a,a),(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$, and

$R_1 = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$ $R_2 \subseteq B \times A$ on $X \times X$,

$R_1^{-1} = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle \subseteq B \times A$ and

$R_2^{-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle \subseteq B \times A$.

Example 5.2

Let $X = \{a,b,c,d,e,f\}$, $A = \langle \{a,b,c,d\}, \{e\}, \{f\} \rangle$,

$D = \langle \{a,b\}, \{e,c\}, \{f,d\} \rangle$ be a NCS-Type 2,

$B = \langle \{a,b,c\}, \{\phi\}, \{d,e\} \rangle$ be a NCS-Type1.

$C = \langle \{a,b\}, \{c,d\}, \{e,f\} \rangle$ be a NCS-Type 3.Then

$A \times D = \langle \{(a,a),(a,b),(b,a),(b,b),(b,b),(c,a),(c,b),(d,a),(d,b)\}, \{(e,e),(e,c)\}, \{(f,f),(f,d)\} \rangle$

$D \times C = \langle \{(a,a),(a,b),(b,a),(b,b)\}, \{(e,c),(e,d),(c,c),(c,d)\}, \{(f,e),(f,f),(d,e),(d,f)\} \rangle$

We can construct many types of relations on products.

We can define the operations of neutrosophic crisp relation.

Definition 5.3

Let R and S be two neutrosophic crisp relations between X and Y for every $(x,y) \in X \times Y$ and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y Then we can defined the following operations

i) $R \subseteq S$ may be defined as two types

a) Type1: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_2 \subseteq B_2$, $A_{3R} \supseteq B_{3S}$

b) Type2: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_{2R} \supseteq B_{2S}$, $B_{3S} \subseteq A_{3R}$

ii) $R \cup S$ may be defined as two types

a) Type1: $R \cup S = \langle A_{1R} \cup B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cap B_{3S} \rangle$,

b) Type2:

$R \cup S = \langle A_{1R} \cup B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cap B_{3S} \rangle$.

iii) $R \cap S$ may be defined as two types

a) Type1: $R \cap S = \langle A_{1R} \cap B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cup B_{3S} \rangle$,

b) Type2:

$R \cap S = \langle A_{1R} \cap B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cup B_{3S} \rangle$.

Theorem 5.1

Let R, S and Q be three neutrosophic crisp relations between X and Y for every $(x, y) \in X \times Y$, then

i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$.

ii) $(R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1}$.

iii) $(R \cap S)^{-1} \Rightarrow R^{-1} \cap S^{-1}$.

iv) $(R^{-1})^{-1} = R$.

v) $R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q)$.

vi) $R \cup (S \cap Q) = (R \cup S) \cap (R \cup Q)$.

vii) If $S \subseteq R$, $Q \subseteq R$, then $S \cup Q \subseteq R$

Proof

Clear

Definition 5.4

The neutrosophic crisp relation $I \in NCR(X \times X)$, the neutrosophic crisp relation of identity may be defined as two types

i) Type1: $I = \{< \{A \times A\}, \{A \times A\}, \phi >\}$

ii) Type2: $I = \{< \{A \times A\}, \phi, \phi >\}$

Now we define two composite relations of neutrosophic crisp sets.

Definition 5.5

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$. Then the composition of R and S, $R \circ S$ be a neutrosophic crisp relation in $X \times Z$ as a definition may be defined as two types

i) Type 1:

$R \circ S \leftrightarrow (R \circ S)(x, z) = \cup\{< \{(A_1 \times B_1)_R \cap (A_2 \times B_2)_S\}$, $\{(A_2 \times B_2)_R \cap (A_2 \times B_2)_S\}$, $\{(A_3 \times B_3)_R \cap (A_3 \times B_3)_S\} >$.

ii) Type 2:

$R \circ S \leftrightarrow (R \circ S)(x, z) = \cap\{< \{(A_1 \times B_1)_R \cup (A_2 \times B_2)_S\}$, $\{(A_2 \times B_2)_R \cup (A_2 \times B_2)_S\}$, $\{(A_3 \times B_3)_R \cup (A_3 \times B_3)_S\} >$.

Example 5.3

Let $X = \{a, b, c, d\}$, $A = \langle \{a, b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d, b\} \rangle$ then the product of two events given by $A \times B = \langle \{(a, a), (b, a)\}, \{(c, c)\}, \{(d, d), (d, b)\} \rangle$, and $B \times A = \langle \{(a, a), (a, b)\}, \{(c, c)\}, \{(d, d), (b, d)\} \rangle$, and

$R_1 = \langle \{(a, a)\}, \{(c, c)\}, \{(d, d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a, b)\}, \{(c, c)\}, \{(d, d), (b, d)\} \rangle$ $R_2 \subseteq B \times A$ on $X \times X$.

$R_1 \circ R_2 = \cup \langle \{(a, a)\} \cap \{(a, b)\}, \{(c, c)\}, \{(d, d)\} \rangle$

$= \langle \{\phi\}, \{(c, c)\}, \{(d, d)\} \rangle$ and

$I_{A1} = \langle \{(a, a).(a, b).(b.a)\}, \{(a, a).(a, b).(b, a)\}, \{\phi\} \rangle$,

$I_{A2} = \langle \{(a, a).(a, b).(b.a)\}, \{\phi\}, \{\phi\} \rangle$

Theorem 5.2

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof

Let $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$, $S \subseteq B \times D$ on $Y \times Z$ then $S^{-1} \subseteq D \times B$, from Definition 5.4 and similarly we can $I_{(R \circ S)^{-1}}(x, z) = I_{S^{-1}}(x, z)$ and $I_{R^{-1}}(x, z)$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

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