



Evaluation of Agronomic Performance of two Varieties of Faba Bean (*Vicia faba* L.), INIAP 440 - Quitumbe and INIAP 442 - Sultana, using Neutrosophic Superhypersoft sets

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Abstract. The yield and earliness of broad bean varieties have depended on key factors to enhance agricultural productivity in various regions. Therefore, the objective of the study focused on evaluating the agronomic yield and earliness of the INIAP 442-SULTANA and INIAP 440-QUITUMBE broad bean varieties under specific edaphoclimatic conditions to determine their feasibility and potential for improving regional crop productivity. For this purpose, a combination of varietal attributes and leaf stages was used to determine eight treatments through modeling with Neutrosophic SuperHyperSoft Sets. Among the results, INIAP 442-SULTANA demonstrated higher earliness and yield compared to INIAP 440-QUITUMBE. In conclusion, it was observed that the INIAP 442-SULTANA-green treatment is a more viable option for improving broad bean crop productivity due to its higher yield and faster harvest.

Keywords: Antimicrobial resistance, *Hedyosmum cuatrecazanum* Occhioni, essential oil, antibacterial, antifungal, Neutrosophic SuperHyperSoft Sets, Fuzzy Extension SuperHyperSoft Sets, Single-Valued Triangular Neutrosophic Number.

1 Introduction

Broad bean cultivation in Ecuador is paramount to nutrition due to its high protein, carbohydrate, vitamin, and mineral content, making it an essential food for the population's diet [1]. This crop represents a promising alternative to other traditional crops in the central region of the country, such as potatoes, beans, and maize [2]. Crop rotation, in which broad beans can play a strategic role, not only diversifies income sources for farmers but also contributes to soil sustainability by preventing overexploitation and promoting enrichment with essential nutrients.

However, broad bean production faces various challenges [3], including limited access to modern technologies and updated knowledge, which restrict the crop's optimization potential. Additionally, adverse climatic factors such as heavy rains, prolonged droughts, frosts, pests, and diseases negatively impact its yield. This limitation reduces opportunities for integration into broader and more efficient markets [4], thereby decreasing the crop's profitability and competitiveness at both local and international levels, despite demand from strategic external markets.

Consequently, strengthening the agroalimentary system through broad bean cultivation is an important step for achieving food sovereignty in Ecuador [5]. In fact, it not only generates employment and additional income in rural areas, where agriculture is vital for subsistence but also reinforces the economic resilience of farming communities [6]. To maximize this potential, it is necessary to train farmers in advanced management techniques and develop varieties that combine characteristics such as earliness, disease resistance [7], quality, and high yields.

Inter-institutional cooperation among entities such as the National Institute of Agricultural Research (INIAP), the Technical University of Cotopaxi (UTC), and other organizations has made progress

in developing new broad bean varieties possible [8]. Therefore, this study focuses on evaluating the agronomic yield and earliness of the INIAP 442-SULTANA and INIAP 440-QUITUMBE broad bean varieties under specific edaphoclimatic conditions to determine the best treatment for their viability in improving productivity in the crops of the San José de Chanchaló sector, in the Cotopaxi province. For this purpose, Neutrosophic SuperHyperSoft Sets modeling was used to calculate the attributes and potential treatments.

2 Preliminaries

For the evaluation of each attribute or parameter associated with the linguistic term used within the single-valued neutrosophic set (SVNS), the decision-maker is able to assign, for each of the criteria considered and for each treatment, a value within the neutrosophic set of choice. Therefore, the following guidelines are outlined to be considered in Table 1.

Table 1: Relationship between measurement ranges and neutrosophic scales. Source: Own elaboration.

Attributes(a_i)/treatments	SVNN	Attributes (b_i)
Very important (VI)	(0.95,0,0)	Extremely High
	(0.85,0.05,0.12)	Very Very High
Important (I)	(0.75,0.15,0.22)	Very High
	(0.65,0.25,0.32)	High
Medium (M)	(0.55,0.35,0.42)	Moderately Moderate
	(0.45,0.45,0.52)	Moderate
	(0.35,0.55,0.62)	Medium Low
Not important (NI)	(0.25,0.65,0.72)	Low
	(0.15,0.75,0.82)	Very Low
Very Not Important (VNI)	(0.05,0.85,0.92)	Very Very Low
	(0,0.95,1)	Extremely Low

2.1 Neutrosophic SuperHyperSoft Sets.

This section serves the purpose of remembering the basic notions of Fuzzy Extension SuperHyperSoft Sets and neutrosophic theory [9].

Definition 1 ([1, 9, 10]). Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U .

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε -elements of the Soft Set (F, E) or the set of ε -approximate elements of the Soft Set [10] [11].

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α -levels definition of a membership function μ_A , having the following:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, the family F is known, then the function μ_A can be reconstructed by using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0, 1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ soft set.

Given a binary operation $*$ for subsets of the set U , where (F, A) and (G, B) are soft sets over U . Then, the operation $*$ for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B .

Definition 2 ([2, 3, 11, 12]). Let U be a universe set, $\mathcal{P}(U)$ the power set of U . Let a_1, a_2, \dots, a_n , for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets

A_1, A_2, \dots, A_n , with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where: $F: A_1 \times A_2 \times \dots \times A_n \rightarrow \mathcal{P}(U)$ is called a *HyperSoft Set* over U .

Definition 3 ([2, 3, 11, 12]). Let U be a universe set, $\mathcal{P}(U)$ the power set of U . Let a_1, a_2, \dots, a_n , for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U .

Definition 4 ([4, 5, 13-15]). Let U be a universe set, $\mathcal{P}(U)$ the power set of U . Let a_1, a_2, \dots, a_n , for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(U(x(d^0)))$ is called a *Fuzzy Extension SuperHyperSoft Set* over U .

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U . Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc [12].

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set* N is characterized by three membership functions [13], which are the truth-membership function T_A , indeterminacy-membership function I_A , and falsity-membership function F_A , where U is the Universe of Discourse and $\forall x \in U, T_A(x), I_A(x), F_A(x) \subseteq]^{-0}, 1^+[$, and $^{-0} \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [14].

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^{-0}, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^{-0}$ and 1^+ belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in U \}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$ and $F_A: U \rightarrow [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

SVNSs (Single-Valued Neutrosophic Sets) emerged with the idea of applying neutrosophic sets for practical purposes. Some operations between SVNSs are expressed below:

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the sum between A_1 and A_2 is defined as:

$$A_1 \oplus A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \tag{1}$$

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNN, the multiplication between A_1 and A_2 is defined as:

$$A_1 \otimes A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \tag{2}$$

The product of a positive scalar with an SVNN, $A = (a, b, c)$ is defined as:

$$A = (1 - (1 - a), b, c) \tag{3}$$

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 7 ([8, 16, 17]). The *single-valued triangular neutrosophic number*, $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on \mathbb{R} , whose truth, indeterminacy, and falsity membership functions are defined as follows [15]:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 8 ([8, 16, 17]). Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Inversion: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.

4. Multiplication by a scalar number:

$$\lambda \tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a} \tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, \wedge is a t-norm and \vee is a t-conorm.

3 Results

3.1 Details of the experiment

Study area: The research was conducted in the Salcedo canton, specifically in San José de Chanchaló, at an altitude of 3,134 meters above sea level. The climate in the area is cold, with temperatures ranging from 12 to 18 °C, reaching highs of 20 °C and lows of 3 °C. However, annual precipitation varies between 500 and 750 mm, while the average relative humidity is 40%, exceeding 70% during the winter. The predominant soil type is Mollisol, characterized by its excellent properties for agricultural development.

Experimental design: The design included four replications, resulting in 16 experimental units, each with an area of 25 m². The Tukey test at 5% was applied to analyze the differences between means with confidence intervals.

Factors under study: The seeds used were provided by the National Institute of Agricultural Research (INIAP). The INIAP 440-QUITUMBE variety originated in San Isidro, Carchi province, and has been previously evaluated in various locations in the Ecuadorian highlands. Meanwhile, the INIAP 442-SULTANA variety resulted from a cross between the progenitors ECU 8395 and ECU 2522.

Field design and statistical analysis: The trial design included the distribution of treatments in split plots. For statistical analysis, a variance analysis scheme (ANOVA) was used, considering the sources of variation: replications, varieties, plant stages, and their interaction.

3.2 Modeling with Neutrosophic SuperHyperSoft Sets

The common bean (*Phaseolus vulgaris* L.) is one of the most significant crops in agriculture, both in terms of production and its impact on food security. In various agricultural regions, the yield and earliness of bean varieties are key factors for improving productivity, especially in environments with variable edaphoclimatic conditions. In this regard, evaluating local varieties is crucial for determining their adaptability and efficiency under specific conditions. This study addresses the comparative evaluation of two bean varieties, through the modeling of Neutrosophic SuperHyperSoft Sets, to analyze their yield and earliness in a specific region, and how these factors influence the improvement of agricultural productivity. To this end, the following bean varieties have been identified as viable alternatives for bean farmers: INIAP 442-SULTANA and INIAP 440-QUITUMBE. For this purpose, they were interrelated under the formation of the following attributes, as shown in Table 2.

Table 2: Attributes to define possible alternatives according to the varieties and the state of the leaves. Source: Own elaboration.

Attributes	Name	Items
a ₁	Varieties	INIAP 442-Sultana, INIAP 440-Quitumbe
a ₂	Leaf status	Green, dry

In Table 2, the possible alternatives are represented, where the Neutrosophic SuperHyperSoft Sets are defined for the following sets A_i with $i \in \{1,2\}$ corresponding to each a_i . Therefore, the following pairs are defined:

$$P(A_1) = \{\{INIAP 442 - Sultana\}, \{INIAP 440 - Quitumbe\}, \{INIAP 442 - Sultana, NIAP 440 - Quitumbe\}\}$$

$$P(A_2) = \{\{Green\}, \{Dry\}, \{Green, Dry\}\}$$

As the study aims to determine the best bean variety, the analysis proceeds to define all the pairs to be analyzed from $P(U)$:

$$F: P(A_1) \times P(A_2) \rightarrow P(U)$$

$$= \{\{INIAP 442 - Sultana\}, \{INIAP 440 - Quitumbe\}, \{Green\}, dry, \{green, dry\}, \{INIAP 442 - Sultana, Green\}, \{INIAP 442 - Sultana, dry\}, \{INIAP 442 - Sultana, green, dry\}, \{INIAP 440 - Quitumbe, Green\}, \{INIAP 440 - Quitumbe, dry\}, \{NIAP 440 - Quitumbe, green, dry\}, \{INIAP 442 - Sultana, NIAP 440 - Quitumbe\}, \{INIAP 442 - Sultana, NIAP 440 - Quitumbe, verde\}, \{INIAP 442 - Sultana, NIAP 440 - Quitumbe, dry\}, \{INIAP 442 - Sultana, NIAP 440 - Quitumbe, green, dry\}\}$$

Once each element is defined, the pairs to be modeled are selected according to the sets defined as treatments to be evaluated in the study. Subsequently, Table 3 presents the proposed treatments (L_i , where $i \in \{1,2, \dots, 8\}$), along with the neutrosophic membership degree (see Table 1) and the corresponding weight for each alternative, based on the attributes involved.

Table 3: Alternatives depending on the variety and the state of the leaves. Source: Own elaboration.

Treatment (I_i)	Alternative set	SVNN ($x(d^0)$)
T1	{INIAP 442 – Sultana, green, dry}	(VNI, NI, M, I, VI)
T2	{NIAP 440 – Quitumbe, green, dry}	(VNI, NI, M, I, VI)
T3	{INIAP 442 – Sultana, NIAP 440 – Quitumbe, green}	(VNI, NI, M, I, VI)
T4	{INIAP 442 – Sultana, NIAP 440 – Quitumbe, dry}	(VNI, NI, M, I, VI)
T5	{INIAP 442 – Sultana, Green}	(VNI, NI, M, I, VI)
T6	{INIAP 442 – Sultana, dry}	(VNI, NI, M, I, VI)
T7	{INIAP 440 – Quitumbe, Green}	(VNI, NI, M, I, VI)
T8	{INIAP 440 – Quitumbe, dry}	(VNI, NI, M, I, VI)

To determine the best alternative, an evaluation is conducted based on the following attributes (b_i) or parameters represented in Table 4. Each alternative is assessed using the b_i , where $i \in \{1, 2, \dots, 9\}$. Additionally, each value can be represented as $\{[31.33 - 36.25], [(171.25 - 245)]\}$ for one or more sets (see Table 5).

Table 4: Attributes for evaluating each treatment. Source: Own elaboration.

Sub-attribute	Name	Description	Worth
b_1	Number of pods	Average obtained from 10-20 plants randomly selected per plot.	(31.33-36.25)
b_2	Days to harvest	Time elapsed from sowing until 50% of the plants reached maturity in the green and dry states.	(171.25-245)
b_3	Number of plants	Total number of plants counted in each plot.	(53.75-71)
b_4	Pod yield	Total weight of pods harvested in tender state, expressed in kg/ha.	(6.35-23.54)
b_5	Number of seeds	Average number of seeds obtained by shelling 10 randomly selected pods.	(2.9-3.75)
b_6	Pod length	Measurement in centimeters of 10 pods, excluding the tip.	(8.48-12.58)
b_7	Seed length	Dimensions in centimeters obtained from 10 selected seeds.	(1.92-2.85)
b_8	Wide seeds	Dimensions in centimeters obtained from 10 selected seeds.	(1.2-1.92)
b_9	Grain yield	Total weight of seeds per plot, converted to kg/ha.	(3.17-9.47)

Table 5: Evaluation of each treatment according to the attributes b_i . Source: Own elaboration.

Alternatives/attributes	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
T1	34.61	199	71	16.86	3.63	11.86	2.83	1.77	7.71
T2	33.41	217.25	53.75	7.14	3.08	8.64	2.09	1.29	3.35
T3	35.88	180.38	62.38	15.73	3.38	10.69	2.36	1.56	6.32
T4	32.15	235.88	62.38	8.27	3.33	9.81	2.56	1.5	4.74
T5	36.25	171.25	71	23.54	3.5	12.58	2.81	1.92	9.47
T6	32.98	226.75	71	10.19	3.75	11.15	2.85	1.62	5.95
T7	35.5	189.5	53.75	7.93	2.9	8.48	1.92	1.2	3.17
T8	31.33	245	53.75	6.35	3.25	8.8	2.26	1.38	3.52

To evaluate specific differences between the means of the analyzed variables, the Tukey Test at 5% was applied. The results, presented in Table 5, indicate that the INIAP 442-SULTANA variety (T1) achieved a significant range in most of the evaluated variables, surpassing the INIAP 440-QUITUMBE variety (T2). Regarding the plant state factor, the *green* state demonstrated a significant range in days to harvest, pod yield, and grain yield, indicating its superiority over the dry plant state (T3 and T4).

In the green state, the combination INIAP 442-SULTANA GREEN (T5) obtained the highest range, standing out for its superior performance in key attributes such as pod yield (b_4) (23.54 kg) and grain

yield (b_9) (9.47 kg). Conversely, the combination INIAP 440-QUITUMBE DRY (T8) achieved lower ranges, reflecting reduced performance in the analyzed attributes.

Comparison of agronomic parameters: In terms of agronomic parameters, the INIAP 442-SULTANA variety (T1) exhibited superior values in pod length (b_6) (11.86 cm), seed length (b_7) (2.83 cm), and seed width (b_8) (1.77 cm), significantly outperforming INIAP 440-QUITUMBE (T2), whose values were 8.64 cm, 2.09 cm, and 1.29 cm, respectively. Similarly, grain yield (b_9) was notably higher in the INIAP 442-SULTANA variety (7.71 kg/ha) compared to INIAP 440-QUITUMBE (3.35 kg/ha).

Effect of plant state (attribute a_2): The green state (T3) showed better performance in most of the evaluated variables, particularly in grain yield (b_9) (6.32 kg/ha compared to 4.74 kg/ha in the dry state (T4)). However, the interaction between state and variety revealed that the dry state had less impact on the INIAP 442-SULTANA variety, which exhibited intermediate ranges in the evaluations.

The results highlight the superiority of the INIAP 442-SULTANA variety, particularly in the green state, and its consistent performance under dry state conditions. The interaction between factors confirms that both variety and plant state significantly influence productive and agronomic parameters. This positions INIAP 442-SULTANA as a promising option to optimize productivity in crops under different environmental conditions. Table 6 presents the variance calculations for the attributes b_i .

Table 6: Analysis of variance (ANOVA) of each attribute b_i . Source: Own elaboration.

FV	Repetition	Variety	State	Variety*state	Error	Total	CV%
gl	3	1	1	1	9	15	
b1	14.89 ns	5.76 ns	55.5 ns	0.81 ns	15.6		11.61
b2	0.08 ns	1332.25 **	12321 **	0.00 ns	0.69		0.4
b3	31.58 ns	1190.25 **	0.00 ns	0.00 ns	26.97		8.33
b4	1.76 ns	378.40 **	222.83 **	138.71 **	0.67		6.8
b5	0.94 ns	1.21 ns	0.01 ns	0.36 ns	0.53		21.71
b6	0.89 ns	41.54 **	3.08 ns	1.22 ns	2.19		14.45
b7	0.01 ns	2.18 **	0.15 ns	0.09 ns	0.17		16.6
b8	0.06 ns	0.9 **	0.01 ns	0.23 ns	0.07		16.92
b9	0.52 ns	76.26 **	10.06 **	15 **	0.2		8.16

Analysis of Variance (ANOVA): The results of the analysis of variance for the variables evaluated showed statistical significance ($p < 0.05$) for the variety factor in the attributes days to harvest, number of plants, pod yield, pod length, seed length, seed width, and grain yield. These differences highlight the distinguishing effect of the varieties INIAP 442-SULTANA and INIAP 440-QUITUMBE. Additionally, the plant state attribute (a_2) exhibited statistical significance in days to harvest, pod yield, and grain yield, emphasizing the influence of green and dry conditions on these variables.

In the interaction of variety \times state (attributes a_1 and a_2), significant differences were identified in pod yield and grain yield. The coefficient of variation (CV) ranged from a minimum of 0.4% for days to harvest to a maximum of 21.71% for the number of seeds, indicating acceptable variability in the data.

Neutrosophic Membership Degree: Once the evaluation was defined, the neutrosophic membership degree of each treatment was determined, as shown in Tables 7 and 8. Additionally, the most important treatment in the study was identified. To achieve this, the t_{norm} and t_{conorm} were used to determine the intersection $F(l_1, l_2, \dots, l_n) \cap G(b_1, b_2, \dots, b_n)$, through the aggregation operation $(\min_j\{T_{ij}\}, \max_j\{I_{ij}\}, \max_j\{F_{ij}\})$. This value results in the single-valued neutrosophic number, representing the effectiveness and selection of the best treatment (see Table 9).

Table 7: Neutrosophic Membership degree of each alternative based on a_n . Source: Own elaboration.

l_n/a_n	b_1	b_2	b_3	b_4
T1	(0.55,0.35,0.42)	(0.55,0.35,0.42)	(0.95,0,0)	(0.45,0.45,0.52)
T2	(0.35,0.55,0.62)	(0.25,0.65,0.72)	(0,0.95,1)	(0,0.95,1)
T3	(0.85,0.05,0.12)	(0.75,0.15,0.22)	(0.35,0.55,0.62)	(0.45,0.45,0.52)
T4	(0,0.95,1)	(0,0.95,1)	(0.35,0.55,0.62)	(0,0.95,1)
T5	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)
T6	(0.25,0.65,0.72)	(0.15,0.75,0.82)	(0.95,0,0)	(0.15,0.75,0.82)
T7	(0.75,0.15,0.22)	(0.65,0.25,0.32)	(0,0.95,1)	(0,0.95,1)
T8	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)

Table 8: Neutrosophic Membership degree of each alternative based on a_n (continued). Source: Own elaboration.

l_n/a_n	b_5	b_6	b_7	b_8	b_9
T1	(0.75,0.15,0.22)	(0.75,0.15,0.22)	(0.95,0,0)	(0.65,0.25,0.32)	(0.65,0.25,0.32)
T2	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)
T3	(0.45,0.45,0.52)	(0.45,0.45,0.52)	(0.35,0.55,0.62)	(0.35,0.55,0.62)	(0.35,0.55,0.62)
T4	(0.35,0.55,0.62)	(0.25,0.65,0.72)	(0.55,0.35,0.42)	(0.25,0.65,0.72)	(0.15,0.75,0.82)
T5	(0.55,0.35,0.42)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)
T6	(0.95,0,0)	(0.55,0.35,0.42)	(0.95,0,0)	(0.45,0.45,0.52)	(0.35,0.55,0.62)
T7	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)
T8	(0.25,0.65,0.72)	(0,0.95,1)	(0.25,0.65,0.72)	(0.15,0.75,0.82)	(0,0.95,1)

Table 9: Degree of importance of each alternative. Source: Own elaboration.

l_n/a_n	$F(l_1, l_2, \dots, l_n) \cap G(b_1, b_2, \dots, b_n)$	Position
T1	(0.55,0.35,0.42)	M
T2	(0,0.95,1)	NVI
T3	(0.35,0.45,0.52)	M
T4	(0,0.95,1)	NVI
T5	(0.65,0.35,0.42)	I
T6	(0.15,0.75,0.82)	NI
T7	(0,0.99,1)	NVI
T8	(0,0.99,1)	NVI

The results indicate that the variety INIAP 442-SULTANA demonstrates superior agronomic performance (classified as *medium importance*) compared to INIAP 440-QUITUMBE (classified as *very not important*). This advantage is attributed to its earliness, with an average of 199 days to harvest, positioning it as an earlier option compared to other varieties cultivated in the country. Differences in days to harvest relative to previous reports highlight the influence of local edaphoclimatic factors, which affect plant growth and development.

In terms of yield, INIAP 442-SULTANA achieved the best results in green pod yield (classified as *important*, T5) and dry pod yield (classified as *not important*, T6). These outcomes reflect the productive capacity of the T5 treatment of this variety, as well as its adaptability to the experimental environment's conditions.

In contrast, INIAP 440-QUITUMBE did not outperform INIAP 442-SULTANA in any of the variables analyzed. This may be attributed to the fact that INIAP 440-QUITUMBE, being a variety with over three decades in the field, has surpassed the optimal productivity period typically associated with newly released resistant varieties. Consequently, the evaluated treatment T5 {INIAP 442-Sultana,

Green} has been classified as *important* (0.65, 0.35, 0.42), standing out for its earliness and adaptability to diverse edaphoclimatic conditions.

4 Conclusion

The modeling of Neutrosophic SuperHyperSoft Sets has enabled the evaluation and classification of eight combined treatments to optimize agricultural productivity under varying cultivation conditions. The INIAP 442-SULTANA variety (T1) stands out for its earliness and high yields compared to INIAP 440-QUITUMBE (T2). Additionally, the interaction between the attributes variety and plant state significantly influences yields, with INIAP 442-SULTANA demonstrating greater consistency in both states. Furthermore, the evaluation of neutrosophic membership degrees identifies treatment T5 ({INIAP 442-Sultana, Green}) as exhibiting resilient and adaptable performance, positioning it as a preferred option for optimizing agricultural productivity under different cultivation conditions. Finally, it is suggested to continue research to assess the impact of additional variables, such as soil type and pest resistance, to refine the selection of varieties in environments affected by climate change.

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