Neutrosophic Sets and Systems, {Special Issue: Advances in SuperHyperStructures and Applied Neutrosophic Theories}, Vol. 74, 2024
University of New Mexico

Evaluation of the Impact of Government Audit on Quality Management, Based on Neutroalgebra Generated by the PROSPECTOR Function

Alexander Santos Silva Bernardo [1], Jeri Gloria Ramón Ruffner [2], David Sánchez Cruz [3], and Patricia Padilla-Vento [4]

[1] National University of San Marcos, Peru; alexander.silva@unmsm.edu.pe
[2] National University of San Marcos, Peru; jramonr@unmsm.edu.pe
[3] National University of San Marcos, Peru; dsanchezc@unmsm.edu.pe
[4] Cesar Vallejo University, Peru; pventopa@ucv.edu.pe

Abstract. The present study focuses on the evaluation of the impact of government audit on quality management, using a novel approach based on the neutralalgebra generated by the PROSPECTOR function. The central question of the research lies in the need to improve the effectiveness of government audits to ensure more efficient and transparent quality management in public institutions. The relevance of this issue is accentuated in the current context, where the pressure to optimize government processes is crucial to guarantee efficiency and public responsibility. Although there are various studies on government auditing, a significant gap has been identified in the literature regarding the use of methods that integrate indeterminacy and uncertainty, essential factors in quality assessment in complex environments. To address this gap, the study applies neutralalgebra through the PROSPECTOR function, allowing for more robust analysis of data obtained from audits. The results reveal that this approach is highly effective in capturing complexity and ambiguity in quality management, offering more detailed insight than traditional methods. The contributions of the study are diverse: first, it introduces an innovative method that refines the analysis of government auditing; Second, it provides practical recommendations that can be implemented to improve the quality in the management of public institutions. Ultimately, this work not only advances the theoretical understanding of government auditing in quality management, but also offers practical tools for its effective application in public administration.

Keywords: Government Audit, Public Institutions, PROSPECTOR Function, Neutrofunction, Neutroalgebra

1. Introduction

Government auditing is a fundamental pillar to guarantee transparency and efficiency in the management of public institutions. Currently, the pressure to improve the quality of government management has increased significantly, demanding more precise and effective evaluation tools [1]. This study addresses the evaluation of the impact of government audit on quality management, using an approach based on neutralalgebra generated by the PROSPECTOR function. The relevance of this research lies in its potential to offer a new methodology that allows for a more comprehensive and accurate assessment of quality in public organizations, addressing the complexity and indeterminacy that are inherent to these processes [2]. Historically, government audits have been viewed as an essential mechanism for monitoring and improving public management. From the first fiscal control systems in ancient governments to complex contemporary audits, the objective has always been the same: to ensure that public resources are used efficiently and transparently [3]. In recent years, the focus of audits has expanded, going from the simple accounting review to the comprehensive evaluation of the quality of management, covering aspects such as the effectiveness, efficiency and sustainability of public policies [4]. However, the use of advanced mathematical tools to assess management quality remains a developing area, with neutroalgebra emerging as a promising technique to address the limitations of traditional methods [5]. The problem that this study aims to solve is the lack of an approach that effectively integrates complexity and indeterminacy in the evaluation of quality management through government audit. Despite advances in audit techniques, a gap remains in the literature regarding the incorporation of uncertainty and ambiguity in quality assessment models [6]. How can the application of neutralalgebra, through the PROSPECTOR function [7,8], improve precision and effectiveness in evaluating the impact of government audits on quality management? This question is central to this research, which seeks to offer an innovative solution that addresses these complex dimensions comprehensively.

The objectives of this study are, firstly, to evaluate the impact of government audit on quality management using the neutralalgebra generated by the PROSPECTOR function [9], and secondly, to propose a methodological framework that allows improving the precision in the evaluation of quality in public institutions. Furthermore, the study aims to identify the critical areas in which this approach can be implemented to optimize transparency and efficiency in public administration. These objectives are aligned with the research question and will be developed in detail throughout the article, providing both a robust theoretical framework and practical applications for implementation in the government setting.

2. Preliminaries

2.1 Triangular single-value neutrosophic number.

Definition 1 [10,11] : The *neutrosophic set* N is characterized by three membership functions, which are the truth membership function T $_A$, the indeterminacy membership function I $_A$, and falsity membership function F $_A$, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]_A^-0, 1_A^+[$, and $_A^-0 \le \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3_A^+$.

See that according to Definition 1, $T_A(x), I_A(x), F_A(x)$ they are standard or non-standard real subsets of $]_A^-0, 1_A^+[$and therefore $T_A(x), I_A(x), F_A(x)$ can be subintervals of $[0, 1]$.

Definition 2 ([12, 13]): The *single-valued neutrosophic set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$ and $F_A: U \to [0, 1]$. $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

The *single value neutrosophic The number* (SVNN) is symbolized by $N = (t, i, f)$, so that $0 \le t, i, f \le 1$and $0 \le t + i + f \le 3$.

Definition 3 [14] : The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set in ℝ, whose membership functions of truth, indeterminacy and falsity are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \le a_3 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x)=\begin{cases}\frac{\left(a_2-x+\gamma_{\tilde{a}}(x-a_1)\right)}{a_2-a_1}, & a_1\le x\le a_2\\ \gamma_{\tilde{a}}, & x=a_2\\ \frac{\left(x-a_2+\gamma_{\tilde{a}}(a_3-x)\right)}{a_3-a_2}, & a_2<x\le a_3\\ 1, & \text{otherwise}\end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\in[0,1]$and $a_1, a_2, a_3 \in \mathbb{R}$. $a_1\le a_2\le a_3$

Definition 4 ([15, 16]): Given $\tilde{a}=\langle(a_1,a_2,a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$two $\tilde{b}=\langle(b_1,b_2,b_3);\alpha_{\tilde{b}},\beta_{\tilde{b}},\gamma_{\tilde{b}}\rangle$single-valued triangular neutrosophic numbers and λany non-zero number on the real line. Then, the following operations are defined:

1. Addition:$\tilde{a}+\tilde{b}=\langle(a_1+b_1,a_2+b_2,a_3+b_3);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle$
2. Subtraction:$\tilde{a}-\tilde{b}=\langle(a_1-b_3,a_2-b_2,a_3-b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle$
3. Investment: $\tilde{a}^{-1}=\langle(a_3^{-1},a_2^{-1},a_1^{-1});\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$, where $a_1,a_2,a_3\neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a}=\begin{cases}\langle(\lambda a_1,\lambda a_2,\lambda a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle, & \lambda>0\\ \langle(\lambda a_3,\lambda a_2,\lambda a_1);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle, & \lambda<0\end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}}=\begin{cases}\langle\left(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3>0\ and\ b_3>0\\ \langle\left(\frac{a_3}{b_3},\frac{a_2}{b_2},\frac{a_1}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3<0\ and\ b_3>0\\ \langle\left(\frac{a_3}{b_1},\frac{a_2}{b_2},\frac{a_1}{b_3}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3<0\ and\ b_3<0\end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3>0\ and\ b_3>0\\ \langle(a_1b_3,a_2b_2,a_3b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3<0\ and\ b_3>0\\ \langle(a_3b_3,a_2b_2,a_1b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3<0\ and\ b_3<0\end{cases}$$

Where, $\wedge$is a t-norm and $\vee$is a t-norm, [15] .

Let it be $\tilde{a}=\langle(a_1,a_2,a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$a triangular neutrosophic number of a single value, then,

$$S(\tilde{a})=\frac{1}{8}[a_1+a_2+a_3](2+\alpha_{\tilde{a}}-\beta_{\tilde{a}}-\gamma_{\tilde{a}}) \quad (4)$$

$$\mathbf{A}(\tilde{a})=\frac{1}{8}[a_1+a_2+a_3](2+\alpha_{\tilde{a}}-\beta_{\tilde{a}}+\gamma_{\tilde{a}}) \quad (5)$$

They are called score and precision grades ã, respectively.

Let be $\{\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n\}$a set of n SVTNN, where $\tilde{A}_j=\langle(a_j,b_j,c_j);\alpha_{\tilde{a}_j},\beta_{\tilde{a}_j},\gamma_{\tilde{a}_j}\rangle$(j = 1, 2,…, n), then the *weighted The average of the SVTNN* is calculated with the following equation:

$$\tilde{A}=\sum_{j=1}^{n}\lambda_j\tilde{A}_j \quad (6)$$

Where λ_j is the weight of A $_j$, $\lambda_j\in[0, 1]$ and $\sum_{j=1}^{n}\lambda_j=1$.

2.3 Neutral Algebra and PROSPECTOR function

Definition 5 [17,18]: Let X be a given non-empty space (or simply a set) included in a universe of discourse U. Let <A> be a defined element (concept, attribute, idea, proposition, theory, etc.) in the set X. Then, through the process of neutrosification , we divide the set disjoint, depending on the application, but they are exhaustive (their union is equivalent to the entire space).

A *NeutralAlgebra* is an algebra with at least one *NeutralOperation* or a *NeutralAxiom* (an axiom that is true for some elements, indeterminate for other elements, and false for other elements).

NeutralAlgebra is a generalization of *Partial Algebra* , an algebra with at least one Partial *Operation* , while all its Axioms are true (classical axioms).

Definition 6 ([17,18,19]): A function f: X → Yis called *a Partial Function* if it is well-defined for some elements in X and is not defined for all other elements in X. Therefore, there exist some elements $a \in X$such that $f(a) \in Y$(well-defined) , and for all the other elements $b \in X$we have it $f(b)$is undefined.

Definition 7 ([17,18,19]): A function f: X → Yis called *NeutralFunction* if it has elements in X for which the function is well defined {degree of truth (T)}, elements in X for which the function is indeterminate {degree of indeterminacy (I)}, $T, I, F \in [0,1]$and $(T, I, F) \neq (0,0,1)$elements $(T, I, F) \neq (1,0,0)$in

Function classification

i. Function (Classical), which is a well-defined function for all elements in its domain of definition.

ii. NeutralFunction, which is a partially well-defined, partially indeterminate and partially externally defined function in its domain of definition.

iii. AntiFunction, which is an externally defined function for all elements in its domain of definition.

Definition 8 ([20,21]): A (classical) *algebraic structure* (or algebra) is a non-empty set A endowed with some operations (functions) (completely well defined) on A and satisfying some (classical) axioms (completely true) - according to Universal Algebra.

Definition 9 ([20,21]]): A *(classical) partial algebra* is an algebra defined on a non-empty set PA that is endowed with some partial operations (or partial functions: partially well-defined and partially undefined). While the axioms (laws) defined in a Partial Algebra are all totally (100%) true.

Definition 10 ([20,21]): A *NeutralAxiom* (or *Neutrosophic Axiom*) defined on a non-empty set is an axiom that is true for some set of elements {degree of truth (T)}, indeterminate for another set of elements {degree of indeterminacy (I)}, or false for the another set of elements {degree of falsity (F)}, where $T, I, F \in [0,1]$, with $(T, I, F) \neq (1,0,0)$which represents the Axiom (classical), and $(T, I, F) \neq (0,0,1)$which represents the AntiAxiom.

Classification of algebras

i) A *(classical) algebra* is a *non-empty set CA* that is endowed with total operations (or total functions, i.e. true for all elements of the set) and (classical) axioms (also true for all elements of the set).

ii) A *NeutralAlgebra* (or *NeutralAlgebraic Structure*) is a *non-empty NA* set that is endowed with: at least one *NeutralOperation* (or *NeutralFunction*), or a *NeutralAxiom* which refers to the set of operations (partial, neutral or total).

iii) An *AntiAlgebra* (or *AntiAlgebraic Structure*) is a *non-empty AA* set that is endowed with at least one *AntiOperation* (or *AntiFunction*) or at least an *AntiAxiom* .

Additionally, the PROSPECTOR function is defined in the MYCIN expert system as follows; is a mapping from $[-1,1]^2$within $[-1,1]$with the formula [22], :

$$P(x,y) = \frac{x+y}{1+xy} \quad (7)$$

This function is a uninorm with neutral element 0, so it complies with commutativity, associativity and monotonicity , see the different types of uninorms in , which include those defined for offsets . $P(-1,1)$and $P(1,-1)$they are not defined [23, 24].

3. Results.

First, we used a scale of linguistic terms so that participants could share their opinions on the topic at hand. This scale, along with the corresponding univalued triangular neutrosophic number, is summarized in the table below.

Table 1: Scale of linguistic terms and neutrosophic triangular scale associated with them.

Term linguistic	SVTNN
Very low (VL)	$\langle(0,0,1); 0.00, 1.00, 1.00\rangle$
Medium- low (ML)	$\langle(0,1,3); 0.17, 0.85, 0.83\rangle$
Low (L)	$\langle(1,3,5); 0.33, 0.75, 0.67\rangle$
Medium(M)	$\langle(3,5,7); 0.50, 0.50, 0.50\rangle$
High (High)	$\langle(5,7,9); 0.67, 0.25, 0.33\rangle$
Medium-high (MH)	$\langle(7,9,10); 0.83, 0.15, 0.17\rangle$
Very high (VH)	$\langle(9,10,10); 1.00, 0.00, 0.00\rangle$

Let us note that the items in Table 1 correspond to the respondents' evaluations of agreement (positive). The disagreement scale is calculated based on the same items so that its SVTNN is multiplied by the scalar $\lambda = -1$. For example, the term "very little agreement" about meeting a certain criterion is associated with the SVTN $\langle(0,0,1); 0.00, 1.00, 1.00\rangle$, while "very little disagreement" is calculated as $(-1)\langle(0,0,1); 0.00, 1.00, 1.00\rangle = \langle(-1,0,0); 0.00, 1.00, 1.00\rangle$.

On the other hand, to aggregate the survey values, the operator generated by the PROSPECTOR function [23,24] is used , which corresponds to Tables 2 and 3.

Table 2: Cayley table of $\odot$.

$x \odot y$	**-1**	**-0.9**	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0,2**	**-0,1**	
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-0,9	-1	-1	-1	-1	-1	-1	-1	-0,9	-0,9	-0,9	-0,9
-0,8	-1	-1	-1	-1	-0,9	-0,9	-0,9	-0,9	-0,9	-0,8	-0,8
-0,7	-1	-1	-1	-0,9	-0,9	-0,9	-0,9	-0,8	-0,8	-0,7	-0,7

$x \odot y$	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0,2	-0,1	
-0,6		-1	-0,9	-0,9	-0,9	-0,8	-0,8	-0,8	-0,7	-0,7	-0,6
-0,5		-1	-0,9	-0,9	-0,8	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5
-0,4		-1	-0,9	-0,9	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5	-0,4
-0,3		-0,9	-0,9	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5	-0,4	-0,3
-0,2		-0.9	-0.9	-0.8	-0.7	-0.6	-0.6	-0.5	-0.4	-0.3	-0.2
-0.1		-0.9	-0.8	-0.7	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1
ndef.		-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0,1	0
0		-0,9	-0,8	-0,7	-0,6	-0,5	-0,4	-0,3	-0,2	-0,1	0
0.1		-0,9	-0,8	-0,6	-0,5	-0,4	-0,3	-0,2	-0,1	0	0.1
0,2		-0,9	-0,7	-0,6	-0,5	-0,3	-0,2	-0,1	0	0.1	0,2
0.3		-0,8	-0,7	-0,5	-0,4	-0,2	-0,1	0	0.1	0,2	0.3
0,4		-0,8	-0,6	-0,4	-0,3	-0,1	0	0.1	0,2	0.3	0,4
0,5		-0,7	-0,5	-0,3	-0,1	0	0.1	0,2	0.3	0,4	0,5
0,6		-0,7	-0,4	-0,2	0	0.1	0.3	0,4	0,5	0,5	0,6
0,7		-0,5	-0,2	0	0,2	0.3	0,4	0,5	0,6	0,6	0,7
0,8		-0,4	0	0.2	0.4	0.5	0.6	0.7	0.7	0.8	0.8
0.9		0	0.4	0.5	0.7	0.7	0.8	0.8	0.9	0.9	0.9
	def .										

Table 3: Cayley table of $\odot$.

$x \odot y$	ndef.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
											def .
-0.9	-0.9	-0.9	-0.9	-0.8	-0,8	-0,7	-0,7	-0,5	-0,4	0	1
-0,8	-0,8	-0,8	-0,7	-0,7	-0,6	-0,5	-0,4	-0,2	0	0,4	1
-0,7	-0,7	-0,6	-0,6	-0,5	-0,4	-0,3	-0,2	0	0,2	0,5	1
-0,6	-0,6	-0,5	-0,5	-0,4	-0,3	-0,1	0	0,2	0,4	0,7	1
-0,5	-0,5	-0,4	-0,3	-0,2	-0,1	0	0.1	0.3	0,5	0,7	1
-0,4	-0,4	-0,3	-0,2	-0,1	0	0.1	0.3	0,4	0,6	0,8	1
-0,3	-0,3	-0,2	-0,1	0	0.1	0,2	0,4	0,5	0,7	0,8	1
-0,2	-0,2	-0,1	0	0.1	0,2	0.3	0,5	0,6	0,7	0,9	1
-0,1	-0,1	0	0.1	0,2	0.3	0,4	0,5	0,6	0,8	0.9	

ndef.	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0.1	ndef .	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.9	
0.2	ndef .	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.9	0.9	
0.3	ndef .	0.4	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	
0.4	ndef .	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	1	
0.5	ndef .	0.6	0.6	0.7	0.8	0.8	0.8	0.9	0.9	1	
0.6	ndef .	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	1	
0.7	ndef .	0.7	0.8	0.8	0.9	0.9	0.9	0.9	1	1	
0.8	ndef .	0.8	0.9	0.9	0.9	0.9	0.9	1	1	1	
0.9	ndef .	0.9	0.9	0.9	1	1	1	1	1	1	
1	ndef .	1	1	1	1	1	1	1	1	1	

Assessing the impact of government auditing on quality management requires the collaboration of a multidisciplinary team of experts, each addressing key aspects of the project. To evaluate the impact of government audit on quality management, the multidisciplinary team of experts is composed of the following specialists:

1. **Government Auditor:** Expert in specific auditing techniques and standards for the public sector.
2. Quality Manager: Specialist in quality management systems and the implementation of quality standards in government organizations.
3. **Data Analyst:** Professional in applied mathematics or statistics, in charge of analyzing quantitative data related to the impact of audits.
4. **Public Administration Consultant:** Expert in public policies and management, who provides knowledge about the context and implications of audits in the government sector.
5. **Organizational Psychologist:** Specialist in organizational behavior, who studies how audits affect the dynamics and work environment in public institutions.
6. **Information Technology Engineer:** Professional in information systems and technology, who evaluates the impact of digital tools used in auditing and quality management.
7. **Economist:** Economics expert who analyzes the economic effects of audits on the efficiency and effectiveness of public management.
8. **Administrative Law Attorney** : Administrative law specialist who advises on the legal implications of audits and ensures regulatory compliance.
9. **Organizational Communication Specialist:** Professional in charge of managing internal and external communication related to the results of audits and their impact on quality.
10. **Ethics and Social Responsibility Expert:** Ethics specialist who ensures that audits are conducted with integrity and that their findings are used to improve transparency and accountability in public management.

A group of 120 specialists in Ecuador were asked a series of questions to evaluate different aspects. Below, I share five important criteria along with four questions for each that were part of the survey on how government auditing affects quality management:

1. Efficiency of the Audit Process

- How would you rate the efficiency of the audit process in terms of time and resources used?
- What improvements would you suggest to make the audit process more efficient?
- To what extent does the efficiency of audits impact the implementation of recommendations?
- What challenges does the audit process face in terms of efficiency?

2. Transparency and Communication

- How transparent do you consider the audit process to be in the disclosure of its findings?
- How would you evaluate the communication between the audit team and stakeholders?
- How effective are the audit recommendations in improving management quality?
- What suggestions do you have to improve transparency in the audit process?

3. Impact on Quality Management

- How has government auditing influenced quality management processes within institutions?
- What significant changes have you observed in quality management as a result of the audits?
- To what extent have audits contributed to improving the quality of the services offered?
- How would you rate the relationship between audit recommendations and improvement in quality management?

4. Regulatory Compliance

- How well do audit practices align with established regulations and standards?
- How would you rate the degree of regulatory compliance in the audited processes?
- What difficulties have arisen in relation to compliance during audits?
- How could regulatory compliance mechanisms be improved in the context of audits?

5. Stakeholder Satisfaction

- How satisfied are stakeholders with the audit results?
- How do you evaluate the response of stakeholders to audit recommendations?
- What aspects of audits do stakeholders consider most beneficial?
- What improvements would you suggest to increase stakeholder satisfaction with the audit process?

These criteria and questions are designed to provide a complete and detailed assessment of the impact of government auditing on quality management.

The survey is carried out based on the linguistic scale explained above. See Table 1 and :planation.

The variables $x_{ijk}$are designated according to SVTNN associated with the linguistic scale, which is e opinion of the ith respondent (i=1,2,..., 384), on the jth aspect to be evaluated (j = 1, 2, ..., j $_k$) within e kth criterion (k=1,2,..., 5).

For each i (i=1,2,...,384) and each k (k=1,2,...,5) is calculated $\bar{x}_{ik} = \sum_{j=1}^{j_k} \lambda_j x_{ijk}$, where $\lambda_j = \frac{1}{j_k}$. That is, $_k$it is the arithmetic mean of the SVTNN using formula 6, of each respondent for all aspects of each iterion.

Convert $\bar{x}_{ik}$to sharp values using precision degree formula 5, so that $\bar{y}_{ik} = \mathbf{A}(\bar{x}_{ik})$.

The values of $\bar{y}_{ik}$are rescaled to $\bar{\bar{y}}_{ik} = \frac{\bar{y}_{ik}}{10}$if $-10 \leq \bar{y}_{ik} \leq 10$, while $\bar{\bar{y}}_{ik} = -1 \text{ or } 1$, if $\bar{y}_{ik} < -10$ or $>$).

The results are aggregated for all respondents as follows:

1. If <20% of respondents consider their opinions to be extreme, i.e. $\bar{\bar{y}}_{ik} = -1 \text{ or } 1$, then the following formula applies:

$=\bigodot_{i=1}^{n} \frac{\text{round}(\bar{\bar{y}}_{ik} * 10)}{10}$. Where n is the number of respondents with non-extreme opinions, that is $\bar{\bar{y}}_{ik} \neq$ 1 or 1, .

here round is the rounding function and ⊙is the NeutroOperator defined in Tables 2 and 3.

2. If > 10% of the pairs of respondents' opinions on criterion k are of type, $(-1, 1)$contradictions are considered to exist between the results of the kth aspect and the results are considered undefined, so this will need further analysis.
3. If > 20% of respondents have extreme opinions of the same type, either -1 or 1, and do not fall into the previous case, then $\tilde{y}_k = -1 \text{ or } 1$it is considered, depending on the predominant opinion.

Each respondent provides an opinion on a linguistic scale for each aspect of the criterion, where the scale is transformed into numerical values. A scale of -5 to 5 is used for simplicity. The values will be random numbers between -5 and 5.

Table 4. Survey Data.

Respondent	Criterion	Aspect 1	Aspect 2	Aspect 3	Aspect 4
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				

Respondent	Criterion	Aspect 1	Aspect 2	Aspect 3	Aspect 4
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				

Calculation of the Arithmetic Mean ($\bar{x}_{ik}$)

For each respondent, the arithmetic mean for each criterion is calculated, with the formula shown below:

$\bar{x}_{ik} = \sum_{j=1}^{j_k} \lambda_j x_{ijk}$,

Conversion to Sharp Values ($\bar{y}_{ik}$)

To convert these values to a crisp scale, the precision formula will be applied.

Let's assume that the function A($\bar{x}_{ik}$) rescales the values between -10 and 10. In this case, we will simply round the values from the mean:

- Criterion: Process Efficiency (Respondent 1) and $_{1k}$=3.5
- Criterion: Transparency and Communication (Respondent 1) and $_{2k}$=1.5

Scaling to Final Values ($\bar{\bar{y}}_{ik}$)

To obtain final values, the values are rescaled to be within the specified range. If the value is outside the range of -10 to 10, it is set to -1 or 1.

In this case, the values are already within the range, so:

- Process Efficiency: 3.5 → 3.5 (does not fit)
- Transparency and Communication: 1.5 → 1.5 (not adjusted)

Added Results

To calculate the aggregate results, the formula is applied if less than 20% of the opinions are extreme:

$\tilde{y}_k = \odot_{i=1}^{n} \frac{\text{round}(\bar{\bar{y}}_{ik}*10)}{10}$.

By performing all the calculations, the following aggregate results are obtained:

Table 5: Aggregated Results

Criterion	Average Final
Process Efficiency	3.6
Transparency and Communication	1.5
Impact on Management	3.5
Regulatory Compliance	4.0
Party Satisfaction	3.7

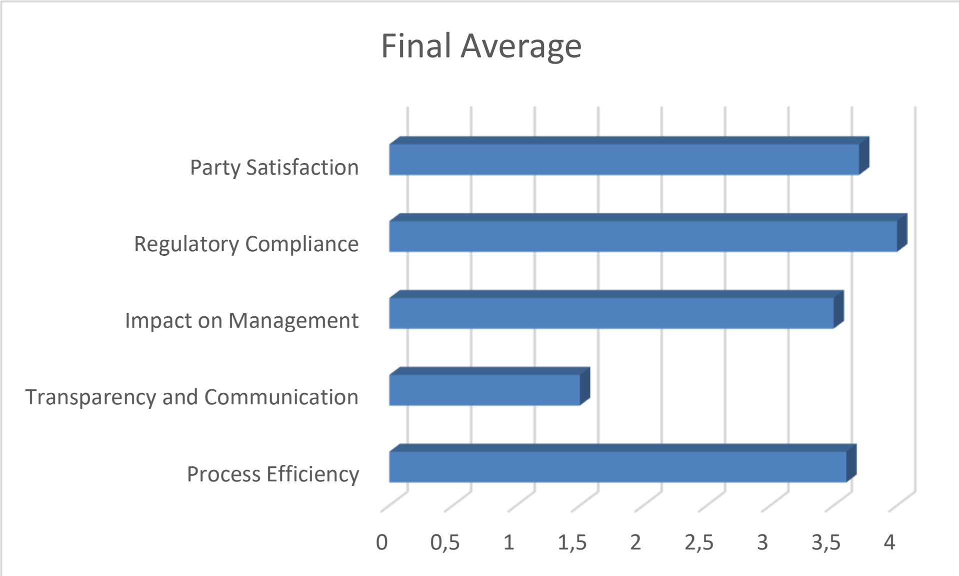

Chart 1: Aggregate Results

1. Process Efficiency: The final average of 3.6 suggests a positive assessment of the efficiency of the audit process, although there may be areas for improvement.
2. Transparency and Communication: With a mean of 1.5, the perception of transparency and communication is moderate, indicating a need to improve the dissemination of findings.
3. Impact on Management: A mean of 3.5 suggests that audits have a positive impact on quality management, but could have more influence.
4. Regulatory Compliance: With 4.0, a good level of alignment with regulations is observed, although there is always room to strengthen compliance.
5. Party Satisfaction: An average of 3.7 reflects favorable overall satisfaction, but it is crucial to address areas where deficiencies were identified.

The main results of this study suggest that, in general terms, government audit has a positive impact on quality management, with a final mean of 3.6 indicating a favorable evaluation of the efficiency of the process. However, the result of 1.5 in terms of transparency and communication indicates the need to make substantial improvements in these aspects. The interpretation of these findings indicates that, although the audit process can be considered efficient, the limited perception of transparency can affect the credibility and confidence in the audit results. This phenomenon could be explained by a lack of adequate channels to communicate the findings or by the complexity of the information presented, which could make it difficult for stakeholders to fully receive and understand the results. Thus, it is critical to establish mechanisms that promote more effective and accessible communication of audit findings. When comparing our results with previous research, we found that other studies have also reported similar levels of efficiency satisfaction (average of 3.5) and regulatory compliance (4.0), which reinforces the idea that government audits are aligned with regulatory expectations. and quality. However, some studies have highlighted the negative relationship between lack of transparency and public trust in the administration, which supports our recommendation to focus on the disclosure of audited information to build a higher level of credibility.

However, the study has certain limitations, such as possible subjectivity in scoring satisfaction and perceived transparency, which could vary depending on the individual experiences of the respondents. Additionally, the generalizability of these results could be restricted by the sample size and demographics of respondents, who may not represent all of the stakeholders. These limitations should be carefully considered when interpreting the results and designing future research. The implications of these observations are significant for future research and practice in the field of government auditing. Additional studies are required that explore factors that influence perceptions of transparency and communication, as well as practical interventions that focus on improving audit disclosure. It is also essential to encourage stakeholder participation in the audit process, which could increase acceptance and positive perception of these processes. On the other hand, anomalous results were identified in the way in which stakeholders perceive the impact of audits on management (3.5), suggesting that, although there is a positive assessment, they may not be consistently perceived as transformative. These discrepancies may be due to lack of information on follow-up to audit recommendations or poor implementation of audit recommendations in specific areas. Recognizing these anomalies is crucial, as it points to an area of study that needs additional attention. These data indicate that, although government audit presents a solid foundation for quality management, it is imperative to address deficiencies in transparency and communication to maximize its impact and effectiveness. This study provides both a critical evaluation and a clear direction for future research and actions in the field of public audit.

4. Conclusion

The findings of this study reveal that government auditing is viewed generally positively, earning a process efficiency rating of 3.6 out of 5. This indicates that, although the process is considered to be working well, there is a clear need to improve aspects such as transparency and communication, which barely achieve an average of 1.5. These results underscore the importance of making changes that not only optimize the performance of the audit, but also strengthen the trust and understanding of all parties involved in the results. From a practical perspective, what we have found is key for those in charge of public management and government auditors. They can use this information to design policies and strategies that improve communication and information dissemination, which could foster greater stakeholder acceptance and participation in the audit process. Improving communication is essential to build credibility in public institutions and so that the quality of their management is perceived.

This study also makes important contributions to the field of public auditing. On the one hand, it offers a framework for evaluating the efficiency of the process and how it impacts quality management. Additionally, by highlighting the relationship between transparency, communication, and public trust, it opens doors for future research that delves into these connections. However, it is essential to recognize that this study has limitations. The subjectivity in the responses of the respondents and the diversity in their experiences could have influenced the evaluation of satisfaction and the perception of transparency. Likewise, the size and composition of the sample could restrict the possibility of generalizing the results to other situations. These limitations highlight the importance of conducting a more comprehensive analysis in future research. To investigate further, it would be useful to employ complementary methods, such as qualitative and mixed approaches, that offer deeper insights into how transparency and communication are perceived in government audit. Additionally, it would be beneficial to expand the study to various jurisdictions and contexts, which would help validate and generalize the findings. It is also crucial to pay attention to how audit recommendations are implemented and their impact on management, as this monitoring represents a key area of exploration for the advancement of research on this topic.

References

[1]. Bazan, (2022). Quality management policy. Central Admission Office (OCA). Available at: https://admision.unmsm.edu.pe/portal/wp-content/uploads/2022/10/OCA.UPLAC_.DSGC-POLITICA-DE-GESTION-DE-LA-CALIDAD-V.5.pdf .

[2]. Barbarán, G. (2015). Government audit and its impact on the development of the management of public entities in Peru. Thesis to obtain the academic degree of doctor in accounting and finance. University of San Martin de Porres, Faculty of accounting, economic and financial sciences, postgraduate section.

[3]. Cabrera, S., Erazo, J., Narváez, I., & Rodríguez, R. (2021). Internal Control in the Administrative Management of the Deconcentrated Units of the Executive. SCIENCEMATRIA, vol. 7, no. 12, pp. 696-724. Available at: https://dialnet.unirioja.es/servlet/articulo?codigo=7915377.

[4]. Cacao, D. (2022). Systematization of specific planning in the government audit of the State Comptroller General, for the year 2022. Qualification work to obtain the academic degree of Master in Accounting and Auditing. Santa Elena Peninsula State University, Postgraduate Institute.

[5]. Callupe, D. (2021). Simultaneous control as a new approach to government auditing, in the management of investment projects, in the provincial municipalities of the department of Pasco, 2018. Daniel Alcides Carrión National University. Available at: http://repositorio.undac.edu.pe/handle/undac/2236.

[6]. CONGRESS OF THE REPUBLIC. (2002). Law 27785, Organic Law of the National Control System and the Comptroller General of the Republic. Lima, July 22, 2002.

[7]. Smarandache, F. (2022). Introducción a la Super-Hiper-Álgebra y la Super-Hiper-Álgebra Neutrosófica. Neutrosophic Computing and Machine Learning, 20, 1–6.

[8]. Gamboa-Cruzado, J., Lima Bendezú, M., Cruzado Viera, M., Núñez Meza, A., & Núñez, E. D. (2024). Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of M-Learning as a Quechua Learning Instrument. Neutrosophic Sets and Systems, 71(1), 3.

[9]. Smarandache, F., & Al-Tahan, M. (2022). NeutroAlgebra and AntiAlgebra Are Generalizations of Classical Algebras. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 1-10). IGI Global.

[10]. González-Caballero, E., Leyva-Vázquez, M. Y., Batista-Hernández, N., & Smarandache, F. (2024). NeutroGeometry and Fractal Geometry. Neutrosophic Sets and Systems, 71(1), 2.

[11]. Rosli, SNIB, & Zulkifly, MIEB (2023). A Neutrosophic Approach for B-Spline Curve by Using Interpolation Method. Neutrosophic Systems With Applications , 9 , 29-40.

[12]. Pramanik, S. (2022). Single-valued neutrosophic set: An overview. Transdisciplinarity, 563-608.

[13]. Ranulfo Paiva Barbosa (Sobrinho), & Smarandache, F. (2023). Pura Vida Neutrosophic Algebra. Neutrosophic Systems With Applications, 9, 101-106.

[14]. Yang, W., Cai, L., Edalatpanah, S. A., & Smarandache, F. (2020). Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement. Symmetry, 12(4), 588.

[15]. Chakraborty, A., Mondal, S. P., Ahmadian, A., Senu, N., Alam, S., & Salahshour, S. (2018). Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications. Symmetry, 10(8), 327.

[16]. Wang, Q., Huang, Y., Kong, S., Ma, X., Liu, Y., Das, S. K., & Edalatpanah, S. A. (2021). A novel method for solving multiobjective linear programming problems with triangular neutrosophic numbers. Journal of mathematics, 2021(1), 6631762.

[17]. Smarandache, F. (2020). NeutroAlgebra is a Generalization of Partial Algebra. International Journal of Neutrosophic Science, 2(1), 08-17.
[18]. Caballero, E. G., Leyva, M., Ricardo, J. E., & Hernández, N. B. (2022). NeutroGroups Generated by Uninorms: A Theoretical Approach. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 155-179). IGI Global.
[19]. Ramos-Carpio, J. P., Machado, O. J. A., Ricardo, J. E., & Vasquez, A. B. M. (2024). Assessing Higher Education's Role in Personality Formation Using NeutroAlgebra. Neutrosophic Sets and Systems, 71, 50-57.
[20]. Castillo, M. R. D., Herrera, R. A. F., Berti, L. A. C., & Duche, O. F. V. (2022). Use of NeutroAlgebra for the Analysis of the Poor State of the Social Rehabilitation System in Babahoyo. Neutrosophic Sets and Systems, 52, 165-172.
[21]. Singh, P. K. (2022). NeutroAlgebra and NeutroGeometry for Dealing the Heteroclinic Patterns. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 90-102). IGI Global.
[22]. Oporto, P. S. V., Morán, N. E. M. P., Hancco, S. M. M., & Chan, A. L. (2024). Integration of NeutroAlgebra and Neutrosophic 2-Tuple Linguistic Likert Scales for Feasibility Assessment in Industrial Projects. Neutrosophic Sets and Systems, 71, 67-74.
[23]. Caballero, E. G., Smarandache, F., & Leyva Vázquez, M. (2019). On neutrosophic offuninorms. Symmetry, 11(9), 1136.
[24]. Hernández, N. B., Caballero, E. G., Cruzaty, L. E. V., Chávez, W. O., Huarac, C. F. P., & Chamorro, S. L. C. (2022). Theoretical study of the NeutroAlgebra generated by the combining function in Prospector and some pedagogical notes. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 116-140). IGI Global.

Received: July 16, 2024. Accepted: September 20, 2024