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Lagrange-Integrated GRA Model for Urban Landscape Sculpture Site Selection with Neutrosophic Decision-Making

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Abstract: Urban landscape sculpture plays a crucial role in preserving cultural heritage, enhancing city vitality, and reflecting the lives of urban citizens. With the continued development of landscape sculpture in modern urban environments, the process of site selection has become increasingly important. A well-chosen location can establish a positive interaction between sculptures and urban activities. Traditionally, site selection methods have been largely theoretical, lacking quantitative foundations, which made practical application challenging for designers. To address this, site selection for urban landscape sculptures can be approached as a multiple-attribute decision-making (MADM) problem. Grey Relational Analysis (GRA) is an effective method for handling MADM. Additionally, the probabilistic simplified Neutrosophic set (PSNS) is a useful tool for expressing uncertainty during site selection evaluations. This study proposes an optimization technique using the Lagrange function to determine attribute weights, along with the PSNN-GRA technique for MADM. A numerical example of site selection for urban landscape sculptures is used to validate the PSNN-GRA method through comparative analysis.

Keywords: Multiple-attributes decision-making (MADM); probabilistic simplified neutrosophic sets (PSNSs); GRA technique; site selection evaluation

1. Background

Landscape sculpture can play a certain role in shaping the image of a city. Therefore, at the beginning of design, landscape sculpture should take the city's image as an important reference, and the expression of landscape sculpture should be adapted to urban culture, economy, and environmental factors [1]. Firstly, focus on the personalized display of urban images. In the design stage of landscape sculpture, it is necessary to understand the profound cultural background of the city. Only by mastering the historical and cultural characteristics of the city can landscape sculpture design stand out and showcase different images that are different from other cities. Secondly, showcases urban culture and vitality [2]. The impact of landscape sculpture on the promotion and promotion of urban image is immeasurable. To design landscape sculptures that are more in line with the city's image, it is necessary to add urban cultural elements to the design to present the spiritual connotation and inexhaustible vitality of the city. Thirdly, make reasonable use of the natural environment. Landscape sculpture design needs to consider environmental factors such as urban climate and geographical location, and its shape and color presentation should be adapted to the urban environment [3]. At the same time, landscape sculpture design should consider the surrounding natural environment and not cause damage to the original natural environment. The application of urban landscape sculpture started relatively late, and there is limited experience in site selection planning for landscape sculpture [4]. Compared with the speed of urban development, the site selection planning for urban landscape sculpture is relatively lagging. In practice, it mainly draws on some foreign experiences in site selection planning for urban landscape sculpture. Due to differences in urban culture, history, development positioning, and functions, it is not advisable to fully draw on the site selection planning experience of others [5]. If planning is not based on the characteristics and needs of the city itself, urban landscape sculpture will find it difficult to play its functions and cannot meet the cultural and spiritual needs of urban landscape architecture. Urban landscape sculpture site selection planning is a process of controlling the relationship between landscape sculpture and urban spatial environment. The quality of site selection planning is related to whether the relationship between landscape sculpture and urban spatial environment is reasonable. The urban spatial environment mainly includes the material environment and the cultural environment. As the carrier space of urban landscape sculpture, the sculpture should be appropriately selected and integrated into the urban spatial environment. As a part of the urban spatial environment, it should coordinate and unify the urban landscape sculpture with the spatial environment [6]. Therefore, before planning the location of urban landscape sculpture, it is necessary to clarify the relationship between the two. The urban spatial environment is the carrying space of landscape sculpture, therefore, urban landscape sculpture acts on the spatial environment [7]. A qualified landscape sculpture site selection plan can create rich urban spatial environmental significance, and fully reflect the spiritual quality and cultural connotations of the urban area. Landscape sculpture is arranged in the form of points, lines, and surfaces in the urban spatial environment, playing a certain role in decorating the urban spatial environment. Therefore, in the overall layout planning process, more attention is paid to urban cultural elements and, the reasonable layout of landscape sculpture, shaping the humanistic culture, historical culture, and spiritual culture of the city, thereby improving the quality of the urban spatial environment[8]. Secondly, through the interpretation of urban landscape sculpture, one can understand urban customs, historical processes, etc., thereby understanding the connotation of urban spatial environment. Therefore, urban landscape sculpture not only originates from the urban spatial environment but also acts on the urban spatial environment [9]. There is a certain constraint relationship between urban spatial environment and landscape sculpture. Firstly, it restricts the site selection and design of urban landscape sculpture. The site selection of urban landscape sculptures should be in line with the functions of urban space. Different functions and forms of urban space result in different locations for landscape sculpture site selection. For example, urban scenic spots are divided into cultural and natural scenic spots, which are further divided into historical and cultural scenic spots, religious and cultural scenic spots, and classical garden scenic spots. The landscape sculptures set up in historical and cultural scenic spots should mainly be sculptures related to the corresponding historical and cultural aspects of the scenic spot [10]. Through the interpretation and appreciation of the sculptures, people can understand the relevant historical and cultural aspects, and thus play the cultural function of the scenic spot. Secondly, the urban spatial environment constrains the color, scale, and theme of landscape sculpture. These elements of landscape sculpture are limited by the characteristics of the urban spatial environment. The color of landscape sculpture should be unified with the urban spatial environment, the scale should be coordinated with the urban spatial environment, and the theme should be consistent with the cultural characteristics of the urban spatial environment.

MADM is a decision-making process that prioritizes alternative solutions in line with existing decision information in a certain way[11, 12]. Its theory and models are widely used in different fields such as venture capital decision-making, project evaluation, and industrial sector development evaluation [13, 14]. In recent decades, MADM has been widely applied to engineering, the economy, technology, and the military. The site selection evaluation for urban landscape sculpture is a MADM. Recently, the GRA model [15-18] was employed and recommended the MADM. The

PSNSs [19] are employed as a technique for characterizing uncertain information during the site selection evaluation for urban landscape sculpture. In this study, the PSNN-GRA model is recommended to solve the MADM with PSNSs. Finally, a numerical study for site selection evaluation for urban landscape sculpture is recommended to validate the PSNN-GRA model.

The study is driven by four key motivations:

(1) Extension of the GRA model to PSNSs: Traditional GRA models are limited in handling uncertainty. By extending GRA to PSNSs, the study aims to improve the ability to manage uncertain and imprecise information during the decision-making process. This enhancement provides a more accurate and comprehensive evaluation of complex scenarios.

(2) Optimization of attribute weights: A new optimization technique based on the Lagrange function is introduced to calculate attribute weights objectively. This method ensures that the weights assigned to different criteria in the decision-making process are more balanced and reflective of the actual importance of each attribute.

(3) Development of the PSNN-GRA model: The PSNN-GRA model is proposed as a solution to MADM problems using PSNSs. This model offers a robust and flexible framework for decision-making, particularly in situations characterized by high levels of uncertainty, making it a valuable tool for complex evaluations.

(4)Validation through a numerical study: To verify the effectiveness of the PSNN-GRA model, a numerical example related to site selection for urban landscape sculptures is conducted. Comparative analysis is used to demonstrate the performance of the approach, highlighting its practical applications in real-world decision-making scenarios.

The overall structure is as follows: The PSNSs are discussed in Sect. 2. The PSNN-GRA model is introduced as the recommended MADM method in Sect. 3. A case study for urban landscape sculpture site selection is presented in Sect. 4, demonstrating the PSN-GRA technique along with a sensitivity analysis. Sect. 5 concludes the paper.

2. Preliminary Remarks

Wang et al. [20] recommended the SVNSs

Definition 1 [20]. The SVNSs are recommended:

$$V=\left\{\left(\theta,T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)\right)\middle|\theta\in\Theta\right\} \quad (1)$$

where $T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)$ is membership, indeterminacy-membership and falsity-membership, $T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)\in\left[0,1\right]$, $0\le T\left(\theta\right)+I\left(\theta\right)+F\left(\theta\right)\le 3$.

Altun, Sahin and Guler [19] recommended the PSNSs.

Definition 2[19]. The PSNSs is recommended:

$$PV=\left\{\left(\theta,T\left(\theta\right)\left(PT\left(\theta\right)\right),I\left(\theta\right)\left(PI\left(\theta\right)\right),F\left(\theta\right)\left(PF\left(\theta\right)\right)\right)\middle|\theta\in\Theta\right\} \quad (2)$$

where $T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)$ is truth-membership, indeterminacy-membership and falsity-membership, $T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)\in\left[0,1\right]$, $0\le T\left(\theta\right)+I\left(\theta\right)+F\left(\theta\right)\le 3$, $0\le PT\left(\theta\right),PI\left(\theta\right),PF\left(\theta\right)\le 1$, the $PT\left(\theta\right),PI\left(\theta\right),PF\left(\theta\right)$ is possibility values of $T\left(\theta\right),I\left(\theta\right),F\left(\theta\right)$. The probabilistic simplified neutrosophic number (PSNN) is listed as $PV=\left(T\left(PT\right),I\left(PI\right),VF\left(PF\right)\right)$.

Definition 3[19]. Let $PV_1 = \left(T_1\left(PT_1\right), I_1\left(PI_1\right), F_1\left(PF_1\right)\right)$, $PV_2 = \left(T_2\left(PT_2\right), I_2\left(PI_2\right), F_2\left(PF_2\right)\right)$, the basic operations are recommended:

(1) $$PV_1 \oplus PV_2 = \begin{pmatrix} T_1 + T_2 - T_1 \cdot T_2 \left(2!\dfrac{PT_1 \cdot PT_2}{PT_1 + PT_2}\right), \\ I_1 \cdot I_2 \left(2!\dfrac{PI_1 \cdot PI_2}{PI_1 + PI_2}\right), F_1 \cdot F_2 \left(2!\dfrac{PF_1 \cdot PF_2}{PF_1 + PF_2}\right) \end{pmatrix};$$

(2) $$PV_1 \otimes PV_2 = \begin{pmatrix} T_1 \cdot T_2 \left(2!\dfrac{PT_1 \cdot PT_2}{PT_1 + PT_2}\right), \\ I_1 + I_2 - I_1 \cdot I_2 \left(2!\dfrac{PI_1 \cdot PI_2}{PI_1 + PI_2}\right), \\ F_1 + F_2 - F_1 \cdot F_2 \left(2!\dfrac{PF_1 \cdot PF_2}{PF_1 + PF_2}\right) \end{pmatrix};$$

(3) $\lambda PV = \left(1-\left(1-T\right)^{\lambda}\left(PT\right), \left(I\right)^{\lambda}\left(PI\right), \left(F\right)^{\lambda}\left(PF\right)\right), \lambda > 0;$

(4) $\left(PV\right)^{\lambda} = \left(\left(T\right)^{\lambda}\left(PT\right), 1-\left(1-I\right)^{\lambda}\left(PI\right), 1-\left(1-F\right)^{\lambda}\left(PF\right)\right), \lambda > 0.$

Definition 4[19]. Let $PV_1 = \left(T_1\left(PT_1\right), I_1\left(PI_1\right), F_1\left(PF_1\right)\right)$ and $PV_2 = \left(T_2\left(PT_2\right), I_2\left(PI_2\right), F_2\left(PF_2\right)\right)$, the probabilistic simplified Neutrosophic number Logarithmic distance (PSNNLD) between $PV_1 = \left(T_1\left(PT_1\right), I_1\left(PI_1\right), F_1\left(PF_1\right)\right)$ and $PV_2 = \left(T_2\left(PT_2\right), I_2\left(PI_2\right), F_2\left(PF_2\right)\right)$ is recommended:

$$PSNNLD\left(PV_1, PV_2\right) = \frac{1}{3}\begin{pmatrix} \left(T_1 \times PT_1\right)\log\dfrac{\left(T_1 \times PT_1\right)}{\dfrac{\left(T_1 \times PT_1\right)+\left(T_2 \times PT_2\right)}{2}} + \left(T_2 \times PT_2\right)\log\dfrac{\left(T_2 \times PT_2\right)}{\dfrac{\left(T_1 \times PT_1\right)+\left(T_2 \times PT_2\right)}{2}} \\ +\left(I_1 \times PI_1\right)\log\dfrac{\left(I_1 \times PI_1\right)}{\dfrac{\left(I_1 \times VI_1\right)+\left(I_2 \times PI_2\right)}{2}} + \left(I_2 \times PI_2\right)\log\dfrac{\left(I_2 \times PI_2\right)}{\dfrac{\left(I_1 \times PI_1\right)+\left(I_2 \times PI_2\right)}{2}} \\ +\left(F_1 \times PF_1\right)\log\dfrac{\left(F_1 \times PF_1\right)}{\dfrac{\left(F_1 \times PF_1\right)+\left(F_2 \times PF_2\right)}{2}} + \left(F_2 \times PF_2\right)\log\dfrac{2\left(F_2 \times PF_2\right)}{\dfrac{\left(F_1 \times PF_1\right)+\left(F_2 \times PF_2\right)}{2}} \end{pmatrix} \quad (3)$$

3. PSN-GRA method for MADM problem with Lagrange function

The PSNN-GRA is recommended for MADM under PSNSs in light of the extended GRA method[21, 22] with a completely unknown weight in line with the Lagrange function. Let be alternatives, and $VG = \left\{VG_1, VG_2, \cdots, VG_n\right\}$ be attributes with weight vw, where $vw_j \in \left[0,1\right]$,

$\sum_{j=1}^{n} vw_j = 1$. Suppose that the decision information is depicted with PSNNs $VR = (VR_{ij})_{m\times n} = \left(T_{ij}\left(PT_{ij}\right), I_{ij}\left(PI_{ij}\right), F_{ij}\left(PF_{ij}\right)\right)_{m\times n}$. Then, the PSNN-GRA technique is recommended MADM with Lagrange function (See Figure 1).

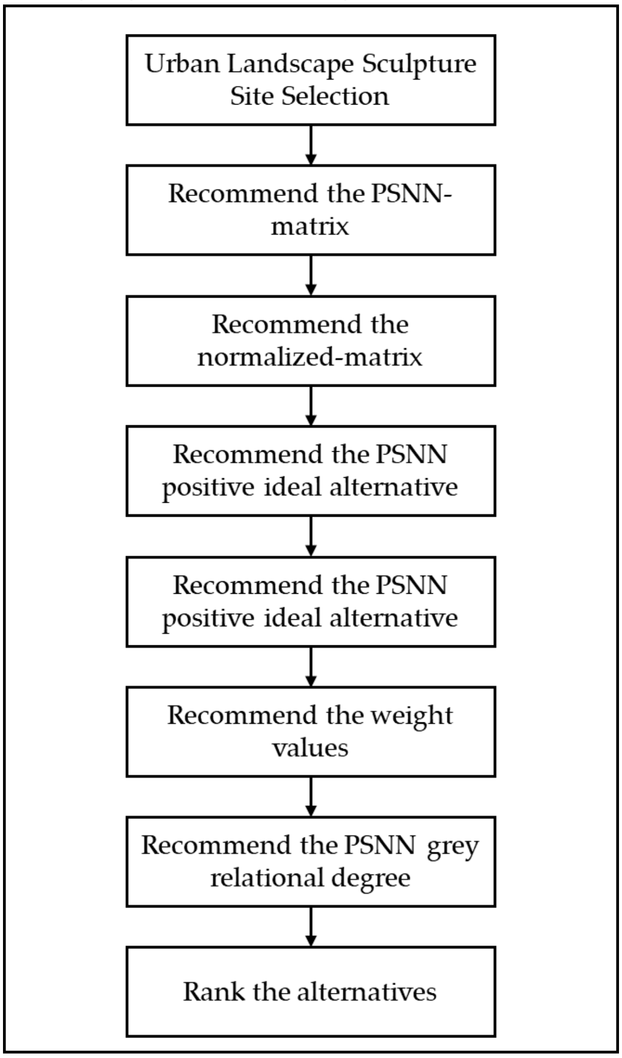

Figure 1. PSNN-GRA framework for MADM with Lagrange function

Step 1. Recommend the PSNN-matrix $VR = (VR_{ij})_{m\times n} = \left(T_{ij}\left(PT_{ij}\right), I_{ij}\left(PI_{ij}\right), F_{ij}\left(PF_{ij}\right)\right)_{m\times n}$.

Step 1. Recommend the normalized matrix $NVR = \left[NVR_{ij}\right]_{m\times n}$ based on $VR = (VR_{ij})_{m\times n}$.

$$NVR_{ij} = \left(T_{ij}^N\left(PT_{ij}^N\right), I_{ij}^N\left(PI_{ij}^N\right), F_{ij}^N\left(PF_{ij}^N\right)\right)$$
$$= \begin{cases} \left(T_{ij}\left(PT_{ij}\right), I_{ij}\left(PI_{ij}\right), F_{ij}\left(PF_{ij}\right)\right), & VG_j \text{ is the benefit attributes} \\ \left(F_{ij}\left(PF_{ij}\right), 1-I_{ij}\left(1-PI_{ij}\right), T_{ij}\left(PT_{ij}\right)\right), & VG_j \text{ is the cost attributes} \end{cases} \quad (4)$$

Step 2. Recommend the PSNN positive ideal alternative (PSNNPIA) [23]:

$$PSNNPIA = \left\{PSNNPIA_j\right\}, \; j = 1, 2, \ldots, n. \quad (5)$$

$$PSNNPIA_j = \left(T_j^N\left(PT_j^N\right), I_j^N\left(PI_j^N\right), F_j^N\left(PF_j^N\right)\right), j = 1, 2, \ldots, n. \tag{6}$$

$$\begin{aligned} & SV\left(T_j^N\left(PT_j^N\right), I_j^N\left(PI_j^N\right), F_j^N\left(PF_j^N\right)\right) \\ & = \max_i SV\left(T_{ij}^N\left(PT_{ij}^N\right), I_{ij}^N\left(PI_{ij}^N\right), F_{ij}^N\left(PF_{ij}^N\right)\right) \end{aligned} \tag{7}$$

where

$$\begin{aligned} & SV\left(T_j^N\left(PT_j^N\right), I_j^N\left(PI_j^N\right), F_j^N\left(PF_j^N\right)\right) \\ & = \frac{T_j^N \cdot PT_j^N + I_j^N \cdot PI_j^N + F_j^N \cdot PF_j^N}{3} \end{aligned} \tag{8}$$

$$\begin{aligned} & SV\left(T_{ij}^N\left(PT_{ij}^N\right), I_{ij}^N\left(PI_{ij}^N\right), F_{ij}^N\left(PF_{ij}^N\right)\right) \\ & = \frac{T_{ij}^N \cdot PT_{ij}^N + I_{ij}^N \cdot PI_{ij}^N + F_{ij}^N \cdot PF_{ij}^N}{3} \end{aligned} \tag{9}$$

Step 3. Recommend the PSNN grey relational coefficients (PSNNGRC) from PSNNPIA:

$$GRC\left(\xi_{ij}^{+}\right) = \frac{\left(\min\limits_{1\le i\le m} PSNNLD\left(NVR_{ij}, PSNNPIA_j\right) + \rho \max\limits_{1\le i\le m} PSNNLD\left(NVR_{ij}, PSNNPIA_j\right)\right)}{\left(PSNNLD\left(NVR_{ij}, PSNNPIA_j\right) + \rho \max\limits_{1\le i\le m} PSNNLD\left(NVR_{ij}, PSNNPIA_j\right)\right)} \tag{10}$$

In generally, $\rho \in [0,1]$, usually $\rho = 0.5$.

Step 4. Recommend the weight values.

If the attribute weights are completely unknown, firstly, the weight should be obtained. Thus, the PSNN deviation sum (PSNNDS) in light with PSNNGRC is recommended:

$$PSNNDS_i\left(vw\right) = \sum_{j=1}^{n}\left[\left(1 - GRC\left(\xi_{ij}^{+}\right)\right)vw_j\right]^2 \tag{11}$$

The multiple-objective optimization technique (MOOT-1) is recommended for the weight values:

$$\left(\text{MOOT-1}\right)\begin{cases} \min PSNNDS_i\left(vw\right) = \sum\limits_{j=1}^{n}\left[\left(1 - GRC\left(\xi_{ij}^{+}\right)\right)vw_j\right]^2, \\ \sum\limits_{j=1}^{n} vw_j = 1 \end{cases} \tag{12}$$

Because every alternative is the same, there is no preference for these alternatives. Then, the MOOT-1 is recommended the same weights in the single-objective optimization technique (SOOT-1):

$$\left(\text{SOOT-1}\right)\begin{cases} \min PSNNDS\left(vw\right) = \sum\limits_{i=1}^{m} PSNNDS_i\left(vw\right) \\ = \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\left[\left(1 - GRC\left(\xi_{ij}^{+}\right)\right)vw_j\right]^2 \\ s.t.\ \sum\limits_{j=1}^{n} vw_j = 1 \end{cases} \tag{13}$$

To cope up with the SOOT-1 technique, the Lagrange function is recommended:

$$LF\left(vw,\lambda\right)=\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(1-GRC\left(\xi_{ij}^{+}\right)\right)vw_j\right]^2+2\lambda\left(\sum_{j=1}^{n}vw_j-1\right)\right) \quad (14)$$

where λ is the Lagrange multiplier.

Differentiating Eq. (12) forward $vw_j\ (j=1,2,\cdots,n)$ and λ, and supposing that these partial derivatives are zero, the weight values are obtained below:

$$vw_j=\frac{\left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(1-GRC\left(\xi_{ij}^{+}\right)\right)^2\right)^{-1}\right]^{-1}}{\sum_{i=1}^{m}\left(1-GRC\left(\xi_{ij}^{+}\right)\right)^2} \quad (15)$$

Step 5. Recommend the grey relational degree (GRD) from PSNNPIA:

$$PSNNGRD\left(\xi_i^{+}\right)=\sum_{j=1}^{n}vw_jGRC\left(\xi_{ij}^{+}\right) \quad (16)$$

Step 6. In light with $GRD\left(\xi_i^{+}\right)$. The larger $GRD\left(\xi_i^{+}\right)$, the better the alternative is.

4. Case Study and Sensitivity Analysis of Urban Sculpture Site Selection Using Neutrosophic Decision-Making

In this section, we conduct a case study and sensitivity analysis to explore the application of Neutrosophic decision-making for urban sculpture site selection. We examine how different decision-making methods and criteria weights influence the ranking of potential sites, providing insights into the robustness of the proposed approach.

4.1. Case study

With the continuous development of China's economy and the acceleration of urbanization, people's production and living standards have been improving year by year, and their pursuit of quality of life has also become increasingly high. As for urban landscape sculpture, it has already become a part of people's daily lives. On the one hand, urban landscape sculptures meet people's emotional needs for the city they rely on for their livelihood; On the other hand, urban landscape sculptures also carry the cultural history of a city. However, at present, there are some problems in the construction of urban landscape sculptures. Some sculptures cannot truly reflect the historical and cultural characteristics of the city, and there is a sense of inconsistency with the local cultural customs. In addition, a single design cannot meet the diverse development of the current era, and it lacks artistic and aesthetic sense. These problems need to be solved. Landscape sculpture is a unique way of presenting urban culture. When appreciating landscape sculpture, people associate it with urban culture and can understand and appreciate it through landscape sculpture. Landscape sculpture design usually presents urban culture in a strong artistic form. Therefore, landscape sculpture design helps to promote the enhancement of urban cultural image. On the one hand, every city has a long history, and a unique urban culture has been formed in the continuous development of cities. Urban culture is an important reference for landscape sculpture design, conveying specific cultural connotations. Therefore, landscape sculpture design has a certain cultural color and has become an important form of showcasing urban culture. On the other hand, landscape sculpture serves as a business card to showcase the cultural image of a city, which also includes the historical connotation of the city. Through landscape sculpture, people can understand the historical customs and changes of the city. Landscape sculpture, like other art or cultural industries, is a product of urban development to a certain stage. The level and quality of landscape sculpture are symbols of a city's comprehensive strength in economy, culture, and other aspects. Only when the urban economy is mature and stable can public cultural construction be gradually promoted to create a good urban cultural atmosphere. The role of landscape sculpture in promoting urban economic image is mainly reflected in its ability to enhance urban attractiveness. Firstly, landscape sculpture can showcase a well-developed urban image, thereby attracting more investment and talent, promoting local economic development, and providing many driving forces for urban economic development. Secondly, landscape sculpture can effectively promote the development of the urban tourism industry. Landscape sculpture itself is a tourism resource with direct economic benefits. Beautifully designed and meaningful landscape sculptures can bring people a profound artistic and cultural impression. Therefore, the artistic and cultural value of landscape sculpture can promote the development of the tourism industry and inject new vitality into urban economic development. At the same time, while promoting the development of the tourism industry, it can also generate income for tourism-related industries such as catering and transportation, drive economic construction in multiple fields of the city, and accelerate the pace of urban modernization. The site selection evaluation for urban landscape sculpture is MADM. In this work, the PSNN-GRA is recommended for site selection evaluation for urban landscape sculpture. Six urban landscape sculpture sites are assessed with 18 attributes as shown in Table 1.

Table 1. Criteria weights.

Criteria		Weights
C_1	Harmony with Surroundings	0.058909
C_2	Public Transportation Connectivity	0.057197
C_3	Operational Costs	0.051321
C_4	Community Engagement	0.055844
C_5	Safety Considerations	0.054315
C_6	Historical Significance	0.055856
C_7	Visibility and Prominence	0.060893
C_8	Environmental Compatibility	0.055626
C_9	Cost of Installation	0.051988
C_{10}	Event Hosting Capability	0.055785
C_{11}	Green Space Preservation	0.054823
C_{12}	Structural Stability	0.054775
C_{13}	Pedestrian Accessibility	0.057391
C_{14}	Weather Resistance	0.055549
C_{15}	Maintenance Requirements	0.053175
C_{16}	Potential Economic Benefits	0.05079
C_{17}	Scenic Value	0.058761
C_{18}	Parking Availability	0.057002

We begin by constructing the decision matrix that links the criteria and alternatives, as shown in Table 2. Next, we normalize the decision matrix, which is presented in Table 3. After that, we identify the positive ideal alternatives. Then, we calculate the criteria weights, as shown in Table 1. Following this, we compute grey relational scores. Finally, we rank the alternatives, as illustrated in Figure 2.

Table 2. The decision matrix.

	A_1	A_2	A_3	A_4	A_5	A_6
C_1	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C_2	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C_3	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C_4	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C_5	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
C_6	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}
C_7	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C_8	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C_9	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}

C10	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C11	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
C12	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}
C13	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C14	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C15	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C16	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C17	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C18	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
	A1	A2	A3	A4	A5	A6
C1	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C2	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C3	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C4	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C5	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C6	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}
C7	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C8	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C9	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C10	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C11	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
C12	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}
C13	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C14	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C15	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C16	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C17	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C18	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
	A1	A2	A3	A4	A5	A6
C1	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C2	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C3	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C4	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C5	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
C6	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C7	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C8	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C9	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C10	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C11	{0.26(0.5),0.39(0.2),0.54(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}
C12	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C13	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.95(0.3),0.86(0.3),0.19(0.4)}
C14	{0.95(0.3),0.86(0.3),0.19(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C15	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}
C16	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.26(0.5),0.39(0.2),0.54(0.3)}
C17	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}
C18	{0.43(0.3),0.81(0.4),0.32(0.3)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.34(0.4),0.53(0.1),0.84(0.5)}	{0.13(0.2),0.74(0.1),0.48(0.7)}	{0.86(0.4),0.16(0.2),0.65(0.4)}	{0.43(0.3),0.81(0.4),0.32(0.3)}

Table 3. The normalized decision matrix.

	A1	A2	A3	A4	A5	A6
C1	-0.17896	-1	-0.34699	-0.4153	0	-0.06967
C2	-0.09574	-1	-0.875	-0.92819	0	-0.9734
C3	-0.30769	-0.88718	0	-1	-0.88718	-0.8
C4	-0.03306	0	-0.92231	-1	-0.41157	-0.31901
C5	-1	0	-0.4782	-0.32771	-0.4782	-0.14627
C6	-1	-0.14925	-0.50959	-0.75906	0	-0.14712
C7	-0.04775	-0.94382	-1	0	-0.4382	-0.01966
C8	-0.06557	-0.8051	-0.59927	-0.66667	0	-1
C9	-0.02023	-0.67341	0	-0.69075	-1	-0.8526
C10	-0.10256	-0.27863	-0.40855	-1	-0.8	0
C11	-0.82017	-0.47395	-1	0	-0.30756	-0.31933
C12	-1	-0.14925	-0.89979	-0.75906	0	-0.14712
C13	-1	-0.16393	-0.35155	-0.4408	-0.6357	0
C14	0	-1	-0.30986	-0.45599	-0.64437	-0.64437
C15	0	-0.58333	-0.60109	-0.81967	-0.33333	-1
C16	0	-0.68223	-0.51881	-0.36706	-0.9144	-1
C17	-0.46753	-0.12987	-1	-0.60519	-0.42338	0
C18	-0.19262	-1	-0.58333	-0.31967	0	-0.35656

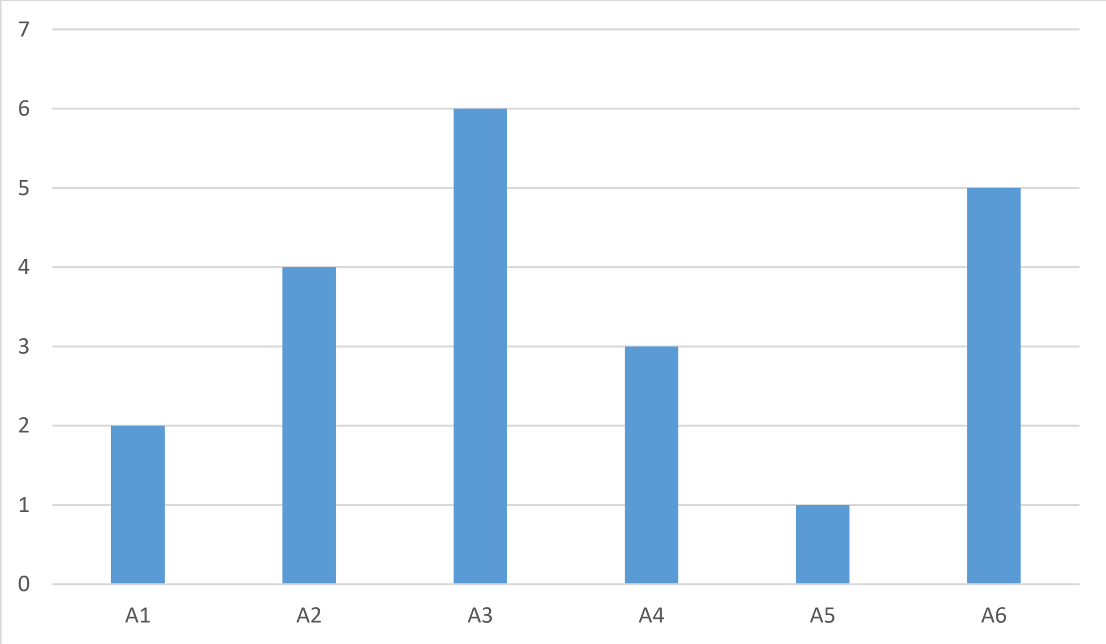

Figure 2. The rank of alternatives.

Table 4. The different criteria weights.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}	Y_{16}	Y_{17}	Y_{18}	Y_{19}
C_1	0.055556	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_2	0.055556	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_3	0.055556	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_4	0.055556	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_5	0.055556	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_6	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_7	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_8	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_9	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_{10}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_{11}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_{12}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118
C_{13}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118	0.054118
C_{14}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118	0.054118
C_{15}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118	0.054118
C_{16}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118	0.054118
C_{17}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08	0.054118
C_{18}	0.055556	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.054118	0.08

4.2. Sensitivity analysis

In real-life decision-making, different decision-makers often have different levels of risk tolerance, which can impact how the identification coefficient is chosen. This, in turn, may change the ranking of the alternatives. To address this, we perform a sensitivity analysis in this section. We adjust the criteria weights across 19 different cases, as shown in Table 4, and then apply the proposed method with these different weights. Finally, we rank the alternatives, as illustrated in Figure 3.

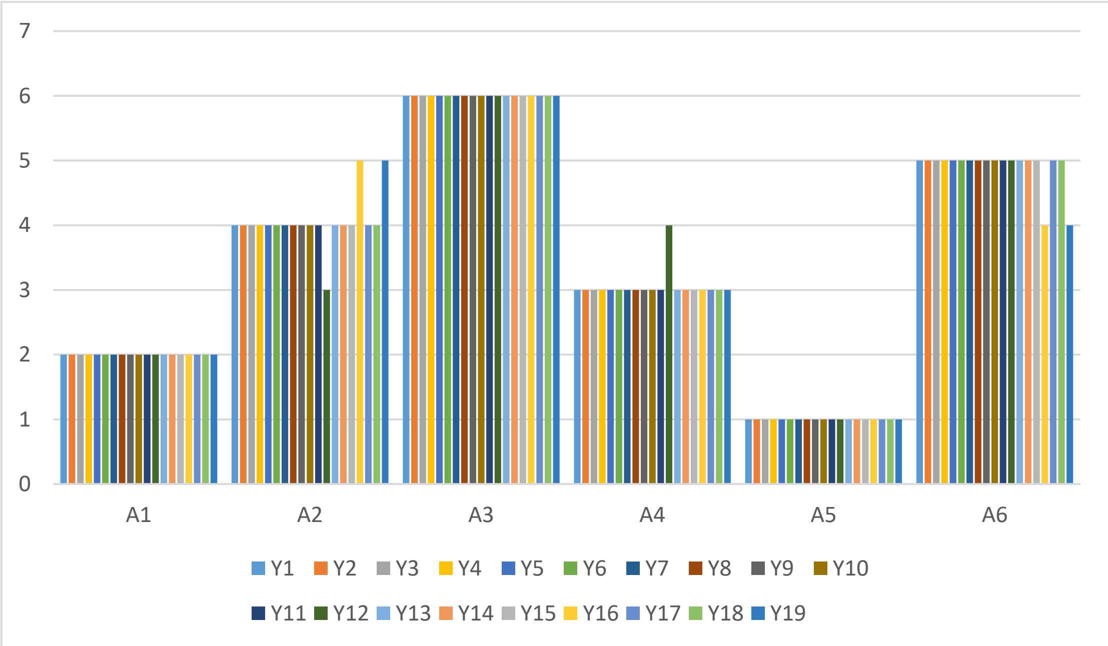

Figure 3. The rank of alternatives under different weights.

5. Conclusions and future research directions

In recent years, economic development has driven continuous progress in politics, culture, technology, and education, and the pace of construction of urbanization is also getting faster and faster. Urban landscape sculpture is an important component of urban construction, and the aesthetics contained in urban landscape sculpture symbolize the essence of urban culture on the one hand; On the other hand, it is also a representative of the city's landmark. At present, the integration of planning urban landscape sculptures and urban environment has become a hot topic of concern in urban construction. The site selection evaluation for urban landscape sculpture is viewed as MADM. In this study, the PSNN-GRA model is recommended to solve the MADM under PSNSs. Finally, a numerical study for site selection evaluation for urban landscape sculpture is recommended to validate the PSNN-GRA technique. The main research motivations of this study are recommended: (1) the GRA model are extended to PSNSs; (2) an optimization technique is recommended to obtain the attribute weights based on Lagrange function; (3) the PSNN-GRA model is recommended to solve the MADM with PSNSs; (4) Finally, a numerical study for site selection evaluation for urban landscape sculpture is recommended to validate the PSNN-GRA technique with comparative analysis.

Although this study provides an effective solution for urban landscape sculpture site selection through the PSNN-GRA method, there are still some research limitations and shortcomings. These limitations mainly lie in the complexity of the model, insufficient adaptability to dynamic environments, and the lack of consideration for social and public participation. To enhance the comprehensiveness of the research and its practical applicability, future studies can explore and improve in the following three directions. Thus, the research limitations are obtained: (1) Balance between model complexity and practical applicability: Although the PSNN-GRA method effectively deals with uncertainty in multi-attribute decision-making, its construction and computational process are relatively complex. This complexity, especially in large-scale decision problems, may pose operational challenges for designers and decision-makers. The intricate computational steps and high technical threshold may limit its widespread application in practical projects. (2) Lack of adaptability to dynamic environments: The proposed method in this study is primarily based on a static decision-making environment. However, in reality, urban environments and social activities are often in a state of dynamic change. The PSNN-GRA model does not account for the influence of time, making it less adaptable to long-term or continuously evolving decision-making scenarios, where flexibility is needed to respond to dynamic environmental changes. (3) Insufficient consideration of social and public participation: Although the study mentions that the site selection of sculptures should promote interaction with urban activities, the specific model lacks sufficient consideration of public participation and social feedback. Public opinions and preferences play a crucial role in actual site selection decisions, and neglecting these factors may lead to outcomes that are inconsistent with real needs, affecting the social acceptance and effectiveness of the sculpture.

Although the PSNN-GRA method proposed in this study offers significant advantages in urban landscape sculpture site selection, there is still room for improvement. Future research can focus on optimizing the model's complexity, adaptability to dynamic environments, and public participation to further enhance its practical applicability and decision-making effectiveness. These improvements aim to provide better decision support in more complex and dynamic real-world scenarios. Thus, future research directions are outlined:

(1) Simplifying model complexity and improving practical applicability

Given the complexity of the current model, future research can focus on simplifying the computational process of the PSNN-GRA method and developing more intuitive and user-friendly tools or algorithms to improve its practicality in real-world applications. By optimizing and simplifying the model, it can be applied in a wider range of scenarios, lowering the barrier to use and making it more accessible for non-expert users.

(2) Introducing dynamic decision-making models

Considering the dynamic nature of urban environments, future research can explore incorporating time factors into the decision-making model and develop methods that adapt to dynamic changes. For example, dynamic grey relational analysis or temporal neutrosophic sets could be integrated into the PSNN-GRA framework to handle the evolving urban environment and social needs, enhancing the model's applicability in long-term decision-making.

(3) Enhancing public participation and social feedback mechanisms

To better reflect the needs and preferences of the public, future research can integrate public participation mechanisms and incorporate public feedback into the decision-making process. By leveraging social feedback data, such as surveys or social media data, more inclusive decision-making models can be developed. This would improve the social acceptance of site selection decisions and ensure that urban landscape sculptures better serve the public, enhancing their overall effectiveness.

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