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Enhanced MADM Strategy with Heptapartitioned Neutrosophic Distance Metrics

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Abstract: In this work, a novel methodological approach to multi-attribute decision-making problems is developed and the notion of Heptapartitioned Neutrosophic Set Distance Measures (HNSDM) is introduced. By averaging the Penta-Partitioned Neutrosophic Distance Measures under particular conditional criteria, the suggested technique ranks the options. Additionally, we explore the fundamental characteristics of HNSDM and provide several illustrative theorems. The Heptapartitioned Neutrosophic Set Distance Measures are formulated based on the Hepta Neutrosophic Set. Lastly, a practical numerical example is provided to illustrate the efficacy of the suggested methodology.

Keywords: Heptapartitioned set, Heptapartitioned Neutrosophic set, Neutrosophic sine distance measure, Heptapartitioned Neutrosophic set and MADM strategy.

1. Introduction

Neutrosophic sets have emerged as a powerful tool for handling uncertainty, indeterminacy, and inconsistency in various fields of research [5]. In recent years, classical set theory has been insufficient for modeling complex uncertain systems [5]. Florentin Smarandache introduced the concept of Neutrosophic set theory, or Neutral knowledge, in 1998 [1, 4], provides a robust framework for handling indeterminacy and inconsistency [6]. Recent studies have explored neutrosophic extensions of distance measures, including sine distance measure (SDM) and Hepta Hypothesis Distance Measures (HHDM) [2]. Mostly we choose the leaders by voting i.e.., by election. Every election gives voters the choice to support candidate A, support opponent B, or abstain from voting altogether. The fraction of voters who choose not to cast ballots may determine the election's outcome by selecting options A through B; this is known as "neutral or indeterminist" [3].

The distance measure, which is used to examine the relationship between two distinct neutrosophic sets, is computed [3]. This research investigates the intersection of these concepts, proposing neutrosophic sine distance measures [NSDM] and Neutrosophic Hepta Hypothesis Distance Measure, Sine Distance Measures [SDM] quantify the similarity or dissimilarity between sets or vectors using the sine function [9], [10], [11], [12], [13], [14].

One of the most crucial clustering methods for figuring out how closely two things are related is the distance measure [11]. Medical diagnosis decision-making is developing into a broad area of study. Distance metrics have played an important role in decision science throughout the past few decades. Over the past few decades, numerous researchers have created a variety of distance metrics that are utilized in medical diagnostics [12].

There has been a lot of interest in hesitant fuzzy linguistic term sets (HELTSs) because of their special capacity and efficacy to convey ambiguity and uncertainty in the decision-making process. We can research and create various distance and similarity metrics for HFLTSs to improve their applicability [14]. A distance measure between intuitionistic fuzzy sets [IFSs] is defined axiomatically. We can calculate IFS's distance and similarity [15J]. Order is simply defined by signed-distance ranking, which allows us to describe ordering using both positive and negative values [21].

2. Preliminaries

Definition 1 [22]

Let X be a non-empty set. A Heptapartitioned neutrosophic set A having the form $A = \{(x, T_A(x), M_A(x), C_A(x), I_A(x), U_A(x), F_A(x), K_A(x): x \in X\}$, where $T_A(x), M_A(x), C_A(x), I_A(x), U_A(x), F_A(x), K_A(x) \in [0,1]$represent an absolute truth-membership (namely $T_A(x)$), a relative truth-membership (namely $M_A(x)$), a contradiction membership (namely $C_A(x)$), a ignorance-membership (namely $I_A(x)$), an unknown membership (namely $U_A(x)$), an absolute falsity-membership (namely $F_A(x)$) and a relative falsity membership (namely $K_A(x)$) respectively of each $x \epsilon X$ to the set A such that $0 \leq T_A(x) + M_A(x) + C_A(x) + I_A(x) + U_A(x) + F_A(x) + K_A(x) \leq 7$ for all $x \in X$.

The collection of all hepta-partitioned neutrosophic sets of X is represented as HNS(X).

Definition 2 [22]

Let $X = \{x_1, x_2, \ldots .. x_n\}$ be a distinct, limited set. If the following axioms are met, a mapping d: $NS(X) \times NS(X) \rightarrow [0,1]$ is considered a distance measure between two neutrosophic sets.

$(i) d(A, B) \geq 0$ for all $A, B \in NS(X)$.

$(ii) d(A, B) = 0$ is and only if $A = B$ for all $A, B \in NS(X)$.

$(iii) d(A, B) = d(B, A)$ for all $A, B \in NS(X)$.

(iv) If $A \subseteq B \subseteq C$ for all $A, B, C \in NS(X)$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.

Definition 3 [22]

The normalized hamming distance of two single-valued neutrosophic sets, A and B, is described as

$$d_1(A, B) = \frac{1}{3n} \sum_{j=1}^{n} \left(\left| \mu_A(x_j) - \mu_B(x_j) \right| + \left| \sigma_A(x_j) - \sigma_B(x_j) \right| + \left| \gamma_A(x_j) - \gamma_B(x_j) \right| \right).$$

Definition 4 [22]

Two single-valued neutrosophic sets A and B are separated by the normalized Euclidean distance, which is defined as

$$d_2(A,B)=\left\{\frac{1}{3n}\sum_{j=1}^{n}(\mu_A(x_j)-\mu_B(x_j))^2+(\sigma_A(x_j)-\sigma_B(x_j))^2+(\gamma_A(x_j)-\gamma_B(x_j))^2\right\}^{\frac{1}{2}}.$$

Definition 5 [23]

Two single-valued neutrosophic sets, A and B, have distance measures that are defined as

$$D_1(A,B)=\frac{1}{3n}\sum_{j=1}^{n}\left(\left|\mu_A(x_j)^2-\mu_B(x_j)^2\right|+\left|\sigma_A(x_j)^2-\sigma_B(x_j)^2\right|+\left|\gamma_A(x_j)^2-\gamma_B(x_j)^2\right|\right)$$

and

$$D_2(A,B)=\frac{1}{3n}\sum_{j=1}^{n}\left|\left(\mu_A(x_j)^2-\mu_B(x_j)^2\right)-\left(\sigma_A(x_j)^2-\sigma_B(x_j)^2\right)-\left(\gamma_A(x_j)^2-\gamma_B(x_j)^2\right)\right|$$

Definition 6 [18]

A mapping $S: NS(X)\times NS(X)\to[0,1]$ is considered a measure of similarity between two Neutrosophic sets if it satisfies the subsequent requirements:

$(i) S(A,B)\geq 0$ for all $A,B\in NS(X)$.

$(ii) S(A,B)=1$ if and only if $A=B$ for all $A,B\in NS(X)$.

$(iii) S(A,B)=S(B,A)$ for all $A,B\in NS(X)$.

(iv) If $A\subseteq B\subseteq C$ for all $A,B,C\in NS(X)$, then $S(A,C)\leq S(A,B)$ and $S(A,C)\leq S(B,C)$.

3. Heptapartitioned Neutrosophic Distance Measures

Definition 7

Let $X=\{x_1,x_2\dots\dots.x_n\}$ be a distinct, limited set. A mapping $d: HNS(X)*HNS(X)\to[0,1]$ is said if it meets the following criteria, it can be considered a Heptapartitioned neutrosophic distance measure between two Hepta partitioned neutrosophic sets:

1. $d(A,B)\geq 0$ for all $A,B\in HNS(X)$.

2. $d(A,B)=0$ if and only if $A=B$ for all $A,B\in HNS(X)$.

3. $d(A,B)=d(B,A)$ for all $A,B\in HNS(X)$.

4. If $A\subseteq B\subseteq C$ for all $A,B,C\in HNS(X)$, then $d(A,C)\geq d(A,B)$ and $d(A,C)\geq d(B,C)$.

Definition 8

Let $X = \{x_1, x_2 \ldots \ldots . x_n\}$ be a discrete confined set. Let $A = \{x_j, T_A(x_j), M_A(x_j), C_A(x_j), I_A(x_j), U_A(x_j), F_A(x_j), K_A(x_j): x_j \in X\}$ and $B = \{x_j, T_B(x_j), M_B(x_j), C_B(x_j), I_B(x_j), U_B(x_j), F_B(x_j), K_B(x_j): x_j \in X\}$ be two Heptapartitioned neutrosophic sets. Then for $i = 1,2,3,4,5,6,7$, define a mapping $d_i: HNS(X) * HNS(X) \to [0, 1]$ as

$$1. d_1(A,B) = \frac{1}{7n}\sum_{j=1}^{n}\Big(\left|T_A(x_j) - T_B(x_j)\right| + \left|M_A(x_j) - M_B(x_j)\right| + \left|C_A(x_j) - C_B(x_j)\right| + \left|I_A(x_j) - I_B(x_j)\right| + \left|U_A(x_j) - U_B(x_j)\right| + \left|F_A(x_j) - F_B(x_j)\right| + \left|K_A(x_j) - K_B(x_j)\right|\Big).$$

$$2. d_2(A,B) = \left\{\frac{1}{7n}\sum_{j=1}^{n}\left(\left(T_A(x_j) - T_B(x_j)\right)^2 + \left(M_A(x_j) - M_B(x_j)\right)^2 + \left(C_A(x_j) - C_B(x_j)\right)^2 + \left(I_A(x_j) - I_B(x_j)\right)^2 + \left(U_A(x_j) - U_B(x_j)\right)^2 + \left(F_A(x_j) - F_B(x_j)\right)^2 + \left(K_A(x_j) - K_B(x_j)\right)^2\right)\right\}^{1/2}.$$

$$3. d_3(A,B) = \frac{1}{7n}\sum_{j=1}^{n}\Big(\left|(T_A(x_j))^2 - (T_B(x_j))^2\right| + \left|(M_A(x_j))^2 - (M_B(x_j))^2\right| + \left|(C_A(x_j))^2 - (C_B(x_j))^2\right| + \left|(I_A(x_j))^2 - (I_B(x_j))^2\right| + \left|(U_A(x_j))^2 - (U_B(x_j))^2\right| + \left|(F_A(x_j))^2 - (F_B(x_j))^2\right| + \left|(K_A(x_j))^2 - (K_B(x_j))^2\right|\Big).$$

$$4. d_4(A,B) = \frac{1}{7n}\sum_{j=1}^{n}\Big(\left|(T_A(x_j))^2 - (T_B(x_j))^2\right| + \left|(M_A(x_j))^2 - (M_B(x_j))^2\right| + \left|(C_A(x_j))^2 - (C_B(x_j))^2\right| - \left|\left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2\right| - \left|\left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2\right| - \left|\left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2\right| - \left|(K_A(x_j))^2 - (K_B(x_j))^2\right|\Big).$$

5. $d_5(A,B)$

$$=\frac{7}{5n}\sum_{j=1}^{n}\frac{\left(\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\\sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}\right)}{1+\left(\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\\sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}\right)}$$

$$6.\ d_6(A,B)=\frac{7}{5n}\sum_{J=1}^{n}\frac{\left(\begin{array}{c}sin\left\{\frac{\pi}{6}\left(T_A(x_j)-T_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(M_A(x_j)-M_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(C_A(x_j)-C_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(I_A(x_j)-I_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(U_A(x_j)-U_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(F_A(x_j)-F_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(K_A(x_j)-K_B(x_j)\right)^2\right\}\end{array}\right)}{1+\left(\begin{array}{c}sin\left\{\frac{\pi}{6}\left(T_A(x_j)-T_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(M_A(x_j)-M_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(C_A(x_j)-C_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(I_A(x_j)-I_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(U_A(x_j)-U_B(x_j)\right)^2\right\}+sin\left\{\frac{\pi}{6}\left(F_A(x_j)-F_B(x_j)\right)^2\right\}\\+sin\left\{\frac{\pi}{6}\left(K_A(x_j)-K_B(x_j)\right)^2\right\}\end{array}\right)}$$

7. $d_7(A,B)$

$$=\frac{9}{7n}\sum_{j=1}^{n}\frac{\left(\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\\sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}\right)}{1+\left(\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}\\+\sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\\sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}\right)}$$

Theorem 1: A Hepta-partitioned neutrosophic measure of distance between two Hepta-partitioned Neutrosophic sets B and A is given for each i=1,2,3,4,5,d_i (A,B).

Proof: Only d_4 (A,B) is shown to be a Heptapartitioned neutrosophic distance measure here; the other d_i (A,B) have comparable arguments for i=1,2,5.

1. $d_4(A,B) \geq 0$ is trivially true from the definition of Heptapartitioned neutrosophic sets.

2. $d_4(A,B) = 0$ if and only if $\frac{1}{7n}\sum_{j=1}^{n}\Big(\left|(T_A(x_j))^2 - (T_B(x_j))^2\right| + \left|(M_A(x_j))^2 - (M_B(x_j))^2\right| + \left|(C_A(x_j))^2 - (C_B(x_j))^2\right| - \left|\left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2\right| - \left|\left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2\right| - \left|\left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2\right| - \left|(K_A(x_j))^2 - (K_B(x_j))^2\right|\Big) = 0,$

if and only if, for all $x_j \in X, \Big(\left|\left(T_A(x_j)\right)^2 - \left(T_B(x_j)\right)^2\right| + \left|\left(M_A(x_j)\right)^2 - \left(M_B(x_j)\right)^2\right| + \left|\left(C_A(x_j)\right)^2 - \left(C_B(x_j)\right)^2\right| - \left|\left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2\right| - \left|\left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2\right| - \left|\left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2\right| - \left|\left(K_A(x_j)\right)^2 - \left(K_B(x_j)\right)^2\right|\Big) = 0.$

if, for all $x_j \in X$, $\left(T_A(x_j)\right)^2 - \left(T_B(x_j)\right)^2 = 0;$ $\left(M_A(x_j)\right)^2 - \left(M_B(x_j)\right)^2 = 0;$

$$\left(C_A(x_j)\right)^2 - \left(C_B(x_j)\right)^2 = 0\,;\ \left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2 = 0;\ \left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2 = 0;\ \left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2 = 0;\ \left(K_A(x_j)\right)^2 - \left(K_B(x_j)\right)^2 = 0.$$

if and only if, $T_A(x_j) = T_B(x_j)\,;\ M_A(x_j) = M_B(x_j)\,;\ C_A(x_j) = C_B(x_j)\,;\ I_A(x_j) = I_B(x_j)\,;\ U_A(x_j) = U_B(x_j)\,;\ F_A(x_j) = F_B(x_j)\,;\ K_A(x_j) = K_B(x_j)$, for all $x_j \in X$, iff $A = B$.

$$3.\ d_4(A,B) = \frac{1}{7n}\sum_{j=1}^{n}\Bigg(\Bigg|\left(\left(T_A(x_j)\right)^2 - \left(T_B(x_j)\right)^2\right) + \left(\left(M_A(x_j)\right)^2 - \left(M_B(x_j)\right)^2\right) + \left(\left(C_A(x_j)\right)^2 - \left(C_B(x_j)\right)^2\right) - \left(\left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2\right) - \left(\left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2\right) - \left(\left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2\right) - \left(\left(K_A(x_j)\right)^2 - \left(K_B(x_j)\right)^2\right)\Bigg|\Bigg)$$

$$= \frac{1}{7n}\sum_{j=1}^{n}\Bigg(\Bigg|\left(\left(T_B(x_j)\right)^2 - \left(T_A(x_j)\right)^2\right) + \left(\left(M_B(x_j)\right)^2 - \left(M_A(x_j)\right)^2\right) + \left(\left(C_B(x_j)\right)^2 - \left(C_A(x_j)\right)^2\right) - \left(\left(I_B(x_j)\right)^2 - \left(I_A(x_j)\right)^2\right) - \left(\left(U_B(x_j)\right)^2 - \left(U_A(x_j)\right)^2\right) - \left(\left(F_B(x_j)\right)^2 - \left(F_A(x_j)\right)^2\right) - \left(\left(K_B(x_j)\right)^2 - \left(K_A(x_j)\right)^2\right)\Bigg|\Bigg)$$

$$= d_4(B, A).$$

4. If $A \subseteq B \subseteq C$, then $T_A(x_j) \leq T_B(x_j) \leq T_C(x_j)$; $M_A(x_j) \leq M_B(x_j) \leq M_C(x_j)$; $C_A(x_j) \leq C_B(x_j) \leq C_C(x_j)$; $I_A(x_j) \geq I_B(x_j) \geq I_C(x_j)$; $U_A(x_j) \geq U_B(x_j) \geq U_C(x_j)$; $F_A(x_j) \geq F_B(x_j) \geq F_C(x_j)$; $K_A(x_j) \geq K_B(x_j) \geq K_C(x_j)$, for all $x_j \in X$.

Then, we have the following inequalities:

$$\left(T_A(x_j)\right)^2 - \left(T_C(x_j)\right)^2 \leq \left(T_A(x_j)\right)^2 - \left(T_B(x_j)\right)^2, \left(T_A(x_j)\right)^2 - \left(T_C(x_j)\right)^2 \leq \left(T_B(x_j)\right)^2 - \left(T_C(x_j)\right)^2$$

$$\left(M_A(x_j)\right)^2 - \left(M_C(x_j)\right)^2 \leq \left(M_A(x_j)\right)^2 - \left(M_B(x_j)\right)^2, \left(M_A(x_j)\right)^2 - \left(M_C(x_j)\right)^2 \leq \left(M_B(x_j)\right)^2 - \left(M_C(x_j)\right)^2$$

$$\left(C_A(x_j)\right)^2 - \left(C_C(x_j)\right)^2 \leq \left(C_A(x_j)\right)^2 - \left(C_B(x_j)\right)^2, \left(C_A(x_j)\right)^2 - \left(C_C(x_j)\right)^2 \leq \left(C_B(x_j)\right)^2 - \left(C_C(x_j)\right)^2$$

$$\left(I_A(x_j)\right)^2 - \left(I_C(x_j)\right)^2 \geq \left(I_A(x_j)\right)^2 - \left(I_B(x_j)\right)^2, \left(I_A(x_j)\right)^2 - \left(I_C(x_j)\right)^2 \geq \left(I_B(x_j)\right)^2 - \left(I_C(x_j)\right)^2$$

$$\left(U_A(x_j)\right)^2 - \left(U_C(x_j)\right)^2 \geq \left(U_A(x_j)\right)^2 - \left(U_B(x_j)\right)^2, \left(U_A(x_j)\right)^2 - \left(U_C(x_j)\right)^2 \geq \left(U_B(x_j)\right)^2 - \left(U_C(x_j)\right)^2$$

$$\left(F_A(x_j)\right)^2 - \left(F_C(x_j)\right)^2 \geq \left(F_A(x_j)\right)^2 - \left(F_B(x_j)\right)^2, \left(F_A(x_j)\right)^2 - \left(F_C(x_j)\right)^2 \geq \left(F_B(x_j)\right)^2 - \left(F_C(x_j)\right)^2$$

$$\left(K_A(x_j)\right)^2 - \left(K_C(x_j)\right)^2 \geq \left(K_A(x_j)\right)^2 - \left(K_B(x_j)\right)^2, \left(K_A(x_j)\right)^2 - \left(K_C(x_j)\right)^2 \geq \left(K_B(x_j)\right)^2 - \left(K_C(x_j)\right)^2$$

From these inequalities we have,

$$\frac{1}{7n}\sum_{j=1}^{n}\left(\left|\left(\left(T_A(x_j)\right)^2-\left(T_C(x_j)\right)^2\right)+\left(\left(M_A(x_j)\right)^2-\left(M_C(x_j)\right)^2\right)\right.\right.$$
$$+\left(\left(C_A(x_j)\right)^2-\left(C_C(x_j)\right)^2\right)-\left(\left(I_A(x_j)\right)^2-\left(I_C(x_j)\right)^2\right)$$
$$-\left(\left(U_A(x_j)\right)^2-\left(U_C(x_j)\right)^2\right)-\left(\left(F_A(x_j)\right)^2-\left(F_C(x_j)\right)^2\right)$$
$$\left.\left.-\left(\left(K_A(x_j)\right)^2-\left(K_C(x_j)\right)^2\right)\right|\right)$$

$$\geq \frac{1}{7n}\sum_{j=1}^{n}\left(\left|\left(\left(T_B(x_j)\right)^2-\left(T_C(x_j)\right)^2\right)+\left(\left(M_B(x_j)\right)^2-\left(M_C(x_j)\right)^2\right)\right.\right.$$
$$+\left(\left(C_B(x_j)\right)^2-\left(C_C(x_j)\right)^2\right)-\left(\left(I_B(x_j)\right)^2-\left(I_C(x_j)\right)^2\right)$$
$$-\left(\left(U_B(x_j)\right)^2-\left(U_C(x_j)\right)^2\right)-\left(\left(F_B(x_j)\right)^2-\left(F_C(x_j)\right)^2\right)$$
$$\left.\left.-\left(\left(K_B(x_j)\right)^2-\left(K_C(x_j)\right)^2\right)\right|\right)$$

and

$$\frac{1}{7n}\sum_{j=1}^{n}\left(\left|\left(\left(T_A(x_j)\right)^2-\left(T_C(x_j)\right)^2\right)+\left(\left(M_A(x_j)\right)^2-\left(M_C(x_j)\right)^2\right)\right.\right.$$
$$+\left(\left(C_A(x_j)\right)^2-\left(C_C(x_j)\right)^2\right)-\left(\left(I_A(x_j)\right)^2-\left(I_C(x_j)\right)^2\right)$$
$$-\left(\left(U_A(x_j)\right)^2-\left(U_C(x_j)\right)^2\right)-\left(\left(F_A(x_j)\right)^2-\left(F_C(x_j)\right)^2\right)$$
$$\left.\left.-\left(\left(K_A(x_j)\right)^2-\left(K_C(x_j)\right)^2\right)\right|\right)$$

$$\geq \frac{1}{7n}\sum_{j=1}^{n}\left(\left|\left(\left(T_A(x_j)\right)^2-\left(T_B(x_j)\right)^2\right)+\left(\left(M_A(x_j)\right)^2-\left(M_B(x_j)\right)^2\right)\right.\right.$$
$$+\left(\left(C_A(x_j)\right)^2-\left(C_B(x_j)\right)^2\right)-\left(\left(I_A(x_j)\right)^2-\left(I_B(x_j)\right)^2\right)$$
$$-\left(\left(U_A(x_j)\right)^2-\left(U_B(x_j)\right)^2\right)-\left(\left(F_A(x_j)\right)^2-\left(F_B(x_j)\right)^2\right)$$
$$\left.\left.-\left(\left(K_A(x_j)\right)^2-\left(K_B(x_j)\right)^2\right)\right|\right)$$

Therefore, $d_4(A,C) \geq d_4(B,C)$ and $d_4(A,C) \geq d_4(A,B)$.

Hence $d_4(A,B)$ is a Heptapartitioned neutrosophic distance measure.

Theorem 2: For $i = 1,2,3,4,5,6,7, d_i(A,C) \leq d_i(A,B) + d_i(B,C)$ is true for $A,B,C \in HNS(X)$.

Proof: Here, we only demonstrate the triangle inequality for i=7. In a similar manner, we may demonstrate it for i=1, 2, and 3.

Let $A, B, C \in HNS(X)$ then for the real numbers, the following inequalities hold true.

$$\left|T_A(x_j) - T_C(x_j)\right| \le \left|T_A(x_j) - T_B(x_j)\right| + \left|T_B(x_j) - T_C(x_j)\right|, \sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j) - T_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j) - M_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j) - C_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j) - I_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j) - U_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j) - F_C(x_j)\right|\right)\right\}, \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_C(x_j)\right|\right)\right\}$$

$$\le \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j) - K_C(x_j)\right|\right)\right\}$$

Then,

$$\begin{aligned}
&\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_C(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_C(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_C(x_j)\right|\right)\right\} \\
&\quad + \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_C(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_C(x_j)\right|\right)\right\} \\
&\quad + \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_C(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_C(x_j)\right|\right)\right\}
\end{aligned}$$

$$\begin{aligned}
\le\ &\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j) - T_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j) - M_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j) - C_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j) - I_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j) - U_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j) - F_C(x_j)\right|\right)\right\} \\
&+ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j) - K_C(x_j)\right|\right)\right\}
\end{aligned}$$

So,

$$\frac{1}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}+\end{array}} \geq \frac{1}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}$$

Then,

$$1-\frac{1}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}+\end{array}} \leq 1-\frac{1}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}$$

Hence,

$$\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}$$

$$\leq \frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}+\sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}} \leq$$

$$\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}++\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}}+\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}\Rightarrow$$

$$\frac{7}{5n}\sum_{j=1}^{n}\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_C(x_j)\right|\right)\right\}\end{array}} \le \frac{7}{5n}\sum_{j=1}^{n}\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j)-T_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j)-M_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j)-C_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j)-I_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j)-U_B(x_j)\right|\right)\right\}++\\ \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j)-F_B(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j)-K_B(x_j)\right|\right)\right\}\end{array}}+$$

$$\frac{7}{5n}\sum_{j=1}^{n}\frac{\begin{array}{c}\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}{\begin{array}{c}1+\sin\left\{\frac{\pi}{6}\left(\left|T_B(x_j)-T_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|M_B(x_j)-M_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|C_B(x_j)-C_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|I_B(x_j)-I_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|U_B(x_j)-U_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|F_B(x_j)-F_C(x_j)\right|\right)\right\}+\\ \sin\left\{\frac{\pi}{6}\left(\left|K_B(x_j)-K_C(x_j)\right|\right)\right\}\end{array}}$$

Therefore, $d_5(A,C) \leq d_5(A,B) + d_5(B,C)$ is true for $A,B,C \in HNS(X)$.

4. Methodology:

In this section, a sine metric heptapartitioned neutrosophic distance measure is used to present a systematic approach to solving neutrosophic multiple attributes decision making issues. The steps required to determine the pertinent traits and options are described in the approach below.

Step 1: Selection of the problem field

Table 1 Attributes vs Alternatives

	A_1	A_2	A_m
R_1	(r_{11})	(r_{12})	(r_{1m})
R_2	(r_{21})	(r_{22})	(r_{2m})
R_p	(r_{p1})	(r_{p2})	(r_{pm})

Table 2 Alternatives vs Decision Attributes

	R_1	R_2	R_P
B_1	(a_{11})	(a_{12})	(a_{1p})
B_2	(a_{21})	(a_{22})	(a_{2p})
B_n	(a_{n1})	(a_{n2})	(a_{np})

Table 3 Distance Measure Table

B_kA_j	B_1	B_2	B_n
A_1	$d(A_1,B_1)$	$d(A_1,B_2)$	$d(A_1,B_n)$
A_2	$d(A_2,B_1)$	$d(A_2,B_2)$	$d(A_2,B_n)$
A_m	$d(A_m,B_1)$	$d(A_m,B_2)$	(A_m,B_n)

Select the attribute from the table of distance measures. The optimal attribute for alternative A_j is $B_k, k = 1,2, \dots \dots n$ for the substitutes $A_j, j = 1,2, \dots . m$, which is determined by the options' lowest distance measure value A_j and the attributes B_k,Nd.

In order to demonstrate a practical implementation of our Heptapartitioned Neutrosophic Distance Measure within the suggested approach, we present a numerical illustration of hospital selection for specifically diseases in this section.

4.1 Numerical illustration: Utilization of the sine metric to the hexapartitioned measure of neutrosophic distance

Hospitals play a vital role in managing and treating a wide range of diseases, catering to the diverse needs of patients. General hospitals provide comprehensive care for common illnesses and emergencies, offering essential diagnostic and treatment services. Specialized hospitals, on the other hand, focus on specific medical fields such as cardiology, oncology, orthopaedics, and neurology, making them ideal for treating complex and chronic conditions. The selection of a hospital often depends on factors like its reputation, the expertise of its medical staff, available technology, cost, and proximity. Diseases, whether acute or chronic, require different levels of care acute conditions like appendicitis necessitate immediate intervention, while chronic illnesses like diabetes benefit from long-term management and specialized care. Advances in healthcare, such as telemedicine and multidisciplinary approaches, are further enhancing the treatment landscape, making it easier to address a variety of medical needs effectively.

Step 1 Problem field selection.

Let $A = \{A_1, A_2, A_3\}$Be the set of treatments and $B = \{B_1, B_2, B_3\}$Be the set of a selection of hospitals and $R = \{R_1, R_2, R_3, R_4, R_5\}$ be the set of specified diseases. Table 4 shows the specified set of diseases and Table 5 shows the selection of hospitals and specified diseases.

Step 2 Proximity Analysis for and Attributes Alternatives

The measures of distance between each disease concerning and each hospital

$$d_7(B_k A_j) = \frac{9}{7n}\sum_{j=1}^{n} \frac{\begin{pmatrix} \sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_B(x_j)\right|\right)\right\} \\ + \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_B(x_j)\right|\right)\right\} \\ + \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_B(x_j)\right|\right)\right\} + \\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_B(x_j)\right|\right)\right\} \end{pmatrix}}{1 + \begin{pmatrix} \sin\left\{\frac{\pi}{6}\left(\left|T_A(x_j) - T_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|M_A(x_j) - M_B(x_j)\right|\right)\right\} \\ + \sin\left\{\frac{\pi}{6}\left(\left|C_A(x_j) - C_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|I_A(x_j) - I_B(x_j)\right|\right)\right\} \\ + \sin\left\{\frac{\pi}{6}\left(\left|U_A(x_j) - U_B(x_j)\right|\right)\right\} + \sin\left\{\frac{\pi}{6}\left(\left|F_A(x_j) - F_B(x_j)\right|\right)\right\} + \\ \sin\left\{\frac{\pi}{6}\left(\left|K_A(x_j) - K_B(x_j)\right|\right)\right\} \end{pmatrix}}$$

Table 4 Treatments Vs Specified Diseases

	R_1	R_2	R_3	R_4	R_5
A_1	(0.5,0.5,0.9,0.8, 0.4,0.1,0)	(0.4,0.6,0.2,0.1, 0.5,0.2,0.3)	(0.8,0.2,0.6,0.4, 0.2,0,0.1)	(0.3,0.2,0.1,0.5, 0.3,0.2,0.1)	(0.3,0.3,0.3,0.2, 0.4,0.1,0.1)
A_2	(0.8,0.1,0.2,0.2, 0.4,0.3,0.4)	(0.3,0.6,0.7,0.4, 0.6,0.4,0.3)	(0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	(0.2,0.2,0.4,0.2, 0.1,0.3,0.1)	(0.7,0.8,1,0.4, 0.5,0.2,0.2)
A_3	(0.2,0,0.1,0.2, 0.4,0.4,0.3)	(0.8,0.4,0.1,0, 0.1,0.1,0.2)	(0.3,0.6,0.7,0.1, 0.2,0.1,0.3)	(0.9,0.1,0.4,0.6, 0.7,0.8,0.7)	(0.3,0.2,0.3,0.7, 0.4,0.2,0.1)

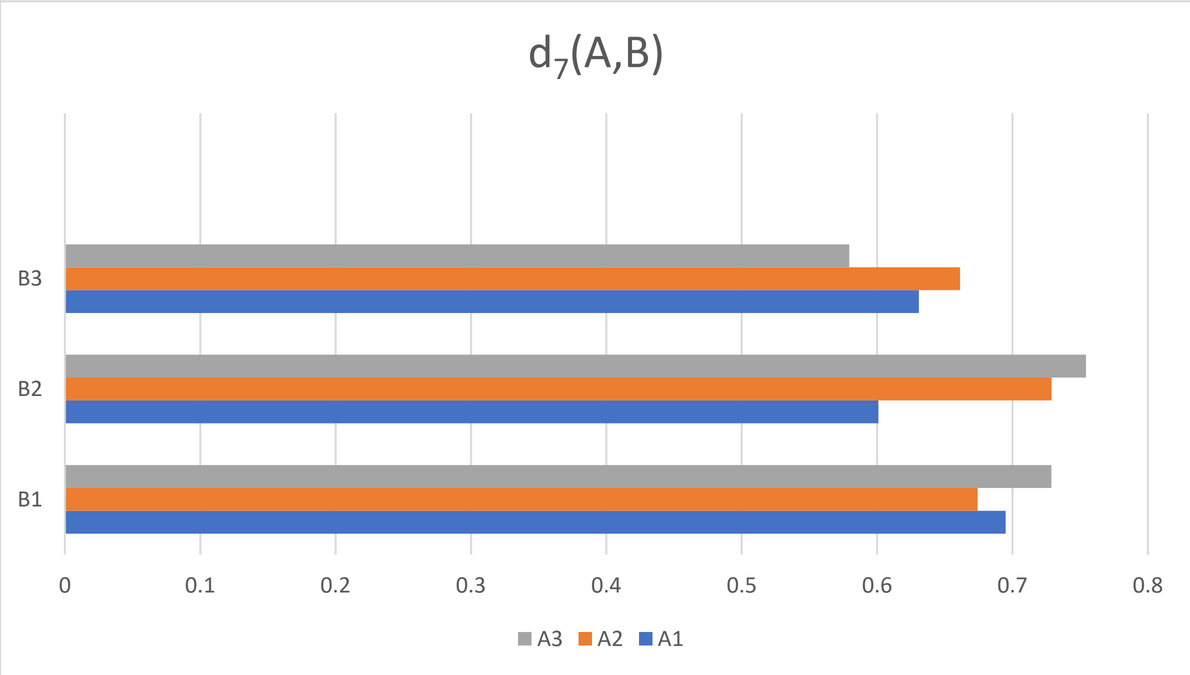
Table 5 Specified Diseases Vs Hospitals

	B_1	B_2	B_3
R_1	(0.8,0.9,0.1,0.7,0.5,0.6,0.2)	(0.4,0.5,1,0.5,0.6,1,0.1)	(0.1,0.1,0.1,0.1,0.1,0.1,0.1)
R_2	(0.7,0.9,0.3,0.8,0.9,0.1,0.3)	(0.8,0.9,0.7,0.2,0.3,0,1)	(0.6,0.3,0.1,0.1,0.5,0.5,0.4)
R_3	(1,1,1,0,0,0,0)	(0.6,0.4,0.2,0.3,0.2,0.1,0)	(0,0,0,0,1,1,1)
R_4	(0,0.7,1,0.1,0.4,0.4,0.3)	(0.9,0.8,0.6,0.4,0.4,0.2,0.2)	(0.8,0.1,0.3,0.3,0.1,0.4,0.2)

R_5	(0.5,0.8,0.9,0.9,0.7,0.6,0.1)	(0.3,0.2,0.2,0.1,0.1,0.3,1)	(0,0,0,0.2,0.2,0.2,0.1)

Table 6 Distance Analysis Matrix

	A_1	A_2	A_3	w_r
B_1	0.69494	0.67421	0.72870	0.699283
B_2	0.60093	0.72888	0.75425	0.694687
B_3	0.63083	0.66120	0.57944	0.623823



4.2 Comparison Study and Simulation

This section presents a comparative analysis and simulation of Tables 6 through 12.

Table 7 Distance Analysis Matrix $d_1(A, B)$

	A_1	A_2	A_3	w_r
B_1	0.337143	$\mathbf{0.317143}$	0.37143	0.34191
B_2	0.38286	0.37143	$\mathbf{0.32286}$	0.35905
B_3	0.53429	0.32	$\mathbf{0.27714}$	0.37714

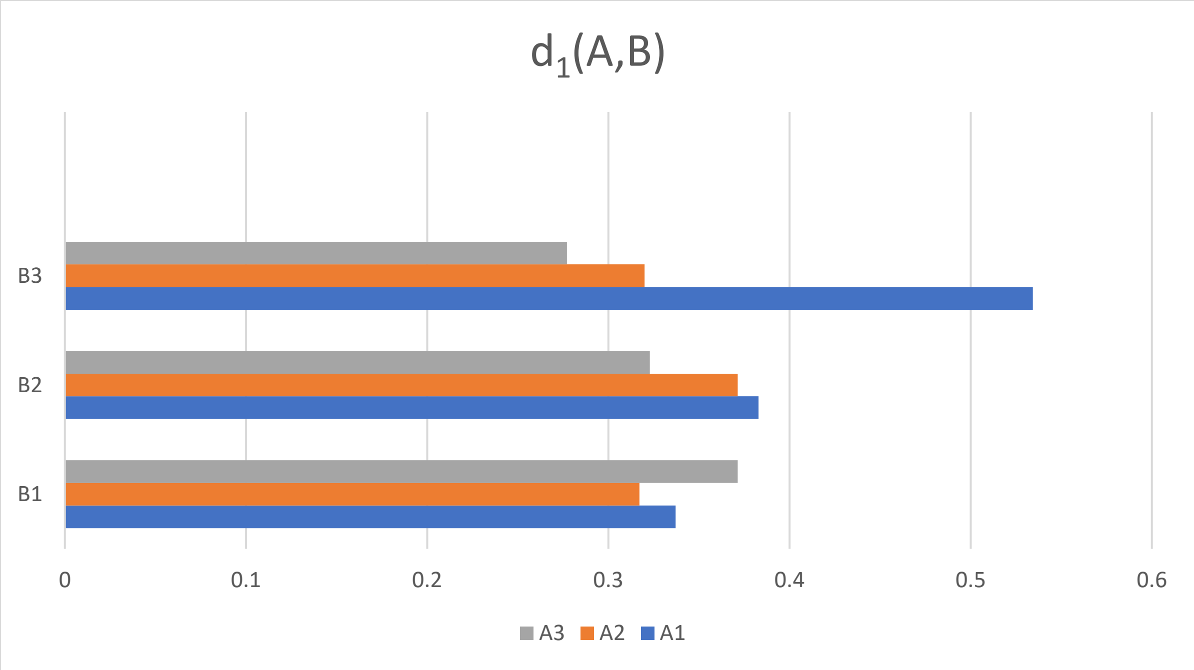


Table 8 Distance Analysis Matrix $d_2(A,B)$

	A_1	A_2	A_3	w_r
B_1	0.170255	$\mathbf{0.13764}$	0.16777	0.158555
B_2	$\mathbf{0.12939}$	0.17113	0.15448	0.151667
B_3	0.13905	0.14676	$\mathbf{0.12858}$	0.13813

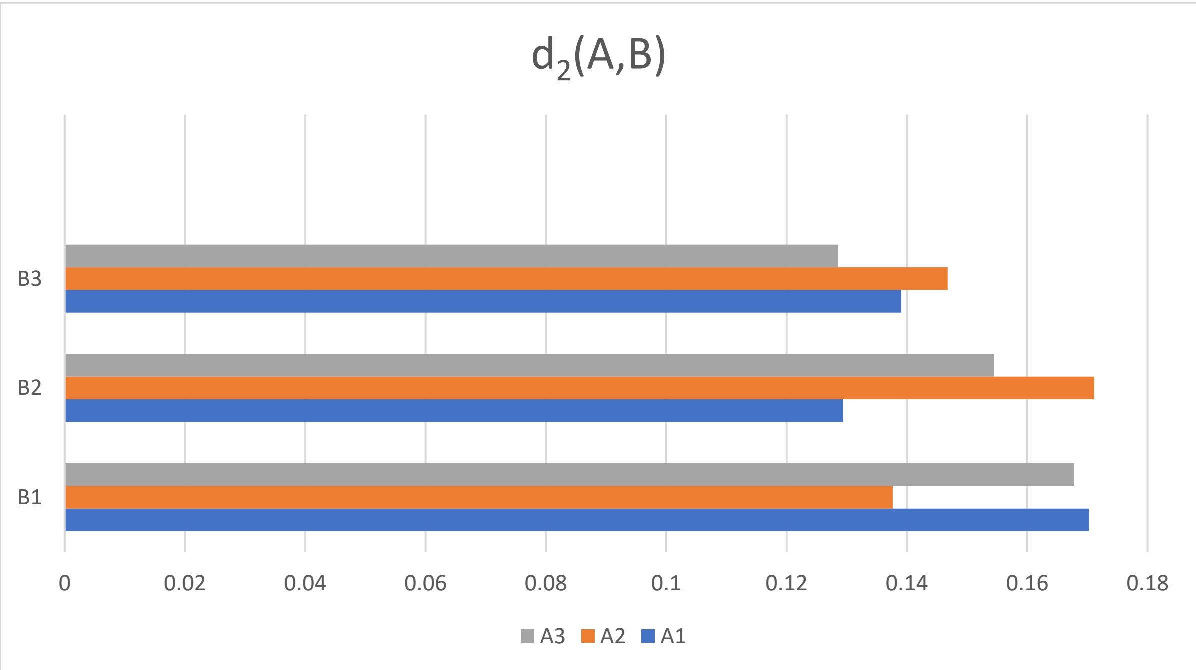


Table 9 Distance Analysis Matrix $d_3(A,B)$

	A_1	A_2	A_3	w_r
B_1	0.32914	**0.30486**	0.36914	0.33438
B_2	**0.24029**	0.34743	0.28943	0.29238
B_3	0.242	0.26	**0.22657**	0.24286

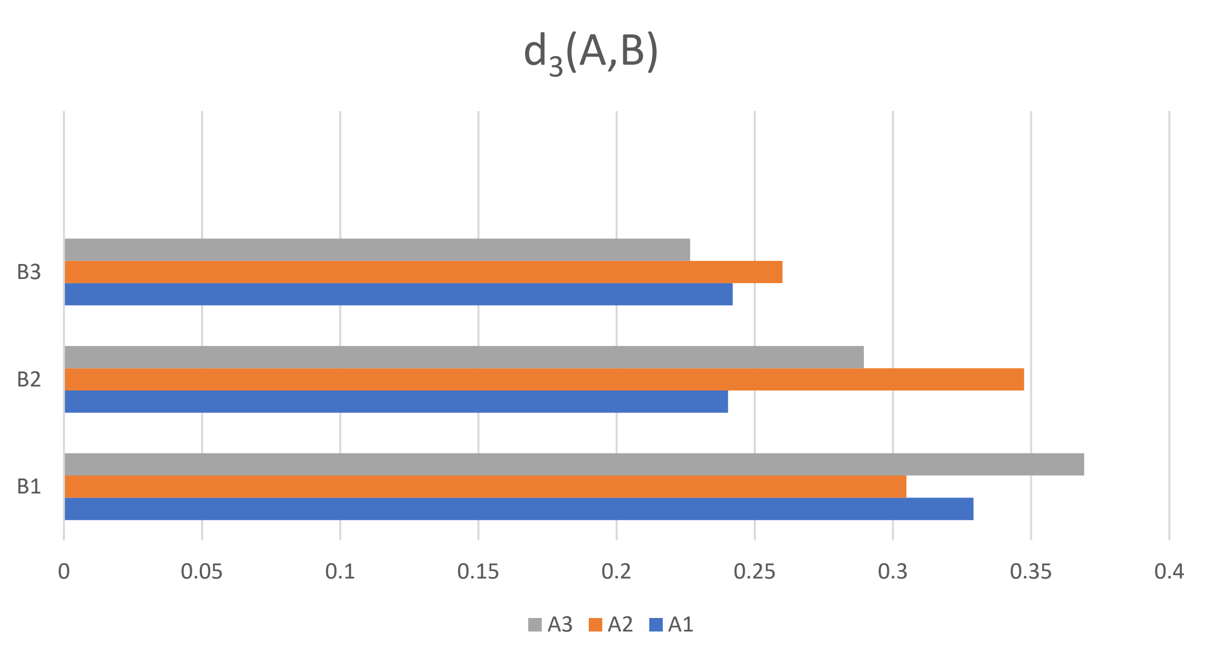


Table 10 Distance Analysis Matrix $d_4(A,B)$

	A_1	A_2	A_3	w_r
B_1	0.12286	**0.09514**	0.12057	0.11286
B_2	0.13743	**0.0826**	0.09114	0.10372
B_3	**0.09057**	0.14057	0.13	0.12038

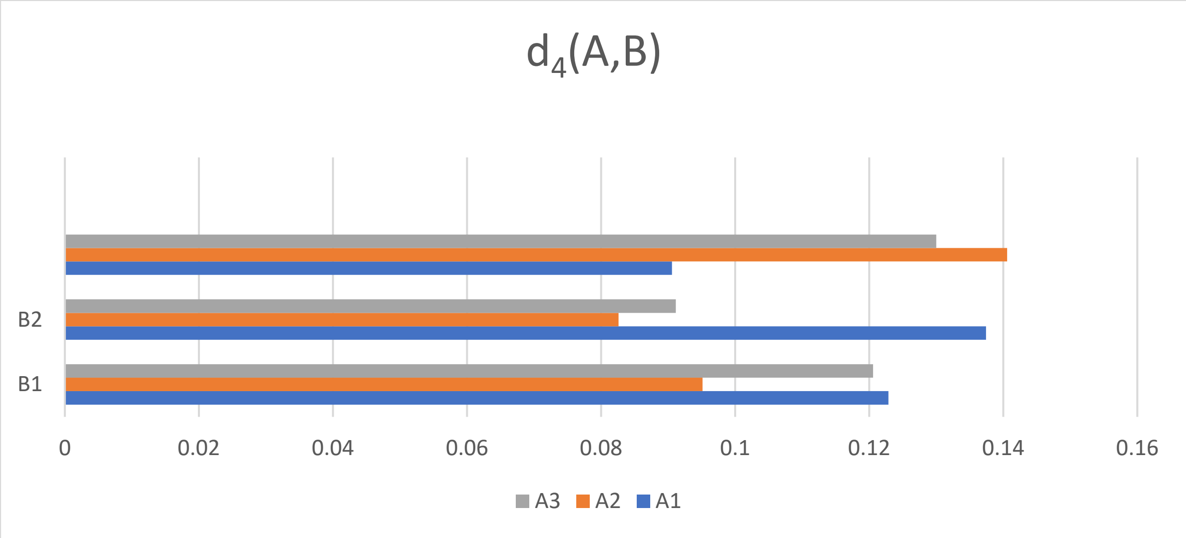


Table 11 Distance Analysis Matrix $d_5(A,B)$

	A_1	A_2	A_3	w_r
B_1	0.75671	**0.73414**	0.79347	0.76144
B_2	**0.65435**	0.79367	0.82129	0.75644
B_3	0.68690	0.71998	**0.63095**	0.67928

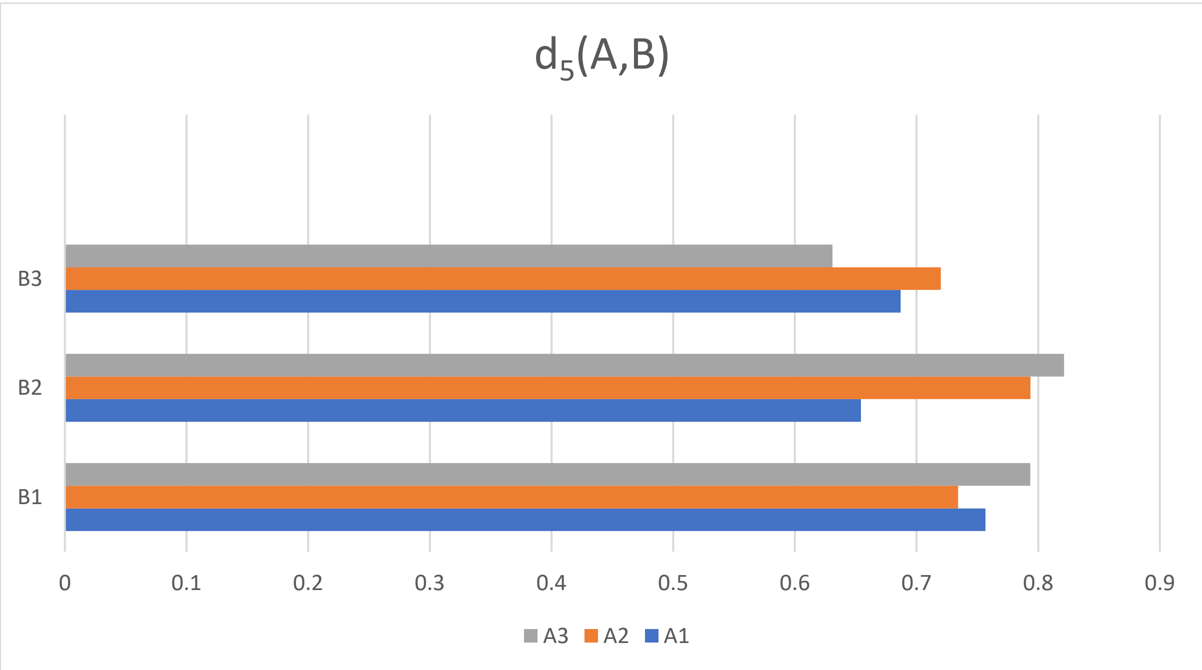


Table 12 Distance Analysis Matrix $d_6(A,B)$

	A_1	A_2	A_3	w_r
B_1	0.423434	0.12848	**0.116632**	0.2228624
B_2	**0.08335**	0.109326	0.1047452	0.0991404
B_3	0.08590	**0.07608**	0.07945	0.080477

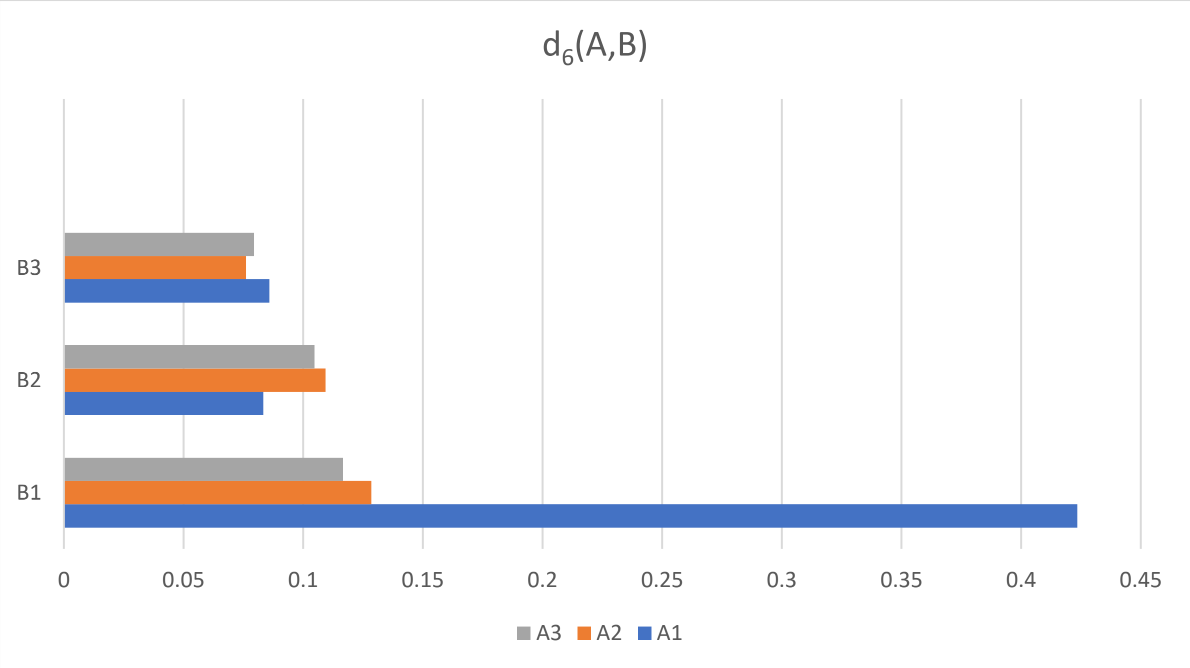


Table 13 Comparison of distance measures

	$d_1(A,B)$	$d_2(A,B)$	$d_3(A,B)$	$d_4(A,B)$	$d_5(A,B)$	$d_6(A,B)$
B_1	A_2	A_2	A_2	A_2	A_2	A_3
B_2	A_3	A_1	A_1	A_2	A_1	A_1
B_3	A_3	A_3	A_3	A_1	A_3	A_2

From the above table 13, we can observe that for B_1, decisions of $d_1(A,B), d_2(A,B), d_3(A,B), d_4(A,B), d_5(A,B)$Are the same, that is A_2 but the decision of $d_6(A,B)$ is A_3.

For B_2, the decision of $d_2(A,B)$, $d_3(A,B)$, $d_5(A,B)$, $d_6(A,B)$Re the same, that is A_1 but the decision of $d_1(A,B)$ is A_3 and $d_4(A,B)$ is A_2.

For B_3, the decision of $d_1(A,B)$, $d_2(A,B)$, $d_3(A,B)$, $d_5(A,B)$Are the same, that is A_3 but the decision of $d_4(A,B)$ is A_1 and $d_6(A,B)$ is A_2.

Thus, the final decision for the hospital is the majority decision, which is appropriate for the respective hospital is A_2.

5. Conclusion:

An expansion of the neutrosophic set, the heptapartitioned Neutrosophic set is a potent mathematical instrument for dealing with inconsistent, unclear, and incomplete data in using several factors to make decisions. In order to ensure compliance with the axioms of distance measures, this work provides a number of measures for heptapartitioned neutrosophic sets, some of which also meet metric axioms.A practical application is presented to identify the best hospital in several factors to make decision context, demonstrating the effectiveness of the proposed approach. These distance measures are pivotal in facilitating accurate decision-making across various criteria. Furthermore, the newly developed measures and methods are expected to stimulate research in heptapartitioned neutrosophic sets, with potential applications in fields such as rough topology, digital topology, and other domains within general topology.

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