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A Cross-Entropy-Based Framework Using HyperSoft Type-2 Neutrosophic Numbers for Innovation Capability Evaluation in Central Cities of Emerging Economies

Xiangke Zhang[1,2*], Shoukui He[1], Xu Chen[3]

[1]School of Economics and Management, Chongqing Jiaotong University, Chongqing, 400074, China

[2]School of Urban Construction Engineering, Chongqing Technology and Business Institute, Chongqing, 400052, China

[3]Division of Infrastructure and Logistics, Chongqing University of Arts and Sciences, Chongqing, 402160, China

*Corresponding author, E-mail: zhangxiangke@cqtbi.edu.cn

https://orcid.org/0009-0001-6001-635X

Shoukui He, E-mail: 1559846793@qq.com

Xu Chen, E-mail: 20170012@cqwu.edu.cn

Abstract: The comprehensive evaluation of innovation capabilities in central cities of emerging countries is essential for promoting regional economic growth. This evaluation examines various factors, such as scientific activities, innovation infrastructure, talent resources, and policy environments, offering insights into the current innovation landscape and future potential. By guiding the allocation of resources and optimizing innovation policies, it fosters the transformation of technological achievements and accelerates urban economic transformation and sustainable development. Moreover, it helps cities attract investment and skilled talent, enhancing their competitiveness and influence. This evaluation is a multi-attribute decision-making (MADM) challenge, for which the cross-entropy method has been recently adopted to handle complex criteria. Type-2 neutrosophic numbers (T2NNs) are used to represent uncertainty in assessing innovation capabilities. This study introduces the Type-2 neutrosophic number cross-entropy (T2NN-HS-CE) approach with HyperSoft to solve MADM problems involving T2NNs. A numerical case study and sensitivity analysis validate the effectiveness of the proposed method in assessing the innovation capacity of cities in developing nations.

Keywords: Multiple-attributes decision-making (MADM); Type-2 neutrosophic number (T2NN); Cross-entropy approach; Comprehensive evaluation of innovation capability; HyperSoft (HS).

1. Introduction

The comprehensive evaluation of innovation capabilities in central cities of emerging countries holds significant practical and theoretical value for policymakers, entrepreneurs, and scholars. Firstly, it provides specific insights for governments on how to promote innovation and development in central cities by optimizing policy environments. By considering technological, economic, and social factors comprehensively, this evaluation helps policymakers identify and strengthen the drivers of innovation in cities, thus more effectively allocating resources and enhancing urban competitiveness. Secondly, for businesses, understanding the results of the innovation capability evaluation of their city helps them make more accurate strategic decisions, such as choosing the best locations for investment, research and development centers, and partnership opportunities. Additionally, this evaluation offers a rich database and analytical framework for academic research, fostering the development of theories and empirical studies on urban innovation systems. Overall, this comprehensive evaluation provides a multi-dimensional perspective that not only enhances the comparability among cities but also promotes global and regional dialogues and cooperation on urban innovation capabilities, supporting the advancement of global and regional economic growth. Over the past years, the study of innovation capacity in central cities within developing nations has gained significant academic interest. Researchers have explored various factors influencing this capacity, ranging from government policies and infrastructure to cultural impacts and international trade. This review examines fifteen pivotal studies published, providing insight into the evolution of methodologies, findings, and theoretical approaches within this research area. Li and Zhang [1] were pioneers in examining the relationship between infrastructure quality and innovation efficiency in Chinese cities, suggesting that robust infrastructure significantly boosts innovation outcomes. Wang and et al. [2] extended this by introducing a composite index to assess the technological innovation capabilities in Indian metropolitan areas, with an emphasis on R&D investments and patent outputs. Morris [3] shifted the focus to Latin America, analyzing how government policies in Brazilian cities shape their innovation landscapes. Chen [4] provided an in-depth look at Shenzhen's innovation ecosystem, proposing a model that synergizes public and private sector efforts. Concurrently, Thompson and Garcia [5] offered a comparative perspective by examining innovation capacities in emerging economy cities across Africa, employing network analysis to explore inter-city collaborations. Kumar and Singh [6] used advanced econometric models to link economic diversity with innovation rates in Indonesian cities, highlighting the multifaceted nature of urban innovation. In 2020, Zhao [7] explored how cultural dynamics within Eastern Chinese cities influence their innovation capabilities, advocating for a culturally aware innovation framework. Similarly, Brooks and Chen [8] looked at the impact of international trade on innovation in Mexican cities, identifying trade openness as a catalyst for innovation. Zhang [9] and Johnson [10] both emphasized the role of education and infrastructure, respectively. Zhang investigated how higher education resources in Chinese cities enhance local innovation systems, while Johnson focused on the technological readiness and infrastructure challenges in Nigerian cities. Lee and Kim [11] analyzed the sustainability of innovation models in South Korean cities, using Seoul as a case study for green innovation strategies. Petrov [12] assessed the impact of digital transformation in Russian cities, exploring how these initiatives influence both public and private sector innovation. Ming and Liu [13] and Smith and Roberts [14] provided fresh perspectives by incorporating environmental, social metrics, and smart city technologies into the evaluation of innovation capacity. Ming and Liu looked at Vietnamese cities, while Smith and Roberts focused on Egypt. Wu [15] examined the synergistic effects of urban planning and innovation policies in Malaysian cities, offering integrated urban-innovation growth strategies.

As the economy grows rapidly and scientific and technological advancements continue, decision-making in various sectors of social production is increasingly becoming complex [16, 17]. MADM stands out as an effective tool in the domain of decision-making, utilizing specific methodologies to evaluate and choose the optimal solutions among multiple decision attributes [18, 19]. It finds extensive application in areas such as project selection, investment decisions, and comprehensive assessments. Nevertheless, the actual process of decision-making faces challenges due to the dynamic nature of environments, the intricacies of decision problems, and limitations in the expertise of decision-making professionals, which makes it challenging to precisely determine values for scheme attributes and weighting coefficients, thereby complicating the attainment of scientific, rational, and effective decision outcomes [20]. The selection of suitable decision-making approaches offers fresh perspectives and approaches for addressing fuzzy multi-attribute decision issues, and supplies new mathematical tools for tackling such problems in sectors including economics, engineering, and management. This enhances the methodologies and thought processes within multi-level uncertainty decision theory, improving the precision and efficiency of real-world decision-making. Actual decision-making is fraught with substantial uncertainty and fuzziness. In 1965, Zadeh [21] introduced the concept of fuzzy sets, which comprised only membership degrees. Later, in 1986, Atanassov [22]expanded this theory by incorporating non-membership degrees and introduced the notion of intuitionistic fuzzy sets (IFSs), which more accurately represent the fuzziness and uncertainty found in practical decision-making situations. IFSs provide decision-related information through exact numerical values. Following the foundational work on IFSs and neutrosophic sets (NSs) [23], approaches such as the single-valued neutrosophic set (SVNS) [24] and interval-valued neutrosophic set (IVNS) [25] have been developed for practical decision-making applications. Building on earlier research on NSs, Abdel-Basset et al. [26] introduced the type-2 neutrosophic number (T2NN).

The evaluation of innovation capabilities in central cities of emerging nations presents a complex MADM challenge. Recently, the cross-entropy method [6] has been applied to address various MADM issues. The cross-entropy method has the advantage of handling complex uncertainties in multi-criteria decision-making. By measuring the difference between probability distributions, it effectively optimizes decision models, particularly excelling in multi-attribute decision problems. This method is not only computationally simple but also quickly converges to the optimal solution, making it suitable for decision-making in complex environments. Additionally, it accurately handles multidimensional data under different weights, improving decision accuracy and reliability. Type-2 neutrosophic numbers (T2NNs) [26] are highly effective in managing uncertainty and imprecision in decision-making processes. They extend traditional neutrosophic numbers by better capturing the degrees of truth, indeterminacy, and falsity, allowing for more nuanced representation of vague or incomplete information. This makes T2NNs particularly useful in complex environments where information is often ambiguous, providing a more flexible and accurate tool for multi-criteria decision-making problems. In this context, T2NNs serve as a valuable approach for representing uncertain information during the evaluation of innovation capabilities. To date, there has been limited development of approaches utilizing the entropy and cross-entropy methods [27] in conjunction with T2NNs. This paper introduces the Type-2 neutrosophic number cross-entropy (T2NN-HS-CE) method to facilitate MADM processes involving T2NNs. Additionally, a numerical study is performed to evaluate the innovation capabilities of central cities in emerging countries, along with several sensitivity analyses to demonstrate the validity of the proposed T2NN-HS-CE method.

The HyperSoft Set is an advanced extension of soft sets that provides enhanced tools for handling uncertainty and complex data structures, particularly in MADM and other applications involving high-dimensional data. HyperSoft sets can analyze multiple criteria simultaneously with greater flexibility by accommodating multi-sub attribute parameters under a single attribute. It allows for representing complex and interrelated data structures, making them useful in domains like economics, engineering, and social sciences.

The key contributions of this research are as follows: (1) the integration of the cross-entropy method with T2NNs; (2) the application of the entropy method to establish weight values within the T2NN framework; (3) the implementation of the T2NN-HS-CE method for managing MADM scenarios under T2NNs; and (4) the development of an algorithmic framework for the comprehensive evaluation of innovation capabilities, supported by a numerical example to illustrate the effectiveness of the T2NN-HS-CE method.

The organization of this article is outlined as follows: Section 2 introduces the concept of T2NNs. In Section 3, we define the cross-entropy associated with T2NNs. Section 4 provides a description of the MADM approach utilizing the T2NN-HS-CE approach. Section 5 presents a numerical example that evaluates the innovation capabilities of central cities in emerging countries, along with a sensitivity analysis to illustrate the approaches developed. Finally, Section 6 offers the concluding remarks.

2. Preliminary

Abdel-Basset et al. [26] conducted the T2NNs.

Definition 1[26]. Let Θ be a fix set, the T2NN is defined as:

$$FF = \left\{ \left(\theta, FA(\theta), FB(\theta), FC(\theta) \right) \middle| \theta \in \Theta \right\} \tag{1}$$

where $FA(\theta), FB(\theta), FC(\theta) \in [0,1]$ represent the TM, IM and FM with triangular fuzzy numbers.

$$FA(\theta) = \left(FA^L(\theta), FA^M(\theta), FA^U(\theta) \right), 0 \le FA^L(\theta) \le FA^M(\theta) \le FA^U(\theta) \le 1 \tag{2}$$

$$FB(\theta) = \left(FB^L(\theta), FB^M(\theta), FB^U(\theta) \right), 0 \le FB^L(\theta) \le FB^M(\theta) \le FB^U(\theta) \le 1 \tag{3}$$

$$FC(\theta) = \left(FC^L(\theta), FC^M(\theta), FC^U(\theta) \right), 0 \le FC^L(\theta) \le FC^M(\theta) \le FC^U(\theta) \le 1 \tag{4}$$

We let $FF = \left\{ \begin{array}{l} \left(FA^L, FA^M, FA^U \right), \\ \left(FB^L, FB^M, FB^U \right), \left(FC^L, FC^M, FC^U \right) \end{array} \right\}$ be a T2NN, $0 \le FA^U + FB^U + FC^U \le 3$.

Definition 2[26]**.** There are three TFNNs $FF_1 = \left\{ \begin{array}{l} \left(FA_1^L, FA_1^M, FA_1^U \right), \\ \left(FB_1^L, FB_1^M, FB_1^U \right), \left(FC_1^L, FC_1^M, FC_1^U \right) \end{array} \right\}$, $FF_2 = \left\{ \begin{array}{l} \left(FA_2^L, FA_2^M, FA_2^U \right), \\ \left(FB_2^L, FB_2^M, FB_2^U \right), \left(FC_2^L, FC_2^M, FC_2^U \right) \end{array} \right\}$ and $FF = \left\{ \begin{array}{l} \left(FA^L, FA^M, FA^U \right), \\ \left(FB^L, FB^M, FB^U \right), \left(FC^L, FC^M, FC^U \right) \end{array} \right\}$, the operation laws of them are implemented:

(1) $FF_1 \oplus FF_2 = \left\{ \begin{array}{l} \left(FA_1^L + FA_2^L \text{-} FA_1^L FA_2^L, FA_1^M + FA_2^M \text{-} FA_1^M FA_2^M, FA_1^U + FA_2^U \text{-} FA_1^U FA_2^U \right), \\ \left(FB_1^L FB_2^L, FB_1^M FB_2^M, FB_1^U FB_2^U \right), \left(FC_1^L FC_2^L, FC_1^M FC_2^M, FC_1^U FC_2^U \right) \end{array} \right\}$;

(2) $FF_1 \otimes FF_2 = \left\{ \begin{array}{l} \left(FA_1^L FA_2^L, FA_1^M FA_2^M, FA_1^U FA_2^U \right), \\ \left(FB_1^L + FB_2^L \text{-} FB_1^L FB_2^L, FB_1^M + FB_2^M \text{-} FB_1^M FB_2^M, FB_1^U + FB_2^U \text{-} FB_1^U FB_2^U \right), \\ \left(FC_1^L + FC_2^L \text{-} FC_1^L FC_2^L, FC_1^M + FC_2^M \text{-} FC_1^M FC_2^M, FC_1^U + FC_2^U \text{-} FC_1^U FC_2^U \right) \end{array} \right\}$;

$$(3)\ \xi FF=\begin{Bmatrix}\left(1-\left(1-FA^{L}\right)^{\xi},1-\left(1-FA^{M}\right)^{\xi},1-\left(1-FA^{U}\right)^{\xi}\right),\\ \left(\left(FB^{L}\right)^{\xi},\left(FB^{M}\right)^{\xi},\left(FB^{U}\right)^{\xi}\right),\\ \left(\left(FC^{L}\right)^{\xi},\left(FC^{M}\right)^{\xi},\left(FC^{U}\right)^{\xi}\right)\end{Bmatrix},\xi>0;$$

$$(4)FF^{\xi}=\begin{Bmatrix}\left(\left(FA^{L}\right)^{\xi},\left(FA^{M}\right)^{\xi},\left(FA^{U}\right)^{\xi}\right),\\ \left(1-\left(1-FB^{L}\right)^{\xi},1-\left(1-FB^{M}\right)^{\xi},1-\left(1-FB^{U}\right)^{\xi}\right),\\ \left(1-\left(1-FC^{L}\right)^{\xi},1-\left(1-FC^{M}\right)^{\xi},1-\left(1-FC^{U}\right)^{\xi}\right)\end{Bmatrix},\xi>0.$$

The operation laws have several properties.

$$(1)\ FF_1\oplus FF_2=FF_2\oplus FF_1, FF_1\otimes FF_2=FF_2\otimes FF_1,\left(\left(FF_1\right)^{\xi_1}\right)^{\xi_2}=\left(FF_1\right)^{\xi_1\xi_2}; \tag{5}$$

$$(2)\ \xi\left(FF_1\oplus FF_2\right)=\xi FF_1\oplus\xi FF_2,\left(FF_1\otimes FF_2\right)^{\xi}=\left(FF_1\right)^{\xi}\otimes\left(FF_2\right)^{\xi}; \tag{6}$$

$$(3)\ \xi_1 FF_1\oplus\xi_2 FF_1=\left(\xi_1+\xi_2\right)FF_1,\left(FF_1\right)^{\xi_1}\otimes\left(FF_1\right)^{\xi_2}=\left(FF_1\right)^{\left(\xi_1+\xi_2\right)}. \tag{7}$$

Definition 3[26]. Let $FF=\begin{Bmatrix}\left(FA^{L},FA^{M},FA^{U}\right),\\ \left(FB^{L},FB^{M},FB^{U}\right),\left(FC^{L},FC^{M},FC^{U}\right)\end{Bmatrix}$ be a TFNN, the score and accuracy functions are:

$$SF\left(FF\right)=\frac{1}{12}\begin{bmatrix}8+\left(FA^{L}+2FA^{M}+FA^{U}\right)\\ -\left(FB^{L}+FVB^{M}+FB^{U}\right)\\ -\left(FC^{L}+2FC^{M}+FC^{U}\right)\end{bmatrix},\qquad SF\left(FF\right)\in\left[0,1\right] \tag{8}$$

$$AF\left(FF\right)=\frac{1}{8}\begin{bmatrix}4+\left(FA^{L}+2FA^{M}+FA^{U}\right)\\ -\left(FB^{L}+2FB^{M}+FB^{U}\right)\end{bmatrix},\qquad AF\left(FF\right)\in\left[-1,1\right] \tag{9}$$

For FF_1 and FF_2, based on Definition 3, we have:

(1) $if\ SF\left(FF_1\right)\prec SF\left(FF_2\right), then\ FF_1\prec FF_2;$

(2) $if\ SF\left(FF_1\right)=SF\left(FF_2\right), AF\left(FF_1\right)\prec AF\left(FF_2\right), then\ FF_1\prec FF_2;$

(3) $if\ SF\left(FF_1\right)=SF\left(FF_2\right), AF\left(FF_1\right)=AF\left(FF_2\right), then\ FF_1=FF_2.$

Definition 4[26]. Let $FF_1=\begin{Bmatrix}\left(FA_1^{L},FA_1^{M},FA_1^{U}\right),\\ \left(FB_1^{L},FB_1^{M},FB_1^{U}\right),\left(FC_1^{L},FC_1^{M},FC_1^{U}\right)\end{Bmatrix}$, $FF_2=\begin{Bmatrix}\left(FA_2^{L},FA_2^{M},FA_2^{U}\right),\\ \left(FB_2^{L},FB_2^{M},FB_2^{U}\right),\left(FC_2^{L},FC_2^{M},FC_2^{U}\right)\end{Bmatrix}$ be two TFNNs, the normalized Hamming distance is defined:

$$HD\left(FF_1,FF_2\right) = \frac{1}{9}\begin{pmatrix} \left|FA_1^L-FA_2^L\right|+\left|FA_1^M-FA_2^M\right|+\left|FA_1^U-FA_2^U\right| \\ +\left|FB_1^L-FB_2^L\right|+\left|FB_1^M-FB_2^M\right|+\left|FB_1^U-FB_2^U\right| \\ +\left|FC_1^L-FC_2^L\right|+\left|FC_1^M-FC_2^M\right|+\left|FC_1^U-FC_2^U\right| \end{pmatrix} \tag{10}$$

3. HyperSoft with Cross-Entropy approach between T2NN

We define the operations of HyperSoft as:

We can define the soft set as:

Let U be a universe of discourse (U) the power set of U and X a set of criteria. Then, the pair $(F, U), F: U \rightarrow$ is called a Soft Set over U.

HyperSoft can be defined as:

Let U be a universe of discourse (U) the power set of U. Let $x_1, x_2, \ldots, x_n for\ n \geq 1$, be n distinct criteria, whose corresponding criteria are respectively set of $X_1, X_2, \ldots, X_n for\ n\ with\ X_i \cap A_j = \emptyset, for\ i \neq j\ and\ i, j \in \{1,2, \ldots, n\}$

Then the pair $(F: X_1 \times X_2 \times X_3 \times \ldots X_n), X_n: \rightarrow U$ is called a HyperSoft Set over U.

Then, the cross-entropy approach for T2NNs is conducted based on cross-entropy approach.

Definition 4[6]. Two fuzzy sets are conducted: $a=\left(a\left(F_1\right),a\left(F_2\right),\cdots,a\left(F_n\right)\right)$ and $b=\left(b\left(F_1\right),b\left(F_2\right),\cdots,b\left(F_n\right)\right)$, $F_i \in \left(f_1, f_2, \cdots, f_n\right)$.

$$CE(a,b)=\sum_{j=1}^{n}\begin{pmatrix} a\left(F_i\right)\ln\dfrac{a\left(F_i\right)}{\frac{1}{2}\left(a\left(F_i\right)+b\left(F_i\right)\right)} \\ +\left(1-a\left(F_i\right)\right)\ln\dfrac{1-a\left(F_i\right)}{1-\frac{1}{2}\left(a\left(F_i\right)+b\left(F_i\right)\right)} \end{pmatrix} \tag{11}$$

$CE(a,b)$ is the classical cross-entropy approach of a from b.

The cross-entropy approach could be conducted under T2NNs. Assume there are two T2NNs, $FF_1=\left\{\begin{matrix}\left(FA_1^L,FA_1^M,FA_1^U\right), \\ \left(FB_1^L,FB_1^M,FB_1^U\right),\left(FC_1^L,FC_1^M,FC_1^U\right)\end{matrix}\right\}$ and $FF_2=\left\{\begin{matrix}\left(FA_2^L,FA_2^M,FA_2^U\right), \\ \left(FB_2^L,FB_2^M,FB_2^U\right),\left(FC_2^L,FC_2^M,FC_2^U\right)\end{matrix}\right\}$.

According to Eq. (11), the cross-entropy approach for truth-membership is constructed:

$$
\begin{aligned}
&CE\left(\left(FA_1^L, FA_1^M, FA_1^U\right),\left(FA_2^L, FA_2^M, FA_2^U\right)\right) \\
&= FA_1^L \ln \frac{FA_1^L}{\frac{1}{2}\left(FA_1^L + FA_2^L\right)} + \left(1 - FA_1^L\right)\ln \frac{1 - FA_1^L}{1 - \frac{1}{2}\left(FA_1^L + FA_2^L\right)} \\
&+ FA_1^M \ln \frac{FA_1^M}{\frac{1}{2}\left(FA_1^M + FA_2^M\right)} + \left(1 - FA_1^M\right)\ln \frac{1 - FA_1^M}{1 - \frac{1}{2}\left(FA_1^M + FA_2^M\right)} \\
&+ FA_1^U \ln \frac{FA_1^U}{\frac{1}{2}\left(FA_1^U + FA_2^U\right)} + \left(1 - FA_1^U\right)\ln \frac{1 - FA_1^U}{1 - \frac{1}{2}\left(FA_1^U + FA_2^U\right)}
\end{aligned}
\tag{12}
$$

Similar to the cross-entropy approach of truth-membership, the cross-entropy approach for indeterminacy-membership is conducted:

$$
\begin{aligned}
&CE\left(\left(FB_1^L, FB_1^M, FB_1^U\right),\left(FB_2^L, FB_2^M, FB_2^U\right)\right) \\
&= FB_1^L \ln \frac{FB_1^L}{\frac{1}{2}\left(FB_1^L + FB_2^L\right)} + \left(1 - FB_1^L\right)\ln \frac{1 - FB_1^L}{1 - \frac{1}{2}\left(FB_1^L + FB_2^L\right)} \\
&+ FB_1^M \ln \frac{FB_1^M}{\frac{1}{2}\left(FB_1^M + FB_2^M\right)} + \left(1 - FB_1^M\right)\ln \frac{1 - FB_1^M}{1 - \frac{1}{2}\left(FB_1^M + FB_2^M\right)} \\
&+ FB_1^U \ln \frac{FB_1^U}{\frac{1}{2}\left(FB_1^U + FB_2^U\right)} + \left(1 - FB_1^U\right)\ln \frac{1 - FB_1^U}{1 - \frac{1}{2}\left(FB_1^U + FB_2^U\right)}
\end{aligned}
\tag{13}
$$

Similar to the cross-entropy approach of truth-membership, the cross-entropy approach for falsity-membership is conducted:

$$
\begin{aligned}
&CE\left(\left(FC_1^L, FC_1^M, FC_1^U\right),\left(FC_2^L, FC_2^M, FC_2^U\right)\right) \\
&= \left(
\begin{aligned}
&FA_1^L \ln \frac{FA_1^L}{\frac{1}{2}\left(FA_1^L + FA_2^L\right)} + \left(1 - FA_1^L\right)\ln \frac{1 - FA_1^L}{1 - \frac{1}{2}\left(FA_1^L + FA_2^L\right)} \\
&+ FA_1^M \ln \frac{FA_1^M}{\frac{1}{2}\left(FA_1^M + FA_2^M\right)} + \left(1 - FA_1^M\right)\ln \frac{1 - FA_1^M}{1 - \frac{1}{2}\left(FA_1^M + FA_2^M\right)} \\
&+ FA_1^U \ln \frac{FA_1^U}{\frac{1}{2}\left(FA_1^U + FA_2^U\right)} + \left(1 - FA_1^U\right)\ln \frac{1 - FA_1^U}{1 - \frac{1}{2}\left(FA_1^U + FA_2^U\right)}
\end{aligned}
\right)
\end{aligned}
\tag{14}
$$

So the cross-entropy approach of FF_1 from FF_2 can be conducted:

$$CE\left(FF_1, FF_2\right)$$
$$=\left(\begin{array}{l} CE\left(FA_1^L, FA_1^M, FA_1^U\right),\left(FA_2^L, FA_2^M, FA_2^U\right) \\ +CE\left(FB_1^L, FB_1^M, FB_1^U\right),\left(FB_2^L, FB_2^M, FB_2^U\right) \\ +CE\left(FC_1^L, FC_1^M, FC_1^U\right),\left(FC_2^L, FC_2^M, FC_2^U\right) \end{array}\right)$$
$$=\left(\begin{array}{l} FA_1^L \ln \frac{FA_1^L}{\frac{1}{2}\left(FA_1^L+FA_2^L\right)}+\left(1-FA_1^L\right)\ln \frac{1-FA_1^L}{1-\frac{1}{2}\left(FA_1^L+FA_2^L\right)} \\ +FA_1^M \ln \frac{FA_1^M}{\frac{1}{2}\left(FA_1^M+FA_2^M\right)}+\left(1-FA_1^M\right)\ln \frac{1-FA_1^M}{1-\frac{1}{2}\left(FA_1^M+FA_2^M\right)} \\ +FA_1^U \ln \frac{FA_1^U}{\frac{1}{2}\left(FA_1^U+FA_2^U\right)}+\left(1-FA_1^U\right)\ln \frac{1-FA_1^U}{1-\frac{1}{2}\left(FA_1^U+FA_2^U\right)} \end{array}\right)$$
$$+\left(\begin{array}{l} FB_1^L \ln \frac{FB_1^L}{\frac{1}{2}\left(FB_1^L+FB_2^L\right)}+\left(1-FB_1^L\right)\ln \frac{1-FB_1^L}{1-\frac{1}{2}\left(FB_1^L+FB_2^L\right)} \\ +FB_1^M \ln \frac{FB_1^M}{\frac{1}{2}\left(FB_1^M+FB_2^M\right)}+\left(1-FB_1^M\right)\ln \frac{1-FB_1^M}{1-\frac{1}{2}\left(FB_1^M+FB_2^M\right)} \\ +FB_1^U \ln \frac{FB_1^U}{\frac{1}{2}\left(FB_1^U+FB_2^U\right)}+\left(1-FB_1^U\right)\ln \frac{1-FB_1^U}{1-\frac{1}{2}\left(FB_1^U+FB_2^U\right)} \end{array}\right)$$
$$+\left(\begin{array}{l} FC_1^L \ln \frac{FC_1^L}{\frac{1}{2}\left(FC_1^L+FC_2^L\right)}+\left(1-FC_1^L\right)\ln \frac{1-FC_1^L}{1-\frac{1}{2}\left(FC_1^L+FC_2^L\right)} \\ +FC_1^M \ln \frac{FC_1^M}{\frac{1}{2}\left(FC_1^M+FC_2^M\right)}+\left(1-FC_1^M\right)\ln \frac{1-FC_1^M}{1-\frac{1}{2}\left(FC_1^M+FC_2^M\right)} \\ +FC_1^U \ln \frac{FC_1^U}{\frac{1}{2}\left(FC_1^U+FC_2^U\right)}+\left(1-FC_1^U\right)\ln \frac{1-FC_1^U}{1-\frac{1}{2}\left(FC_1^U+FC_2^U\right)} \end{array}\right) \quad (15)$$

The larger the difference between FF_1 and FF_2, the larger $CE\left(FF_1, FF_2\right)$ is.

In line with the Shannon's inequality[28], it is easily conducted that $0 \le \text{T2NN-CE}\left(FF_1, FF_2\right) \le 1$, and $\text{T2NN-CE}\left(FF_1, FF_2\right) = 0$ if and only if

$\left(FA_1^L, FA_1^M, FA_1^U\right)=\left(FA_2^L, FA_2^M, FA_2^U\right), \left(FB_1^L, FB_1^M, FB_1^U\right)=\left(FB_2^L, FB_2^M, FB_2^U\right)$,
$\left(FC_1^L, FC_1^M, FC_1^U\right)=\left(FC_2^L, FC_2^M, FC_2^U\right)$.

4. Models for MADM

Let $FA=\{FA_1, FA_2, \cdots, FA_m\}$ be alternatives, $FC=\{FC_1, FC_2, \cdots, FC_n\}$ be attributes. $f\omega=\left(f\omega_1, f\omega_2, \cdots, f\omega_n\right)$ is the weight information for attributes, $f\omega_j$ is the weight of FC_j, where $f\omega_j \in [0,1]$, and $\sum_{j=1}^{n} f\omega_j = 1$. Then, the T2NN-HS-CE approach is conducted to solve the MADM with information entropy.

Step 1. Obtain the decision matrix $FR=\left(FR_{ij}\right)_{m\times n}=\begin{Bmatrix}\left(FA_{ij}^L, FA_{ij}^M, FA_{ij}^U\right),\\ \left(FB_{ij}^L, FB_{ij}^M, FB_{ij}^U\right),\\ \left(FC_{ij}^L, FC_{ij}^M, FC_{ij}^U\right)\end{Bmatrix}_{m\times n}$, $FA_{ij}^L, FA_{ij}^M, FA_{ij}^U \in [0,1]$, $FB_{ij}^L, FB_{ij}^M, FB_{ij}^U \in [0,1]$, $\left(FC_{ij}^L, FC_{ij}^M, FC_{ij}^U\right) \in [0,1]$. $FA_{ij}^L < FA_{ij}^M < FA_{ij}^U$, $FB_{ij}^L < FB_{ij}^M < FB_{ij}^U$, $FC_{ij}^L < FC_{ij}^M < FC_{ij}^U$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

Step 2. Normalize the $FR=\left(FR_{ij}\right)_{m\times n}=\begin{Bmatrix}\left(FA_{ij}^L, FA_{ij}^M, FA_{ij}^U\right),\\ \left(FB_{ij}^L, FB_{ij}^M, FB_{ij}^U\right),\\ \left(FC_{ij}^L, FC_{ij}^M, FC_{ij}^U\right)\end{Bmatrix}_{m\times n}$ into

$$NFR=\left(FR_{ij}\right)_{m\times n}=\begin{Bmatrix}\left(NFA_{ij}^L, NFA_{ij}^M, NFA_{ij}^U\right),\\ \left(NFB_{ij}^L, NFB_{ij}^M, NFB_{ij}^U\right),\\ \left(NFC_{ij}^L, NFC_{ij}^M, NFC_{ij}^U\right)\end{Bmatrix}_{m\times n}.$$

For benefit attributes:

$$\begin{aligned} NFR&=\left(NFR_{ij}\right)_{m\times n}\\ &=\begin{Bmatrix}\left(NFA_{ij}^L, NFA_{ij}^M, NFA_{ij}^U\right),\\ \left(NFB_{ij}^L, NFB_{ij}^M, NFB_{ij}^U\right),\\ \left(NFC_{ij}^L, NFC_{ij}^M, NFC_{ij}^U\right)\end{Bmatrix}=\begin{Bmatrix}\left(FA_{ij}^L, FA_{ij}^M, FA_{ij}^U\right),\\ \left(FB_{ij}^L, FB_{ij}^M, FB_{ij}^U\right),\\ \left(FC_{ij}^L, FC_{ij}^M, FC_{ij}^U\right)\end{Bmatrix}_{m\times n} \end{aligned} \tag{16}$$

For cost attributes:

$$NFR = \left(NFR_{ij}\right)_{m\times n}
= \begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} = \begin{Bmatrix} \left(FC_{ij}^{L}, FC_{ij}^{M}, FC_{ij}^{U}\right), \\ \left(FB_{ij}^{L}, FB_{ij}^{M}, FB_{ij}^{U}\right), \\ \left(FA_{ij}^{L}, FA_{ij}^{M}, FA_{ij}^{U}\right) \end{Bmatrix}_{m\times n} \tag{17}$$

Step 3. Conduct the attributes weight through employing information entropy.

The information entropy is employed to put forward the weight values under different environment[29-33]. Entropy [27] is an efficient approach to put forward the weight values. Then, the normalized T2NN decision matrix $NT2NNDM_{ij}$ is conducted:

$$NT2NNDM_{ij} = \frac{\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix}\right)}{\sum_{i=1}^{m}\left(\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix}\right)\right)}, \tag{18}$$

Then, the T2NN Shannon decision entropy $T2NNSDE = \left(T2NNSDE_1, T2NNSDE_2, \cdots, T2NNSDE_n\right)$ is constructed:

$$T2NNSDE_j = -\frac{1}{\ln m}\sum_{i=1}^{m} NT2NNDM_{ij} \ln NT2NNDM_{ij}$$

$$= -\frac{1}{\ln m}\sum_{i=1}^{m}\left(\frac{\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} \right)}{\sum_{i=1}^{m}\left(\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} \right)\right)} \times \ln \frac{\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} \right)}{\sum_{i=1}^{m}\left(\left(SF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} + AF\begin{Bmatrix} \left(NFA_{ij}^{L}, NFA_{ij}^{M}, NFA_{ij}^{U}\right), \\ \left(NFB_{ij}^{L}, NFB_{ij}^{M}, NFB_{ij}^{U}\right), \\ \left(NFC_{ij}^{L}, NFC_{ij}^{M}, NFC_{ij}^{U}\right) \end{Bmatrix} \right)\right)} \right) \tag{19}$$

and $NT2NNDM_{ij} \ln NT2NNDM_{ij} = 0$ if $NT2NNDM_{ij} = 0$.

Then, the weight values $f\omega = \left(f\omega_1, f\omega_2, \cdots, f\omega_n\right)$ is constructed:

$$f\omega_j = \frac{1 - T2NNSDE_j}{\sum_{j=1}^{n}\left(1 - T2NNSDE_j\right)}, \quad j = 1, 2, \cdots, n. \tag{20}$$

Step 4. Conduct the T2NN positive ideal decision solution (T2NNPIDS) and NFR_i:

$$T2NNPIDS_j = \left\{ \begin{array}{l} \left(NFA_j^L, NFA_j^M, NFA_j^U\right), \\ \left(NFB_j^L, NFB_j^M, NFB_j^U\right), \\ \left(NFC_j^L, NFC_j^M, NFC_j^U\right) \end{array} \right\} \tag{21}$$

$$SF\left\{ \begin{array}{l} \left(NFA_j^L, NFA_j^M, NFA_j^U\right), \\ \left(NFB_j^L, NFB_j^M, NFB_j^U\right), \\ \left(NFC_j^L, NFC_j^M, NFC_j^U\right) \end{array} \right\} = \underset{i}{Max}\, SF\left\{ \begin{array}{l} \left(NFA_{ij}^L, NFA_{ij}^M, NFA_{ij}^U\right), \\ \left(NFB_{ij}^L, NFB_{ij}^M, NFB_{ij}^U\right), \\ \left(NFC_{ij}^L, NFC_{ij}^M, NFC_{ij}^U\right) \end{array} \right\} \tag{22}$$

$$NFR_i = \left\{ \begin{array}{l} \left(NFA_{ij}^L, NFA_{ij}^M, NFA_{ij}^U\right), \\ \left(NFB_{ij}^L, NFB_{ij}^M, NFB_{ij}^U\right), \\ \left(NFC_{ij}^L, NFC_{ij}^M, NFC_{ij}^U\right) \end{array} \right\} \tag{23}$$

Step 5. Conduct the T2NN-CE between $T2NNPIDS$ and NFR_i as:

$$
\begin{aligned}
&T2NN\text{-}CE\left(NFR_i, T2NNPIDS\right)\\
&=\sum_{j=1}^{n}\left(f\omega_j \left(\begin{array}{l}
FA_{ij}^{L}\ln\frac{FA_{ij}^{L}}{\frac{1}{2}\left(FA_{ij}^{L}+FA_{j}^{L}\right)}+\left(1-FA_{ij}^{L}\right)\ln\frac{1-FA_{ij}^{L}}{1-\frac{1}{2}\left(FA_{ij}^{L}+FA_{j}^{L}\right)}\\
+FA_{ij}^{M}\ln\frac{FA_{ij}^{M}}{\frac{1}{2}\left(FA_{ij}^{M}+FA_{j}^{M}\right)}+\left(1-FA_{ij}^{M}\right)\ln\frac{1-FA_{ij}^{M}}{1-\frac{1}{2}\left(FA_{ij}^{M}+FA_{j}^{M}\right)}\\
+FA_{ij}^{U}\ln\frac{FA_{ij}^{U}}{\frac{1}{2}\left(FA_{ij}^{U}+FA_{j}^{U}\right)}+\left(1-FA_{ij}^{U}\right)\ln\frac{1-FA_{ij}^{U}}{1-\frac{1}{2}\left(FA_{ij}^{U}+FA_{j}^{U}\right)}
\end{array}\right)\right)\\
&+\sum_{j=1}^{n}\left(f\omega_j \left(\begin{array}{l}
FB_{ij}^{L}\ln\frac{FB_{ij}^{L}}{\frac{1}{2}\left(FB_{ij}^{L}+FB_{j}^{L}\right)}+\left(1-FB_{ij}^{L}\right)\ln\frac{1-FB_{ij}^{L}}{1-\frac{1}{2}\left(FB_{ij}^{L}+FB_{j}^{L}\right)}\\
+FB_{ij}^{M}\ln\frac{FB_{ij}^{M}}{\frac{1}{2}\left(FB_{ij}^{M}+FB_{j}^{M}\right)}+\left(1-FB_{ij}^{M}\right)\ln\frac{1-FB_{ij}^{M}}{1-\frac{1}{2}\left(FB_{ij}^{M}+FB_{j}^{M}\right)}\\
+FB_{ij}^{U}\ln\frac{FB_{ij}^{U}}{\frac{1}{2}\left(FB_{ij}^{U}+FB_{j}^{U}\right)}+\left(1-FB_{ij}^{U}\right)\ln\frac{1-FB_{ij}^{U}}{1-\frac{1}{2}\left(FB_{ij}^{U}+FB_{j}^{U}\right)}
\end{array}\right)\right)\\
&+\sum_{j=1}^{n}\left(f\omega_j \left(\begin{array}{l}
FC_{ij}^{L}\ln\frac{FC_{ij}^{L}}{\frac{1}{2}\left(FC_{ij}^{L}+FC_{j}^{L}\right)}+\left(1-FC_{ij}^{L}\right)\ln\frac{1-FC_{ij}^{L}}{1-\frac{1}{2}\left(FC_{ij}^{L}+FC_{j}^{L}\right)}\\
+FC_{ij}^{M}\ln\frac{FC_{ij}^{M}}{\frac{1}{2}\left(FC_{ij}^{M}+FC_{j}^{M}\right)}+\left(1-FC_{ij}^{M}\right)\ln\frac{1-FC_{ij}^{M}}{1-\frac{1}{2}\left(FC_{ij}^{M}+FC_{j}^{M}\right)}\\
+FC_{ij}^{U}\ln\frac{FC_{ij}^{U}}{\frac{1}{2}\left(FC_{ij}^{U}+FC_{j}^{U}\right)}+\left(1-FC_{ij}^{U}\right)\ln\frac{1-FC_{ij}^{U}}{1-\frac{1}{2}\left(FC_{ij}^{U}+FC_{j}^{U}\right)}
\end{array}\right)\right)
\end{aligned} \tag{24}
$$

Step 6. In line with the $T2NN\text{-}CE\left(NFR_i, T2NNPIDS\right)$, the smaller $T2NN\text{-}CE\left(NFR_i, T2NNPIDS\right)$ is, the better alternative is.

5. Case Study

In the context of globalization, the comprehensive evaluation of innovation capability in the central cities of emerging countries is particularly important as these cities are the frontiers of economic and technological development. This evaluation not only influences the formulation of city policies but is also a crucial factor in attracting domestic and international investments. Typically, such evaluations involve multiple dimensions, including technological innovation, industrial upgrading capability, the concentration of high-tech enterprises, the efficiency of innovation resource allocation, and the optimization of the innovation environment. Firstly, technological innovation is at the core of the evaluation, involving indicators such as the number of patent applications, the influence of research institutions, and the rate of technological achievement transformation. These indicators vividly reflect the level of a city's technological innovation activities and the practical application of its technological outcomes. Secondly, the capability of industrial upgrading focuses on the optimization of the city's industrial structure and the growth rate of high-tech industries. By analyzing the effectiveness of industrial structure adjustments and the growth rate of emerging industries, one can assess a city's ability to adapt to the global economic trends. Furthermore, the concentration of high-tech enterprises is another important indicator of a city's innovation capability. A high density of technology enterprises can foster a regional innovation atmosphere, promoting rapid knowledge and technology turnover and renewal. Additionally, the efficiency of innovation resource allocation involves the effective configuration and utilization of resources such as talent, capital, and information. Efficient resource allocation can significantly enhance a city's innovation efficiency and accelerate the production of innovative outcomes. Lastly, the optimization of the innovation environment includes aspects such as government policy support, intellectual property protection, and cooperation mechanisms between research and industry. An optimized innovation environment can provide enterprises and research institutions with favorable external conditions, facilitating the incubation and growth of technological achievements. Evaluating these dimensions comprehensively not only provides a thorough understanding of a central city's innovation capability but also offers decision support for policymakers, helping them to identify strengths and weaknesses and formulate corresponding enhancement strategies. The comprehensive evaluation of innovation capability in central cities of emerging countries can be seen as a MADM problem. In this section, a numerical example is conducted for comprehensive evaluation of innovation capability in central cities of emerging countries through employing the T2NN-HS-CE approach. Assume that seven central cities of emerging countries are selected with 8 attributes.

5.1 Identify a Set of Criteria and Alternatives

Choice of the best alternatives based on a set of criteria. We defined 8 criteria and seven alternatives with several values as shown in Table 1.

Table 1. The set of criteria.

	Criteria	Values	Weights
C_1	Tech Transfer Efficiency	100, 250, and 500projects/year	0.126719
C_2	Environmental Sustainability: Green Innovation	10%, 25%, and 50%	0.069933
C_3	Governance and Policy: Innovation-Friendly Policies	10, 25, and 50 policies	0.104498
C_4	Infrastructure and Connectivity: Urban Mobility	5,10, and 20	0.064706
C_5	Economic: Startup Density	0–50 startups and 10,000 residents	0.198644
C_6	Patent Output	100, 250, and 500 patents	0.147166
C_7	Technological: Digital Infrastructure	10 Mbps, 100, and 500 Mbps	0.162071
C_8	Human Capital and Social: Educational Attainment	10%, 50%, and 70%.	0.126263

5.2 HyperSoft with Cross Entropy Results

Let $C = C_1 \times C_2 \times C_3 \times C_4 \times C_5 \times C_6 \times C_7 \times C_8$ $and\ the\ values\ of\ crietria\ are\ (V_1, V_2, \ldots.)\ we\ select\ the\ 8\ values\ of\ crietria$

We bult the decision matrix using the T2NNs as shown in Table 2.

Then we compute the criteria by using the entropy method as shown in Table 1.

Then we normalize the decision matrix.

Then we conduct the T2NNPIDS.

Then we Conduct the T2NN-CE$\left(NFR_i, TFNNPIDS\right)$.

Then we rank the alternatives as shown in Figure 1.

Table 2. The T2NNs with criteria and alternatives.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))
A_2	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))
A_3	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))
A_4	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))
A_5	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))
A_6	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))
A_7	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))
A_2	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))
A_3	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))
A_4	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))
A_5	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))
A_6	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))
A_7	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))

A_2	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))
A_3	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))
A_4	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))
A_5	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))
A_6	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.50,0.30,0.50), (0.50,0.35,0.45), (0.45,0.30,0.60))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))
A_7	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))	((0.70,0.75,0.80), (0.15,0.20,0.25), (0.10,0.15,0.20))	((0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15))	((0.40,0.45,0.50), (0.40,0.45,0.50), (0.35,0.40,0.45))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95), (0.10,0.10,0.05), (0.05,0.05,0.05))

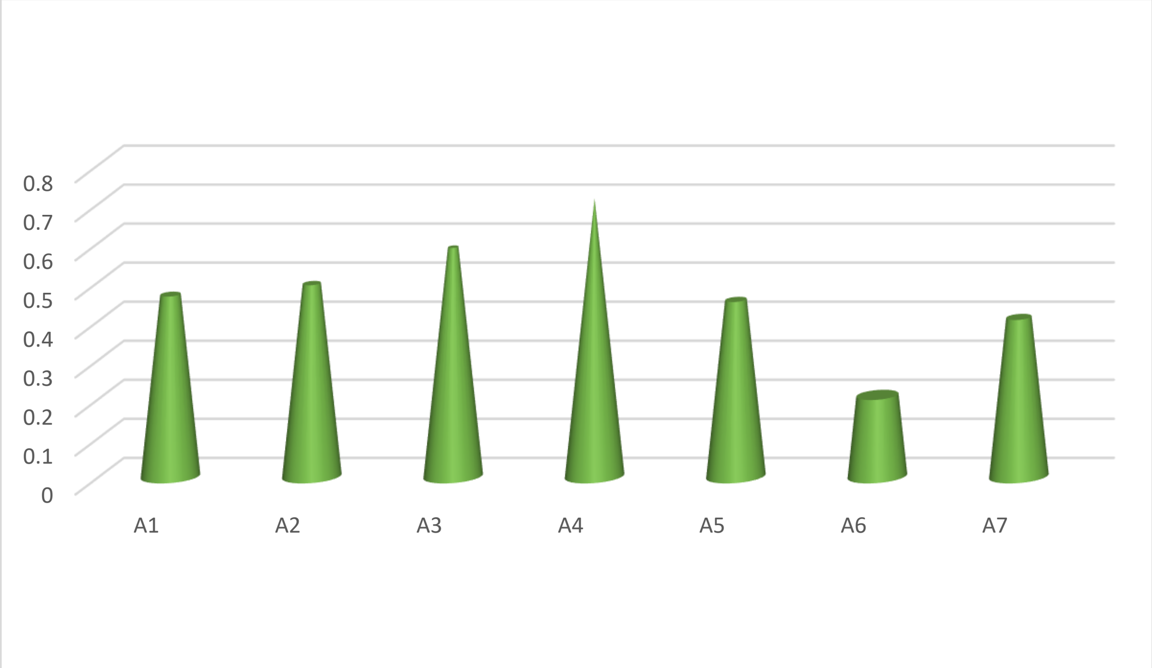

Figure 1. The rank of alternatives.

5.3 Sensitivity Analysis

This section shows the rank of alternatives under different weights of criteria. We change the criteria weights by 9 cases as shown in Figure 2. The first case show the equal weight of criteria. The second case shows the first attribute has 0.2 weights and other attributes are equal weights. Figure 3 shows the rank of alternatives under different weights. We show the alternative 4 is the best under different weights.

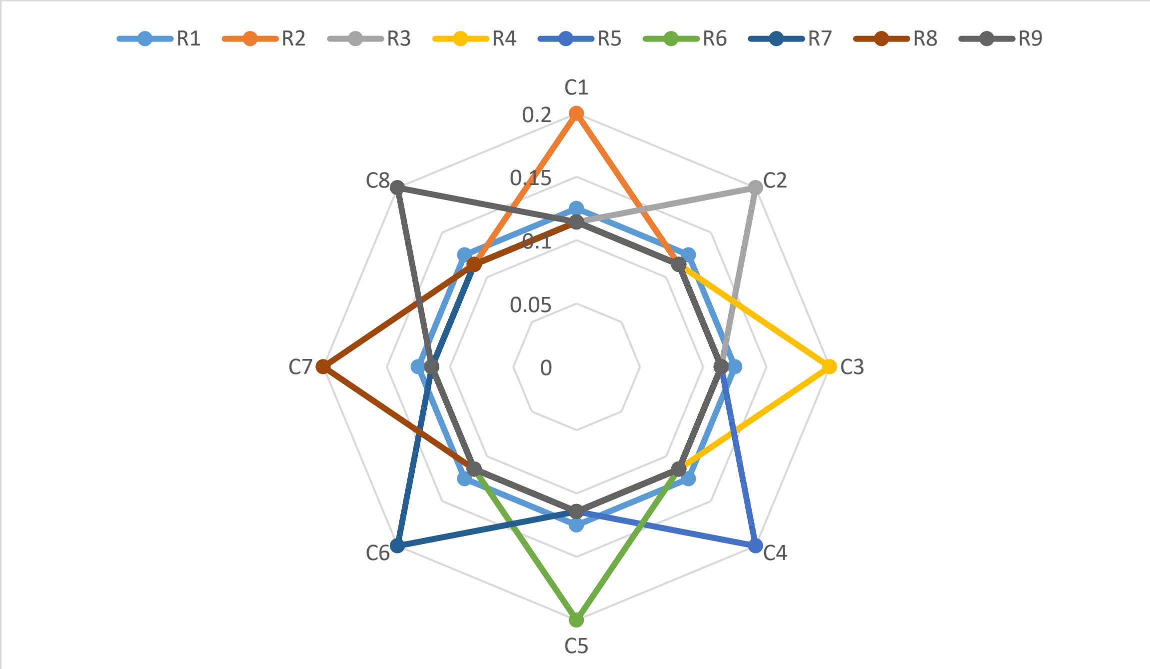

Figure 2. The different weights of criteria.

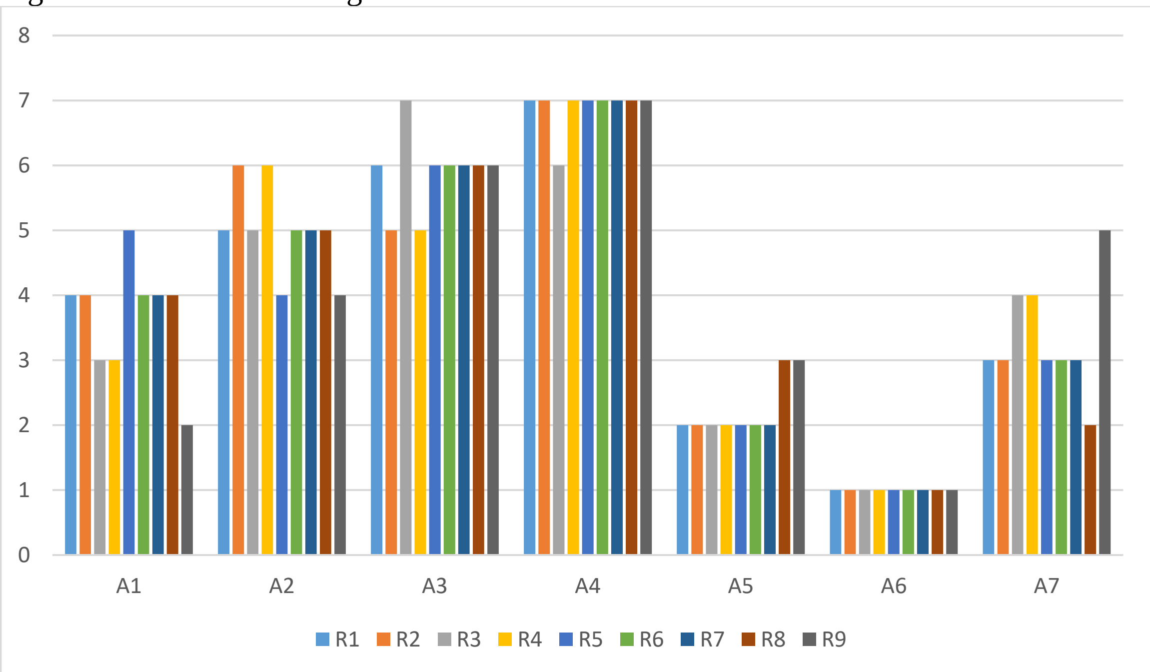

Figure 3. The rank of alternatives under different weights.

6. Conclusion

The comprehensive evaluation of innovation capacity in central cities of growing countries is a critical analytical and decision-making tool designed to measure and compare the innovation capabilities and potential of different central cities in innovation-driven development. This evaluation integrates multiple key indicators—such as scientific activities, enterprise innovation, policy support, and talent aggregation—to build a comprehensive model of innovation capacity assessment. Its purpose is to identify strengths and weaknesses in each city's efforts to promote technological progress and the development of high-tech industries, thereby providing data support for policymakers to optimize resource allocation and develop more effective regional innovation promotion strategies. Through this evaluation, cities can better position themselves within the national and global innovation networks, promote the optimization and upgrading of economic structures, and accelerate the transition from "growth-oriented" to "innovation-oriented" cities. The comprehensive evaluation of innovation capability in central cities of emerging countries is a MADM problem. Lately, the cross-entropy method has been adapted to address such MADM challenges. T2NNs have been utilized effectively to represent uncertain data throughout the evaluation process. This study introduces the application of the T2NN-HS-CE method to advance MADM analysis using T2NNs with HyperSoft. The HyperSoft is used to deal with complex decision making process. Additionally, this research includes a numerical study and multiple sensitivity analyses to assess and confirm the effectiveness of the proposed T2NN-HS-CE method. The seminal contributions of this research are enumerated as follows: (1) Implementation of the cross-entropy method alongside T2NNs with HyperSoft; (2) Application of the entropy method to establish weight values when using T2NNs and HyperSoft; (3) Development and application of the T2NN-HS-CE method for handling MADM problems characterized by T2NNs; (4) Creation of an algorithmic framework for a comprehensive evaluation of innovation capabilities in central cities of emerging countries, supported by a numerical example and sensitivity analyses to demonstrate the utility of the T2NN-HS-CE method.

The paper introduces a new MADM framework using T2NNs and T2NN-HS-CE method to evaluate the innovation capabilities of central cities in emerging countries. **While this research is innovative and potentially very practical, it has some shortcomings that require further improvements and exploration.** Firstly, the complexity of the T2NN-HS-CE method could pose a barrier to its practical application, especially in policymaking and urban planning settings. The high data requirements and complex processing involved may limit its popularity among non-specialists. Therefore, simplifying and optimizing this methodology to make it more understandable and operable is an important direction for future research. Secondly, this study primarily targets central cities in emerging nations, and its universality and adaptability have not been fully tested across different cultural and economic contexts. **Future research could expand the scope to include more countries and cities, exploring the effectiveness and flexibility of the model in varying environments.** A deeper analysis of data applicability and sensitivity would also help assess the potential for global application of the model. Lastly, integrating the T2NN-HS-CE method with other tools and indicators for assessing urban innovation capabilities, such as the Global Innovation Index, could offer a more comprehensive evaluation perspective. Developing a multi-level assessment framework that combines quantitative and qualitative analyses would provide a more thorough and in-depth evaluation of innovation capabilities. These suggested research directions can further refine and extend the methods and applications of this study, providing more effective tools and strategies for assessing the innovation capabilities of central cities in emerging countries.

Conflict of interest

The authors declare that they have no conflict of interest.

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