Neutrosophic Sets and Systems, Vol. 78, 2025

University of New Mexico

A Novel Approach to Assessing Art Education Teaching Quality in Vocational Colleges Based on Double-Valued Neutrosophic Numbers and Multi-Attribute Decision-Making with Tree Soft Sets

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Abstract: To promote stable and sustainable growth in vocational education, it is crucial to innovate and adapt the teaching methods of art education to align with modern educational trends. A comprehensive analysis of the art education curriculum is necessary to drive reform and modernize teaching methodologies, ensuring the continued relevance and effectiveness of art instruction. By updating art education, students can enhance their aesthetic appreciation, develop a refined understanding of high-quality artworks, and experience consistent growth in their artistic development. Moving beyond traditional teaching models is essential for the ongoing advancement of art education in vocational colleges. The evaluation of teaching quality in these institutions is approached using a well-established Multi-Attribute Decision Making (MADM) framework. Recently, the CoCoSo method, combined with average techniques, has been applied to address the complexities of MADM. To manage uncertain or ambiguous information during the evaluation process, Double-Valued Neutrosophic Sets (DVNSs) have also been utilized. This study introduces the integration of the CoCoSo method with DVNSs, leading to the development of the Double-Valued Neutrosophic Number CoCoSo (DVNN-CoCoSo) method for MADM. We used the Tree Soft Set (TSS) with the CoCoSo method to deal with the main and sub criteria to obtain the relation between them (DVNN-TSS-CoCoSo). To illustrate the practical application of this approach, a numerical example is provided to assess the quality of art education in vocational colleges. The DVNN-TSS-CoCoSo method offers a significant improvement in evaluating teaching quality, marking an important step toward refining evaluation processes and enhancing educational strategies in vocational settings.

Keywords: Multiple-attribute decision-making (MADM); DVNSs; CoCoSo approach; teaching quality evaluation; Tree Soft Set.

1. Introduction

Art is a special course. In order for students to learn art well, art teachers need to improve their art appreciation, cognitive ability, and artistic creation ability, in order to achieve comprehensive development of students[1]. Therefore, art teachers need to have a longer-term perspective when carrying out teaching activities. In the past teaching models, some teachers placed too much emphasis on conducting classroom teaching and imparting theoretical knowledge to students. Although this form can improve students' painting skills, it is difficult to enhance their practical abilities and develop their artistic literacy[2]. Therefore, teachers need to actively guide students to participate in social practice activities, so that students can improve their artistic literacy and develop their abilities in practical activities[3]. For example, college teachers could encourage students to participate in local or international painting competitions. By participating in art related competition activities, students can enhance their competitive awareness, actively learn art knowledge, and improve their own literacy and abilities. In addition, teachers can also lead or encourage students to visit art galleries and museums, allowing them to accumulate humanistic knowledge during the visit process, broaden their horizons by appreciating famous works, and improve their painting skills and artistic literacy[4]. Teachers should practice new educational concepts in art teaching in vocational colleges, in order to enrich students' learning experience and enable them to master art knowledge in a joyful environment. The new curriculum reform advocates more flexible forms of art teaching activities, which can create a good atmosphere for students[5]. In the process of promoting the new curriculum reform, teachers should clarify the purpose of art teaching, understand the key and difficult points of art subjects in vocational colleges, and demonstrate the new value of art courses. There are obvious differences between art courses in vocational colleges and ordinary high schools. Art teaching in vocational colleges has strong operability, and students can participate in creative practice, relying on visual and tactile senses to feel and create art works[6]. Therefore, teachers should comprehensively study art teaching in order to enhance the art teaching effectiveness. Currently, with the rapid development for information technology, electronic products have played an important role for people's lives and become an indispensable part of their daily lives. Numerous sectors have embraced modern information technology, with its impact being particularly noticeable in the education industry. The integration of modern information technology in educational settings has notably enhanced teaching effectiveness [7]. Art teachers in vocational colleges need to use multimedia technology in the classroom, which not only attracts students' attention, but also allows them to better experience the charm of art works, thereby improving their artistic literacy. In the process of teaching, art teachers should pay attention to combining theory with practice, allowing students to master theoretical knowledge while improving their practical abilities[8]. In addition to using multimedia technology, teachers can also introduce micro lessons in art teaching. Micro lessons mainly present targeted content to students in the form of short videos. This teaching method is not only concise, but also helps students consolidate their knowledge. Art teachers in vocational colleges can make reasonable use of micro courses to help students master relevant art theory knowledge. This not only saves more classroom time and allows students to engage in more practical activities, but also enables students to apply the theoretical knowledge they have learned to practice in a timely manner, thereby helping students improve their practical abilities and ultimately improve classroom teaching effectiveness[9]. Designing micro courses requires a certain level of computer operation ability from art teachers. Therefore, teachers should improve their computer application level and design more valuable and targeted micro courses[10]. In the process of teaching, art teachers for vocational colleges not only need to strengthen students' artistic foundation, but also need to integrate folk art into teaching and set more distinctive teaching content. Art is an art discipline, and in order to deepen students' learning of art, teachers should understand their learning interests and integrate content closely related to their daily lives into art teaching, in order to better enhance students' artistic literacy[11]. Teachers should avoid teaching fixed content according to textbooks, as this can limit students' thinking and vision. Teachers can explore the integration of folk art into art teaching, allowing students to be exposed to diverse art works and styles. In order to better demonstrate the importance of folk art, teachers can play movies related to folk art for students during classroom teaching, so that students can understand the development background of folk art, feel the characteristics of folk art, and better learn relevant art knowledge and skills.

MADM is a decision analysis technique primarily used to select the best option from multiple alternatives, especially when these alternatives vary across multiple evaluation dimensions[12, 13]. This method is applicable to various complex decision-making situations such as project selection, resource allocation, strategy formulation, and spans multiple fields including industry, economics, management, healthcare, environmental protection, and policy analysis[14, 15]. The core of multi-attribute decision-making lies in how to handle and synthesize multiple attributes or criteria to achieve a comprehensive evaluation and rational choice. In traditional decision-making processes, decision-makers might subjectively evaluate each option, whereas multi-attribute decision-making offers a systematic and quantitative framework that makes the decision-making process more objective, transparent, and verifiable[16]. The basic steps of MADM include establishing a decision matrix that lists all the alternatives and their performances on different attributes; determining the weight of each attribute to reflect its importance to the overall decision; and using appropriate decision rules (such as weighted sum, ideal solution distance, etc.) to assess and rank the options. Since the mid-20th century, when multi-attribute decision-making began to develop, various methods and techniques have emerged. Initially, the focus was on how to appropriately assign weights to attributes and how to handle quantitative data[17]. As theory and application evolved, researchers began exploring how to handle qualitative data, how to consider changes in decision-makers' preferences, and how to make decisions when facing uncertainty and fuzzy information. For example, the Analytic Hierarchy Process (AHP) introduced by Saaty in the 1970s, uses a hierarchical structure model to analyze and address decision issues, allowing complex decisions to be broken down into smaller, more manageable parts. Additionally, techniques such as TOPSIS and fuzzy comprehensive evaluation have been developed to handle more types of data and more complex decision environments. With the improvement of computational capabilities and the development of information technology, multi-attribute decision-making methods continue to evolve. Modern multi-attribute decision-making tools and software can handle large data sets, support more complex models and algorithms. Moreover, the development of artificial intelligence and machine learning provides new analytical tools for multi-attribute decision-making, such as automatically identifying attribute weights through data mining techniques or finding optimal decisions through optimization algorithms [18, 19]. Today, multi-attribute decision-making has become an indispensable tool for decision-makers facing complex issues. It not only helps decision-makers evaluate the pros and cons of different options but also promotes the scientification and standardization of the decision-making process, enhancing the rationality and effectiveness of decisions. With further globalization and technological advancement, MADM is expected to demonstrate its unique value in more fields and a wider range of application scenarios.

The art education teaching quality evaluation in vocational colleges is a well-established field of research in the context of MADM. Recently, the CoCoSo approach [20] and entropy [21] have been widely adopted to address MADM challenges. To characterize fuzzy information inherent in art education teaching quality evaluation, the utilization of DVNSs [22] has gained attention. However, most existing approaches have independently employed CoCoSo [20] and entropy [21] without considering their integration under DVNSs. Therefore, in this study, we propose the establishment of the DVNN-TSS-CoCoSo approach to effectively manage MADM. This model fills the gap by combining the CoCoSo [20] and entropy [21] under the framework of DVNSs.

The major motivations of this research are illustrated: (1) Introducing a novel MADM approach based on the integration of CoCoSo and average approaches under DVNSs with the TSS to deal with the criteria and sub criteria, thereby enhancing the accuracy and robustness of the evaluation process. (2) Incorporating objective weights through the utilization of the average approach, enabling a more comprehensive and unbiased assessment of the art education teaching quality in vocational colleges. (3) Proposing a new MADM approach, the DVNN-TSS-CoCoSo approach, specifically tailored for the evaluation of art education teaching quality in vocational colleges, considering the unique characteristics of DVNSs. (4) Demonstrating the effectiveness of the DVNN-TSS-CoCoSo model through a practical numerical example, thereby providing empirical evidence and validation for its applicability in the field of art education teaching quality evaluation in vocational colleges.

The framework of this study has been established and consists of the following sections: In Section 2, the concept of DVNSs is introduced. Section 3 focuses on the construction of the DVNN-TSS-CoCoSo approach. Section 4, numerical example, is presented to prove the practical application of DVNN-TSS. Finally, the study concludes in Section 5.

2. Preliminaries

Wang et al. [23] coped with the SVNSs.

Definition 1 [23]. The SVNSs is illustrated:

$$CA=\left\{\left(y,CT_A(y),CI_A(y),CF_A(y)\right)\middle|,y\in Y\right\} \tag{1}$$

where $CT_A(y),CI_A(y),CF_A(y)$ is truth-membership (TM), indeterminacy membership (IM) and falsity-membership (FM), $CT_A(y),CI_A(y),CF_A(y)\in[0,1]$, $0\le CT_A(y)+CI_A(y)+CF_A(y)\le 3$.

Kandasamy [22] illustrated the DVNSs.

Definition 2 [22]. The DVNSs is put forward:

$$CA = \{(\theta, CT_A(\theta), CIT_A(\theta), CIF_A(\theta), CF_A(\theta)) | \theta \in \Theta\}. \quad (2)$$

where $CT_A(\theta), CIT_A(\theta), CIF_A(\theta), CF_A(\theta)$ is TM, IM leaning towards TM, IM leaning towards FM, FM, $CT_A(\theta), CIT_A(\theta), CIF_A(\theta), CF_A(\theta) \in [0,1]$, $0 \le CT_A(\theta) + CIT_A(\theta) + CIF_A(\theta) + CF_A(\theta) \le 4$.

The DVNN is illustrated as $CA = (CT_A, CIT_A, CIF_A, CF_A)$, where $(CT_A, CIT_A, CIF_A, CF_A) \in [0,1]$, $0 \le CT_A + CIT_A + CIF_A + CF_A \le 4$.

Definition 3[22]. Let $CA = (CT_A, CIT_A, CIF_A, CF_A)$ be DVNN, score value is illustrated:

$$SV(CA) = \frac{(2 + CT_A + CIT_A - CIF_A - CF_A)}{4}, \ SV(CA) \in [0,1]. \quad (3)$$

Definition 4[22]. Let $CA = (CT_A, CIT_A, CIF_A, CF_A)$ be DVNN, accuracy value is illustrated:

$$AV(CA) = \frac{(CT_A + CIT_A + CIF_A + CF_A)}{4}, \ AV(CA) \in [0,1] \ . \quad (4)$$

The order for different DVNNs is illustrated.

Definition 5[22]. Let $CA = (CT_A, CIT_A, CIF_A, CF_A)$ and $CB = (CT_B, CIT_B, CIF_B, CF_B)$, $SV(CA) = \frac{(2 + CT_A + CIT_A - CIF_A - CF_A)}{4}$, $SV(CB) = \frac{(2 + CT_B + CIT_B - CIF_B - CF_B)}{4}$, $AV(CA) = \frac{(CT_A + CIT_A + CIF_A + CF_A)}{4}$, $AV(CB) = \frac{(CT_B + CIT_B + CIF_B + CF_B)}{4}$, $SV(CA) < SV(CB)$, $CA < CB$; $SV(CA) = SV(CB)$, (1) $AV(CA) = AV(CB)$, $CA = CB$; (2) $AV(CA) < AV(CB)$, $CA < CB$.

Definition 6[22]. Let $CA = (CT_A, CIT_A, CIF_A, CF_A)$ and $CB = (CT_B, CIT_B, CIF_B, CF_B)$ be DVNNs, the operations are illustrated:

(1) $CA \oplus CB = (CT_A + CT_B - CT_A CT_B, CIT_A + CIT_B - CIT_A CIT_B, CIF_A CIF_B, CF_A CF_B)$;

(2) $CA \otimes CB = (CT_A CT_B, CIT_A CIT_B, CIF_A + CIF_B - CIF_A CIF_B, CF_A + CF_B - CF_A CF_B)$;

(3) $\lambda CA = \left(1-(1-CT_A)^\lambda, 1-(1-CIT_A)^\lambda, (CIF_A)^\lambda, (CF_A)^\lambda\right), \lambda > 0$;

(4) $(CA)^\lambda = \left((CT_A)^\lambda, (CIT_A)^\lambda, 1-(1-CIF_A)^\lambda, 1-(1-CF_A)^\lambda\right), \lambda > 0.$

Definition 7[22]. Let $CA = (CT_A, CIT_A, CIF_A, CF_A)$ and $CB = (CT_B, CIT_B, CIF_B, CF_B)$, the Hamming distance for $CA = (CT_A, CIT_A, CIF_A, CF_A)$ and $CB = (CT_B, CIT_B, CIF_B, CF_B)$ is illustrated:

$$HAMMD(CA,CB)=\frac{1}{4}\begin{pmatrix}|CT_A-CT_B|+|CIT_A-CIT_B|\\+|CIF_A-CIF_B|+|CF_A-CF_B|\end{pmatrix} \tag{5-a}$$

Definition 8 [22]. Let $CA=(CT_A,CIT_A,CIF_A,CF_A)$ and $CB=(CT_B,CIT_B,CIF_B,CF_B)$, the Euclidean distance for $CA=(CT_A,CIT_A,CIF_A,CF_A)$ and $CB=(CT_B,CIT_B,CIF_B,CF_B)$ is illustrated:

$$EUCLD(CA,CB)=\sqrt{\frac{1}{4}\begin{pmatrix}|CT_A-CT_B|^2+|CIT_A-CIT_B|^2\\+|CIF_A-CIF_B|^2+|CF_A-CF_B|^2\end{pmatrix}} \tag{5-b}$$

Definition 9. Let $CA=(CT_A,CIT_A,CIF_A,CF_A)$ and $CB=(CT_B,CIT_B,CIF_B,CF_B)$, the Hellinger distance for $CA=(CT_A,CIT_A,CIF_A,CF_A)$ and $CB=(CT_B,CIT_B,CIF_B,CF_B)$ is illustrated:

$$HELLD(CA,CB)=\sqrt{\frac{1}{4}\begin{pmatrix}\left|\sqrt{CT_A}-\sqrt{CT_B}\right|^2+\left|\sqrt{CIT_A}-\sqrt{CIT_B}\right|^2\\+\left|\sqrt{CIF_A}-\sqrt{CIF_B}\right|^2+\left|\sqrt{CF_A}-\sqrt{CF_B}\right|^2\end{pmatrix}} \tag{5-c}$$

Definition 10

The TSS methodology is used to show the relationship between the criteria and sub criteria. The basic idea of this methodology can be expressed as:

Let P be universe of discourse, and Y non-empty subset of P with the power set of Y $P(Y)$.

Let X be a set of criteria for main nodes as $X=\{x_1,x_2,\ldots,x_n\}$ $where\ n\geq 1$ this is called the first level.

The sub criteria are located in the second level of TSS as $\{x_{1-1},x_{2-1},\ldots,x_{n-1}\}$.

3. DVNN-TSS-CoCoSo approach for MADM with average

The DVNN-TSS-CoCoSo approach is illustrated for MADM. Let $CA=\{CA_1,CA_2,\cdots,CA_m\}$ be different alternatives, $CG=\{CG_1,CG_2,\cdots,CG_n\}$ be different attributes with weight uw, $cw_j\in[0,1]$, $\sum_{j=1}^{n}cw_j=1$. The DVNNs are illustrated: $CR=(CR_{ij})_{m\times n}=(CT_{ij},CIT_{ij},CIF_{ij},CF_{ij})_{m\times n}$.

Step 1. Illustrate the DVNN-matrix $CR=(CR_{ij})_{m\times n}=(CT_{ij},CIT_{ij},CIF_{ij},CF_{ij})_{m\times n}$.

$$CR = \left[CR_{ij}\right]_{m\times n} = \begin{bmatrix} CR_{11} & CR_{12} & \dots & CR_{1n} \\ CR_{21} & CR_{22} & \dots & CR_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ CR_{m1} & CR_{m2} & \dots & CR_{mn} \end{bmatrix} \tag{6}$$

Step 2. Normalize the decision matrix

$$N_{ij} = \frac{CR_{ij} - \min CR_{ij}}{\max CR_{ij} - \min CR_{ij}} \text{ for beneficial criteria.} \tag{7}$$

$$N_{ij} = \frac{\max CR_{ij} - CR_{ij}}{\max CR_{ij} - \min CR_{ij}} \text{ for cost criteria.} \tag{8}$$

Step 3. Compute the criteria weights by using the average method.

Step 4. Compute the total of the weighted comparability sequence (weighted normalized decision matrix) and the whole of the power weight of comparability of each alternative.

$$S_i = \sum_{j=1}^{n} w_j N_{ij} \tag{9}$$

$$P_i = \sum_{j=1}^{n} \left(N_{ij}\right)^{w_j} \tag{10}$$

Step 5. Compute the relative weights of the alternatives as:

$$U_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m}(P_i + S_i)} \tag{11}$$

$$U_{ib} = \frac{S_i}{\min S_i} + \frac{P_i}{\min P_i} \tag{12}$$

$$U_{ic} = \frac{\gamma(S_i) + (1-\gamma)(P_i)}{\gamma \max S_i + (1-\gamma) \max P_i}; \quad 0 \leq \gamma \leq 1 \tag{13}$$

$$U_i = (U_{ia} U_{ib} U_{ic})^{\frac{1}{3}} + \frac{1}{3}(U_{ia} + U_{ib} + U_{ic}) \tag{14}$$

Step 7. Rank the alternatives.

4. Example study for art education teaching quality evaluation of vocational colleges

Art teachers in vocational colleges are mostly graduates from normal universities. Some teachers directly enter the workforce after graduation. Although they have mastered certain professional abilities and qualities during their university years and achieved certain results, they lack teaching experience and are difficult to effectively impart the knowledge they have mastered to students, making it difficult to provide effective guidance to students. Some vocational colleges have overlooked the improvement and further training of teacher abilities, and some teachers have not yet formulated clear career plans, resulting in limited opportunities for teachers to participate in training. At the same time, some art teachers lack learning ability, making it difficult to recognize the importance of improving teaching ability, which limits the improvement of art teaching quality. Currently, many vocational colleges have offered art courses, but some vocational colleges focus on cultivating and enhancing students' painting abilities in art courses, without combining the current hot topics of art education and the specific learning situation of students to carry out teaching. The teaching form and content are relatively single. This not only causes waste of art teaching resources, but also to some extent affects the learning effectiveness of students. There is a high demand for diversified art talents in the current society. However, in teaching, some art teachers only rely on textbooks to explain knowledge, without combining it with current art hotspots, which is detached from the actual teaching needs of vocational colleges and not conducive to cultivating students' social adaptability. This also affects the improvement of the quality of art education to a certain extent. The educational function of art is reflected in improving students' personality and thinking, enhancing their aesthetic abilities, and helping to develop their comprehensive qualities. In the current social context, the demand for art professionals in society is constantly increasing, and the previous concept of talent cultivation is no longer able to adapt to the development needs of the new era. In this situation, vocational college teachers need to explore education models that adapt to social development, improve professional course teaching, and enhance students' professional abilities and literacy through art course education. Although the current art courses have achieved good results and trained art professionals with good qualities and abilities, some vocational colleges find it difficult to integrate art courses with the content of other courses, and the actual teaching effect is not satisfactory, which to some extent limits the development of students and leads to a relative lack of innovative and practical abilities in the trained talents. The art education teaching quality evaluation of vocational colleges is MADM. Six possible art education colleges are depicted with 16 attributes as shown in Table 1.

Table 1. The relationship between criteria and sub criteria.

	Criteria	Sub criteria	weights
N_1	Faculty Professional Experience	N_{11}<50%, N_{12}>50%	0.062484
N_2	Facility Quality	N_{21}<50%, N_{22}>50%	0.061977
N_3	Creative Skill Development	N_{31}<50%, N_{32}>50%	0.062611
N_4	Employment in Art-Related Fields	N_{41}<50%, N_{42}>50%	0.062484
N_5	Student-Centered Teaching Approaches	N_{51}<50%, N_{52}>50%	0.062636
N_6	Curriculum Relevance	N_{61}<60%, N_{62}>60%	0.063042
N_7	Innovation in Teaching Materials	N_{71}<50%, N_{72}>50%	0.06289
N_8	Practical Components	N_{81}<50%, N_{82}>50%	0.062814
N_9	Community Projects	N_{91} 2, N_{92} 5, N_{93} 10 projects	0.062332
N_{10}	Ongoing Training	$N_{10,1}$<50%, $N_{10,2}$>50%	0.061825
N_{11}	Student Participation in Exhibitions	$N_{11,1}$<50%, $N_{11,2}$>50%	0.061977
N_{12}	Student Satisfaction	$N_{12,1}$ 3, $N_{12,2}$ 4, and $N_{12,3}$ 5 rate	0.062484
N_{13}	Graduation Rate	$N_{13,1}$<50%, $N_{13,2}$>50%	0.062611
N_{14}	Use of Technology in Teaching	$N_{14,1}$<50%, $N_{14,2}$>50%	0.062687
N_{15}	Teacher-Student Ratio	$N_{15,1}$ <10, $N_{15,2}$ >10	0.062256
N_{16}	Interdisciplinary Integration	$N_{16,1}$ 3, $N_{16,2}$ 7, and $N_{16,3}$ 10 courses	0.06289

Table 2. The opinions of three experts.

	A_1	A_2	A_3	A_4	A_5	A_6
N_{12}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.73, 0.18, 0.06, 0.03)	(0.65, 0.22, 0.08, 0.05)	(0.68, 0.15, 0.12, 0.05)
N_{21}	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.59, 0.26, 0.10, 0.05)	(0.68, 0.15, 0.12, 0.05)	(0.59, 0.26, 0.10, 0.05)	(0.52, 0.33, 0.08, 0.07)
N_{31}	(0.73, 0.18, 0.06, 0.03)	(0.76, 0.11, 0.07, 0.06)	(0.76, 0.11, 0.07, 0.06)	(0.65, 0.22, 0.08, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.59, 0.26, 0.10, 0.05)
N_{42}	(0.71, 0.20, 0.07, 0.02)	(0.59, 0.26, 0.10, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.73, 0.18, 0.06, 0.03)	(0.47, 0.38, 0.09, 0.06)	(0.76, 0.11, 0.07, 0.06)
N_{52}	(0.80, 0.10, 0.06, 0.04)	(0.52, 0.33, 0.08, 0.07)	(0.76, 0.11, 0.07, 0.06)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)	(0.47, 0.38, 0.09, 0.06)
N_{61}	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)
N_{71}	(0.76, 0.11, 0.07, 0.06)	(0.65, 0.22, 0.08, 0.05)	(0.71, 0.20, 0.07, 0.02)	(0.47, 0.38, 0.09, 0.06)	(0.73, 0.18, 0.06, 0.03)	(0.71, 0.20, 0.07, 0.02)
N_{82}	(0.59, 0.26, 0.10, 0.05)	(0.73, 0.18, 0.06, 0.03)	(0.73, 0.18, 0.06, 0.03)	(0.76, 0.11, 0.07, 0.06)	(0.65, 0.22, 0.08, 0.05)	(0.73, 0.18, 0.06, 0.03)
N_{93}	(0.52, 0.33, 0.08, 0.07)	(0.71, 0.20, 0.07, 0.02)	(0.65, 0.22, 0.08, 0.05)	(0.59, 0.26, 0.10, 0.05)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
$N_{10,2}$	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.68, 0.15, 0.12, 0.05)	(0.52, 0.33, 0.08, 0.07)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)
$N_{11,1}$	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.59, 0.26, 0.10, 0.05)	(0.68, 0.15, 0.12, 0.05)	(0.59, 0.26, 0.10, 0.05)	(0.52, 0.33, 0.08, 0.07)
$N_{12,3}$	(0.73, 0.18, 0.06, 0.03)	(0.76, 0.11, 0.07, 0.06)	(0.76, 0.11, 0.07, 0.06)	(0.65, 0.22, 0.08, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.59, 0.26, 0.10, 0.05)
$N_{13,1}$	(0.71, 0.20, 0.07, 0.02)	(0.59, 0.26, 0.10, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.73, 0.18, 0.06, 0.03)	(0.47, 0.38, 0.09, 0.06)	(0.76, 0.11, 0.07, 0.06)
$N_{14,2}$	(0.80, 0.10, 0.06, 0.04)	(0.52, 0.33, 0.08, 0.07)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)	(0.47, 0.38, 0.09, 0.06)
$N_{15,1}$	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.73, 0.18, 0.06, 0.03)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)
$N_{16,3}$	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.73, 0.18, 0.06, 0.03)	(0.71, 0.20, 0.07, 0.02)
	A_1	A_2	A_3	A_4	A_5	A_6
N_{12}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
N_{21}	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
N_{31}	(0.73, 0.18, 0.06, 0.03)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.59, 0.26, 0.10, 0.05)
N_{42}	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.76, 0.11, 0.07, 0.06)
N_{52}	(0.80, 0.10, 0.06, 0.04)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)
N_{61}	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)

N_{71}	(0.76, 0.11, 0.07, 0.06)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.71, 0.20, 0.07, 0.02)
N_{82}	(0.59, 0.26, 0.10, 0.05)	(0.73, 0.18, 0.06, 0.03)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
N_{93}	(0.52, 0.33, 0.08, 0.07)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.65, 0.22, 0.08, 0.05)
$N_{10,2}$	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)
$N_{11,1}$	(0.65, 0.22, 0.08, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.52, 0.33, 0.08, 0.07)
$N_{12,3}$	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.59, 0.26, 0.10, 0.05)
$N_{13,1}$	(0.71, 0.20, 0.07, 0.02)	(0.59, 0.26, 0.10, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
$N_{14,2}$	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
$N_{15,1}$	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)
$N_{16,3}$	(0.76, 0.11, 0.07, 0.06)	(0.52, 0.33, 0.08, 0.07)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.73, 0.18, 0.06, 0.03)	(0.71, 0.20, 0.07, 0.02)
	A_1	A_2	A_3	A_4	A_5	A_6
N_{12}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.73, 0.18, 0.06, 0.03)	(0.65, 0.22, 0.08, 0.05)	(0.68, 0.15, 0.12, 0.05)
N_{21}	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.52, 0.33, 0.08, 0.07)
N_{31}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
N_{42}	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.47, 0.38, 0.09, 0.06)	(0.76, 0.11, 0.07, 0.06)
N_{52}	(0.80, 0.10, 0.06, 0.04)	(0.52, 0.33, 0.08, 0.07)	(0.76, 0.11, 0.07, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
N_{61}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.80, 0.10, 0.06, 0.04)
N_{71}	(0.76, 0.11, 0.07, 0.06)	(0.65, 0.22, 0.08, 0.05)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
N_{82}	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.73, 0.18, 0.06, 0.03)
N_{93}	(0.52, 0.33, 0.08, 0.07)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.65, 0.22, 0.08, 0.05)
$N_{10,2}$	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)
$N_{11,1}$	(0.65, 0.22, 0.08, 0.05)	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.52, 0.33, 0.08, 0.07)
$N_{12,3}$	(0.73, 0.18, 0.06, 0.03)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.76, 0.11, 0.07, 0.06)	(0.59, 0.26, 0.10, 0.05)
$N_{13,1}$	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
$N_{14,2}$	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
$N_{15,1}$	(0.47, 0.38, 0.09, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.65, 0.22, 0.08, 0.05)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)
$N_{16,3}$	(0.76, 0.11, 0.07, 0.06)	(0.68, 0.15, 0.12, 0.05)	(0.80, 0.10, 0.06, 0.04)	(0.71, 0.20, 0.07, 0.02)	(0.73, 0.18, 0.06, 0.03)	(0.71, 0.20, 0.07, 0.02)

This study has 16 criteria and six alternatives, and three experts have evaluated the criteria and alternatives as shown in Table 2. Then we selected the highest related sub criteria as $N_{12} \times N_{21} \times N_{31} \times N_{42} \times N_{52} \times N_{61} \times N_{71} \times N_{82} \times N_{93} \times N_{10,2} \times N_{11,1} \times N_{12,3} \times N_{13,1} \times N_{14,2} \times N_{15,1} \times N_{16,3}$

Then we obtained the normalized decision matrix as shown in Table 3. Then we obtained the weighted comparability sequence as shown in Table 4. Then we obtained the relative weights of the alternatives. Then we obtained the final rank as shown in Figure 1.

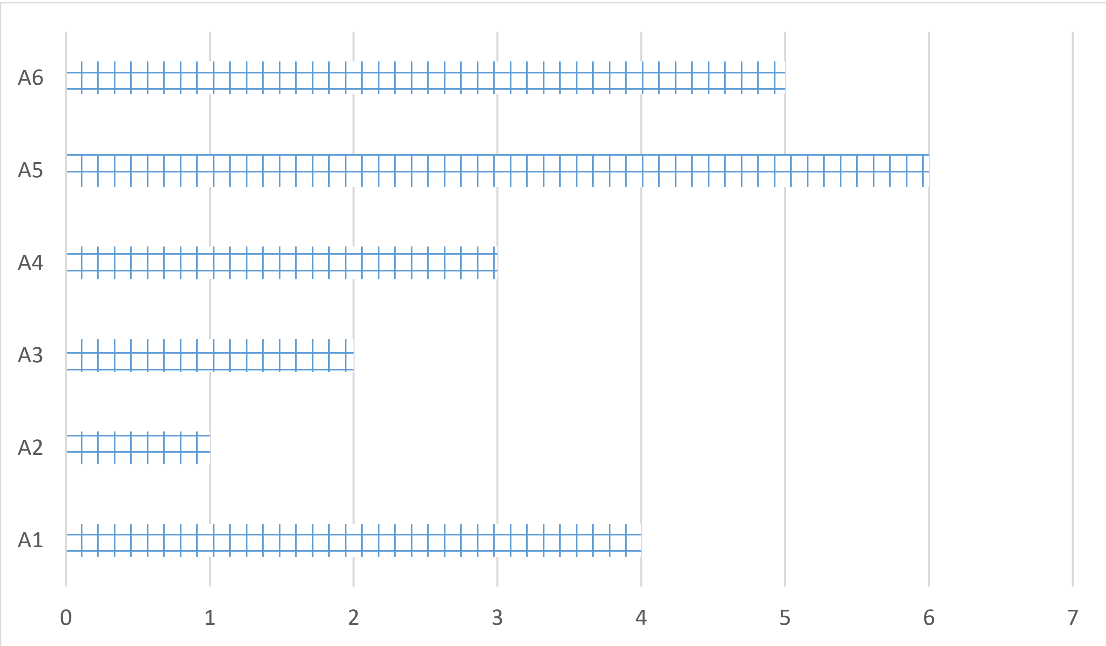

Figure 1. The rank of the alternatives.

Table 3. The normalized decision matrix.

	A_1	A_2	A_3	A_4	A_5	A_6
N_{12}	0	1	0.952381	0.857143	0.380952	0.190476
N_{21}	1	0	0	0.5	0.666667	0.333333
N_{31}	1	0.916667	0.833333	0	0.916667	0.666667
N_{42}	1	0	0.3125	1	0.125	0.5
N_{52}	1	0	0.153846	0	0.769231	0.307692
N_{61}	0	0.166667	0.5	0.666667	1	0.833333

N_{71}	0.454545	0.454545	0.727273	0	0.772727	1
N_{82}	0	1	0.421053	0.473684	0.421053	0.842105
N_{93}	0	1	0	0.214286	0.428571	0.428571
$N_{10,2}$	0.25	1	0	0.8125	0.75	0
$N_{11,1}$	0.8	0.4	0	0.3	1	0.2
$N_{12,3}$	1	0	0.357143	0.714286	0.428571	0
$N_{13,1}$	1	0.285714	0.714286	0.142857	0	1
$N_{14,2}$	0.111111	0.111111	1	0	0.444444	0.444444
$N_{15,1}$	0.315789	0	0.631579	0.526316	0.789474	1
$N_{16,3}$	0.4	0	0.15	0.6	1	1

Table 4. The weighted comparability sequence.

	A_1	A_2	A_3	A_4	A_5	A_6
N_{12}	0	0.062484	0.059509	0.053558	0.023803	0.011902
N_{21}	0.061977	0	0	0.030989	0.041318	0.020659
N_{31}	0.062611	0.057393	0.052176	0	0.057393	0.041741
N_{42}	0.062484	0	0.019526	0.062484	0.007811	0.031242
N_{52}	0.062636	0	0.009636	0	0.048182	0.019273
N_{61}	0	0.010507	0.031521	0.042028	0.063042	0.052535
N_{71}	0.028586	0.028586	0.045738	0	0.048597	0.06289
N_{82}	0	0.062814	0.026448	0.029754	0.026448	0.052896
N_{93}	0	0.062332	0	0.013357	0.026714	0.026714
$N_{10,2}$	0.015456	0.061825	0	0.050233	0.046369	0
$N_{11,1}$	0.049582	0.024791	0	0.018593	0.061977	0.012395
$N_{12,3}$	0.062484	0	0.022316	0.044632	0.026779	0
$N_{13,1}$	0.062611	0.017889	0.044722	0.008944	0	0.062611
$N_{14,2}$	0.006965	0.006965	0.062687	0	0.027861	0.027861
$N_{15,1}$	0.01966	0	0.03932	0.032766	0.049149	0.062256
$N_{16,3}$	0.025156	0	0.009433	0.037734	0.06289	0.06289

Sensitivity Analysis

This part shows the sensitivity analysis by changing in the lambda values between 0 to 1. Then we obtained different ranks of alternatives as shown in Figure 2. The different values of lambdas show the optimal alternatives is stable under different values. The results show alternative 5 is the best under different values.

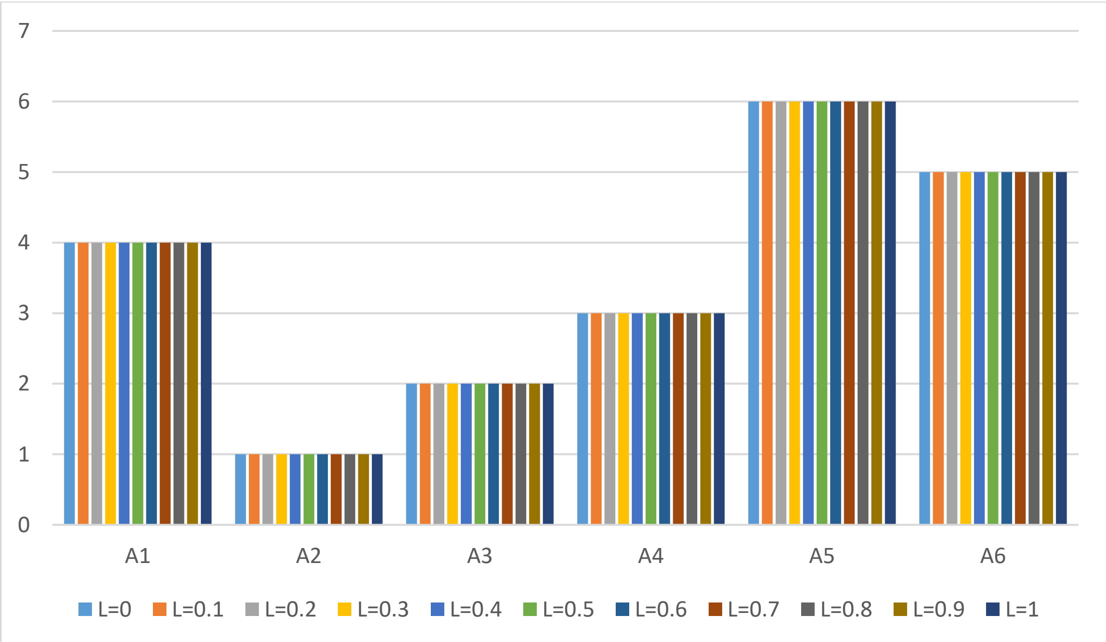

Figure 2. The rank of alternatives under different values of lambda.

5. Conclusion

In art education, teachers tailor their instruction to match the varying levels and circumstances of their students, a practice that demands robust teaching skills. Universities, reflecting their unique visions for student futures and differing art faculty competencies, set distinct benchmarks for training art educators. Art teachers are expected not only to possess advanced practical artistic skills but also to have a comprehensive understanding of art theory, effectively merging theory with practice during instruction. Given that art teachers interpret and convey artistic knowledge in diverse ways, students are encouraged to selectively engage with teachings that resonate with their personal learning styles. To enhance the sustainability of art education, it is essential to deeply explore both the content and methodologies of art instruction, identify the most appropriate courses for vocational college students, and subsequently refine the overall art curriculum. The evaluation of art education teaching quality in vocational colleges is approached through MADM. In this study, we introduce the application of the CoCoSo method for MADM within the framework of DVNSs. We further develop the DVNN-TSS-CoCoSo model for MADM. To illustrate the practical application of this model, we provide the numerical example that assesses the quality of art education in vocational colleges.

The primary research contributions of this paper are summarized: (1)The creation of a novel MADM approach that integrates the CoCoSo approach and average strategies using DVNSs with the Tree Soft Set to deal with the relationship between criteria and sub criteria; (2) The incorporation of objective weights with the average approach; (3) The introduction of the DVNN-TSS-CoCoSo approach specifically tailored for evaluating the quality of art education in vocational colleges; (4) The validation of the DVNN-TSS-CoCoSo model through a practical numerical example, underscoring its efficacy in assessing the quality of art education.

In assessing the quality of art education in vocational colleges, it's crucial to recognize potential shortcomings that could be explored in future studies. Several key areas offer valuable opportunities for deeper investigation: (1) Consensus Management: Future research could aim to develop strategies for achieving consensus in evaluating the quality of art education in vocational colleges, within the scope of DVNSs. This would involve integrating the diverse opinions and perspectives of various stakeholders to reach a unified conclusion. Developing and refining methodologies that support consensus building could significantly improve the reliability and validity of the evaluation processes. (2) Regret Theory: Another promising area for research is the application of regret theory to the evaluation of teaching quality in art education at vocational colleges, also employing DVNSs. Regret theory examines the decision-making process by accounting for the potential regret that might follow from choosing a specific option. By integrating regret theory into the evaluative framework, researchers can gain a more nuanced understanding of the compromises and dynamics involved in making quality assessments. Addressing these areas in subsequent research would not only augment the efficacy and thoroughness of the evaluation methods used for art education teaching quality in vocational colleges but also push forward the field, offering critical insights to educational institutions and policymakers.

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Received: Aug 27, 2024. Accepted: Nov 25, 2024