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A Triangular Fuzzy Neutrosophic Model for Assessing and Optimizing Educational Quality in University Badminton Initiatives: A comprehensive study

Yi Zheng[1], Jun Chen[2]*
[1.]College of Physical Education and Health, Wenzhou University, Wenzhou, 325035, Zhejiang, China
(E-mail: 18957725115@163.com)
[2.]Wenzhou University of Technology, Wenzhou, 325000, Zhejiang, China
*Corresponding author, E-mail: 20210574@wzut.edu.cn; 13819720353@163.com

Abstract: The evaluation of teaching quality in public badminton classes at general universities primarily considers students' learning outcomes, teachers' teaching abilities, class organization, and the appropriateness of the teaching content. Evaluation criteria include students' mastery of skills, participation, interest development, and teachers' teaching methods, classroom management, and interaction effectiveness. By combining quantitative and qualitative approaches, the evaluation aims to assess teaching quality, promote improvements, enhance students' physical fitness and sports skills, and support teachers' professional development and improvement of teaching standards. The teaching quality evaluation of public badminton classes at general universities is a multi-attribute decision-making (MADM). In this paper, a novel technique is introduced that combines triangular fuzzy neutrosophic numbers and PROMETHEE II (TFNN-PROMETHEE II) to address this issue. The TFNN-PROMETHEE II method leverages the properties of TFNN- PROMETHEE II and triangular fuzzy neutrosophic sets (TFNSs) to solve MADM problems effectively. A detailed numerical example is provided to demonstrate the application of the TFNN-PROMETHEE II technique in evaluating the teaching quality of public badminton classes at general universities. Additionally, several comparisons are made to highlight the advantages of this approach. The primary contributions of this paper are outlined: (1) The TFNN-PROMETHEE II technique is developed within the framework of TFNSs; (2) sensitivity analysis was conducted to show the stability of the rank of alternatives; (3) The technique is specifically applied to evaluate the teaching quality of public badminton classes at general universities; (4) Comparative analysis demonstrates that the TFNN-PROMETHEE II technique is also effective in assessing the teaching quality of public badminton classes at general universities.

Keywords: MADM; TFNSs; PROMETHEE II technique; TFNN-PROMETHEE II approach; teaching quality evaluation

1. Introduction

The evaluation of teaching quality in public badminton courses at ordinary universities is a comprehensive and ongoing process aimed at improving classroom effectiveness and student satisfaction. The evaluation primarily focuses on four aspects: the teacher's teaching ability, the design of the course content, the adequacy of teaching facilities, and the students' learning outcomes. Firstly, the teacher's teaching ability directly impacts the quality of the class. Teachers need not only a solid grasp of badminton techniques but also scientific teaching methods. In the classroom, teachers should adjust their teaching strategies flexibly according to the different levels and characteristics of the students. By combining demonstrations and guidance, they can enhance students' interest and participation. Additionally, the teacher's ability to organize the class is crucial, as good classroom management ensures that every student can learn efficiently within the limited time. Secondly, the design of the course content determines the students' learning outcomes. A well-designed badminton course should cover basic techniques, common tactics, and competition rules. Throughout the teaching process, the difficulty of the course should gradually increase, allowing students to have clear goals and achievements at each stage. The course should not only focus on skill training but also on fostering students' teamwork and competitive spirit. Meanwhile, the quality of teaching facilities and the environment is also an important factor. Adequate badminton courts and equipment ensure the smooth progress of the class, and good facility conditions can enhance students' interest and enthusiasm for learning. The school should regularly inspect and maintain the courts and equipment to provide a safe and comfortable learning environment for students. Finally, students' learning outcomes are a crucial measure of teaching quality. After the teaching period, observing students' skill improvement and collecting their feedback can help understand the actual effectiveness of the course and identify any shortcomings. Student satisfaction not only reflects their level of recognition on the course but also provides valuable references for teachers' improvements. In conclusion, the evaluation of teaching quality in public badminton courses at ordinary universities is not only an assessment of current teaching outcomes but also an important means of driving continuous improvement in teaching. Through scientific evaluation and ongoing enhancement, teachers can continually improve the quality of the course, providing students with richer learning experience.

MADM is an important branch of decision science, primarily used to address complex decision problems that involve multiple evaluation criteria or attributes[1-3]. In real life, many decision-making problems not only involve a single objective or attribute but often require consideration of several independent or interrelated factors simultaneously. For example, when a company selects a supplier, it needs to evaluate not only the price but also factors such as quality, delivery time, and service level. Such problems are typically solved using MADM methods [4]. The core of MADM is to evaluate alternatives based on different attributes and, through certain decision-making methods, select the optimal solution or rank the alternatives. Common MADM methods include the weighted sum method, Analytic Hierarchy Process (AHP), fuzzy set theory, and the TOPSIS method. These methods usually require assigning weights to each attribute to reflect their relative importance in the decision-making process. MADM methods are widely applied in various fields such as engineering, management, economics, environmental decision-making, and educational evaluation. They help decision-makers systematically analyze complex problems and can handle uncertainty and ambiguity, providing more comprehensive and rational bases for decision-making. In the era of information and big data, the application of MADM continues to expand, becoming a crucial tool for addressing complex decision problems. The evaluation of teaching quality for public badminton classes at general universities is a MADM problem. TFNSs [5] have been introduced as an effective approach to represent uncertain information in this context. TFNSs are an extension of fuzzy sets, incorporating the concept of indeterminacy and offering a more flexible framework for dealing with uncertainty in decision-making. Here are two key advantages of TFNS: (1) Better handling of uncertainty and indeterminacy: Unlike traditional fuzzy sets that only account for membership degrees, TFNS also consider non-membership and indeterminacy degrees. This allows for a more comprehensive representation of uncertainty, particularly in complex situations where information is incomplete, ambiguous, or contradictory. The triangular form further simplifies the representation of these degrees, making it easier to model real-world problems where precise values are difficult to obtain. (2) Enhanced computational efficiency: The triangular representation in TFNS reduces computational complexity compared to other forms of fuzzy neutrosophic sets (such as trapezoidal or general fuzzy neutrosophic sets). The triangular structure requires fewer parameters (only three points: lower, middle, and upper bounds), allowing for faster calculations in decision-making processes, while still effectively capturing the range of uncertainty, indeterminacy, and membership. In previous studies, several techniques have employed PROMETHEE II [6-10] and entropy [11] separately to determine the optimal solution.

Herein, this study introduces the use of TFNSs as a robust solution to handle the inherent uncertainties between attributes and alternatives in MADM processes. Specifically, the TFNSs, a specialized form of neutrosophic set, is employed to manage this complexity. In real-world MADM scenarios, the variability and unpredictability of initial data often make it challenging for DMs to accurately convey their assessments[12]. Traditional methods struggle with capturing the nuances of vague, incomplete, or contradictory information. To address this, Smarandache's neutrosophic sets (NSs) [13] were developed, offering a more flexible framework that can simultaneously represent degrees of truth, indeterminacy, and falsity. This triad enables a more comprehensive representation of ambiguous data, far beyond the capabilities of classical and fuzzy sets. By leveraging this expanded framework, the study applies the PROMETHEE II method within the context of TFNSs, offering a more nuanced approach to evaluating and ranking alternatives. This integration of PROMETHEE II with TFNSs allows for a more precise reflection of the complexities within the decision-making process, ensuring better-informed and more reliable outcomes in uncertain environments. To address this gap, the triangular fuzzy neutrosophic number PROMETHEE II (TFNN-PROMETHEE II) technique is proposed, utilizing both PROMETHEE II and TFNSs. This TFNN-PROMETHEE II method is designed to tackle the MADM problem. A numerical example is provided to demonstrate its application in evaluating the teaching quality of public badminton classes at general universities, and comparative analysis is conducted to highlight the advantages of this approach.

Some significant Highlights of the work

(1) To introduce the TFNN-PROMETHEE II technique within the framework of TFNSs.

(2) To apply the TFNN-PROMETHEE II technique for MADM problems under TFNSs.

(3) To implement the TFNN-PROMETHEE II technique specifically for the evaluation of teaching quality in public badminton classes at general universities.

(4) Through several comparative analyses, it is shown that the TFNN-PROMETHEE II technique is highly effective for this purpose.

2. Literature review

The evaluation of teaching quality in public badminton courses at regular universities should be based on multiple dimensions to ensure the improvement of both course effectiveness and student experience. First, the scientific and reasonable design of the teaching content is fundamental. The course should balance technical skills, physical fitness, and tactical training, progressively enhancing students' badminton abilities. Second, the teaching methods employed by the instructor are crucial. Emphasis should be placed on making the lessons engaging and interactive, stimulating students' interest in learning, while also providing individualized instruction to meet the needs of students at different skill levels. Third, students' learning outcomes and mastery of skills are central to the evaluation. The effectiveness of the teaching can be assessed through students' performance in practice, technical progress, and their understanding of badminton rules. Additionally, student participation and course satisfaction are important reference indicators. A comprehensive evaluation across these dimensions provides valuable insights for improving teaching, thereby enhancing the overall quality of public badminton courses in universities. In 2017, Sun [14] emphasized the importance of core strength training in badminton teaching at universities. He pointed out that traditional badminton teaching overly focused on training activities while neglecting theoretical guidance, and that core strength training could effectively enhance students' athletic abilities and improve teaching outcomes. The following year, Wang [15] explored the construction and innovation of badminton teaching methods in universities. By analyzing current teaching issues, he proposed innovative teaching suggestions aimed at improving teaching quality and promoting the development of badminton. Yu [16] studied badminton teaching methods in public physical education courses, suggesting that teachers should break away from traditional methods to enhance student interest and, in turn, improve course quality. Luo [17] investigated the innovation of teaching and training methods in university badminton courses, analyzing the popularity of badminton in universities and proposing suggestions for improving both teaching and training methods to enhance teaching quality. Simultaneously, Zhang [18] examined the application of dynamic layered teaching methods in badminton courses, noting that this approach could cater to different student needs and proposed solutions to the constraints in its implementation. In 2020, Guan [19] studied the restructuring of badminton courses in physical education programs under the background of MOOCs. He pointed out the conflict between MOOCs and physical practice courses and suggested that combining MOOCs with flipped classrooms could break the limitations of time and space, thereby improving teaching quality. Following that, in 2021, Cao [20] explored the innovation of badminton teaching models in the "Internet+" era. She proposed a blended teaching model that integrates online and offline elements, supported by big data, to enhance interactivity and flexibility in badminton teaching. By 2022, Li [21] conducted practical research on the flipped classroom in university badminton teaching, analyzing its feasibility and proposing recommendations for optimizing the teaching scheme. In the same year, Fu [22] constructed a "MOOC + Flipped Classroom" teaching model for university badminton courses, analyzing its value in curriculum reform and offering application suggestions to improve teaching quality. In 2023, You [23] studied the application of the OMO (Online-Merge-Offline) teaching model in university badminton courses, analyzing both its advantages and limitations, and proposed optimization strategies to improve teaching quality. Finally, in 2024, Ye and Tang [24] explored the current state of badminton teaching in universities under the background of physical education integration and proposed strategies for improvement, while also outlining future development directions to promote the comprehensive development of badminton teaching in universities.

3. Preliminaries

Biswas et al. [5] initiated the TFNSs.

Definition 1[5]. Let X be a fixed set, the TFNSs Z is initiated:

$$Z = \left\{ \left(\theta, A(\theta), B(\theta), C(\theta) \right) \middle| \theta \in \Theta \right\} \tag{1}$$

where $A(\theta), B(\theta), C(\theta) \in [0,1]$ initiated truth-membership, indeterminacy membership and falsity-membership which are initiated through triangular fuzzy numbers.

$$A(\theta) = \left(A^L(\theta), A^M(\theta), A^U(\theta) \right), 0 \le A^L(\theta) \le A^M(\theta) \le A^U(\theta) \le 1 \tag{2}$$

$$B(\theta) = \left(B^L(\theta), B^M(\theta), B^U(\theta) \right), 0 \le B^L(\theta) \le B^M(\theta) \le B^U(\theta) \le 1 \tag{3}$$

$$C(\theta) = \left(C^L(\theta), C^M(\theta), C^U(\theta) \right), 0 \le C^L(\theta) \le C^M(\theta) \le C^U(\theta) \le 1 \tag{4}$$

For convenience, we let $Z = \left\{ \left(A^L, A^M, A^U \right), \left(B^L, B^M, B^U \right), \left(C^L, C^M, C^U \right) \right\}$ be an TFNN, $0 \le A^U + B^U + C^U \le 3$.

Definition 2[5]. There are three TFNNs $Z_1 = \left\{ \left(A_1^L, A_1^M, A_1^U \right), \left(B_1^L, B_1^M, B_1^U \right), \left(C_1^L, C_1^M, C_1^U \right) \right\}$, $Z_2 = \left\{ \left(A_2^L, A_2^M, A_2^U \right), \left(B_2^L, B_2^M, B_2^U \right), \left(C_2^L, C_2^M, C_2^U \right) \right\}$ and $Z = \left\{ \left(A^L, A^M, A^U \right), \left(B^L, B^M, B^U \right), \left(C^L, C^M, C^U \right) \right\}$, the operation laws of TFNNs are initiated:

$$(1)\ Z_1 \oplus Z_2 = \begin{Bmatrix} \left(A_1^L + A_2^L - A_1^L A_2^L, A_1^M + A_2^M - A_1^M A_2^M, A_1^U + A_2^U - A_1^U A_2^U\right), \\ \left(B_1^L B_2^L, B_1^M B_2^M, B_1^U B_2^U\right), \left(C_1^L C_2^L, C_1^M C_2^M, C_1^U C_2^U\right) \end{Bmatrix};$$

$$(2)\ Z_1 \otimes Z_2 = \begin{Bmatrix} \left(A_1^L A_2^L, A_1^M A_2^M, A_1^U A_2^U\right), \\ \left(B_1^L + B_2^L - B_1^L B_2^L, B_1^M + B_2^M - B_1^M B_2^M, B_1^U + B_2^U - B_1^U B_2^U\right), \\ \left(C_1^L + C_2^L - C_1^L C_2^L, C_1^M + C_2^M - C_1^M C_2^M, C_1^U + C_2^U - C_1^U C_2^U\right) \end{Bmatrix};$$

$$(3)\ \lambda Z = \begin{Bmatrix} \left(1-\left(1-A^L\right)^\lambda, 1-\left(1-A^M\right)^\lambda, 1-\left(1-A^U\right)^\lambda\right), \\ \left(\left(B^L\right)^\lambda, \left(B^M\right)^\lambda, \left(B^U\right)^\lambda\right), \\ \left(\left(C^L\right)^\lambda, \left(C^M\right)^\lambda, \left(C^U\right)^\lambda\right) \end{Bmatrix}, \lambda > 0;$$

$$(4)\ Z^\lambda = \begin{Bmatrix} \left(\left(A^L\right)^\lambda, \left(A^M\right)^\lambda, \left(A^U\right)^\lambda\right), \\ \left(1-\left(1-B^L\right)^\lambda, 1-\left(1-B^M\right)^\lambda, 1-\left(1-B^U\right)^\lambda\right), \\ \left(1-\left(1-C^L\right)^\lambda, 1-\left(1-C^M\right)^\lambda, 1-\left(1-C^U\right)^\lambda\right) \end{Bmatrix}, \lambda > 0.$$

From Definition 2, the operation laws for TFNNs have some properties.

$$(1)\ Z_1 \oplus Z_2 = Z_2 \oplus Z_1; \quad (5)$$

$$(2) Z_1 \otimes Z_2 = Z_2 \otimes Z_1, \left(\left(Z_1\right)^{\lambda_1}\right)^{\lambda_2} = \left(Z_1\right)^{\lambda_1 \lambda_2}; \quad (6)$$

$$(3)\ \lambda\left(Z_1 \oplus Z_2\right) = \lambda Z_1 \oplus \lambda Z_2, \left(Z_1 \otimes Z_2\right)^\lambda = \left(Z_1\right)^\lambda \otimes \left(Z_2\right)^\lambda; \quad (7)$$

$$(4)\ \lambda_1 Z_1 \oplus \lambda_2 Z_1 = \left(\lambda_1 + \lambda_2\right) Z_1, \left(Z_1\right)^{\lambda_1} \otimes \left(Z_1\right)^{\lambda_2} = \left(Z_1\right)^{\left(\lambda_1 + \lambda_2\right)}. \quad (8)$$

Definition 3[5]. Let $Z = \left\{\left(A^L, A^M, A^U\right), \left(B^L, B^M, B^U\right), \left(C^L, C^M, C^U\right)\right\}$ be TFNN, the fuzzy score functions (FSF) and fuzzy accuracy functions (FAF) are initiated:

$$FSF(Z) = \frac{1}{12}\begin{bmatrix} 8 + \left(A^L + 2A^M + A^U\right) - \left(B^L + 2B^M + B^U\right) \\ -\left(C^L + 2C^M + C^U\right) \end{bmatrix}, FSF(Z) \in [0,1] \quad (9)$$

$$FAF(Z) = \frac{1}{4}\left[\left(A^L + 2A^M + A^U\right) + \left(B^L + 2B^M + B^U\right)\right], FAF(Z) \in [0,1] \quad (10)$$

For Z_1 and Z_2, in line with the Definition 3, then

(1) $if\ FSF(Z_1) \prec FSF(Z_2),\ Z_1 \prec Z_2;$
(2) $if\ FSF(Z_1) \succ FSF(Z_2),\ Z_1 \succ Z_2;$
(3) $if\ FSF(Z_1) = FSF(Z_2), FAF(Z_1) \prec FAF(Z_2),\ Z_1 \prec Z_2;$
(4) $if\ FSF(Z_1) = FSF(Z_2), FAF(Z_1) \succ FAF(Z_2), Z_1 \succ Z_2;$
(5) $if\ FSF(Z_1) = FSF(Z_2), FAF(Z_1) = FAF(Z_2), Z_1 = Z_2.$

4. Proposed TFNN-PROMETHEE II approach for MADM

This section outlines the steps of the PROMETHEE II method used to solve a decision-making problem within the MADM process. The steps are as follows:

Step 1: Identify criteria and alternatives

A team of experts and decision-makers is formed to define the criteria and alternatives relevant to the MADM problem.

Step 2: Conduct interviews and surveys

Experts participate in interviews and complete questionnaires designed for the PROMETHEE II method. These experts, equipped with the necessary knowledge and experience, evaluate the criteria and alternatives. To enhance the evaluation, they use Triangular Fuzzy Number Networks (TFNNs).

Step 3. The decision matrix is built

Step 4. Normalize decision matrix is computed as:

$$U_{ij} = \frac{e_{ij} - \min e_{ij}}{\max e_{ij} - \min e_{ij}} \text{ for positive criteria} \quad (11)$$

$$U_{ij} = \frac{\max e_{ij} - e_{ij}}{\max e_{ij} - \min e_{ij}} \text{ for negative criteria} \quad (12)$$

Where $i = 1,2, \dots, m; j = 1,2, \dots, n$

Step 5. Compute the alternative difference of i^{th} alternative with respect to other alternatives.

Step 6. Compute the criteria weights.

The criteria weights are determined using the mean method.

Step 7. Compute the preference function.

$$P_j(x, y) = 0\ if\ U_{xj} \leq U_{yj} \quad (13)$$

$$P_j(x, y) = (U_{xj} - U_{yj})\ if\ U_{xj} > U_{yj} \quad (14)$$

Step 8. Compute the combined preference functions.

$$CP = \frac{\left(\sum_{j=1}^{n} w_j P_j(x,y)\right)}{\sum_{j=1}^{n} w_j} \quad (15)$$

Step 9. Compute the leavening and entering outranking flows

$$Q^{+} = \frac{1}{m-1}\sum_{i=i}^{m} CP \quad (16)$$

$$Q^{-} = \frac{1}{m-1}\sum_{j=i}^{n} CP \quad (17)$$

Step 10. Commute the net outranking

$$Q = Q^{+} - Q^{-} \quad (18)$$

Step 11. Rank the alternatives.

5. Application for Triangular Fuzzy Neutrosophic Sets

The evaluation of teaching quality in public badminton classes at general universities is a systematic process of analyzing and assessing the teaching process and outcomes, aimed at ensuring the effectiveness of teaching activities and the holistic development of students. As an essential part of university physical education, badminton classes not only enhance students' physical health but also foster their interest in sports and promote lifelong exercise habits. Therefore, evaluating the teaching quality of these classes is of great significance. First, the core of teaching quality evaluation lies in the overall planning and implementation of the teaching process. The effective delivery of badminton classes depends on whether the instructor can design a reasonable teaching plan based on the actual situation of the students. The scientific, structured, and practical design of the curriculum directly affects whether students can grasp the basic skills and tactics of badminton within the limited class time. Additionally, the ability of teachers to adapt their teaching plans to meet the needs of students at different levels is a crucial aspect of evaluating teaching quality. Second, the organization and management of classroom teaching are key factors affecting teaching quality. Badminton classes typically require a high level of court utilization and active student participation, making the teachers' organizational and leadership abilities vital. Good classroom management helps increase student engagement and learning efficiency, ensuring that the teaching content is effectively conveyed. Moreover, how the teacher handles unexpected situations in class and establishes effective interaction with students are also important elements in assessing teaching quality. In addition, the professional competence and teaching attitude of the teacher directly influence the quality of teaching. Badminton classes require instructors to not only possess solid professional knowledge and skills but also have rich teaching experience and the ability to tailor their instruction to individual students. The teacher's attitude, sense of responsibility, and commitment to the course directly affect students' satisfaction and their enthusiasm for learning. Whether the teacher demonstrates sufficient passion and patience in teaching and whether they can stimulate students' interest in learning are important factors to consider in the evaluation of teaching quality. Lastly, the evaluation of teaching quality should also take into account the overall effectiveness of the course. The goal of badminton classes is not only to impart sports skills but also to cultivate students' interest in sports, enhance teamwork, and improve their mental resilience. Therefore, the ultimate goal of teaching quality evaluation is to determine whether the course has achieved its expected educational outcomes and whether it has contributed to the comprehensive development of the students. In summary, the evaluation of teaching quality in public badminton classes at general universities is a comprehensive and systematic process. It requires attention to the rationality of the teaching plan and course implementation, as well as an assessment of the teacher's ability and attitude. The ultimate aim is to ensure that students gain well-rounded benefits from the class, both physically and mentally. The teaching quality evaluation of public badminton classes at general universities is MADM. Nine general universities are assessed with 12 attributes as shown in Table 1.

Step 1. The List of criteria is shown in Table 1.

Step 2. Three experts are evaluated the criteria and alternatives with expertise and knowledge in this kind of problem.

Step 3. Table 2 shows the decision matrix with TFNNS. Then we applied the score function to obtain crisp values. Then we combined these numbers into one matrix.

Step 4. Eq. (11) is used to normalize the TFNNSs as shown in Table 3. All criteria in this study are positive.

Step 5. Then we compute the alternative difference of i^{th} alternative with respect to other alternatives.

Step 6. Criteria weights are computed as shown in Table 1.

Step 7. Then we used Eqs. (13 and 14) to compute the preference function

Step 8. Then we compute the combined preference functions by using Eq. (15) as shown in Table 4.

Step 9. Then we used Eqs. (16 and 17) to compute the leavening and entering outranking flows

Step 10. Then we used Eq. (18) to commute to the net outranking

Step 11. Rank the alternatives as shown in Figure 1.

Table 1. List of criteria.

Criteria	Weights
C1: Student Satisfaction	0.083362
C2: Program Accessibility and Inclusiveness	0.08387
C3: Community Engagement and Social Impact	0.081413
C4: Outreach Programs	0.080989
C5: Sustainability and Long-Term Development	0.082218
C6: Integration with Academic Goals	0.082684
C7: Student Performance and Development	0.085225
C8: Coaching and Instruction Quality	0.086623
C9: Facilities and Equipment	0.085691
C10: Promotion of Health and Fitness	0.078744
C11: Collaboration and Networking	0.08243
C12: Program Curriculum and Content	0.08675

Table 2. The TFNNs with three experts.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_2	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
A_3	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_4	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
A_5	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
A_6	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_7	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)

A_8	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_9	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_{10}	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_2	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
A_3	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_4	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
A_5	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
A_6	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
A_7	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_8	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_9	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_{10}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}

A_1	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_2	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
A_3	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_4	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_5	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_6	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
A_7	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)
A_8	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_9	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
A_{10}	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.4,0.6,0.8),(0.3,0.4,0.5),(0.2,0.3,0.6)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)

Table 3. The normalized TFNNs.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	1	0.645161	0	0.604167	0.797619	0.461538	0.157895	0.965517	0	0
A_2	0.27907	0.935484	0.375	0.208333	0.452381	1	0.657895	0.241379	0.95	0.473684
A_3	0.232558	0	0.125	0.1875	0.785714	0.551282	0.434211	0.977011	0	0.157895
A_4	0.813953	0.967742	0.777778	0	0.452381	0.115385	0.5	0.666667	0.3	1
A_5	0.232558	0.129032	0.625	0.395833	0.285714	0	0	0.54023	0.483333	0.807018
A_6	0.325581	0.387097	1	0.5625	0.369048	0.74359	0.236842	0	0	0.666667
A_7	0.55814	1	0.736111	1	0	0.307692	1	0.287356	0.283333	0.77193
A_8	0.465116	0	0	0.3125	1	0.153846	0.473684	1	0.1	0.473684
A_9	0	0.580645	0.930556	0	0	0.230769	0.986842	0.517241	1	0.315789
A_{10}	1	0.645161	0	0.604167	0.797619	0.461538	0.157895	0.965517	0	0

Table 4. The combined preference functions.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0	0.260721	0.054575	0.234778	0.307019	0.163959	0.314889	0.092409	0.256965	0
A_2	0.226381	0	0.091129	0.187809	0.283979	0.122258	0.233768	0.155815	0.101445	0.226381
A_3	0.249176	0.320069	0	0.310821	0.161532	0.274662	0.433381	0.159324	0.269514	0.249176
A_4	0.234084	0.221454	0.115526	0	0.241896	0.18677	0.207761	0.185802	0.143473	0.234084
A_5	0.307019	0.283979	0.161532	0.241896	0	0.194788	0.312991	0.212181	0.210212	0.307019
A_6	0.219607	0.212246	0.13571	0.243113	0.194788	0	0.268957	0.191119	0.210648	0.219607
A_7	0.220589	0.173808	0.144481	0.114156	0.312991	0.119009	0	0.187324	0.097156	0.220589
A_8	0.170724	0.26847	0.043038	0.264811	0.212181	0.213786	0.35994	0	0.251675	0.170724
A_9	0.374047	0.252866	0.191995	0.261249	0.210212	0.272082	0.308537	0.290441	0	0.374047
A_{10}	0	0.260721	0.054575	0.234778	0.307019	0.163959	0.314889	0.092409	0.256965	0

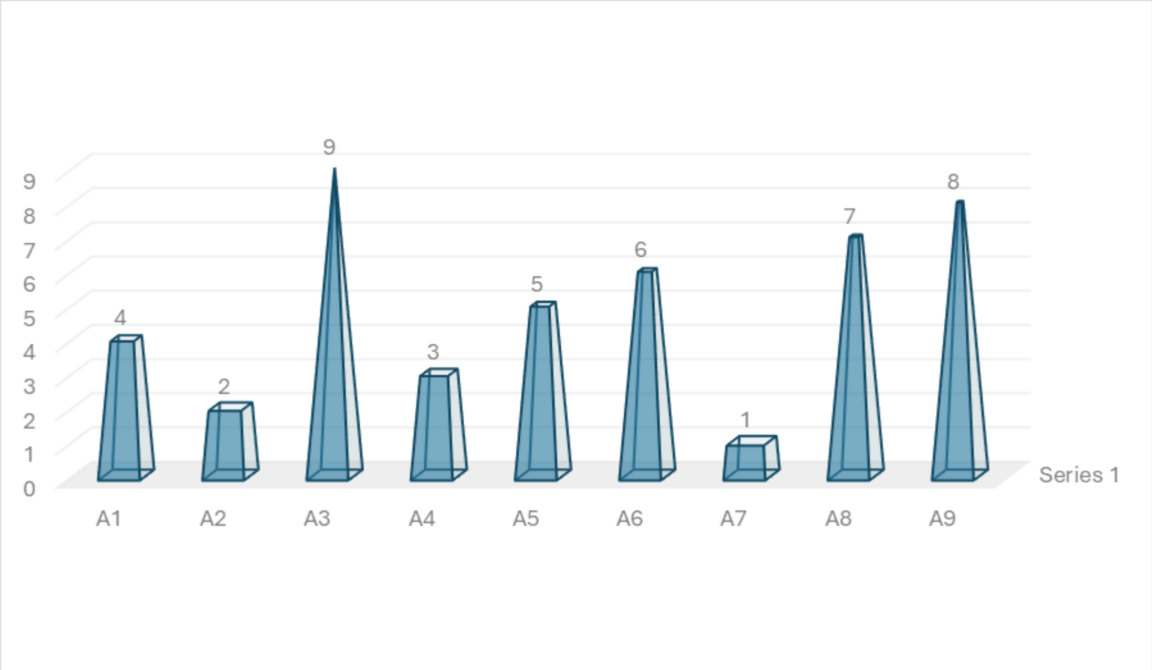

Figure 1. The rank of alternatives.

5.1 Sensitivity analysis

This section changes the criteria weights by different values as shown in Figure 2. We put all criteria with equal weights. Then we put the first criterion with 0.2 and the second criterion with 0.1 weights. Then we applied the proposed method to show the rank of alternatives as shown in Figure 3. The results show the rank of alternatives is stable under different weights.

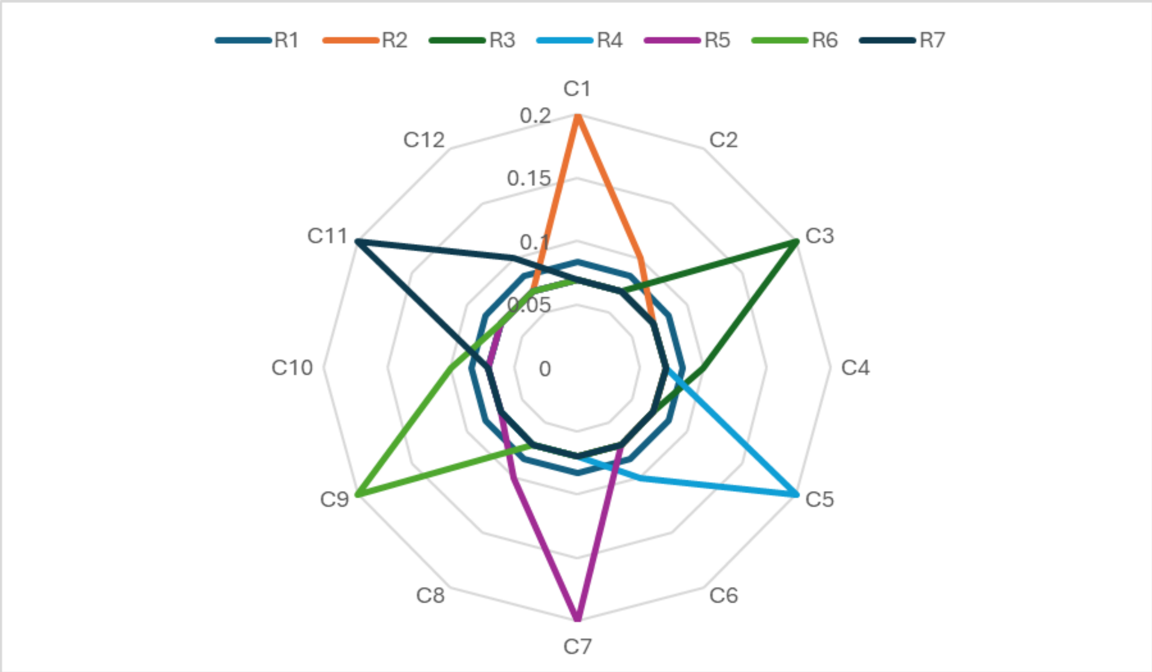

Figure 2. Different criteria weights.

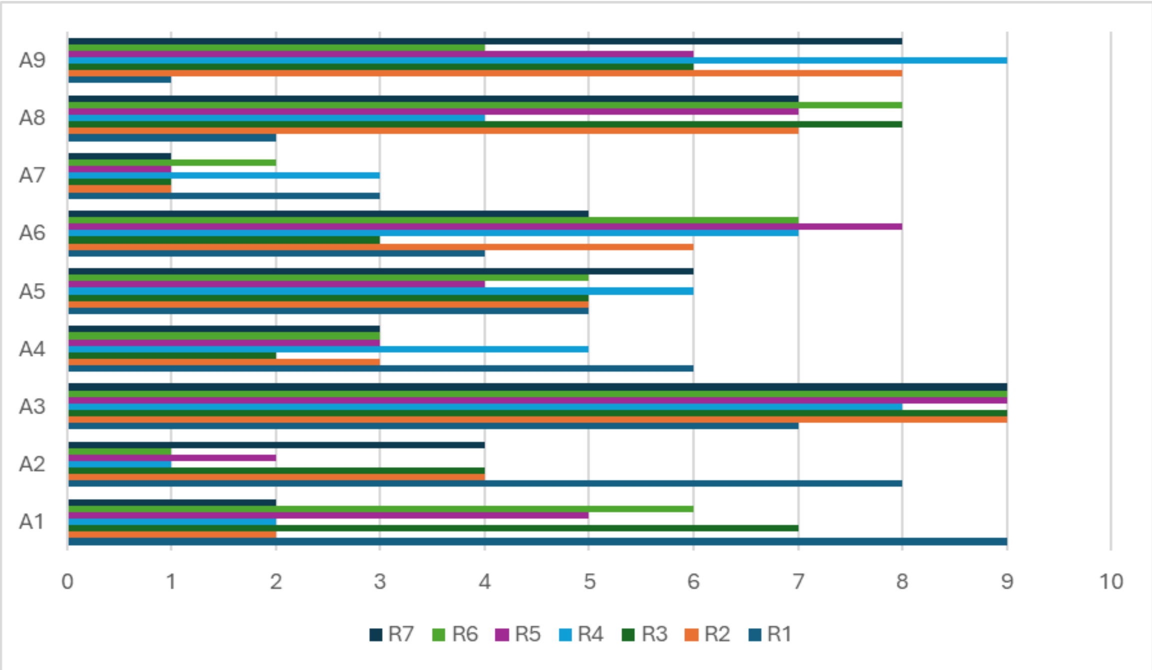

Figure 3. Different ranks of alternatives.

5.2. Comparative analysis among TFNSs

This section presents a comparative analysis of different MADM methods, including WASPAS, MABAC, and EDAS, to demonstrate the robustness of the proposed method. These methods were applied using the same weighting criteria as the proposed approach and were implemented within the framework of Triangular Fuzzy Number Networks (TFNNs). The ranking of alternatives is illustrated in Figure 4. The results indicate that the proposed method exhibits superior performance compared to the other MADM methods.

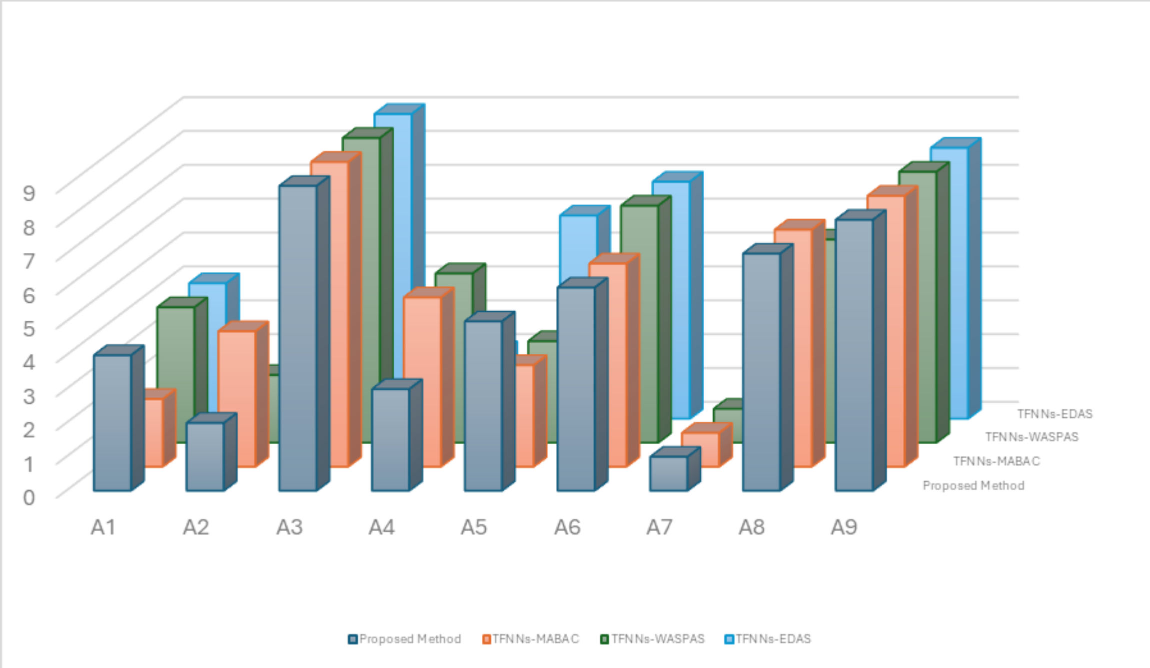

Figure 4. Comparative analysis results.

6. Results Discussion and Interpretation

In comparison with existing methods, the TFNN-PROMETHEE II approach proposed in this paper demonstrates notable advantages, alongside certain limitations. Established methods such as TFNN-EDAS, TFNN-WASPAS, and TFNN-MABAC, though widely used in MADM, have the following drawbacks:

Limited ability to handle fuzziness and uncertainty: These methods are less effective in managing complex fuzzy and uncertain data.

Strong reliance on weight allocation: The dependence on decision-makers' preferences can introduce bias, potentially compromising the reliability of evaluations.

High computational complexity: While these methods perform well in specific scenarios, their computational processes are intricate. This complexity poses challenges for large-scale decision-making problems, making them less efficient and slower in generating results.

In contrast, the TFNN-PROMETHEE II method offers three key advantages:

Enhanced capability to handle fuzziness: By integrating the strengths of triangular fuzzy neutrosophic numbers and PROMETHEE II, this approach effectively manages highly uncertain decision-making problems.

6.1 Managerial Implications of TFNN-PROMETHEE II in Teaching Quality Evaluation

The TFNN-PROMETHEE II method offers several key managerial benefits for university administrators:

By handling uncertainty and providing detailed data, this approach allows managers to make informed decisions about curriculum changes, teacher training, and resource allocation.

The method helps identify areas where instructors need improvement, enabling targeted professional development and improving teaching effectiveness.

Student Engagement: Including student feedback ensures that teaching methods can be adjusted to boost participation and satisfaction.

By assessing class organization and resource utilization, administrators can optimize the use of facilities and equipment, leading to better learning experiences.

This method ensures transparent and objective evaluations, fostering a culture of accountability among teachers and students.

Insights from the evaluation can guide improvements in course content and teaching strategies, aligning them better with student needs.

Comparing results with other institutions helps identify best practices, driving continuous improvements in teaching quality.

6.2 Implications for Practice

The findings from this study have several practical implications for the evaluation of teaching quality in public badminton classes at general universities. The use of the TFNN-PROMETHEE II method allows university administrators and instructors to make more informed decisions regarding course design, teaching methods, and resource allocation.

Teachers can benefit from the insights into their teaching effectiveness, enabling them to focus on areas that need improvement, such as student engagement and classroom management. This approach also helps improve the overall learning experience by tailoring teaching methods to better meet students' needs and preferences.

For administrators, the method provides a clearer understanding of how resources, such as equipment and court space, are being utilized, helping them make more efficient decisions. Additionally, by incorporating student feedback into the evaluation, universities can continually optimize their programs, leading to higher levels of student satisfaction and better learning outcomes.

7. Conclusion

The evaluation of teaching quality in public badminton classes at general universities primarily focuses on teaching effectiveness, student learning outcomes, teacher competence, and course implementation. First, students' mastery of techniques and their level of participation are key to the evaluation, with the emphasis on whether students can grasp basic badminton skills and tactics within the limited course time. Second, the teacher's teaching methods, classroom management skills, and ability to guide students directly impact on the quality of instruction. The ability of the teacher to tailor instruction to individual needs and effectively manage the classroom is an important evaluation criterion. Additionally, the curriculum design and efficient use of teaching resources, including court and equipment allocation, also affect teaching outcomes. Finally, student feedback and satisfaction provide critical insights for assessing teaching quality. Students' opinions and suggestions help continually optimize course content and methods, ensuring that teaching objectives are met and promoting course improvement. The teaching quality evaluation of public badminton classes at general universities is MADM. In this paper, a PROMETHEE II technique under TFNSs is introduced based on the traditional PROMETHEE II method. First, TFNSs are outlined. Then, by combining the fuzzy PROMETHEE II technique with TFNS information, the TFNN-PROMETHEE II technique is developed for MADM under TFNSs. Finally, a numerical example is provided to evaluate the teaching quality of public badminton classes at general universities, and several comparisons are made to demonstrate the advantages of the TFNN-PROMETHEE II technique with TFNSs. The key contributions of this paper are as follows: (1) The TFNN-PROMETHEE II technique is developed under TFNSs; (2) the TFNN-PROMETHEE II technique is applied to MADM under TFNSs; (3) the TFNN-PROMETHEE II technique is proposed for evaluating the teaching quality of public badminton classes at general universities; and (4) through several efficient comparisons, the TFNN-PROMETHEE II technique is shown to be effective for teaching quality evaluation in public badminton classes.

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