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Neutrosophic Left Almost Semigroup

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Abstract. In this paper we extend the theory of neutrosophy to study left almost semigroup shortly LA-semigroup. We generalize the concepts of LA-semigroup to form that for neutrosophic LA-semigroup. We also extend the ideal theory of LA-semigroup to neutrosophy and discuss different kinds of neutrosophic ideals. We also find some new type of neutrosophic ideal which is related to the strong or pure part of neutrosophy. We have given many examples to illustrate the theory of neutrosophic LA-semigroup and display many properties of neutrosophic LA-semigroup in this paper.

Keywords: LA-semigroup,sub LA-semigroup, ideal, neutrosophic LA-semigroup, neutrosophic sub LA- semigroup, neutrosophic ideal.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in . Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup abbreviated as LA-semigroup is an algebraic structure which was introduced by M .A. Kazim and M. Naseeruddin $[3]$ in 1972. This structure is basically a midway structure between a groupoid and a commutative semigroup. This structure is also termed as Able-Grassmann's groupoid abbreviated as AG -groupoid $[6]$. This is a non associative and non commutative algebraic structure which closely resemble to commutative semigroup. The generalization of semigroup theory is an LA-semigroup and this structure has wide applications in collaboration with semigroup. We have tried to develop the ideal theory of LA-semigroups in a logical manner. Firstly, preliminaries and basic concepts are given for LA-semigroups. Section 3 presents the newly defined notions and results in neutrosophic LA-semigroups. Various types of ideals are defined and elaborated with the help of examples. Furthermore, the homomorphisms of neutrosophic LA-semigroups are discussed at the end.

2 Preliminaries

Definition 1. A groupiod $(S,*)$ is called a left almost semigroup abbreviated as LA-semigroup if the left invertive law holds, i.e.

$(a*b)*c=(c*b)*a$ for all $a,b,c\in S$.

Similarly $(S,*)$ is called right almost semigroup denoted as RA-semigroup if the right invertive law holds, i.e.

$a*(b*c)=c*(b*a)$ for all $a,b,c\in S$.

Proposition 1. In an LA-semigroup S , the medial law holds. That is

$(ab)(cd)=(ac)(bd)$ for all $a,b,c,d\in S$.

Proposition 2. In an LA-semigrup S , the following statements are equivalent:

1) $(ab)c=b(ca)$
2) $(ab)c=b(ac)$. For all $a,b,c\in S$.

Theorem 1. An LA-semigroup S is a semigroup if and only if $a(bc)=(cb)a$, for all $a,b,c\in S$.

Theorem 2. An LA-semigroup with left identity satisfies the following Law,

$(ab)(cd)=(db)(ca)$ for all $a,b,c,d\in S$.

Theorem 3. In an LA-semigroup S , the following holds, $a(bc)=b(ac)$ for all $a,b,c\in S$.

Theorem 4. If an LA-semigroup S has a right identity, then S is a commutative semigroup.

Definition 2. Let S be an LA-semigroup and H be a proper subset of S . Then H is called sub LA-semigroup of S if $H.H\subseteq H$.

Definition 3. Let S be an LA-semigroup and K be a subset of S . Then K is called Left (right) ideal of S if $SK\subseteq K$, $(KS\subseteq K)$.

If K is both left and right ideal, then K is called a two sided ideal or simply an ideal of S .

Lemma 1. If K is a left ideal of an LA-semigroup S with left identity e , then aK is a left ideal of S for all $a\in S$.

Definition 4. An ideal P of an LA-semigroup S with left identity e is called prime ideal if $AB\subseteq P$ implies either

$A\subseteq P$ or $B\subseteq P$, where A,B are ideals of S .

Definition 5. An LA-semigroup S is called fully prime LA-semigroup if all of its ideals are prime ideals.

Definition 6 An ideal P is called semiprime ideal if $T.T\subseteq P$ implies $T\subseteq P$ for any ideal T of S .

Definition 7. An LA-semigroup S is called fully semiprime LA-semigroup if every ideal of S is semiprime ideal.

Definition 8. An ideal R of an LA-semigroup S is called strongly irreducible ideal if for any ideals H,K of S , $H\cap K\subseteq R$ implies $H\subseteq R$ or $K\subseteq R$.

Proposition 3. An ideal K of an LA-semigroup S is prime ideal if and only if it is semiprime and strongly irreducible ideal of S .

Definition 9. Let S be an LA-semigroup and Q be a non-empty subset of S . Then Q is called Quasi ideal of S if $QS\cap SQ\subseteq Q$.

Theorem 5. Every left $(right)$ ideal of an LA-semigroup S is a quasi-ideal of S .

Theorem 6. Intersection of two quasi ideals of an LA-semigroup is again a quasi ideal.

Definition 10. A sub LA-semigroup B of an LA-semigroup is called bi-ideal of S if $(BS)B \subseteq B$.

Definition 11. A non-empty subset A of an LA-semigroup S is termed as generalized bi-ideal of S if $(AS)A \subseteq A$.

Definition 12. A non-empty subset L of an LA-semigroup S is called interior ideal of S if $(SL)S \subseteq L$.

Theorem 7. Every ideal of an LA-semigroup S is an interior ideal.

3 Neutrosophic LA-semigroup

Definition 13. Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a+bI : a,b \in S\}$. The neutrosophic LA-semigroup is generated by S and I under denoted as $N(S) = \{\langle S \cup I \rangle, *\}$, where I is called the neutrosophic element with property $I^2 = I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 1. Let $S = \{1,2,3\}$ be an LA-semigroup with the following table

*	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Then the neutrosophic LA-semigroup $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ with the following table

*	1	2	3	1I	2I	3I
1	1	1	1	1I	1I	1I
2	3	3	3	3I	3I	3I
3	1	1	1	1I	1I	1I
1I	1I	1I	1I	1I	1I	1I
2I	3I	3I	3I	3I	3I	3I
3I	1I	1I	1I	1I	1I	1I

Similarly we can define neutrosophic RA-semigroup on the same lines.

Theorem 9. All neutrosophic LA-semigroups contains corresponding LA-semigroups.

Proof : straight forward.

Proposition 4. In a neutrosophic LA-semigroup $N(S)$, the medial law holds. That is

$$(ab)(cd) = (ac)(bd) \text{ for all } a,b,c,d \in N(S).$$

Proposition 5. In a neutrosophic LA-semigrup $N(S)$, the following statements are equivalent.

1) $(ab)c = b(ca)$
2) $(ab)c = b(ac)$. For all $a,b,c \in N(S)$.

Theorem 9. A neutrosophic LA-semigroup $N(S)$ is a neutrosophic semigroup if and only if $a(bc) = (cb)a$, for all $a,b,c \in N(S)$.

Theorem 10. Let $N(S_1)$ and $N(S_2)$ be two neutrosophic LA-semigroups. Then their cartesian product $N(S_1) \times N(S_2)$ is also a neutrosophic LA-semigroups.

Proof : The proof is obvious.

Theorem 11. Let S_1 and S_2 be two LA-semigroups. If $S_1 \times S_2$ is an LA-semigroup, then $N(S_1) \times N(S_2)$ is also a neutosophic LA-semigroup.

Proof : The proof is straight forward.

Definition 14. Let $N(S)$ be a neutrosophic LA-semigroup. An element $e \in N(S)$ is said to be left identity if $e * s = s$ for all $s \in N(S)$. Similarly e is called right identity if $s * e = s$.

e is called two sided identity or simply identity if e is left as well as right identity.

Example 2. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,4,5,1I,2I,3I,4I,5I\}$

with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	1I	2I	3I	4I	5I
1	4	5	1	2	3	4I	5I	1I	2I	3I
2	3	4	5	1	2	3I	4I	5I	1I	2I
3	2	3	4	5	1	2I	3I	4I	5I	1I
4	1	2	3	4	5	1I	2I	3I	4I	5I
5	5	1	2	3	4	5I	1I	2I	3I	4I
1I	4I	5I	1I	2I	3I	4I	5I	1I	2I	3I
2I	3I	4I	5I	1I	2I	3I	4I	5I	1I	2I
3I	2I	3I	4I	5I	1I	2I	3I	4I	5I	1I
4I	1I	2I	3I	4I	5I	1I	2I	3I	4I	5I
5I	5I	1I	2I	3I	4I	5I	1I	2I	3I	4I

Proposition 6. If $N(S)$ is a neutrosophic LA-semigroup with left identity e, then it is unique.

Proof : Obvious.

Theorem10. A neutrosophic LA-semigroup with left identity satisfies the following Law,

$(ab)(cd) = (db)(ca)$ for all $a,b,c,d \in N(S)$.

Theorem 11. In a neutrosophic LA-semigroup $N(S)$, the following holds,

$a(bc) = b(ac)$ for all $a,b,c \in N(S)$.

Theorem 12. If a neutrosophic LA-semigroup $N(S)$ has a right identity, then $N(S)$ is a commutative semigroup.

Proof. Suppose that e be the right identity of $N(S)$. By definition $ae = a$ for all $a \in N(S)$.So $ea = (e.e)a = (a.e)e = a$ for all $a \in N(S)$. Therefore e is the two sided identity. Now let $a,b \in N(S)$, then $ab = (ea)b = (ba)e = ba$ and hence $N(S)$ is commutative. Again let $a,b,c \in N(S)$, So $(ab)c = (cb)a = (bc)a = a(bc)$ and hence $N(S)$ is commutative semigroup.

Definition 15. Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Example 3. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$

be a neutrosophic LA-semigroup as in example (1). Then $\{1\}$, $\{1,3\}$, $\{1,1I\}$, $\{1,3,1I,3I\}$ etc are neutrosophic sub LA-semigroups but $\{2,3,2I,3I\}$ is not neutrosophic sub LA-semigroup of $N(S)$.

Theorem 13 Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is a neutrosophic sub LA-semigroup of $N(S)$ if $N(H).N(H)\subseteq N(H)$.

Theorem 14 Let H be a sub LA-semigroup of an LA-semigroup S, then $N(H)$ is aneutrosophic sub LA-semigroup of the neutrosophic LA-semigroup $N(S)$, where $N(H)=\langle H\cup I\rangle$.

Definition 16. A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Example 4. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1I,3I\}$ is a strong neutrosophic sub LA-semigroup or pure neutrosophicsub LA-semigroup of $N(S)$.

Theorem 15. All strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups are trivially neutrosophic sub LA-semigroup but the converse is not true.

Example 5. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1\}$, $\{1,3\}$ are neutrosophic sub LA-semigroups but not strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups of $N(S)$.

Definition 17 Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K)\subseteq N(K)$, $\{N(K)N(S)\subseteq N(K)\}$.

If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Example 6. Let $S=\{1,2,3\}$ be an LA-semigroup with the following table.

*	1	2	3
1	3	3	3
2	3	3	3
3	1	3	3

Then the neutrosophic LA-semigroup $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Then clearly $N(K_1)=\{3,3I\}$ is a neutrosophic left ideal and $N(K_2)=\{1,3,1I,3I\}$ is a neutrosophic left as well as right ideal.

Lemma 2. If $N(K)$ be a neutrosophic left ideal of a neutrosophic LA-semigroup $N(S)$ with left identity e, then $aN(K)$ is a neutrosophic left ideal of $N(S)$ for all $a\in N(S)$.

Proof : The proof is satraight forward.

Theorem 16. $N(K)$ is a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ if K is an ideal of an LA-semigroup S, where $N(K)=\langle K\cup I\rangle$.

Definition 18 A neutrosophic ideal $N(K)$ is called strong neutrosophic ideal or pure neutrosophic ideal if all of its elements are neutrosophic elements.

Fxample 7. Let $N(S)$ be a neutrosophic LA-semigroup as in example (5), Then $\{1I,3I\}$ and $\{1I,2I,3I\}$ are strong neutrosophic ideals or pure neutrosophic ideals of $N(S)$.

Theorem 17. All strong neutrosophic ideals or pure neutrosophic ideals are neutrosophic ideals but the converse is not true.

To see the converse part of above theorem, let us take an example.

Example 7 Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) . Then $N(K_1)=\{2,3,2I,3I\}$ and $N(K_2)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$ but clearly these are not strong neutrosophic ideals or pure neutrosophic ideals.

Definition 19 : A neutorophic ideal $N(P)$ of a neutrosophic LA-semigroup $N(S)$ with left identity e is called prime neutrosophic ideal if $N(A)N(B)\subseteq N(P)$ implies either $N(A)\subseteq N(P)$ or $N(B)\subseteq N(P)$, where $N(A),N(B)$ are neutrosophic ideals of $N(S)$.

Example 8. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(A)=\{2,3,2I,3I\}$ and $N(B)=\{1,3,1I,3I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B)\subseteq N(P)$ implies $N(A)\subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a prime neutrosophic ideal of $N(S)$.

Theorem 18. Every prime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

Theorem19. If P is a prime ideal of an LA-semigoup S, Then $N(P)$ is prime neutrosophic ideal of $N(S)$ where $N(P)=\langle P\cup I\rangle$.

Definition 20. A neutrosophic LA-semigroup $N(S)$ is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

Definition 21. A prime neutrosophic ideal $N(P)$ is called strong prime neutrosophic ideal or pure neutrosophic ideal if x is neutrosophic element for all $x\in N(P)$.

Example 9. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(A)=\{2I,3I\}$ and

$N(B)=\{1I,3I\}$ and $N(P)=\{1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B)\subseteq N(P)$ implies $N(A)\subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a strong prime neutrosophic ideal or pure neutrosophic ideal of $N(S)$.

Theorem 20. Every prime strong neutrosophic ideal or pure neutrosophic ideal is neutrosophic ideal but the converse is not true.

Theorem 21. Every prime strong neutrosophic ideal or pure neutrosophic ideal is a prime neutrosophic ideal but the converse is not true.

For converse, we take the following example.

Example 10. In example (6), $N(P)=\{1,3,1I,3I\}$ is a prime neutrosophic ideal but it is not strong neutrosophic ideal or pure neutrosophic ideal.

Definition 22. A neutrosophic ideal $N(P)$ is called semiprime neutrosophic ideal if $N(T).N(T)\subseteq N(P)$ implies $N(T)\subseteq N(P)$ for any neutrosophic ideal $N(T)$ of $N(S)$.

Example 11. Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1,1I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a semiprime neutrosophic ideal of $N(S)$.

Theorem 22. Every semiprime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

Definition 23. A neutrosophic semiprime ideal $N(P)$ is said to be strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal if every element of $N(P)$ is neutrosophic element.

Example 12. Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1I,3I\}$ and $N(P)=\{1I,2I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal of $N(S)$.

Theorem 23. All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are trivially neutrosophic ideals but the converse is not true.

Theorem 24. All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are semiprime neutrosophic ideals but the converse is not true.

Definition 24. A neutrosophic LA-semigroup $N(S)$ is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of $N(S)$ is semiprime neutrosophic ideal.

Definition 25. A neutrosophic ideal $N(R)$ of a neutrosophic LA-semigroup $N(S)$ is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals $N(H),N(K)$ of $N(S)$, $N(H)\cap N(K)\subseteq N(R)$ implies $N(H)\subseteq N(R)$ or $N(K)\subseteq N(R)$.

Example 13. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(H)=\{2,3,2I,3I\}$, $N(K)=\{1I,3I\}$ and $N(R)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(H)\cap N(K)\subseteq N(R)$ implies $N(K)\subseteq N(R)$ but $N(H)$ is not contained in $N(R)$. Hence $N(R)$ is a strong irreducible neutrosophic ideal of $N(S)$.

Theorem 25. Every strongly irreducible neutrosohic ideal is a neutrosophic ideal but the converse is not true.

Theorem 26. If R is a strong irreducible neutrosophic ideal of an LA-semigoup S, Then $N(I)$ is a strong irreducible neutrosophic ideal of $N(S)$ where $N(R)=\langle R\cup I\rangle$.

Proposition 7. A neutrosophic ideal $N(I)$ of a neutrosophic LA-semigroup $N(S)$ is prime neutrosophic ideal if and only if it is semiprime and strongly irreducible neutrosophic ideal of $N(S)$.

Definition 26. Let $N(S)$ be a neutrosophic ideal and

$N(Q)$ be a non-empty subset of $N(S)$. Then $N(Q)$ is called quasi neutrosophic ideal of $N(S)$ if

$$N(Q)N(S)\cap N(S)N(Q)\subseteq N(Q).$$

Example 14. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5). Then $N(K)=\{3,3I\}$ be a non-empty subset of $N(S)$ and $N(S)N(K)=\{3,3I\}$, $N(K)N(S)=\{1,3,1I,3I\}$ and their intersection is $\{3,3I\}\subseteq N(K)$. Thus clearly $N(K)$ is quasi neutrosophic ideal of $N(S)$.

Theorem27. Every left $(right)$ neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is a quasi neutrosophic ideal of $N(S)$.

Proof : Let $N(Q)$ be a left neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then $N(S)N(Q)\subseteq N(Q)$ and so

$$N(S)N(Q)\cap N(Q)N(S)\subseteq N(Q)\cap N(Q)\subseteq N(Q)$$

which proves the theorem.

Theorem 28. Intersection of two quasi neutrosophic ideals of a neutrosophic LA-semigroup is again a quasi neutrosophic ideal.

Proof : The proof is straight forward.

Definition 27. A quasi-neutrosophic ideal $N(Q)$ of a neutrosophic LA-semigroup $N(Q)$ is called quasi-strong neutrosophic ideal or quasi-pure neutosophic ideal if all the elements of $N(Q)$ are neutrosophic elements.

Example 15. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5). Then $N(K)=\{1I,3I\}$ be a quasi-neutrosophic ideal of $N(S)$. Thus clearly $N(K)$ is quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal of $N(S)$.

Theorem 29. Every quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal is qausi-neutrosophic ideal but the converse is not true.

Definition 28. A neutrosophic sub LA-semigroup $N(B)$ of a neutrosophic LA-semigroup is called bi-neutrosophic ideal of $N(S)$ if

$$(N(B)N(S))N(B)\subseteq N(B).$$

Example 16. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1,3,1I,3I\}$ is a neutrosophic sub LA-semigroup of $N(S)$. Then Clearly $N(B)$ is a bi-neutrosophic ideal of $N(S)$.

Theorem 30. Let B be a bi-ideal of an LA-semigroup S, then $N(B)$ is bi-neutrosophic ideal of $N(S)$ where $N(B)=\langle B\cup I\rangle$.

Proof : The proof is straight forward.

Definition 29. A bi-neutrosophic ideal $N(B)$ of a neutrosophic LA-semigroup $N(S)$ is called bi-strong neutrosophic ideal or bi-pure neutrosophic ideal if every element of $N(B)$ is a neutrosophic element.

Example 17. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1I,3I\}$ is a bi-neutrosophic ideal of $N(S)$. Then Clearly $N(B)$ is a bi-strong neutrosophic ideal or bi-pure neutosophic ideal of $N(S)$.

Theorem 31. All bi-strong neutrosophic ideals or bi-pure neutrosophic ideals are bi-neutrosophic ideals but the converse is not true.

Definition 30. A non-empty subset $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is termed as generalized bi-neutrosophic ideal of $N(S)$ if $(N(A)N(S))N(A)\subseteq N(A)$.

Example 18. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(A)=\{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(A)$ is a generalized bi-neutrosophic ideal of $N(S)$.

Theorem 32. Every bi-neutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-ideal but the converse is not true.

Definition 31. A generalized bi-neutrosophic ideal $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is called generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$ if all the elements of $N(A)$ are neutrosophic elements.

Example 19. Let

$N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be

a neutrosophic LA-semigroup as in example (1) and

$N(A)=\{1I,3I\}$ is a generalized bi-neutrosophic ideal of $N(S)$. Then clearly $N(A)$ is a generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$.

Theorem 33. All generalized bi-strong neutrosophic ideals or generalized bi-pure neutrosophic ideals are generalized bi-neutrosophic ideals but the converse is not true.

Theorem 34. Every bi-strong neutrosophic ideal or bi-pureneutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal but the converse is not true.

Definition 32. A non-empty subset $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior neutrosophic ideal of $N(S)$ if $(N(S)N(L))N(S) \subseteq N(L)$.

Example 20. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(L) = \{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Theorem 35. Every neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior neutrosophic ideal.

Proof : Let $N(L)$ be a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then by definition $N(L)N(S) \subseteq N(L)$ and $N(S)N(L) \subseteq N(L)$. So clearly $(N(S)N(L))N(S) \subseteq N(L)$ and hence $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Definition 33. An interior neutrosophic ideal $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior strong neutrosophic ideal or interior pure neutrosophic ideal if every element of $N(L)$ is a neutrosophic element.

Example 21. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(L) = \{1I,3I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal of $N(S)$.

Theorem 36. All interior strong neutrosophic ideals or interior pure neutrosophic ideals are trivially interior neutrosophic ideals of a neutrosophic LA-semigroup $N(S)$ but the converse is not true.

Theorem 37. Every strong neutrosophic ideal or pure neutosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal.

Neutrosophic homomorphism

Definition 34. Let S,T be two LA-semigroups and $\phi : S \to T$ be a mapping from S to T. Let $N(S)$ and $N(T)$ be the corresponding neutrosophic LA-semigroups of S and T respectively. Let $\theta : N(S) \to N(T)$ be another mapping from $N(S)$ to $N(T)$. Then θ is called neutrosophic homomorphis if ϕ is homomorphism from S to T.

Example 22. Let Z be an LA-semigroup under the operation $a*b = b-a$ for all $a,b \in Z$. Let Q be another LA-semigroup under the same operation defined above. For some fixed non-zero rational number x, we define $\phi : Z \to Q$ by $\phi(a) = a/x$ where $a \in Z$. Then ϕ is a homomorphism from Z to Q. Let $N(Z)$ and $N(Q)$ be the corresponding neutrosophic LA-semigroups of Z and Q respectively. Now Let $\theta : N(Z) \to N(Q)$ be a map from neutrosophic LA-semigroup $N(Z)$ to the neutrosophic LA-semigroup $N(Q)$. Then clearly θ is the corresponding neutrosophic homomorphism of $N(Z)$ to $N(Q)$ as ϕ is homomorphism.

Theorem 38. If ϕ is an isomorphism, then θ will be the neutrosophic isomorphism.

Proof : It's easy.

Conclusion

In this paper we extend the neutrosophic group and subgroup,pseudo neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup and respectively soft pseudo neutrosophic group and soft pseudo neutrosophic subgroup. The normal neutrosophic subgroup is extended to soft normal neutrosophic subgroup.

We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

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