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Mixed Graph in Fuzzy, Neutrosophic, and Plithogenic Graphs

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Abstract. Graph theory examines networks consisting of nodes (vertices) and the connections (edges) between them. Mixed graphs, which combine both undirected and directed edges, provide a versatile framework for representing relationships with symmetric and asymmetric connections across various systems. In this work, we introduce and analyze the concept of mixed graphs within the contexts of Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. By exploring their properties, interrelations, and potential applications, we aim to advance the understanding of uncertain graphs.

Keywords: Mixed Graph, Neutrosophic graph, Fuzzy graph, Plithogenic Graph

1. **Introduction**

1.1. *Uncertain Set and Graph Theory*

Graph theory provides a fundamental framework for analyzing networks, composed of nodes (vertices) and their connections (edges). It offers insights into the structure, connectivity, and properties of diverse networks [11]. Among the various types of graphs, this paper focuses on mixed graphs, which integrate both undirected and directed edges. This integration allows the representation of both symmetric and asymmetric relationships, making mixed graphs a versatile tool for modeling complex systems. Consequently, mixed graphs have garnered significant attention in both theoretical and practical research [4, 38, 42, 45, 46, 67].

To address uncertainty in real-world scenarios, concepts like fuzzy sets have been developed [70]. A fuzzy set quantifies the degree to which an element belongs to a set using a membership function. Neutrosophic sets [50–52] further extend this idea by incorporating degrees of truth, indeterminacy, and falsity, enabling more nuanced modeling of uncertain phenomena. Below are intuitive examples illustrating these concepts in practical scenarios. Note that the following are merely intuitive examples.

Example 1.1 (Fuzzy Set)**.** Consider a scenario where A represents the set of ”short buildings” and B represents the set of ”tall buildings.”

Let x be a building with a height of 30 meters. Its membership degree in each set might be defined as follows:

$$\mu_A(x) = 0.6 \quad \text{(short buildings)}, \quad \mu_B(x) = 0.4 \quad \text{(tall buildings)}.$$

This representation reflects that the building x exhibits characteristics of both short and tall buildings, quantifying the overlap between these categories.

Example 1.2 (Neutrosophic Set)**.** Consider the classification of weather as ”Sunny,” ”Cloudy,” or ”Rainy.” The weather state might not fully belong to any single category, making it suitable for representation using a Neutrosophic Set.

Let the weather condition x be characterized as follows:

- *Truth Membership Degree ($T(x)$):* The degree to which the weather is ”Sunny,” $T(x) = 0.7$.
- *Indeterminacy Membership Degree ($I(x)$):* The degree of uncertainty about the weather, $I(x) = 0.2$.
- *Falsity Membership Degree ($F(x)$):* The degree to which the weather is ”Rainy,” $F(x) = 0.1$.

Thus, the weather condition can be represented as:

$$\text{Weather} = \{(\text{Sunny}, T(x) = 0.7), (\text{Uncertain}, I(x) = 0.2), (\text{Rainy}, F(x) = 0.1)\}.$$

This paper investigates uncertain graph models, such as Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs, designed to address uncertainties in various applications. These models, collectively referred to as uncertain graphs, extend classical graph theory by incorporating different levels and types of uncertainty [3,5,6,13,16–21,23,24,44,53,57,59,60].

For example, a fuzzy graph assigns membership degrees in $[0, 1]$ to vertices and edges, representing uncertainties in connections. Neutrosophic graphs go further, assigning truth, indeterminacy, and falsity membership degrees to vertices and edges, thus capturing more complex uncertainties. These models are especially valuable in decision-making scenarios(ex. [3]). Below are examples illustrating the application of these concepts in real-world scenarios. Note that the following are merely intuitive examples.

Example 1.3 (Fuzzy Graph)**.** Consider a scenario where A represents the set of ”short buildings” and B represents the set of ”tall buildings.”

Let x be a building with a height of 30 meters. Its membership degree in each set might be defined as follows:

$$\mu_A(x) = 0.6 \quad \text{(short buildings)}, \quad \mu_B(x) = 0.4 \quad \text{(tall buildings)}.$$

This representation reflects that the building x exhibits characteristics of both short and tall buildings, quantifying the overlap between these categories.

Now, consider a fuzzy graph $G = (V, E)$, where $V = \{x_1, x_2, x_3\}$ represents buildings and E represents connections based on architectural similarity. For instance, the edge (x_1, x_2) might have a membership degree $\mu((x_1, x_2)) = 0.8$, indicating a high degree of similarity in design, materials, or function between the two buildings. This could represent relationships such as ”similar construction style” or ”shared purpose.”

Example 1.4 (Neutrosophic Graph)**.** Consider the classification of weather as ”Sunny,” ”Cloudy,” or ”Rainy.” The state of the weather might not fully belong to any single category, making it suitable for representation using a Neutrosophic Set.

Let the weather condition x be characterized as follows:

- *Truth Membership Degree ($T(x)$):* The degree to which the weather is ”Sunny,” $T(x) = 0.7$.
- *Indeterminacy Membership Degree ($I(x)$):* The degree of uncertainty about the weather, $I(x) = 0.2$.
- *Falsity Membership Degree ($F(x)$):* The degree to which the weather is ”Rainy,” $F(x) = 0.1$.

In the context of a neutrosophic graph $G = (V, E)$, let $V = \{x_1, x_2, x_3\}$, where each vertex represents a weather condition. Edges E represent relationships such as transitions between weather states. For instance, (x_1, x_2) might have the following degrees:

$$T((x_1, x_2)) = 0.5, \quad I((x_1, x_2)) = 0.3, \quad F((x_1, x_2)) = 0.2,$$

indicating the truth, indeterminacy, and falsity degrees of the transition between x_1 (Sunny) and x_2 (Cloudy). Such edges could represent the likelihood of weather transitions based on historical data or meteorological patterns.

1.2. *Our Contribution*

Research on uncertain sets and graphs, as illustrated above, is crucial for modeling real-world phenomena. However, their mathematical structures remain partially unexplored. In this work, we propose and analyze the concept of mixed graphs within the frameworks of Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. We explore their properties, relationships, and potential applications, contributing to the broader understanding of uncertain graphs. Notably, the Turiyam Neutrosophic Set is a specific case of the Quadruple Neutrosophic Set, obtained by replacing the ”Contradiction” component with ”Liberal” [49].

2. Preliminaries and definitions

In this section, we present a brief overview of the definitions and notations used throughout this paper.

2.1. *Basic Graph Concepts*

We outline some basic graph concepts here. Note that this is not an exhaustive list. For more foundational graph concepts and notations, please refer to [10, 11, 29, 69].

Definition 2.1 (Graph)**.** [11] A graph G is a mathematical structure consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ that connect pairs of vertices, representing relationships or connections between them. Formally, a graph is defined as $G = (V, E)$, where V is the vertex set and E is the edge set.

Definition 2.2 (Subgraph)**.** [11] A graph $H = (V_H, E_H)$ is called a *subgraph* of a graph $G = (V, E)$ if:

- $V_H \subseteq V$, i.e., the vertex set of H is a subset of the vertex set of G,
- $E_H \subseteq E$, i.e., the edge set of H is a subset of the edge set of G,
- For each edge $e \in E_H$, if $e = \{u, v\}$, then $u, v \in V_H$.

In other words, a subgraph H of G consists of a subset of the vertices and edges of G, with the condition that all edges in E_H connect vertices in V_H.

Definition 2.3 (Degree)**.** [11] Let $G = (V, E)$ be a graph. The *degree* of a vertex $v \in V$, denoted $\deg(v)$, is the number of edges incident to v. Formally, for undirected graphs:

$$\deg(v) = |\{e \in E \mid v \in e\}|.$$

In the case of directed graphs, the *in-degree* $\deg^-(v)$ is the number of edges directed into v, and the *out-degree* $\deg^+(v)$ is the number of edges directed out of v.

Definition 2.4 (Induced Subgraph)**.** (cf. [8]) Given a graph $G = (V, E)$ and a subset of vertices $S \subseteq V$, the *induced subgraph* of G on S, denoted by $G[S]$, is defined as:

$$G[S] = (S, E_S),$$

where $E_S = \{\{u, v\} \in E \mid u, v \in S\}$.

The induced subgraph $G[S]$ satisfies the following properties:

- The vertex set of $G[S]$ is exactly S.
- The edge set E_S contains all edges in G whose endpoints are both in S.

Definition 2.5 (Directed Graph)**.** [11] A *directed graph* $G = (V, A)$ is a mathematical structure consisting of:

- V: A set of vertices (or nodes),
- $A \subseteq V \times V$: A set of directed edges (or arcs), where each arc $(u,v) \in A$ has an orientation from vertex u (the tail) to vertex v (the head).

2.2. *Fuzzy, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs*

In this subsection, we examine Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. Fuzzy graphs are frequently discussed in comparison with crisp graphs, which represent the classical form of graphs [35, 44].

Definition 2.6. (cf. [35, 44]) A *crisp graph* is an ordered pair $G = (V, E)$, where:

- V is a finite, non-empty set of vertices.
- $E \subseteq V \times V$ is a set of edges, where each edge is an unordered pair of distinct vertices.

Formally, for any edge $(u,v) \in E$, the following holds:

$$(u,v) \in E \iff u \neq v \quad \text{and} \quad u,v \in V$$

This implies that there are no loops (i.e., no edges of the form (v,v)) and edges represent binary relationships between distinct vertices.

Taking the above into consideration, we define Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Turiyam Neutrosophic Graphs as follows. Please note that the definitions have been consolidated for simplicity.

Definition 2.7 (Unified Graphs Framework: Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Turiyam Neutrosophic Graphs)**.** (cf. [16]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, and falsity.

(1) *Fuzzy Graph [43, 44, 68]:*
   - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0,1]$, representing the degree of participation of v in the fuzzy graph.
   - Each edge $e = (u,v) \in E$ is assigned a membership degree $\mu(u,v) \in [0,1]$, representing the strength of the connection between u and v.

(2) *Intuitionistic Fuzzy Graph (IFG) [1, 65, 71]:*
   - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $v_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
   - Each edge $e = (u,v) \in E$ is assigned two values: $\mu_B(u,v) \in [0,1]$ (degree of membership) and $v_B(u,v) \in [0,1]$ (degree of non-membership), such that $\mu_B(u,v) + v_B(u,v) \leq 1$.

(3) *Neutrosophic Graph [2, 7, 14, 30, 33, 57, 59]:*
- Each vertex $v \in V$ is assigned a triple $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where:
  - $\sigma_T(v) \in [0,1]$ is the truth-membership degree,
  - $\sigma_I(v) \in [0,1]$ is the indeterminacy-membership degree,
  - $\sigma_F(v) \in [0,1]$ is the falsity-membership degree,
  - $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
- Each edge $e = (u,v) \in E$ is assigned a triple $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$, representing the truth, indeterminacy, and falsity degrees for the connection between u and v.

(4) *Turiyam Neutrosophic Graph [16, 25–27, 48]:*
- Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where:
  - $t(v) \in [0,1]$ is the truth value,
  - $iv(v) \in [0,1]$ is the indeterminacy value,
  - $fv(v) \in [0,1]$ is the falsity value,
  - $lv(v) \in [0,1]$ is the liberal state value,
  - $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
- Each edge $e = (u,v) \in E$ is similarly assigned a quadruple representing the same parameters for the connection between u and v.

2.3. *Plithogenic Graphs*

Plithogenic Graphs have recently emerged as a generalization of Fuzzy Graphs and Turiyam Neutrosophic Graphs, extending the concept to represent Plithogenic Sets [55]. These graphs are a focus of ongoing research and development [16, 34, 47, 62, 63]. The formal definition is presented below.

Definition 2.8. [63] Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A *Plithogenic Graph PG* is defined as:

$$PG = (PM, PN)$$

where:

(1) *Plithogenic Vertex Set* $PM = (M, l, Ml, adf, aCf)$:
- $M \subseteq V$ is the set of vertices.
- l is an attribute associated with the vertices.
- Ml is the range of possible attribute values.
- $adf : M \times Ml \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* for vertices.
- $aCf : Ml \times Ml \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* for vertices.

(2) *Plithogenic Edge Set* $PN = (N, m, Nm, bdf, bCf)$:
- $N \subseteq E$ is the set of edges.
- m is an attribute associated with the edges.
- Nm is the range of possible attribute values.
- $bdf : N \times Nm \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* for edges.
- $bCf : Nm \times Nm \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* for edges.

The Plithogenic Graph PG must satisfy the following conditions:

(1) *Edge Appurtenance Constraint*: For all $(x,a),(y,b) \in M \times Ml$:

$$bdf\left((xy),(a,b)\right) \leq \min\{adf(x,a), adf(y,b)\}$$

where $xy \in N$ is an edge between vertices x and y, and $(a,b) \in Nm \times Nm$ are the corresponding attribute values.

(2) *Contradiction Function Constraint*: For all $(a,b),(c,d) \in Nm \times Nm$:

$$bCf\left((a,b),(c,d)\right) \leq \min\{aCf(a,c), aCf(b,d)\}$$

(3) *Reflexivity and Symmetry of Contradiction Functions*:

$$\begin{aligned} aCf(a,a) &= 0, && \forall a \in Ml \\ aCf(a,b) &= aCf(b,a), && \forall a,b \in Ml \\ bCf(a,a) &= 0, && \forall a \in Nm \\ bCf(a,b) &= bCf(b,a), && \forall a,b \in Nm \end{aligned}$$

Example 2.9. (cf. [14]) The following examples are provided.

- When $s = t = 1$, PG is called a *Plithogenic Fuzzy Graph*.
- When $s = 2, t = 1$, PG is called a *Plithogenic Intuitionistic Fuzzy Graph*.
- When $s = 3, t = 1$, PG is called a *Plithogenic Neutrosophic Graph*.
- When $s = 4, t = 1$, PG is called a *Plithogenic Turiyam Neutrosophic Graph*.

3. **Result in this paper**

In this section, we present the results of this paper.

3.1. *Uncertain Mixed Graph*

We examine the concept of an Uncertain Mixed Graph, which integrates the ideas of Uncertain Graphs and Mixed Graphs. The definitions, including related concepts, are provided below.

Definition 3.1 (Mixed Graph)**.** [42,45,46,67] A *mixed graph* $G = (V, E, A)$ is a mathematical structure in graph theory consisting of three components:

- V: A set of vertices (or nodes),
- E: A set of undirected edges, where an edge $uv \in E$ (or equivalently, $[u, v]$) connects two vertices u and v without any orientation,
- A: A set of directed edges, also called arcs, where an arc $\overrightarrow{uv} \in A$ (or equivalently, (u, v)) connects two vertices u and v with a specific orientation, making u the tail and v the head.

Properties of Mixed Graphs:

(1) *Vertices and Edges:* Each vertex $u \in V$ can be connected to another vertex $v \in V$ either by an undirected edge or a directed arc. There can be multiple edges between the same pair of vertices, but for simplicity, loops (edges/arcs that start and end at the same vertex) are not considered in this definition.

(2) *Walks in Mixed Graphs:* A *walk* in a mixed graph is a sequence of vertices and edges/arcs denoted as

$$v_0, c_1, v_1, c_2, v_2, \ldots, c_k, v_k,$$

where for each index i, c_i is either an undirected edge $v_i v_{i+1}$ or a directed arc $\overrightarrow{v_i v_{i+1}}$.

(3) *Paths and Cycles:* A *path* is a walk in which no vertices, edges, or arcs are repeated, except possibly the starting and ending vertices. A *closed path* (where the start and end vertices are the same) is called a *cycle*. A mixed graph is called *acyclic* if it does not contain any cycles.

In summary, a mixed graph generalizes both undirected and directed graphs by allowing the presence of both undirected edges and directed arcs, making it a versatile model for representing complex networks with mixed types of relationships.

Proposition 3.2. *An undirected graph is a special case of a mixed graph where the set of directed edges A is empty.*

Proof. Let $G = (V, E)$ be an undirected graph, where:

- V is the set of vertices,
- E is the set of undirected edges.

Define a mixed graph $G' = (V', E', A')$, where:

- $V' = V$,
- $E' = E$,
- $A' = \emptyset$.

Since A' is empty, all edges in G' are undirected, and G' retains the same structure as G. Thus, G can be represented as a mixed graph G'. □

Definition 3.3 (Fuzzy Mixed Graph)**.** (cf. [9, 28, 39–41]) A *Fuzzy Mixed Graph* $G = (V, E, A, \sigma, \mu_E, \mu_A)$ is a mixed graph where:

- V is the set of vertices.
- E is the set of undirected edges.
- A is the set of directed edges (arcs).
- $\sigma : V \to [0, 1]$ is the *vertex membership function*, assigning to each vertex $v \in V$ a membership degree $\sigma(v)$.
- $\mu_E : E \to [0, 1]$ is the *edge membership function* for undirected edges.
- $\mu_A : A \to [0, 1]$ is the *arc membership function* for directed edges.

These functions satisfy the following conditions:

(1) For every undirected edge $e = \{u, v\} \in E$, the edge membership degree satisfies:

$$\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}.$$

(2) For every directed edge $a = (u, v) \in A$, the arc membership degree satisfies:

$$\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}.$$

Theorem 3.4. *A Fuzzy Mixed Graph generalizes a Mixed Graph by assigning membership degrees to vertices, undirected edges, and directed arcs.*

Proof. Let $G = (V, E, A)$ be a Mixed Graph, where:

- V: Set of vertices.
- E: Set of undirected edges.
- A: Set of directed edges.

Define a Fuzzy Mixed Graph $G' = (V', E', A', \sigma, \mu_E, \mu_A)$, where:

- $V' = V$,
- $E' = E$,
- $A' = A$,
- $\sigma(v) = 1$ for all $v \in V$,
- $\mu_E(e) = 1$ for all $e \in E$,
- $\mu_A(a) = 1$ for all $a \in A$.

Since all membership functions are set to 1, G' behaves identically to G. Thus, any Mixed Graph can be represented as a special case of a Fuzzy Mixed Graph.

Conversely, by allowing membership degrees in $[0, 1]$, Fuzzy Mixed Graphs extend the capability of Mixed Graphs to model partial relationships and uncertainties. □

Definition 3.5 (Intuitionistic Fuzzy Mixed Graph)**.** An *Intuitionistic Fuzzy Mixed Graph*

$$G = (V, E, A, \mu_V, \nu_V, \mu_E, \nu_E, \mu_A, \nu_A)$$

consists of:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- $\mu_V : V \to [0, 1]$ and $\nu_V : V \to [0, 1]$ are the *vertex membership* and *non-membership functions*, respectively, satisfying for each $v \in V$:

$$0 \leq \mu_V(v) + \nu_V(v) \leq 1.$$

- $\mu_E : E \to [0, 1]$ and $\nu_E : E \to [0, 1]$ are the *edge membership* and *non-membership functions* for undirected edges, satisfying for each $e = \{u, v\} \in E$:

$$0 \leq \mu_E(e) + \nu_E(e) \leq 1,$$

$$\mu_E(e) \leq \min\{\mu_V(u), \mu_V(v)\},$$

$$\nu_E(e) \geq \max\{\nu_V(u), \nu_V(v)\}.$$

- $\mu_A : A \to [0, 1]$ and $\nu_A : A \to [0, 1]$ are the *arc membership* and *non-membership functions* for directed edges, satisfying for each $a = (u, v) \in A$:

$$0 \leq \mu_A(a) + \nu_A(a) \leq 1,$$

$$\mu_A(a) \leq \min\{\mu_V(u), \mu_V(v)\},$$

$$\nu_A(a) \geq \max\{\nu_V(u), \nu_V(v)\}.$$

Theorem 3.6. *An Intuitionistic Fuzzy Mixed Graph generalizes both Fuzzy Mixed Graphs and Mixed Graphs by incorporating membership and non-membership degrees.*

Proof. Let $G = (V, E, A, \sigma, \mu_E, \mu_A)$ be a Fuzzy Mixed Graph, where:

- V: Set of vertices.
- E: Set of undirected edges.
- A: Set of directed arcs.
- σ, μ_E, and μ_A are the vertex, edge, and arc membership functions, respectively.

Define an Intuitionistic Fuzzy Mixed Graph $G' = (V', E', A', \mu_V, \nu_V, \mu_E, \nu_E, \mu_A, \nu_A)$, where:

- $V' = V$,
- $E' = E$,
- $A' = A$,
- $\mu_V(v) = \sigma(v)$ for all $v \in V$,
- $\nu_V(v) = 1 - \sigma(v)$ for all $v \in V$,

- $\mu_E(e) = \mu_E(e)$, $\nu_E(e) = 1 - \mu_E(e)$ for all $e \in E$,
- $\mu_A(a) = \mu_A(a)$, $\nu_A(a) = 1 - \mu_A(a)$ for all $a \in A$.

By including non-membership functions ν_V, ν_E, and ν_A, G' retains the structure of G while extending it to account for non-membership degrees. If all non-membership degrees are set to 0, G' reduces to a Fuzzy Mixed Graph. Furthermore, if all membership degrees are set to 1, G' reduces to a Mixed Graph.

Therefore, Intuitionistic Fuzzy Mixed Graphs generalize both Fuzzy Mixed Graphs and Mixed Graphs. □

Definition 3.7 (Neutrosophic Mixed Graph)**.** (cf. [32]) A *Neutrosophic Mixed Graph*

$$G = (V, E, A, T_V, I_V, F_V, T_E, I_E, F_E, T_A, I_A, F_A)$$

is defined as:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- $T_V, I_V, F_V : V \to [0,1]$ are the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* functions for vertices, respectively, satisfying for each $v \in V$:

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3.$$

- Similarly, $T_E, I_E, F_E : E \to [0,1]$ are the membership functions for undirected edges, and $T_A, I_A, F_A : A \to [0,1]$ are the membership functions for directed edges, satisfying analogous conditions.
- For each undirected edge $e = \{u, v\} \in E$, the edge membership functions satisfy:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad I_E(e) \geq \max\{I_V(u), I_V(v)\}, \quad F_E(e) \geq \max\{F_V(u), F_V(v)\}.$$

- For each directed edge $a = (u, v) \in A$, the arc membership functions satisfy:

$$T_A(a) \leq \min\{T_V(u), T_V(v)\}, \quad I_A(a) \geq \max\{I_V(u), I_V(v)\}, \quad F_A(a) \geq \max\{F_V(u), F_V(v)\}.$$

Theorem 3.8. *A Neutrosophic Mixed Graph generalizes Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, and Mixed Graphs.*

Proof. Let

$$G = (V, E, A, T_V, I_V, F_V, T_E, I_E, F_E, T_A, I_A, F_A)$$

be a Neutrosophic Mixed Graph. To show that it generalizes other types of graphs, we consider the following cases:

An Intuitionistic Fuzzy Mixed Graph is defined by vertex membership μ_V, vertex non-membership ν_V, edge membership μ_E, edge non-membership ν_E, arc membership μ_A, and arc non-membership ν_A, satisfying $0 \leq \mu + \nu \leq 1$.

Set:

$$T_V(v) = \mu_V(v), \quad I_V(v) = 0, \quad F_V(v) = \nu_V(v),$$

$$T_E(e) = \mu_E(e), \quad I_E(e) = 0, \quad F_E(e) = \nu_E(e),$$

$$T_A(a) = \mu_A(a), \quad I_A(a) = 0, \quad F_A(a) = \nu_A(a).$$

The neutrosophic membership functions now satisfy the Intuitionistic Fuzzy Mixed Graph conditions:

$$0 \leq T + F = \mu + \nu \leq 1, \quad I = 0.$$

Hence, a Neutrosophic Mixed Graph reduces to an Intuitionistic Fuzzy Mixed Graph under these mappings.

A Fuzzy Mixed Graph is defined by vertex membership σ, edge membership μ_E, and arc membership μ_A, satisfying $\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}$ for edges and $\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}$ for arcs.

Set:

$$T_V(v) = \sigma(v), \quad I_V(v) = 0, \quad F_V(v) = 1 - \sigma(v),$$

$$T_E(e) = \mu_E(e), \quad I_E(e) = 0, \quad F_E(e) = 1 - \mu_E(e),$$

$$T_A(a) = \mu_A(a), \quad I_A(a) = 0, \quad F_A(a) = 1 - \mu_A(a).$$

The neutrosophic membership functions now satisfy the Fuzzy Mixed Graph conditions:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad T_A(a) \leq \min\{T_V(u), T_V(v)\}.$$

Hence, a Neutrosophic Mixed Graph reduces to a Fuzzy Mixed Graph under these mappings.

A Mixed Graph is defined by vertices V, undirected edges E, and directed arcs A, without any membership functions.

Set:

$$T_V(v) = 1, \quad I_V(v) = 0, \quad F_V(v) = 0,$$

$$T_E(e) = 1, \quad I_E(e) = 0, \quad F_E(e) = 0,$$

$$T_A(a) = 1, \quad I_A(a) = 0, \quad F_A(a) = 0.$$

The neutrosophic membership functions become constant, making the Neutrosophic Mixed Graph equivalent to a Mixed Graph.

By appropriately setting the neutrosophic membership functions, a Neutrosophic Mixed Graph can reduce to an Intuitionistic Fuzzy Mixed Graph, a Fuzzy Mixed Graph, or a Mixed Graph. Therefore, a Neutrosophic Mixed Graph generalizes all these graph types. □

Definition 3.9 (Turiyam Neutrosophic Mixed Graph)**.** A *Turiyam Neutrosophic Mixed Graph*

$$G = (V, E, A, t_V, iv_V, fv_V, lv_V, t_E, iv_E, fv_E, lv_E, t_A, iv_A, fv_A, lv_A)$$

consists of:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- For each vertex $v \in V$, the membership functions $t_V(v), iv_V(v), fv_V(v), lv_V(v) \in [0, 1]$ represent the truth, indeterminacy, falsity, and liberal state values, respectively, satisfying:

$$0 \leq t_V(v) + iv_V(v) + fv_V(v) + lv_V(v) \leq 4.$$

- Similar membership functions are defined for edges and arcs, satisfying analogous conditions.
- For each undirected edge $e = \{u, v\} \in E$, the edge membership functions satisfy:

$$t_E(e) \leq \min\{t_V(u), t_V(v)\},$$

$$iv_E(e) \geq \max\{iv_V(u), iv_V(v)\},$$

$$fv_E(e) \geq \max\{fv_V(u), fv_V(v)\},$$

$$lv_E(e) \geq \max\{lv_V(u), lv_V(v)\}.$$

- For each directed edge $a = (u, v) \in A$, the arc membership functions satisfy similar conditions.

Theorem 3.10. *A Turiyam Neutrosophic Mixed Graph generalizes Neutrosophic Mixed Graphs, Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, and Mixed Graphs.*

Proof. Let

$$G = (V, E, A, t_V, iv_V, fv_V, lv_V, t_E, iv_E, fv_E, lv_E, t_A, iv_A, fv_A, lv_A)$$

be a Turiyam Neutrosophic Mixed Graph. To prove that it generalizes other types of graphs, we consider the following cases:

A Neutrosophic Mixed Graph is defined with truth-membership (T), indeterminacy-membership (I), and falsity-membership (F) functions for vertices, edges, and arcs.

Set:

$$t_V(v) = T_V(v), \quad iv_V(v) = I_V(v), \quad fv_V(v) = F_V(v), \quad lv_V(v) = 0,$$

$$t_E(e) = T_E(e), \quad iv_E(e) = I_E(e), \quad fv_E(e) = F_E(e), \quad lv_E(e) = 0,$$

$$t_A(a) = T_A(a), \quad iv_A(a) = I_A(a), \quad fv_A(a) = F_A(a), \quad lv_A(a) = 0.$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to a Neutrosophic Mixed Graph by ignoring the liberal membership component (lv).

An Intuitionistic Fuzzy Mixed Graph is defined with membership (μ) and non-membership (ν) functions for vertices, edges, and arcs, satisfying $0 \leq \mu + \nu \leq 1$.

Set:

$$t_V(v) = \mu_V(v), \quad iv_V(v) = 0, \quad fv_V(v) = \nu_V(v), \quad lv_V(v) = 0,$$
$$t_E(e) = \mu_E(e), \quad iv_E(e) = 0, \quad fv_E(e) = \nu_E(e), \quad lv_E(e) = 0,$$
$$t_A(a) = \mu_A(a), \quad iv_A(a) = 0, \quad fv_A(a) = \nu_A(a), \quad lv_A(a) = 0.$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to an Intuitionistic Fuzzy Mixed Graph.

A Fuzzy Mixed Graph is defined with membership functions for vertices (σ), edges (μ_E), and arcs (μ_A).

Set:

$$t_V(v) = \sigma(v), \quad iv_V(v) = 0, \quad fv_V(v) = 1 - \sigma(v), \quad lv_V(v) = 0,$$
$$t_E(e) = \mu_E(e), \quad iv_E(e) = 0, \quad fv_E(e) = 1 - \mu_E(e), \quad lv_E(e) = 0,$$
$$t_A(a) = \mu_A(a), \quad iv_A(a) = 0, \quad fv_A(a) = 1 - \mu_A(a), \quad lv_A(a) = 0.$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to a Fuzzy Mixed Graph.

A Mixed Graph is defined by vertices (V), undirected edges (E), and directed arcs (A), without any membership functions.

Set:

$$t_V(v) = 1, \quad iv_V(v) = 0, \quad fv_V(v) = 0, \quad lv_V(v) = 0,$$
$$t_E(e) = 1, \quad iv_E(e) = 0, \quad fv_E(e) = 0, \quad lv_E(e) = 0,$$
$$t_A(a) = 1, \quad iv_A(a) = 0, \quad fv_A(a) = 0, \quad lv_A(a) = 0.$$

The Turiyam Neutrosophic Mixed Graph becomes equivalent to a Mixed Graph under these mappings.

By appropriately setting the membership functions, a Turiyam Neutrosophic Mixed Graph can reduce to Neutrosophic Mixed Graphs, Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, or Mixed Graphs. Thus, it generalizes all these types of graphs. □

Definition 3.11 (Plithogenic Mixed Graph)**.** A *Plithogenic Mixed Graph*

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

is defined as:

- V: the set of vertices.
- E: the set of undirected edges.

- A: the set of directed edges (arcs).
- $adf_V : V \to [0,1]^s$ is the *attribute degree function* for vertices, assigning to each vertex $v \in V$ an s-tuple $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$.
- $adf_E : E \to [0,1]^s$ is the *attribute degree function* for undirected edges.
- $adf_A : A \to [0,1]^s$ is the *attribute degree function* for directed edges.
- $aCf_V : V \times V \to [0,1]^t$ is the *contradiction degree function* for vertices.
- aCf_E and aCf_A are the contradiction degree functions for undirected edges and arcs, respectively.

These functions satisfy the following conditions:

(1) *Edge Attribute Degree Constraint*:
- For each undirected edge $e = \{u, v\} \in E$:

$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$

where $\wedge$ denotes the minimum operation taken component-wise.
- For each directed edge $a = (u, v) \in A$:

$$adf_A(a) \leq adf_V(u) \wedge adf_V(v).$$

(2) *Contradiction Function Constraints*:
- The contradiction functions satisfy:

$$aCf_V(u,u) = 0, \quad aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V.$$

- Similar properties hold for aCf_E and aCf_A.

Additionally, plithogenic operations and aggregation functions can be defined to handle complex relationships among the attributes.

Example 3.12. (cf. [14]) The following examples of Plithogenic Mixed Graph are provided.

- When $s = t = 1$, G is called a *Plithogenic Fuzzy Mixed Graph*.
- When $s = 2, t = 1$, G is called a *Plithogenic Intuitionistic Fuzzy Mixed Graph*.
- When $s = 3, t = 1$, G is called a *Plithogenic Neutrosophic Mixed Graph*.
- When $s = 4, t = 1$, G is called a *Plithogenic Turiyam Neutrosophic Mixed Graph*.

We explore the mathematical properties of Uncertain Mixed Graphs. The theorems are presented below.

Theorem 3.13. *A Plithogenic Mixed Graph generalizes the Neutrosophic Mixed Graph, Turiyam Neutrosophic Mixed Graph, and Fuzzy Mixed Graph. Specifically, by choosing appropriate dimensions and components of the attribute degree functions in the Plithogenic Mixed Graph, the Plithogenic Mixed Graph reduces to these specific types of mixed graphs.*

Proof. Consider a Plithogenic Mixed Graph

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

where:

- $adf_V : V \to [0,1]^s$, assigning to each vertex $v \in V$ an s-tuple $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$.
- $adf_E : E \to [0,1]^s$, assigning to each undirected edge $e \in E$ an s-tuple $adf_E(e) = (a_1(e), a_2(e), \ldots, a_s(e))$.
- $adf_A : A \to [0,1]^s$, assigning to each directed edge $a \in A$ an s-tuple $adf_A(a) = (a_1(a), a_2(a), \ldots, a_s(a))$.

To recover specific Mixed Graphs:

(1) *Fuzzy Mixed Graph*:

Set $s = 1$. Let the attribute degree function for vertices and edges be:

$$adf_V(v) = \sigma(v), \quad adf_E(e) = \mu_E(e), \quad adf_A(a) = \mu_A(a),$$

where $\sigma(v)$, $\mu_E(e)$, and $\mu_A(a)$ are the vertex membership function, edge membership function, and arc membership function in the Fuzzy Mixed Graph.

The edge attribute degree constraints in the Plithogenic Mixed Graph become:

$$\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}, \quad \text{for } e = \{u, v\} \in E,$$

$$\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}, \quad \text{for } a = (u, v) \in A,$$

which match the conditions in the Fuzzy Mixed Graph definition.

(2) *Neutrosophic Mixed Graph*:

Set $s = 3$. Let the attribute degree functions be:

$$adf_V(v) = (T_V(v), I_V(v), F_V(v)),$$

$$adf_E(e) = (T_E(e), I_E(e), F_E(e)),$$

$$adf_A(a) = (T_A(a), I_A(a), F_A(a)),$$

where $T_V(v), I_V(v), F_V(v)$ are the truth, indeterminacy, and falsity membership functions of vertices, and similarly for edges and arcs.

The edge attribute degree constraints become:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad I_E(e) \geq \max\{I_V(u), I_V(v)\}, \quad F_E(e) \geq \max\{F_V(u), F_V(v)\},$$

which correspond to the conditions in the Neutrosophic Mixed Graph.

(3) *Turiyam Neutrosophic Mixed Graph*:

Set $s = 4$. Let the attribute degree functions be:

$$adf_V(v) = (t_V(v), iv_V(v), fv_V(v), lv_V(v)),$$

$$adf_E(e) = (t_E(e), iv_E(e), fv_E(e), lv_E(e)),$$

$$adf_A(a) = (t_A(a), iv_A(a), fv_A(a), lv_A(a)),$$

where $t_V(v), iv_V(v), fv_V(v), lv_V(v)$ are the truth, indeterminacy, falsity, and liberal state values for vertices, and similarly for edges and arcs.

The edge attribute degree constraints become:

$$t_E(e) \leq \min\{t_V(u), t_V(v)\},$$

$$iv_E(e) \geq \max\{iv_V(u), iv_V(v)\},$$

$$fv_E(e) \geq \max\{fv_V(u), fv_V(v)\},$$

$$lv_E(e) \geq \max\{lv_V(u), lv_V(v)\},$$

which match the conditions in the Turiyam Neutrosophic Mixed Graph definition.

Thus, by appropriately choosing the dimension s and mapping the components of the attribute degree functions, the Plithogenic Mixed Graph reduces to the specific types of Mixed Graphs: Fuzzy Mixed Graph, Neutrosophic Mixed Graph, and Turiyam Neutrosophic Mixed Graph. □

Theorem 3.14. *In a Plithogenic Mixed Graph G, the contradiction degree function $aCf_V : V \times V \to [0,1]^t$ is symmetric and satisfies $aCf_V(u,u) = 0$ for all $u \in V$, i.e.,*

$$aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V,$$

$$aCf_V(u,u) = 0, \quad \forall u \in V.$$

Proof. By the definition of a Plithogenic Mixed Graph, the contradiction degree function aCf_V must satisfy:

$$aCf_V(u,u) = 0, \quad \forall u \in V,$$

$$aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V.$$

These conditions ensure that the contradiction degree function is reflexive and symmetric, as required. □

Theorem 3.15. *In a Plithogenic Mixed Graph G, for any undirected edge $e = \{u, v\} \in E$, the attribute degree function $adf_E(e)$ satisfies:*

$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$

where $\wedge$ denotes the component-wise minimum of the attribute degree functions of u and v.

Proof. Let $adf_V(u) = (a_1(u), a_2(u), \ldots, a_s(u))$ and $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$. Similarly, let $adf_E(e) = (a_1(e), a_2(e), \ldots, a_s(e))$. By the edge attribute degree constraint in the definition of a Plithogenic Mixed Graph, for each component i ($1 \leq i \leq s$):

$$a_i(e) \leq \min\{a_i(u), a_i(v)\}.$$

This implies:

$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$

where the inequality holds component-wise. Therefore, the attribute degree function of an edge is bounded above by the component-wise minimum of the attribute degree functions of its incident vertices. □

Theorem 3.16. *Any induced subgraph of a Plithogenic Mixed Graph G is also a Plithogenic Mixed Graph.*

Proof. Let

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

be a Plithogenic Mixed Graph. Consider an induced subgraph

$$G' = (V', E', A', adf'_V, adf'_E, adf'_A, aCf'_V, aCf'_E, aCf'_A)$$

, where $V' \subseteq V$, $E' \subseteq E \cap \binom{V'}{2}$, and $A' \subseteq A \cap (V' \times V')$.

Define the attribute degree functions adf'_V, adf'_E, adf'_A and contradiction functions aCf'_V, aCf'_E, aCf'_A of G' by restricting those of G to V', E', A'. Since the original functions satisfy the attribute degree and contradiction function constraints on G, their restrictions satisfy the same constraints on G'. Hence, G' inherits the properties of a Plithogenic Mixed Graph and is itself a valid instance of the same structure. □

Theorem 3.17. *The underlying crisp graph of a Plithogenic Mixed Graph G is a mixed graph $G' = (V, E, A)$.*

Proof. The crisp graph G' is obtained by ignoring the attribute degree functions adf_V, adf_E, adf_A and the contradiction functions aCf_V, aCf_E, aCf_A in G. These functions do not alter the structure of the vertex set V, the undirected edges E, or the directed edges A. Thus, removing them results in a mixed graph $G' = (V, E, A)$, which retains the original structure of G without its additional plithogenic properties. □

Theorem 3.18. *A Plithogenic Mixed Graph generalizes the Plithogenic Graph.*

Proof. A Plithogenic Graph is a special case of a Plithogenic Mixed Graph where the set of directed edges A is empty. By setting $A = \emptyset$ in a Plithogenic Mixed Graph, we recover a Plithogenic Graph. Therefore, the Plithogenic Mixed Graph generalizes the Plithogenic Graph. □

4. Future Directions of This Research

This section outlines the future directions of this research.

We aim to extend the concept of Mixed Hypergraphs [12, 36, 37, 64, 66] to the domain of Uncertain Graphs, delving into their mathematical properties and exploring potential applications. Furthermore, we intend to define Mixed Superhypergraphs (cf. [15, 22, 31, 56, 57]), investigating their characteristics and potential uses. Mixed Superhypergraphs expand the concept of Mixed Graphs to superhypergraphs, incorporating both undirected and directed superhypergraphs.

Additionally, this research aims to integrate its core concepts with those of hypersoft sets [54] and superhypersoft sets [58, 61], facilitating a deeper examination of their theoretical foundations and practical implications.

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

References

[1] Muhammad Akram and Noura Omair Alshehri. Intuitionistic fuzzy cycles and intuitionistic fuzzy trees. *The Scientific World Journal*, 2014(1):305836, 2014.

[2] Muhammad Akram, Hafsa M Malik, Sundas Shahzadi, and Florentin Smarandache. Neutrosophic soft rough graphs with application. *Axioms*, 7(1):14, 2018.

[3] Muhammad Akram, Danish Saleem, and Talal Al-Hawary. Spherical fuzzy graphs with application to decision-making. *Mathematical and Computational Applications*, 25(1):8, 2020.

[4] Ravindra B Bapat, Jerrold W Grossman, and Devadatta M Kulkarni. Generalized matrix tree theorem for mixed graphs. *Linear and Multilinear Algebra*, 46(4):299–312, 1999.

[5] Said Broumi et al. Secure dominance in neutrosophic graphs. *Neutrosophic Sets and Systems*, 56(1):7, 2023.

[6] Said Broumi, R Sundareswaran, M Shanmugapriya, Assia Bakali, and Mohamed Talea. Theory and applications of fermatean neutrosophic graphs. *Neutrosophic sets and systems*, 50:248–286, 2022.

[7] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Interval valued neutrosophic graphs. *Critical Review, XII*, 2016:5–33, 2016.

[8] Yixin Cao, Luciano N Grippo, and Martín D Safe. Forbidden induced subgraphs of normal helly circular-arc graphs: Characterization and detection. *Discrete Applied Mathematics*, 216:67–83, 2017.

[9] Kousik Das, Usman Naseem, Sovan Samanta, Shah Khalid Khan, and Kajal De. Fuzzy mixed graphs and its application to identification of covid19 affected central regions in india. *Journal of Intelligent & Fuzzy Systems*, 40(1):1051–1064, 2021.

[10] Reinhard Diestel. Graph theory 3rd ed. *Graduate texts in mathematics*, 173(33):12, 2005.

[11] Reinhard Diestel. *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.

[12] Zdeněk Dvořák et al. On planar mixed hypergraphs. *the electronic journal of combinatorics*, pages R35–R35, 2001.

[13] Takaaki Fujita. Note for neutrosophic incidence and threshold graph. *SciNexuses*, 1:97–125, 2024.

[14] Takaaki Fujita. A review of the hierarchy of plithogenic, neutrosophic, and fuzzy graphs: Survey and applications. *ResearchGate(Preprint)*, 2024.

[15] Takaaki Fujita. Short note of supertree-width and n-superhypertree-width. *Neutrosophic Sets and Systems*, 77:54–78, 2024.

[16] Takaaki Fujita. Survey of intersection graphs, fuzzy graphs and neutrosophic graphs. *ResearchGate*, July 2024.

[17] Takaaki Fujita. Survey of planar and outerplanar graphs in fuzzy and neutrosophic graphs. *ResearchGate*, July 2024.

[18] Takaaki Fujita. Survey of trees, forests, and paths in fuzzy and neutrosophic graphs. July 2024.

[19] Takaaki Fujita. Advancing uncertain combinatorics through graphization, hyperization, and uncertainization: Fuzzy, neutrosophic, soft, rough, and beyond. 2025.

[20] Takaaki Fujita. A comprehensive discussion on fuzzy hypersoft expert, superhypersoft, and indetermsoft graphs. *Neutrosophic Sets and Systems*, 77:241–263, 2025.

[21] Takaaki Fujita and Florentin Smarandache. Antipodal turiyam neutrosophic graphs. *Neutrosophic Optimization and Intelligent Systems*, 5:1–13, 2024.

[22] Takaaki Fujita and Florentin Smarandache. A concise study of some superhypergraph classes. *Neutrosophic Sets and Systems*, 77:548–593, 2024.

[23] Takaaki Fujita and Florentin Smarandache. A short note for hypersoft rough graphs. *HyperSoft Set Methods in Engineering*, 3:1–25, 2024.

[24] Takaaki Fujita and Florentin Smarandache. Study for general plithogenic soft expert graphs. *Plithogenic Logic and Computation*, 2:107–121, 2024.

[25] GA Ganati, VNS Rao Repalle, MA Ashebo, and M Amini. Turiyam graphs and its applications. *Information Sciences Letters*, 12(6):2423–2434, 2023.

[26] Gamachu Adugna Ganati, VN Srinivasa Rao Repalle, and Mamo Abebe Ashebo. Social network analysis by turiyam graphs. *BMC Research Notes*, 16(1):170, 2023.

[27] Gamachu Adugna Ganati, VN Srinivasa Rao Repalle, and Mamo Abebe Ashebo. Relations in the context of turiyam sets. *BMC Research Notes*, 16(1):49, 2023.

[28] Pabitra Kumar Gouri, Bharti Saxena, Rajesh Kedarnath Navandar, Pranoti Prashant Mane, Ramakant Bhardwaj, Jambi Ratna Raja Kumar, Surendra Kisanrao Waghmare, and Antonios Kalampakas. Competitions on fuzzy mixed graph and its application in countries for health and disaster. *International Journal of Fuzzy Logic and Intelligent Systems*, 24(2):93–104, 2024.

[29] Jonathan L Gross, Jay Yellen, and Mark Anderson. *Graph theory and its applications*. Chapman and Hall/CRC, 2018.

[30] Muhammad Gulistan, Naveed Yaqoob, Zunaira Rashid, Florentin Smarandache, and Hafiz Abdul Wahab. A study on neutrosophic cubic graphs with real life applications in industries. *Symmetry*, 10(6):203, 2018.

[31] Mohammad Hamidi, Florentin Smarandache, and Elham Davneshvar. Spectrum of superhypergraphs via flows. *Journal of Mathematics*, 2022(1):9158912, 2022.

[32] Thempaavai Jayaprakash and Antony Cripsin Sweety Charles Selvaraj. Inverse neutrosophic mixed graphs. *Journal of Fuzzy Extension and Applications*, 5(3):395–415, 2024.

[33] Vasantha Kandasamy, K Ilanthenral, and Florentin Smarandache. *Neutrosophic graphs: a new dimension to graph theory*. Infinite Study, 2015.

[34] WB Vasantha Kandasamy, K Ilanthenral, and Florentin Smarandache. *Plithogenic Graphs*. Infinite Study, 2020.

[35] Anjaly Kishore and MS Sunitha. Chromatic number of fuzzy graphs. *Annals of fuzzy mathematics and informatics*, 7(4):543–551, 2014.

[36] Daniel Král. On feasible sets of mixed hypergraphs. *the electronic journal of combinatorics*, 11(1):R19, 2004.

[37] Daniel Král'. Mixed hypergraphs and other coloring problems. *Discrete mathematics*, 307(7-8):923–938, 2007.

[38] Jianxi Liu and Xueliang Li. Hermitian-adjacency matrices and hermitian energies of mixed graphs. *Linear Algebra and its Applications*, 466:182–207, 2015.

[39] Rahul Mondal and Ganesh Ghorai. Inverse fuzzy mixed planar graphs with application. *International Journal of Applied and Computational Mathematics*, 10(4):131, 2024.

[40] Rahul Mondal and Ganesh Ghorai. Wiener index of inverse fuzzy mixed graphs with application in education system. *Journal of Applied Mathematics and Computing*, pages 1–18, 2024.

[41] Soumitra Poulik and Ganesh Ghorai. New concepts of inverse fuzzy mixed graphs and its application. *Granular Computing*, pages 1–11, 2021.

[42] Thomas Richardson. Markov properties for acyclic directed mixed graphs. *Scandinavian Journal of Statistics*, 30(1):145–157, 2003.

[43] David W Roberts. Analysis of forest succession with fuzzy graph theory. *Ecological Modelling*, 45(4):261–274, 1989.

[44] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77–95. Elsevier, 1975.

[45] Kayvan Sadeghi. Stable mixed graphs. 2013.

[46] Kayvan Sadeghi and Steffen Lauritzen. Markov properties for mixed graphs. 2014.

[47] Prem Kumar Singh et al. Dark data analysis using intuitionistic plithogenic graphs. *International Journal of Neutrosophic Sciences*, 16(2):80–100, 2021.

[48] Prem Kumar Singh, Naveen Surathu, Ghattamaneni Surya Prakash, et al. Turiyam based four way unknown profile characterization on social networks. *Full Length Article*, 10(2):27–7, 2024.

[49] Florentin Smarandache. Ambiguous set (as) is a particular case of the quadripartitioned neutrosophic set (qns). *nidus idearum*, page 16.

[50] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In *Philosophy*, pages 1–141. American Research Press, 1999.

[51] Florentin Smarandache. Neutrosophic physics: More problems, more solutions. 2010.

[52] Florentin Smarandache. n-valued refined neutrosophic logic and its applications to physics. *Infinite study*, 4:143–146, 2013.

[53] Florentin Smarandache. *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics*. Infinite Study, 2016.

[54] Florentin Smarandache. Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets and Systems*, 22:168–170, 2018.

[55] Florentin Smarandache. Plithogeny, plithogenic set, logic, probability, and statistics. *arXiv preprint arXiv:1808.03948*, 2018.

[56] Florentin Smarandache. n-superhypergraph and plithogenic n-superhypergraph. *Nidus Idearum*, 7:107–113, 2019.

[57] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.

[58] Florentin Smarandache. Foundation of the superhypersoft set and the fuzzy extension superhypersoft set: A new vision. *Neutrosophic Systems with Applications*, 11:48–51, 2023.

[59] Florentin Smarandache and Said Broumi. *Neutrosophic graph theory and algorithms*. IGI Global, 2019.

[60] Florentin Smarandache, Said Broumi, Mohamed Talea, Assia Bakali, and Kishore Kumar. Shortest path problem on single valued neutrosophic graphs. In *International Symposium on Networks, Computers and Communications (ISNCC-2017), Marrakech, Morocco, May 16-18, 2017, www. isncc-conf. org.* ISNCC, 2017.

[61] Florentin Smarandache, A. Saranya, A. Kalavathi, and S. Krishnaprakash. Neutrosophic superhypersoft sets. *Neutrosophic Sets and Systems*, 77:41–53, 2024.

[62] S Sudha, Nivetha Martin, and Florentin Smarandache. *Applications of Extended Plithogenic Sets in Plithogenic Sociogram*. Infinite Study, 2023.

[63] Fazeelat Sultana, Muhammad Gulistan, Mumtaz Ali, Naveed Yaqoob, Muhammad Khan, Tabasam Rashid, and Tauseef Ahmed. A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, 14(10):13139–13159, 2023.

[64] Zsolt Tuza and Vitaly Voloshin. Uncolorable mixed hypergraphs. *Discrete Applied Mathematics*, 99(1-3):209–227, 2000.

[65] Vakkas Uluçay and Memet Şahin. Intuitionistic fuzzy soft expert graphs with application. *Uncertainty discourse and applications*, 1(1):1–10, 2024.

[66] VI Voloshin. The mixed hypergraphs. *Computer Science Journal of Moldova*, 1(1):1, 1993.

[67] Yi Wang, Bo-Jun Yuan, Shuang-Dong Li, and Chong-Jun Wang. Mixed graphs with h-rank 3. *Linear Algebra and its Applications*, 524:22–34, 2017.

[68] Tong Wei, Junlin Hou, and Rui Feng. Fuzzy graph neural network for few-shot learning. In *2020 International joint conference on neural networks (IJCNN)*, pages 1–8. IEEE, 2020.

[69] Douglas Brent West et al. *Introduction to graph theory*, volume 2. Prentice hall Upper Saddle River, 2001.

[70] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.

[71] Hua Zhao, Zeshui Xu, Shousheng Liu, and Zhong Wang. Intuitionistic fuzzy mst clustering algorithms. *Computers & Industrial Engineering*, 62(4):1130–1140, 2012.

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