Neutrosophic Sets and Systems, Vol. 78, 2025

University of New Mexico

An Efficient Neutrosophic Weighted Sum Approach with Insights into Engineering Project Management Performance Assessment

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Abstract: Project engineering management performance evaluation is a systematic assessment of various tasks in the project management process to measure performance in areas such as efficiency, quality, schedule, and cost control. Through evaluation, strengths and weaknesses in management can be identified, ensuring the achievement of project objectives and optimizing resource allocation. This evaluation not only helps improve the overall management level of engineering projects but also provides valuable experience and insights for future projects, thereby facilitating smooth implementation and successful delivery. The project engineering management performance evaluation is multiple-attribute decision-making (MADM). Recently, the weighted sum method (WSM) method has been established to cope with MADM issues. The triangular fuzzy neutrosophic sets (TFNSs) are established as a tool for characterizing uncertain data during the project engineering management performance evaluation. In this manuscript, the triangular fuzzy neutrosophic number WSM (TFNN-WSM) method is established to solve the MADM under TFNSs. In the end, a numerical case study for project engineering management performance evaluation is given to validate the proposed method.

Keywords: MADM; TFNSs; WSM approach; Project engineering management performance evaluation

1. Introduction

Project engineering management performance evaluation is a systematic process of analyzing and assessing the management activities and outcomes throughout the entire lifecycle of a project. The core objective is to comprehensively measure the efficiency and effectiveness of project management by considering key dimensions such as cost control, time management, quality assurance, safety management, resource utilization, and team performance[1, 2]. This evaluation provides a scientific basis for improving future project management practices. Specifically, project engineering management performance evaluation includes the following steps: First, establish the evaluation criteria system. Common criteria include time schedule, cost control, quality standards, safety records, resource utilization efficiency, and team collaboration. Second, collect and organize relevant data, ensuring its accuracy and completeness. Next, choose appropriate evaluation methods and tools, such as fuzzy set theory and entropy weight method, to quantify each criterion[3, 4]. Then, integrate the evaluation results of each criterion to form an overall performance assessment report. Finally, based on the evaluation results, propose improvement suggestions and optimization measures to help project managers enhance their management practices in future projects. Through systematic performance evaluation, project managers can not only gain a comprehensive understanding of the actual operation of the project but also identify strengths and weaknesses in management. This enables targeted improvements and optimizations in future projects, thereby increasing the success rate of projects and enhancing the overall management capability and competitiveness of the organization. Therefore, project engineering management performance evaluation is a critical component of project management, with significant theoretical and practical implications.

In modern project management, performance evaluation often employs MADM methods such as TODIM [5]and VIKOR [6]. These methods can comprehensively consider multiple evaluation criteria and conduct multidimensional analysis, providing a thorough performance assessment [7-10]. Additionally, due to the inherent uncertainty and fuzziness in project management data, tools such as Fuzzy Sets and Neutrosophic Sets are widely used. These tools effectively handle uncertain and fuzzy data, enhancing the accuracy and reliability of the evaluation results[11-15]. The project engineering management performance evaluation is MADM. Recently, the Exponential TODIM method [16-20] and the VIKOR method [6, 21-23] have been developed to address multi-attribute decision-making (MADM) problems. To handle uncertain information in project engineering management performance evaluation, TFNSs [24] have been introduced as a tool for characterizing such uncertainties. In this paper, the TFNN-WSM method is proposed to solve MADM issues in the context of TFNSs. Finally, a numerical case study is presented to validate the effectiveness of the proposed approach in evaluating project engineering management performance.

2. Literature review

In 2019, Chen, Wang and Zhang [25] introduced a performance evaluation method for large-scale engineering project management based on the HSE (Health, Safety, and Environment) perspective and Choquet fuzzy integral. Their method integrated the independent HSE evaluation system into a comprehensive performance evaluation system, addressing the complexity of internal management and data processing challenges in large projects. Xu and Zhou [26] conducted a meta-analysis on the relationship between trust and engineering project performance. By synthesizing data from 42 empirical studies, they found a significant positive correlation between trust and project performance. They also explored the varying impacts of different dimensions of trust on project performance, concluding that the presence of mediating variables and multi-dimensional trust measures positively influenced the correlation between trust and performance. This study provided evidence for the importance of trust in project management and offered theoretical support for trust-based management approaches. Moving to 2020, Ji [27] focused on the performance evaluation of water conservancy project management. He combined the BP neural network with fuzzy optimization theory to establish a comprehensive evaluation model for management performance. The model was applied to the GuanYinGe Reservoir water transfer project in Liaoning Province, demonstrating its feasibility and effectiveness in evaluating the performance of water conservancy projects. This study provided a novel method for assessing water project management and offered practical insights into improving performance. Also in 2020, Wang [28] explored performance evaluation in municipal road engineering management. He highlighted the limitations of traditional, rough management techniques and proposed a comprehensive evaluation system using the fuzzy analytic hierarchy process (AHP). This system combined quantitative and qualitative evaluation indicators to assess the performance of municipal road projects, addressing the urgent need for more refined management practices in this area. Yang [29] examined the management and performance evaluation of highway reconstruction projects. She analyzed the complex and multi-faceted nature of highway reconstruction and proposed strategies for optimizing performance management. Zhang [30] discussed performance evaluation in construction project management, with a focus on the petrochemical industry. He analyzed the industry-specific characteristics of petrochemical projects and offered suggestions for improving their management performance. His study underscored the importance of optimizing management and performance evaluation processes to enhance project outcomes, especially in the context of China’s energy and resource development goals. Wang [31] proposed a graded management system for performance evaluation in a corporate engineering project. He classified projects based on their nature, type, scope, and difficulty, and developed a tiered performance evaluation mechanism. This approach optimized internal performance management, improving resource allocation and management efficiency within the organization. Wang [32] focused on the performance indicators for the management of hydroelectric resettlement projects. He designed key performance indicators (KPIs) using methods such as logic frameworks and hierarchical analysis. His study emphasized replacing singular financial metrics with comprehensive performance indicators that account for stakeholder satisfaction, suggesting this approach would drive more sustainable project management practices. Lu [33] developed a comprehensive management model for sponge city engineering based on the earned value method (EVM). He studied management issues in sponge city projects and introduced innovative construction techniques such as modular water storage, proposing solutions for cost and schedule deviations. His research contributed to improving the management quality of sponge city projects, which are crucial for urban ecological sustainability. Zhang [34] addressed the limitations of traditional engineering management methods under the EPC (Engineering, Procurement, and Construction) contracting model. He proposed a collaborative management platform based on BIM (Building

Information Modeling) technology, which significantly improved project coordination and performance in an EPC context, demonstrating high feasibility and effectiveness in real-world applications. Also in 2023, Shen et al. [35] explored the formation mechanisms of cooperation and competition networks in complex project groups. Using social network theory and social exchange theory, they identified three factors influencing the formation of such networks: endogenous structure, cross-network effects, and project attributes. Their study revealed that cooperation networks were denser than competition networks and that cooperation and competition mutually reinforced each other, offering insights into resource allocation strategies for improving project team collaboration. Min and Zhang [36] proposed a comprehensive performance evaluation method for project engineering management based on an improved entropy weight method. They addressed the limitations of cost-benefit analysis in evaluating project performance and developed a more accurate model by refining data standardization and weight calculation processes. Their findings showed that the proposed method outperformed traditional evaluation techniques in terms of accuracy, making it more suitable for comprehensive performance evaluations. In 2024, Guo [37] analyzed the impact of risk-sharing on engineering project management performance. He explored the intrinsic relationship between risk-sharing and performance, highlighting both the positive and negative effects of risk-sharing mechanisms. His study concluded that, while risk-sharing can reduce potential losses, it requires strict control during implementation to ensure the effectiveness of performance management and quality improvement. Finally, in 2024, Wu [38] studied the progress management of intelligent building projects. He applied critical chain technology to develop a dynamic construction schedule management process, ensuring that projects were completed efficiently and ahead of schedule. His findings demonstrated that the performance indices of various construction stages exceeded expectations, indicating that the proposed management techniques effectively improved project efficiency.

3. Concepts and Definitions

Biswas et al. [24] introduced the concept of Triangular Fuzzy Neutrosophic Sets (TFNSs) to enhance the representation and processing of uncertainty and imprecision in decision-making scenarios. Their work focused on the aggregation of TFNS information and its application to multi-attribute decision-making problems, providing a robust framework for handling complex problems and we will present some important definitions that will assist us in this paper, as follows:

Definition 1.

The TFNSs

$$FF=\left\{\left(\theta,FA(\theta),FB(\theta),FC(\theta)\right)\middle|\theta\in\Theta\right\} \tag{1}$$

where $FA(\theta),FB(\theta),FC(\theta)\in[0,1]$ represent truth-membership, indeterminacy-membership and falsity-membership which is established through triangular fuzzy numbers (TFNs).

$$FA(\theta)=\left(FA^{L}(\theta),FA^{M}(\theta),FA^{U}(\theta)\right),0\le FA^{L}(\theta)\le FA^{M}(\theta)\le FA^{U}(\theta)\le 1 \quad (2)$$

$$FB(\theta)=\left(FB^{L}(\theta),FB^{M}(\theta),FB^{U}(\theta)\right),0\le FB^{L}(\theta)\le FB^{M}(\theta)\le FB^{U}(\theta)\le 1 \quad (3)$$

$$FC(\theta)=\left(FC^{L}(\theta),FC^{M}(\theta),FC^{U}(\theta)\right),0\le FC^{L}(\theta)\le FC^{M}(\theta)\le FC^{U}(\theta)\le 1 \quad (4)$$

We let $FF=\left\{\left(FA^{L},FA^{M},FA^{U}\right),\left(FB^{L},FB^{M},FB^{U}\right),\left(FC^{L},FC^{M},FC^{U}\right)\right\}$ be TFNN, $0\le FA^{U}+FB^{U}+FC^{U}\le 3$.

Definition 2.

Let TFNNs $FF_1=\left\{\left(FA_1^{L},FA_1^{M},FA_1^{U}\right),\left(FB_1^{L},FB_1^{M},FB_1^{U}\right),\left(FC_1^{L},FC_1^{M},FC_1^{U}\right)\right\}$, $FF_2=\left\{\left(FA_2^{L},FA_2^{M},FA_2^{U}\right),\left(FB_2^{L},FB_2^{M},FB_2^{U}\right),\left(FC_2^{L},FC_2^{M},FC_2^{U}\right)\right\}$ and $FF=\left\{\left(FA^{L},FA^{M},FA^{U}\right),\left(FB^{L},FB^{M},FB^{U}\right),\left(FC^{L},FC^{M},FC^{U}\right)\right\}$ be TFNNs, and the operation laws are :

$$(1)\ FF_1\oplus FF_2=\begin{Bmatrix}\left(FA_1^{L}+FA_2^{L}\text{-}FA_1^{L}FA_2^{L},FA_1^{M}+FA_2^{M}\text{-}FA_1^{M}FA_2^{M},FA_1^{U}+FA_2^{U}\text{-}FA_1^{U}FA_2^{U}\right),\\ \left(FB_1^{L}FB_2^{L},FB_1^{M}FB_2^{M},FB_1^{U}FB_2^{U}\right),\left(FC_1^{L}FC_2^{L},FC_1^{M}FC_2^{M},FC_1^{U}FC_2^{U}\right)\end{Bmatrix};$$

$$(2)\ FF_1\otimes FF_2=\begin{Bmatrix}\left(FA_1^{L}FA_2^{L},FA_1^{M}FA_2^{M},FA_1^{U}FA_2^{U}\right),\\ \left(FB_1^{L}+FB_2^{L}\text{-}FB_1^{L}FB_2^{L},FB_1^{M}+FB_2^{M}\text{-}FB_1^{M}FB_2^{M},FB_1^{U}+FB_2^{U}\text{-}FB_1^{U}FB_2^{U}\right),\\ \left(FC_1^{L}+FC_2^{L}\text{-}FC_1^{L}FC_2^{L},FC_1^{M}+FC_2^{M}\text{-}FC_1^{M}FC_2^{M},FC_1^{U}+FC_2^{U}\text{-}FC_1^{U}FC_2^{U}\right)\end{Bmatrix};$$

$$(3)\ \lambda FF=\begin{Bmatrix}\left(1-\left(1-FA^{L}\right)^{\lambda},1-\left(1-F^{M}\right)^{\lambda},1-\left(1-FA^{U}\right)^{\lambda}\right),\\ \left(\left(FB^{L}\right)^{\lambda},\left(FB^{M}\right)^{\lambda},\left(FB^{U}\right)^{\lambda}\right),\left(\left(FC^{L}\right)^{\lambda},\left(FC^{M}\right)^{\lambda},\left(FC^{U}\right)^{\lambda}\right)\end{Bmatrix},\lambda>0;$$

$$(4)\ FF^{\lambda}=\begin{Bmatrix}\left(\left(FA^{L}\right)^{\lambda},\left(FA^{M}\right)^{\lambda},\left(FA^{U}\right)^{\lambda}\right),\left(1-\left(1-FB^{L}\right)^{\lambda},1-\left(1-FB^{M}\right)^{\lambda},1-\left(1-FB^{U}\right)^{\lambda}\right),\\ \left(1-\left(1-FC^{L}\right)^{\lambda},1-\left(1-FC^{M}\right)^{\lambda},1-\left(1-FC^{U}\right)^{\lambda}\right)\end{Bmatrix},\lambda>0.$$

The operational laws have serval properties.

$$(1)\ FF_1\oplus FF_2=FF_2\oplus FF_1,FF_1\otimes FF_2=FF_2\otimes FF_1,\left(\left(FF_1\right)^{\lambda_1}\right)^{\lambda_2}=\left(FF_1\right)^{\lambda_1\lambda_2}; \quad (5)$$

$$(2)\ \lambda\left(FF_1\oplus FF_2\right)=\lambda FF_1\oplus\lambda FF_2,\left(FF_1\otimes FF_2\right)^{\lambda}=\left(FF_1\right)^{\lambda}\otimes\left(FF_2\right)^{\lambda}; \quad (6)$$

$$(3)\ \lambda_1 FF_1\oplus\lambda_2 FF_1=\left(\lambda_1+\lambda_2\right)FF_1,\left(FF_1\right)^{\lambda_1}\otimes\left(FF_1\right)^{\lambda_2}=\left(FF_1\right)^{\left(\lambda_1+\lambda_2\right)}. \quad (7)$$

Definition 3.

Let $FF=\left\{\left(FA^{L},FA^{M},FA^{U}\right),\left(FB^{L},FB^{M},FB^{U}\right),\left(FC^{L},FC^{M},FC^{U}\right)\right\}$ be TFNN, the score and accuracy functions are:

$$SF\left(FF\right)=\frac{1}{12}\begin{bmatrix}8+\left(FA^{L}+2FA^{M}+FA^{U}\right)\\ -\left(FB^{L}+2FB^{M}+FB^{U}\right)\\ -\left(FC^{L}+2FC^{M}+FC^{U}\right)\end{bmatrix},SF\left(FF\right)\in\left[0,1\right] \tag{8}$$

$$AF\left(FF\right)=\frac{1}{4}\begin{bmatrix}\left(FA^{L}+2FA^{M}+FA^{U}\right)\\ -\left(FB^{L}+2FB^{M}+FB^{U}\right)\end{bmatrix},AF\left(FF\right)\in\left[-1,1\right] \tag{9}$$

For the TFNNs FF_1 and FF_2, from Definition 3, we have:

(1) $if\ \ SF\left(FF_1\right)\prec SF\left(FF_2\right), then\ FF_1\prec FF_2;$

(2) $if\ \ SF\left(FF_1\right)=SF\left(FF_2\right), AF\left(FF_1\right)\prec AF\left(FF_2\right), then\ FF_1\prec FF_2;$

(3) $if\ \ SF\left(FF_1\right)=SF\left(FF\right), AF\left(FF_1\right)=AF\left(FF_2\right), then\ FF_1=FF_2.$

Definition 4

Let $FF_1=\left\{\left(FA_1^{L},FA_1^{M},FA_1^{U}\right),\left(FB_1^{L},FB_1^{M},FB_1^{U}\right),\left(FC_1^{L},FC_1^{M},FC_1^{U}\right)\right\}$, $FF_2=\left\{\left(FA_2^{L},FA_2^{M},FA_2^{U}\right),\left(FB_2^{L},FB_2^{M},FB_2^{U}\right),\left(FC_2^{L},FC_2^{M},FC_2^{U}\right)\right\}$ be TFNNs, the TFNN Hamming distance (TFNNHD) is:

$$TFNNHD\left(FF_1,FF_2\right)=\frac{1}{9}\begin{pmatrix}\left|FA_1^{L}-FA_2^{L}\right|+\left|FA_1^{M}-FA_2^{M}\right|+\left|FA_1^{U}-FA_2^{U}\right|\\ +\left|FB_1^{L}-FB_2^{L}\right|+\left|FB_1^{M}-FB_2^{M}\right|+\left|FB_1^{U}-FB_2^{U}\right|\\ +\left|FC_1^{L}-FC_2^{L}\right|+\left|FC_1^{M}-FC_2^{M}\right|+\left|FC_1^{U}-FC_2^{U}\right|\end{pmatrix} \tag{10}$$

4. TFNN-WSM method

Weighted sum model (WSM) is a MCDM method used for evaluating a number of alternatives in the decision making. The following steps of the WSM method under the TFNN:

1. Develop a decision and weight matrix.

We invited experts to develop a decision and weight matrix between criteria and alternatives such as:

$$Y=\begin{bmatrix}y_{11} & \cdots & y_{1n}\\ \vdots & \ddots & \vdots\\ y_{m1} & \cdots & y_{mn}\end{bmatrix} \tag{11}$$

The following present the weight vector

$$w=[w_1,\dots,w_n], where\ \sum_{j=1}^{n} w=1 \tag{12}$$

The criteria weights are computed using the average method.

2. Apply the score function.

The score function is applied to obtain crisp values.

3. Combined the decision matrix.

The decision matrix is combined into a single matrix.

4. Make the choice matrix normal

$$u_{ij} = \begin{cases} \frac{y_{ij}}{\max y_{ij}} \, for\ positive\ criteria \\ \frac{\min y_{ij}}{y_{ij}} \, for\ negative\ criteria \end{cases} \tag{13}$$

5. Normalized weighted decision matrix.

$$T_{ij} = w_j u_{ij} \tag{14}$$

6. Rank of alternatives.

$$L_i = \sum_{j=1}^{n} T_{ij} \tag{15}$$

The alternatives are ranked from the highest to lowest of L.

Table 1. Importance Weights of Criteria for Decision Making

Criteria	**Weights**
C_1: Quality Assurance and Control	0.052549
C_2: Training and Development of Team Members	0.048093
C_3: Resource Allocation Efficiency	0.052589
C_4: Documentation and Reporting Quality	0.048575
C_5: Stakeholder Communication and Collaboration	0.051224
C_6: Adaptability to Changes and Challenges	0.048936
C_7: Compliance with Safety Standards	0.050703
C_8: Technology Integration and Innovation	0.049739
C_9: Post-Project Evaluation and Feedback Implementation	0.048214
C_{10}: Client Satisfaction with Deliverables	0.048334
C_{11}: Team Leadership and Coordination	0.051224
C_{12}: Budget Management and Cost Control	0.052589
C_{13}: Project Planning and Scheduling	0.05283
C_{14}: Effective Conflict Resolution	0.04729
C_{15}: Use of Engineering Standards and Best Practices	0.050703
C_{16}: Environmental Sustainability Practices	0.048695
C_{17}: Overall Project Outcomes and Impact	0.045484
C_{18}: Adherence to Project Timelines	0.051666
C_{19}: Risk Identification and Mitigation	0.051345
C_{20}: Supply Chain and Procurement Management	0.049217

5. A Case Study with Comparative Evaluation

Project engineering management performance evaluation is a critical step in systematically and scientifically assessing both the project management process and its outcomes. Its aim is to analyze the execution of the project, identify successful experiences and potential issues in management, and provide a basis for improving future projects. Performance evaluation not only focuses on the final results of the project but also examines various stages throughout the entire project management process. Through this evaluation, the project management team can gain a comprehensive understanding of the overall operation, including the effectiveness of each project phase, the validity of management decisions, and the efficiency of resource allocation and utilization. The performance evaluation process typically involves multiple stakeholders, such as the project manager, team members, suppliers, and clients, whose feedback and opinions play a crucial role in improving project management. By collecting and analyzing this information, managers can identify which management strategies were effective and which areas require improvement. This not only helps optimize the current project but also provides valuable insights and lessons for the management of future projects. In modern project management, performance evaluation is seen as a continuous improvement process. After project completion, through systematic review and analysis, the project team can summarize lessons learned and develop more reasonable and effective plans for future project management. Additionally, performance evaluation provides foundational data to help organizations achieve greater efficiency and better resource utilization throughout the project lifecycle. In summary, project engineering management performance evaluation is not only a key step in ensuring project success but also an essential tool for continuously enhancing an organization's project management capabilities. The evaluation of project engineering management performance is a MADM problem. In this study, six potential engineering projects are assessed based on 20 distinct attributes, as outlined in Table 1. The evaluation process follows these systematic steps:

- Developing the Decision Matrix: Using Eq. (11), the decision matrix was constructed by applying the Triangular Fuzzy Neutrosophic Numbers, with the resulting matrix detailed in Table A1.
- Converting to Crisp Values: A score function was applied to the TFNN elements to obtain their corresponding crisp values, simplifying further analysis.
- Aggregating Criteria Weights: The decision matrix was consolidated into a single unified matrix. Subsequently, the criteria's weights were computed, as presented in Table 1.
- Normalization: Eq. (13) was employed to normalize the decision matrix, ensuring all attribute values were scaled appropriately for comparison.
- Weighted Normalized Matrix: Using Eq. (14), a weighted normalized decision matrix was derived by incorporating the computed criteria weights.
- Ranking Alternatives: Finally, Eq. (15) was applied to rank the six engineering project alternatives based on their performance, with the rankings visually represented in Figure 1.

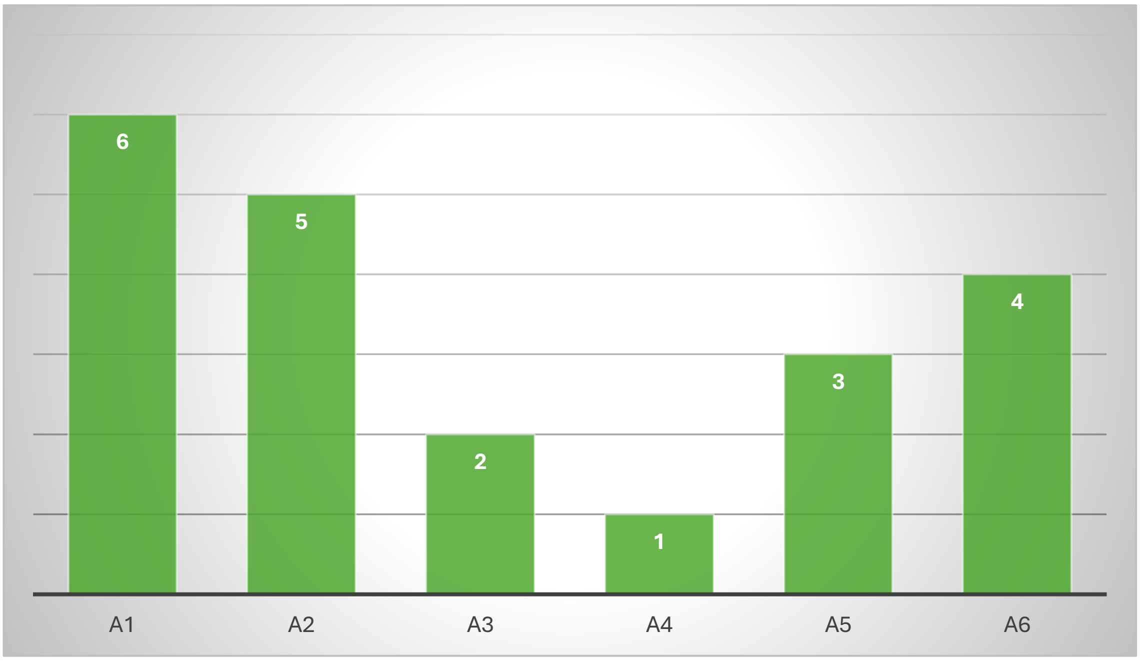

Figure 1. The rank of the alternatives.

5.1. Comparative Analysis of Alternatives and Methods

The comparative analysis of the proposed method (TFNN-WSM) with TFNN-VIKOR [40], TFNN-MABAC [39], and TFNN-EDAS [41] reveal critical insights into their performance in multi-attribute decision-making problems, specifically in project engineering management performance evaluation. The results, depicted in Figure 2, demonstrate that all models consistently identify the same optimal engineering project and rank the six projects similarly. This consistency confirms the reliability and effectiveness of the TFNN-WSM approach.

Limitations of Existing Methods:

TFNN-VIKOR and TFNN-MABAC Approaches:

These methods are based on traditional triangular fuzzy number models that address uncertainty only partially.

They fail to fully account for the three essential dimensions of fuzzy logic: truth-membership, indeterminacy-membership, and falsity-membership.

Consequently, they struggle with decision problems involving conflicting preferences and high levels of uncertainty.

These approaches inadequately capture the complexities of fuzzy logic, limiting their robustness when handling ambiguous or uncertain information.

TFNN-EDAS Approach:

The TFNN-EDAS method relies on evaluating the distance from an average solution, offering a computationally straightforward process.

However, it is primarily suited to linear decision problems and performs poorly with complex, non-linear preferences.

This method lacks flexibility in addressing uncertainty and risk, reducing its ability to manage the variable and multifaceted risks encountered in engineering management scenarios.

Advantages of the Proposed TFNN-WSM Method:

Enhanced Capability to Handle Uncertainty:

By incorporating the complete spectrum of fuzzy dimensions (truth, indeterminacy, and falsity), the TFNN-WSM method effectively manages uncertainty and ambiguity.

Ability to Address Non-Linear Preferences and Risk Attitudes:

The method supports non-linear decision-making, making it suitable for complex engineering scenarios with diverse risk profiles and stakeholder preferences.

Comprehensive Consideration of Objectives and Trade-Offs:

The TFNN-WSM approach ensures a holistic evaluation by balancing multiple objectives and effectively managing trade-offs among competing criteria.

This analysis highlights the superior performance of the TFNN-WSM method in addressing complex and uncertain engineering management performance evaluations, offering a robust alternative to traditional MADM approaches.

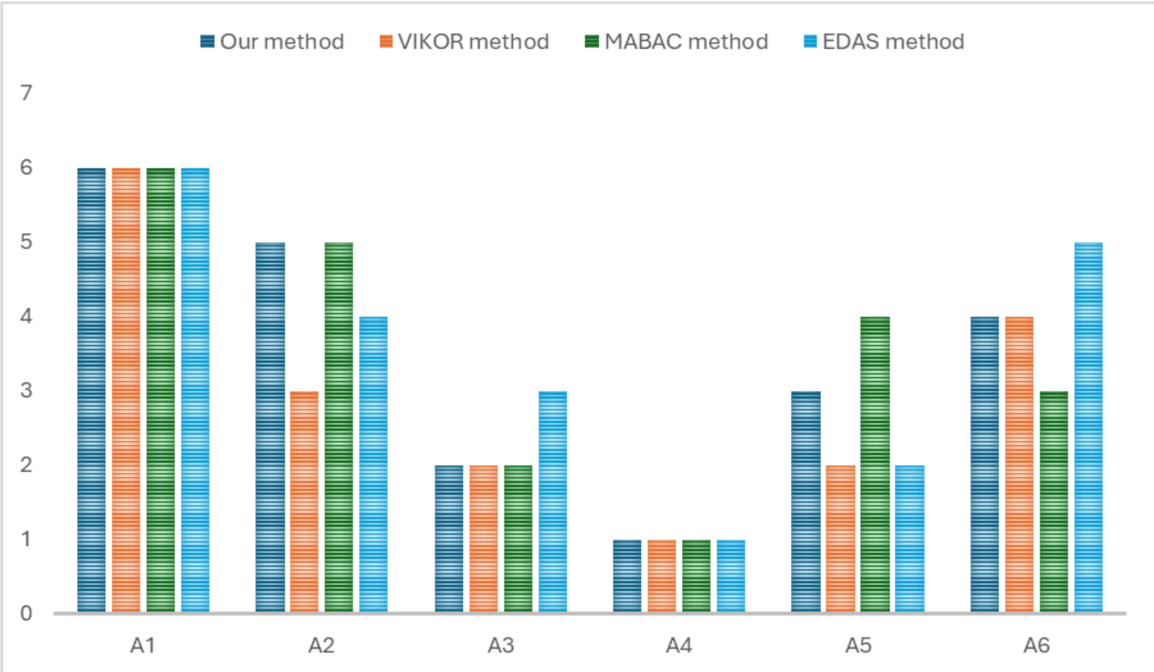

Figure 2. Alternative Ranks in Comparative Perspective

5.2. Sensitivity Analysis

Sensitivity analysis was conducted by modifying the criteria weights across ten different cases, as illustrated in Figure 3. The primary objective of this analysis was to validate the stability of the ranking results under varying weight conditions.

The proposed model was applied to each case with the adjusted criteria weights. The results, presented in Figure 4, indicate that Alternative 1 consistently ranks as the best option, while Alternative 4 consistently ranks as the worst, regardless of the weight changes.

These findings confirm that the ranking of alternatives remains stable, demonstrating the robustness and reliability of the proposed model in addressing changes in criteria weights. This stability is a critical feature for decision-makers, ensuring confidence in the model's applicability to real-world scenarios.

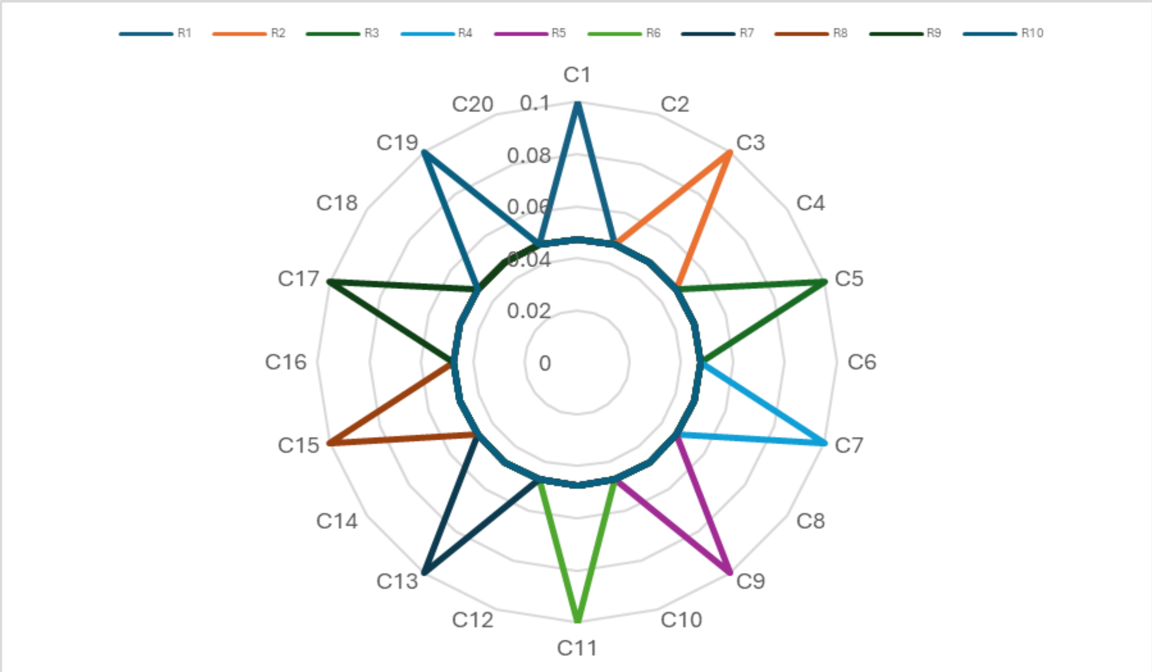

Figure 3. The criteria weights under different cases.

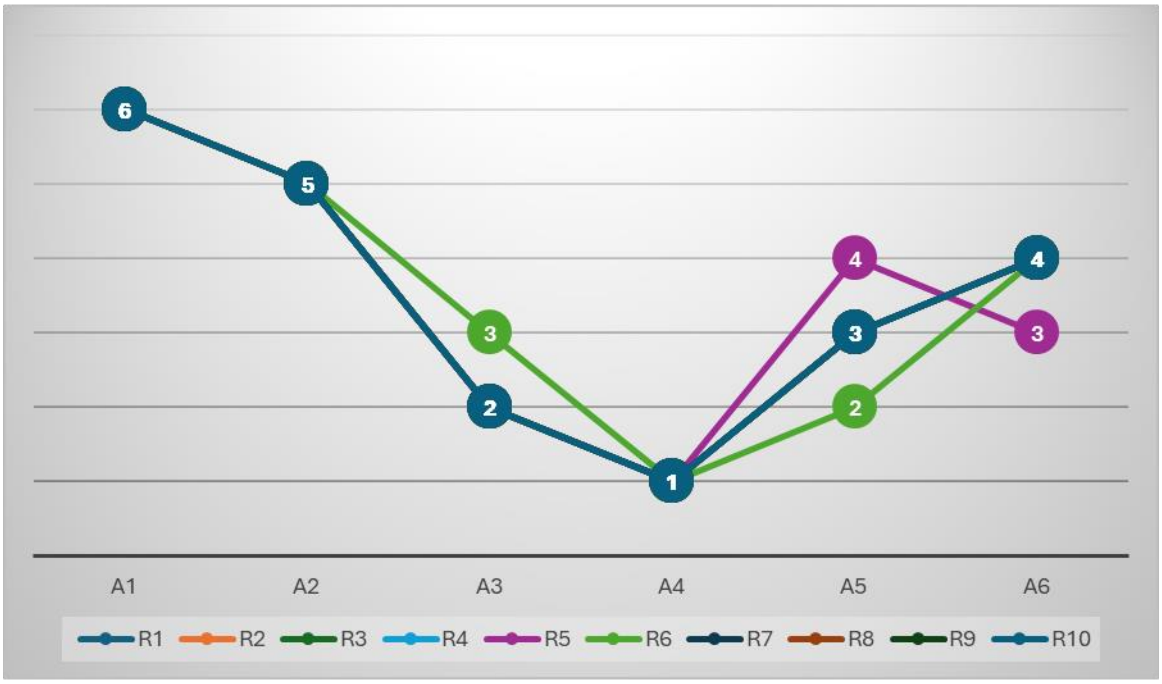

Figure 4. Variations in Alternative Rankings

6. Conclusions

Project engineering management performance evaluation is a crucial process for systematically assessing both the project management process and its outcomes, with the goal of ensuring the achievement of project objectives and improving management efficiency. By analyzing the execution of the project, performance evaluation helps identify successful practices as well as potential issues, thus providing a basis for improvements in future projects. This evaluation focuses not only on the final outcome but also on various stages throughout the project lifecycle. Key factors such as cost, schedule, quality, and risk are typically involved, helping the project team optimize resource utilization and management decisions. Through continuous review and feedback, performance evaluation enhances project execution efficiency and provides valuable insights for the long-term development of the organization, ensuring the smooth implementation and successful delivery of future projects. The project engineering management performance evaluation is an MADM problem. TFNSs are used to characterize uncertain data during the evaluation process. In this study, a TFNN-WSM model is proposed to solve MADM under TFNSs. Finally, a numerical case study is provided to validate the effectiveness of the proposed method.

7. Study Limitations and Suggestions for Future Work

Despite the strengths of the TFNN-WSM method in addressing multi-attribute decision-making problems under uncertainty, there are still several limitations that need to be acknowledged: (1) High Computational Complexity (2) Sensitivity to Parameter Settings (3) Limited Empirical Validation

To address the limitations mentioned above, there are several promising avenues for future research that could enhance the TFNN- WSM method and broaden its applicability: (1) Improving Computational Efficiency (2) Reducing Sensitivity to Parameters (3) Extensive Empirical Validation Across Multiple Domains: Lastly, future research should aim to conduct broader empirical validation of the method across multiple domains. While the current case study demonstrates the potential of the TFNN-WSM approach, testing the method on a wider range of projects in various industries (e.g., construction, transportation, energy) would provide more comprehensive insights into its effectiveness and adaptability. Additionally, comparative studies with other advanced multi-attribute decision-making techniques could help identify areas for improvement, enhancing the universality and applicability of the method. Such validation would strengthen the credibility of the approach and demonstrate its value in different real-world scenarios.

Acknowledgment

The work was supported by the Research on the reconstruction of the ecosystem of high-level professional groups in vocational colleges under the three-integration framework (No. EJA230465).

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Appendix

Table A1. Analysis of Three Decision Matrices

	A_1	A_2	A_3	A_4	A_5	A_6
C_1	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_2	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_3	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_4	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_5	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_6	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
C_7	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_8	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)

C_9	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{10}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_{11}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_{12}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
C_{13}	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_{14}	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{15}	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{16}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{17}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{18}	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_{19}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_{20}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
	A_1	A_2	A_3	A_4	A_5	A_6
C_1	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_2	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_3	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_4	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_5	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_6	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_7	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_8	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_9	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{10}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{11}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{12}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
C_{13}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_{14}	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{15}	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{16}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{17}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{18}	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_{19}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{20}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
	A_1	A_2	A_3	A_4	A_5	A_6
C_1	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_2	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_3	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_4	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_5	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_6	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
C_7	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_8	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)

C_9	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{10}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_{11}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{12}	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)
C_{13}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)
C_{14}	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{15}	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)
C_{16}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{17}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)
C_{18}	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)
C_{19}	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)
C_{20}	(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)	(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)	(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)	(0.4,0.5,0.8),(0.5,0.7,0.9),(0.2,0.3,0.4)

Received: Sep 5, 2024. Accepted: Dec 3, 2024