REGIME Framework for Performance Ability Evaluation of Excellent Football Players with Probabilistic Simplified Neutrosophic Sets

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Abstract
The evaluation of an excellent football player's performance abilities involves assessing their overall capabilities based on in-game performance. Key evaluation dimensions include technical skills (such as passing, shooting, dribbling, etc.), tactical awareness, physical fitness, mental resilience, and teamwork. Through data analysis, video review, and coach feedback, the player's impact and contribution to the game are comprehensively evaluated, helping to determine their actual level in matches and provide guidance for future development. The performance ability evaluation of excellent football players is multiple-attribute decision-making (MADM). The REGIME proves to be an effective method for tackling MADM challenges. The Probabilistic Simplified Neutrosophic Sets (PSNSs) is particularly adept at handling the uncertainties inherent in the performance ability evaluation of excellent football players. In this research, the average technique is introduced to determine the weights of various attributes, and the Probabilistic Simplified Neutrosophic Number REGIME (PSNN-REGIME) method is advanced for MADM applications. To validate the effectiveness of the PSNN-REGIME method, a numerical case study involving the performance ability evaluation of excellent football players is conducted, complete with comparative analysis. This approach not only underscores the method's applicability but also enhances the decision-making process in evaluating and improving the performance ability evaluation of excellent football players.

Keywords: Multiple-attributes decision-making (MADM); probabilistic simplified neutrosophic sets (PSNSs); REGIME technique; performance ability evaluation of excellent football players

1. Overview and Background

The evaluation of an excellent football player's performance abilities is a systematic process that analyzes and assesses a player's overall performance in matches. First, in terms of technical skills, the evaluation includes basic techniques such as passing, shooting, dribbling, and ball control, examining the precision and practicality of these actions during the game. Secondly, tactical awareness is crucial, as players need to have a good understanding of the game, make quick decisions, and adjust their positioning and actions based on the match situation. Physical fitness, including speed, endurance, and explosiveness, is another important indicator, directly influencing a player's performance and consistency throughout the game. Additionally, mental resilience is equally important. Top players need to remain calm, confident, and able to handle pressure in critical moments. Lastly, teamwork is an indispensable part of modern football. Players must not only perform individually but also have the ability to work seamlessly with teammates to achieve collective goals. By evaluating these aspects comprehensively, coaches and evaluators can gain a deeper understanding of a player's abilities, identify their strengths and weaknesses, and provide targeted advice and guidance for their training and development. Since 2000, research on the match running ability of Chinese football players, especially female players, has gradually increased, focusing on overall ability, positional physical characteristics, high-intensity running, and its relationship with technical performance. In chronological order, Sun et al. [1] investigated the match performance of elite Chinese female football players, highlighting differences between domestic and international teams. They found that Chinese players spent 40.9%-54.4% of match time running, with a run-walk-stop ratio of 1:1.7:0.28 and an average activity distance of 5128 meters. This study provided a foundation for maintaining China's international competitiveness. Later, Sun et al. [2] further analyzed the performances of 13 Chinese and 8 foreign women's teams, concluding that wing attacks had the highest success rate and passing combinations were most effective. These findings supported tactical and technical training for Chinese women's football teams. In 2009, Li [3] explored the physical characteristics of players in different positions, proposing the concept of "positional fitness." She outlined the specific needs of each position: forwards required speed and agility, midfielders needed speed endurance, and defenders relied on speed and strength. Additionally, she emphasized that running ability was the core of football-specific fitness and highly correlated with match outcomes. In the same year, Yu, Liu and Li [4] studied male football players' running abilities, revealing significant differences across matches and positions, providing a reference for women's football research. In the 2010s, Bu [5] focused on the relationship between running ability and match performance in high-level women's football. He observed a decline in running ability in the second half, particularly in high-intensity activities above the anaerobic threshold. This decline was accompanied by reduced technical abilities, such as ball control and short-pass success rates, highlighting the critical role of high-intensity activity capacity in match performance. Finally, Xu [6] compared the running abilities of Chinese and international female football players. She found that while Chinese players had lower high-intensity running abilities, their total running distances were comparable. Xu developed a standard for evaluating running abilities, providing important guidance for the scientific training of Chinese women's football teams. In conclusion, these studies underscore the importance of running ability in matches. Through longitudinal studies, positional analyses, and international comparisons, they offer valuable insights for improving the training and performance of Chinese football players

MADM is a decision-making process that prioritizes solutions in alignment with existing information through specific methodologies [7-10]. Its theories and models are extensively utilized across various domains such as venture-capital decision-making, project evaluation, and industrial sector development evaluation [11-14]. In recent decades, MADM has seen broad applications in different fields [15-17]. The performance ability evaluation of excellent football players constitutes a MADM. Recently, the TOPSIS approach [18-21] and Cross-Entropy [22] has been employed to enhance MADM. PSNSs [23] are employed as a technique to characterize uncertain data during the performance ability evaluation of excellent football players.

Hinloopen, Nijkamp, and Rietveld first proposed the REGIME technique in 1983. It is a multiple attribute qualitative approach that uses the REGIME matrix to answer the problem and ranks the options. The decision maker's introduction of the weight of attributes in the final ranking is significant and can affect the outcomes. Decision-making challenges are ranked using this method. The following characteristics of the REGIME approach, which is applied in many fields: It's among the compensatory techniques. The qualities don't depend on one another; The qualitative attributes do not have to be transformed into quantitative attributes [24,25].

In this study, the average approach is proposed to obtain the weights under PSNSs, and the PSNN-REGIME model is introduced to address MADM with PSNSs. Finally, a numerical study on the performance ability evaluation of excellent football players is conducted to validate the PSNN- REGIME approach.

1.1 Relevance and Objectives of the Study

The increasing competitiveness of professional football, coupled with the demand for scientific training methods, underscores the importance of systematically evaluating players' match performance. Elite football players require comprehensive assessments that account for both physical and technical abilities to ensure peak performance during matches. This study's primary objective is to develop a robust and systematic evaluation method for assessing the match performance of elite football players. By introducing the PSNN- REGIME model integrated with the average technique, this research aims to enhance decision-making processes in player evaluation. The relevance lies in providing a scientific framework for improving training effectiveness and match strategies, particularly in optimizing high-intensity performance and tactical execution under competitive conditions.

The overall structure is as follows: Section 2 introduces the PSNSs. Section 3 presents the PSNN-REGIME approach for MADM. Section 4 provides a numerical example for performance ability evaluation of excellent football players, demonstrating the effectiveness of the PSNN- REGIME approach through a comparative analysis. Finally, Section 5 concludes the study.

2. Preliminaries

In recent years, MADM techniques have gained significant attention due to their versatility and wide-ranging applications in fields such as engineering, economics, and resource management. Among these approaches, fuzzy set theory has emerged as a powerful tool for addressing uncertainty and vagueness in information. Building on this foundation, Wang et al. [26] introduced the concept of Single-Valued Neutrosophic Sets (SVNSs), representing a groundbreaking advancement in the realm of neutrosophic logic and decision analysis.

The SVNSs model stands out for its ability to capture the inherent vagueness in real-world problems, offering a robust mathematical structure for quantitative and qualitative analysis. It has been effectively applied in various domains to evaluate alternatives across multiple criteria, providing a clear and comprehensive approach to decision-making. By addressing the limitations of traditional fuzzy sets, SVNSs enable a more flexible and realistic representation of uncertain information.

Definition 1 **[28]**. SVNSs were introduced to extend fuzzy and intuitionistic fuzzy sets by incorporating truth, indeterminacy, and falsity memberships, providing a more comprehensive framework for handling uncertainty and vagueness in decision-making as

$$UU = \left\{\left(\theta, UT(\theta), UI(\theta), UF(\theta)\right) \middle| \theta \in \Theta\right\} \quad (1)$$

where $UT(\theta), UI(\theta), UF(\theta)$ is membership, indeterminacy-membership and falsity-membership, $UT(\theta), UI(\theta), UF(\theta) \in [0,1]$, $0 \le UT(\theta) + UI(\theta) + UF(\theta) \le 3$.

Definition 2 **[23]**. The Probabilistic Simplified Neutrosophic Sets (PSNSs) were introduced as a framework to integrate probabilistic concepts into the simplified neutrosophic set theory. PSNSs enhance the modeling of uncertainty by incorporating probability distributions for truth, indeterminacy, and falsity memberships, enabling more robust and precise decision-making in multi-criteria contexts.

$$PU = \left\{\left(\theta, UT(\theta)\left(PUT(\theta)\right), UI(\theta)\left(PUI(\theta)\right), UF(\theta)\left(PUF(\theta)\right)\right) \middle| \theta \in \Theta\right\} \quad (2)$$

where $UT(\theta), UI(\theta), UF(\theta)$ is truth-membership, indeterminacy-membership and falsity-membership, $UT(\theta), UI(\theta), UF(\theta) \in [0,1]$, $0 \le UT(\theta) + UI(\theta) + UF(\theta) \le 3$, $0 \le PUT(\theta), PUI(\theta), PUF(\theta) \le 1$, the $PUT(\theta), PUI(\theta), PUF(\theta)$ is possibility values of $UT(\theta), UI(\theta), UF(\theta)$. The probabilistic simplified neutrosophic number (PSNN) is listed as $PU = \left(UT(PUT), UI(PUI), UF(PUF)\right)$.

Definition 3 **[23].**

Let $PU_1 = \left(UT_1(PUT_1), UI_1(PUI_1), UF_1(PUF_1)\right)$, $PU_2 = \left(UT_2(PUT_2), UI_2(PUI_2), UF_2(PUF_2)\right)$, the basic operations are:

(1) $PU_1 \oplus PU_2 = \begin{pmatrix} UT_1 + UT_2 - UT_1 \cdot UT_2 \left(2! \frac{PUT_1 \cdot PUT_2}{PUT_1 + PUT_2} \right), \\ UI_1 \cdot UI_2 \left(2! \frac{PUI_1 \cdot PUI_2}{PUI_1 + PUI_2} \right), UF_1 \cdot UF_2 \left(2! \frac{PUF_1 \cdot PUF_2}{PUF_1 + PUF_2} \right) \end{pmatrix};$

(2) $PU_1 \otimes PU_2 = \begin{pmatrix} UT_1 \cdot UT_2 \left(2! \frac{PUT_1 \cdot PUT_2}{PUT_1 + PUT_2} \right), \\ UI_1 + UI_2 - UI_1 \cdot UI_2 \left(2! \frac{PUI_1 \cdot PUI_2}{PUI_1 + PUI_2} \right), \\ UF_1 + UF_2 - UF_1 \cdot UF_2 \left(2! \frac{PUF_1 \cdot PUF_2}{PUF_1 + PUF_2} \right) \end{pmatrix};$

(3) $\lambda PU = \left(1 - (1 - UT)^{\lambda} (PUT), (UI)^{\lambda} (PUI), (UF)^{\lambda} (PUF)\right), \lambda > 0;$

(4) $(PU)^{\lambda} = \left((UT)^{\lambda} (PUT), 1 - (1 - UI)^{\lambda} (PUI), 1 - (1 - UF)^{\lambda} (PUF)\right), \lambda > 0.$

Definition 4 **[27].**

The score function of the PSNN can be computed as:

$$S(a) = \frac{UT\ PUT + (1 - UI\ PUI) + (1 - UF\ PUF)}{3} \quad (3)$$

3. PSNN- REGIME Technique for MADM problem

This section shows the steps of the proposed method as:

Step 1. Build the decision matrix

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix} \quad (4)$$

Step 2. Compute criteria weights.

Step 3. Compute the superiority index.

After computing the criteria weights, the set of criteria in which alternative A_f is at least as good as the alternative A_l, displayed by E_{fl}.

Step 4. Compute the superiority identifier.

$$E_{fl} = \sum_{j \in E_{fl}} w_j \,; j = 1, \dots, n \quad (5)$$

Step 5. Build the impacts matrix.

The impact matrix is derived from ordering the alternatives based on the criteria that order the alternative from the decision matrix information.

Step 6. Build the REGIME matrix

The REGIME matrix is built from a pairwise comparison of alternatives.

$If\ two\ alternatives\ of\ A_1, A_2 \in A\ are\ considered, the\ status\ of$

$A_1, A_2 alternatives\ should\ be\ compared\ to\ each\ other\ in\ all\ criteria.$

$$E_{fl_j} = \begin{cases} -1 & if\ r_{fj} < r_{lj} \\ 0 & if\ r_{fj} = r_{lj} \\ +1 & if\ r_{fj} > r_{lj} \end{cases} \tag{6}$$

Step 7. Compute the guide index.

$$E_{fl} = \sum_{j=1}^{n} E_{fl_j} w_j \tag{7}$$

Step 8. Rank the alternatives.

4. Numerical Study and Comparative Analysis: Insights and Results

This section presents a detailed numerical study to demonstrate the practical application and effectiveness of the proposed PSNN- REGIME approach in solving MADM problems.

4.1 Numerical Study

The evaluation of an excellent football player's performance abilities in a match is a comprehensive and multi-faceted process that goes beyond mere statistics and numbers. It involves a detailed analysis of the player's overall impact on the game, their consistency, and their ability to influence the outcome of a match. This evaluation is crucial not only for determining the player's current skill level but also for shaping future development and optimizing their role within a team. To begin with, the evaluation process focuses on how the player contributes to the overall dynamics of the game. A top-level football player is expected to consistently influence the match, whether through direct actions like goals and assists, or indirect contributions such as creating space for teammates, pressing the opposition, or disrupting the opponent's tactics. Their ability to adapt to different phases of the game—whether attacking, defending, or transitioning between the two—is a key consideration. An excellent player stands out not just for individual moments of brilliance, but for their sustained influence over the course of 90 minutes. Another crucial aspect of evaluating a football player's match performance is their decision-making ability. Football is a fast-paced game that requires players to make split-second decisions under pressure. An excellent player is one who consistently makes the right choices—whether it's deciding to pass, dribble, shoot, or hold the ball. Their awareness of the game, understanding of the situation, and ability to make decisions that positively impact their team's chances are critical to their overall performance. Evaluators look at how often the player makes decisions that lead to positive outcomes, as well as how well they balance risk and reward in key moments. Consistency is another key factor in the evaluation of a football player's match performance. It's not enough for a player to have flashes of brilliance; they need to perform at a high level regularly across multiple games and throughout the entirety of a season. Evaluators consider whether the player can maintain their level of play in different conditions—against stronger or weaker opponents, at home or away, and in high-pressure situations such as finals or tournament knockout stages. Consistency is what separates good players from great ones, as top players are able to deliver strong performances week in and week out. Moreover, the evaluation also considers how the player handles pressure and their mental resilience during the match. Football can be an emotionally and physically demanding sport, particularly in high-stakes matches or when things aren't going in the team's favor. An excellent player is one who can remain focused, composed, and determined even in the face of adversity. Whether dealing with a hostile crowd, a tough opponent, or a crucial game moment, the ability to stay mentally strong is vital for top-level performance. How a player responds to setbacks, such as missing a scoring opportunity or conceding a goal, can reveal much about their mental fortitude. Additionally, the evaluation takes into account the player's leadership and influence on their teammates. Even players who are not formally captains can demonstrate leadership qualities by guiding younger teammates, motivating the squad, or setting an example through their work ethic and determination. Players who can inspire those around them often have a greater impact on the overall team performance and are highly valued in any team setup. In conclusion, the evaluation of an excellent football player's match performance is not limited to individual skills or statistics. It encompasses a broader understanding of how the player integrates into the team, how consistently they contribute to the game, their decision-making under pressure, and their overall influence on the match. This holistic approach ensures that the player's true value is recognized, not just in terms of their technical abilities but also in their mental strength, leadership, and capacity to positively affect the team's performance.

To evaluate the performance ability of excellent football players, a MADM framework is employed. Eight excellent football teams are assessed based on 22 attributes, as shown in Table 1.

Table 1. Weight Allocation for 22 Criteria

C	Criteria	Weights
C_1	Creativity and Innovation on the Field	0.04526
C_2	Teamwork and Communication	0.045843
C_3	Performance Under Pressure	0.044677
C_4	Discipline and Sportsmanship	0.044921
C_5	Statistical Performance Metrics	0.044253
C_6	Consistency in Performance	0.045918
C_7	Tactical Awareness and Decision-Making	0.047206
C_8	Positioning and Spatial Awareness	0.045608
C_9	Flexibility in Playing Multiple Roles or Positions	0.044931
C_{10}	Ball Retention and Control Under Opposition Pressure	0.044592
C_{11}	Aerial Ability and Heading Skills	0.044103
C_{12}	Work Rate and Commitment	0.045852
C_{13}	Physical Fitness	0.047206
C_{14}	Mental Toughness and Resilience	0.045767
C_{15}	Proficiency in Set Pieces	0.044592
C_{16}	Defensive Skills	0.045598
C_{17}	Dribbling and One-on-One Situations	0.044376
C_{18}	Leadership and Influence on the Team	0.045354
C_{19}	Technical Skills	0.047611
C_{20}	Game Intelligence and Vision	0.046238
C_{21}	Ability to Adapt to Different Game Scenarios	0.045288
C_{22}	Recovery and Injury Prevention	0.044808

Step 1. We built the decision matrix using the neutrosophic numbers between the criteria and alternatives as shown in Table 2.

Table 2. Neutrosophic Numbers Representation

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))
C_2	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_3	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_4	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_5	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_6	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_7	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_8	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_9	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{10}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{11}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_{12}	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_{13}	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_{14}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_{15}	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{16}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{17}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_{18}	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_{19}	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_{20}	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_{21}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{22}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))
C_2	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_3	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_4	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_5	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_6	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_7	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_8	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_9	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_{10}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{11}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_{12}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_{13}	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_{14}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_{15}	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{16}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{17}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_{18}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))
C_{19}	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))
C_{20}	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_{21}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{22}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))
C_2	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_3	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_4	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_5	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_6	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))
C_7	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))
C_8	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_9	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.4(1),0.1(1),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{10}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{11}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))

C_{12}	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_{13}	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))
C_{14}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))
C_{15}	(0.7(0.9),0.2(1),0.4(0.8))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{16}	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.4(1),0.1(1),0.2(0.9))
C_{17}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))
C_{18}	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.5(0.9),0.3(0.9),0.5(0.8))
C_{19}	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))
C_{20}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))
C_{21}	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.6,(0.8),0.3(0.9),0.2(0.9))
C_{22}	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))	(0.8(1),0.3(0.8),0.5(0.7))	(0.5(0.9),0.3(0.9),0.5(0.8))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(1),0.3(0.8),0.5(0.7))	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.4(1),0.1(1),0.2(0.9))

Step 2: Calculate the weights of the criteria based on the method described and as shown in Table 1. This step helps determine the relative importance of each criterion in the decision-making process.

Step 3: Compute the superiority index. This index helps measure the relative performance or dominance of the alternatives against each other.

Step 4: Use Equation (5) to calculate the superiority identifier. This calculation quantifies the superiority of each alternative more precisely.

Step 5: Construct the impacts matrix. This matrix represents the influence or effect of each criterion on the alternatives.

Step 6: Build the REGIME matrix, which is a decision matrix used to structure and analyze the rankings of the alternatives.

Step 7: Compute the guide index, which provides additional insights into the overall evaluation of alternatives.

Step 8: Finally, rank the alternatives based on the calculations and present the results as shown in Figure 1. This step identifies the most suitable alternative(s).

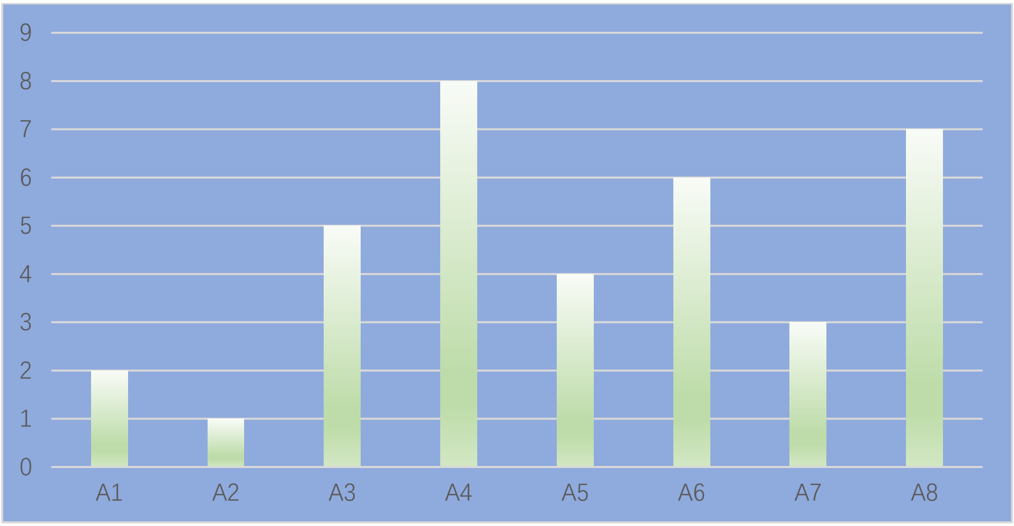

Figure 1. Prioritization of Alternatives

4.2. Comparative analysis

The PSNN-REGIME is compared with the PNN-VIKOR PNN-TOPSIS, and the PNN-PROMETHEE technique [23]. Figure 2 compares different techniques used for ranking and selecting the best category. Each method agreed the best alternative is A4 and the worst alternative is A2.

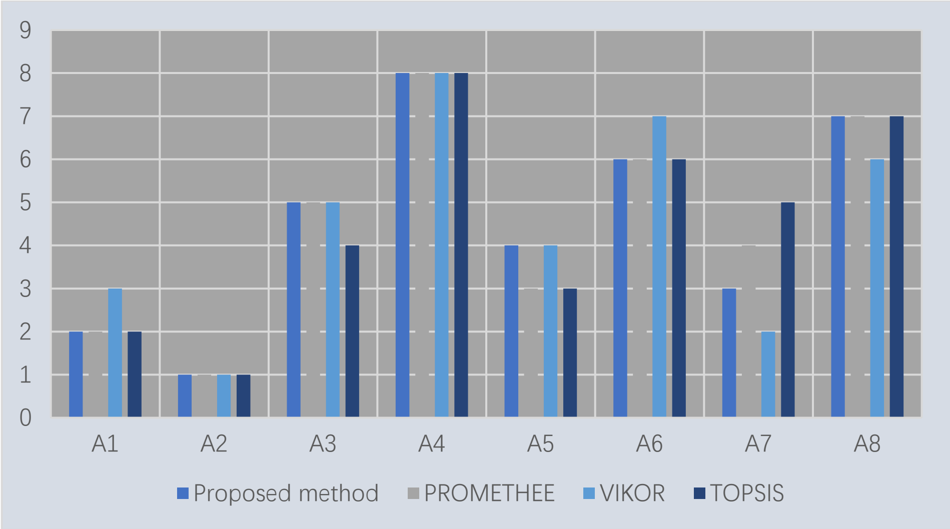

Figure 2. Comparison of Orders Across Different Techniques

Figure 2 represents the ranking orders of categories (A1 to A8) for each technique. Each point shows the rank of a category (along the horizontal axis) according to a specific technique (listed on the vertical axis). A4 consistently ranks the highest across all techniques, it as the optimal choice. The positions of the remaining categories (A1, A2, A3, ….. A8) show slight variations depending on the technique.

From Figure 2, it is clear that all models consistently identify the best and worst football team, despite minor differences in rankings. This highlights the reliability and effectiveness of the PSNN-REGIME method in evaluating the performance ability of excellent football players.

Compared to existing methods like VIKOR, TOPSIS, and PSNN-PROMETHEE, the PSNN-REGIME method offers several key advantages:

PSNN-REGIME effectively processes uncertain data by combining probabilistic analysis and similarity to an ideal solution, providing more reliable results.

It optimizes computations to ensure both high decision accuracy and quick processing, making it suitable for a variety of scenarios.

PSNN-REGIME is designed to handle diverse datasets and complex issues, making it more flexible and adaptable than other methods.

PSNN-REGIME is a robust and versatile tool, excelling in areas such as uncertainty handling, computational balance, and applicability, making it a valuable method for evaluating the performance ability of excellent football players.

We change the criteria weights by 12 cases to show the rank of alternatives under different weights. Figure 3 shows the different criteria for weights. Then we applied the proposed method under different criteria weights as shown in Figure 4. The results show the proposed method is stable under different criteria weights. The best alternative is A4 and the worst alternative is A2.

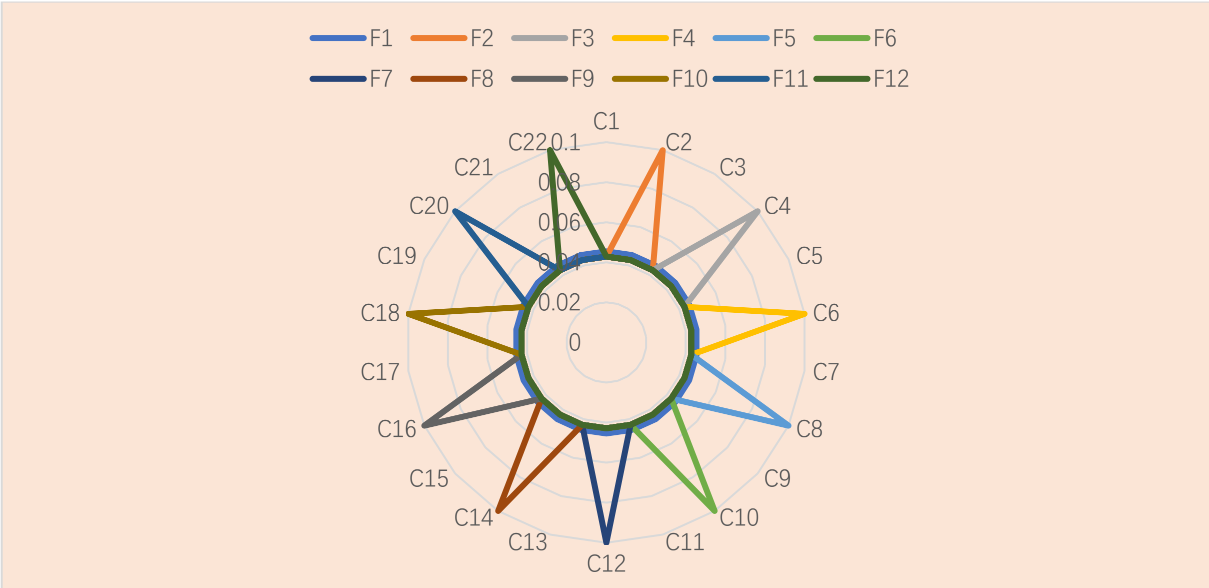

Figure 3. Different criteria weights.

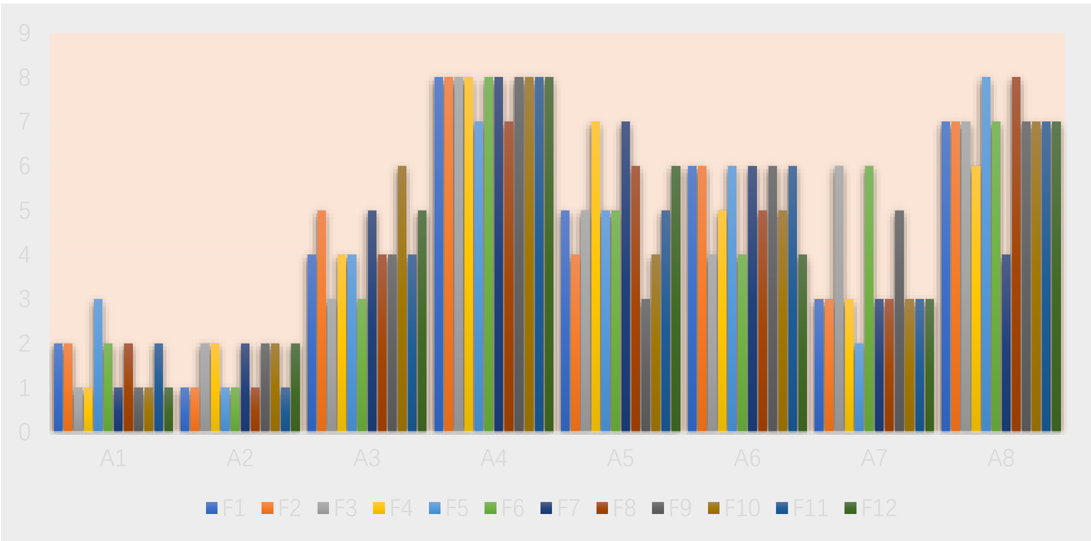

Figure 4. Different ranks of alternatives.

4.3 Practical Implications for Performance Ability of Excellent Football Players

The study's findings offer actionable insights into improving the performance ability of excellent football players:

- The PSNN- REGIME model can be used to identify performance weaknesses in elite football players and allocate training resources effectively to address these areas.
- Emphasizing skill development and continuous education for coaches, analysts, and support staff to enhance the accuracy and effectiveness of player performance evaluations.
- Addressing gaps in performance evaluation by increasing data collection points during matches and optimizing the frequency and timing of assessments.

4.4 Policy Recommendations

- Policymakers can benefit from the study by adopting the following recommendations:

— Develop a standardized framework for performance ability evaluation of excellent football players.
— Enhance coordination between physical training, technical coaching, and sports science systems to streamline the evaluation and improvement of players' match performance.
— Provide financial and logistical support to underperforming teams or training programs to encourage innovation and improvement in evaluating and enhancing players' match performance.

4.5 Challenges and Limitations

This study has several limitations:

— The evaluation focuses on a limited sample of five football teams, which may not fully represent the regional, tactical, and positional variations in elite football match performance.
— The PSNN- REGIME approach, while robust, requires technical expertise, potentially limiting its adoption in smaller or less-resourced institutions. Future studies should address these challenges by expanding datasets and simplifying model implementation.

5. Conclusions and future research

The evaluation of a football player's match performance is crucial for understanding their overall impact on the game and their potential for future development. It goes beyond simply looking at technical skills or statistics; it provides a comprehensive insight into how the player contributes to the team's success. By evaluating key aspects such as decision-making, consistency, mental toughness, and teamwork, coaches and scouts can determine whether the player has the qualities necessary to thrive in competitive environments. The significance of such evaluations lies in their ability to identify strengths and areas for improvement. For coaches, it helps in tailoring training programs to enhance a player's weaknesses, while also leveraging their strengths for team strategy. For scouts, it aids in making informed recruitment decisions by assessing how well a player can adapt to different game situations and their potential to grow. Moreover, performance evaluations help players themselves understand their role in the team and the areas they need to work on to reach higher levels of competition. Overall, evaluating match performance is an essential tool for optimizing both individual and team success, making it a key component in the development of top-level football talent.

This study focuses on evaluating the performance ability of excellent football players using a MADM approach. The average method is proposed to calculate attribute weights under PSNSs, and the PSNN-REGIME model is introduced for decision-making. A numerical study validates the model’s effectiveness. The key contributions of this study include: (1) extending the REGIME model to PSNSs, (2) using the average technique for weight determination, (3) applying the PSNN- REGIME method for MADM under PSNSs, and (4) conducting a comparative analysis to validate the approach.

While the study has made progress in evaluating the match performance of elite football players, it has certain limitations. The data used in the evaluation may lack diversity in terms of leagues, player positions, and match contexts, which could affect the comprehensiveness of the assessment. Additionally, the complexity of the PSNN- REGIME method may pose challenges for practical application in football clubs or training institutions that lack technical expertise or resources. Lastly, the absence of long-term impact evaluations makes it difficult to determine the sustained effectiveness of the proposed framework in enhancing players' match performance and team outcomes.

Future research could address these challenges by: (a) expanding data sources to include diverse leagues, player positions, and competitive levels to ensure a more comprehensive assessment, (b) exploring alternative evaluation methods, such as fuzzy logic or neural networks, to develop more practical and adaptable solutions for real-world applications, and (c) conducting long-term studies to monitor the impact of these evaluation methods on player development, team performance, and match outcomes. These efforts could significantly improve the accuracy and effectiveness of player performance assessments, addressing the evolving demands of modern football.

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