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Interval Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices with Decision Making

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Abstract

The objective of this study is to establish the results concerning Interval-Valued (IV) Secondary k-Range Symmetric (RS) Quadri Partitioned Neutrosophic Fuzzy Matrices (QPNFM). We have applied the RS condition within the neutrosophic environment to explore the relationships between IVQP s–k-RS, s-RS, IVQP k-RS, and IVQP RS matrices. This analysis has yielded significant insights into how these various matrix types interrelate and their structural properties. We have established the necessary and sufficient criteria for IVQP s–k-RS IVQPNFM, along with various generalized inverses of an IVQP s–ks-RS fuzzy matrix to maintain its classification as an IVQP s–k- RS matrix. Furthermore, we have characterized the generalized inverses of an IVQP s–k- RS matrix S corresponding to the sets S = {1,2}, S = {1,2,3}, and S = {1, 2, 4}. This characterization contributes to the foundational understanding of generalized inverses in the context of IVQPNFM. Additionally, a graphical representation of RS, Column symmetric (CS), and kernel symmetric (KS) adjacency and incidence QPNFM is illustrated. It is shown that every adjacency QPNFM is symmetric, RS, CS, and KS, whereas the incidence matrix only satisfies KS conditions. Similarly, every RS adjacency QPNFM is a KS adjacency QPNFM, but a KS adjacency QPNFM does not necessarily imply RS QPNFM. In this paper, we present an application of soft graphs in decision-making through the use of the adjacency matrix of a soft graph. We have developed an algorithm for this purpose and provide an example to demonstrate its application.

Keywords: IV Neutrosophic Fuzzy matrix, IV RS Neutrosophic fuzzy matrix, s-k- RS IV Neutrosophic fuzzy matrix.

1. Introduction

Interval-Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices are an advanced mathematical framework that extends traditional matrix theory to handle uncertainty, indeterminacy, and inconsistency in complex systems. This model integrates multiple mathematical concepts from neutrosophic logic, interval-valued fuzzy sets, and matrix theory to provide a versatile tool for representing and processing data in uncertain environments. Neutrosophic logic is an extension of fuzzy logic that introduces three membership functions—truth (T), falsity (F), and indeterminacy (I). Unlike traditional fuzzy logic, neutrosophic logic allows for the simultaneous existence of truth, falsity, and indeterminacy in different degrees for a given proposition or element. A neutrosophic fuzzy matrix represents a matrix whose elements are neutrosophic fuzzy sets. Each element in the matrix is characterized by an ordered triple (T,I,F) where:

- T represents the degree of truth,
- I represents the degree of indeterminacy,
- F represents the degree of falsity.

Interval-Valued: In this context, instead of single numerical values for T, I, and F, interval values are used to express the uncertainty or range of possibilities. For example, the truth value might be expressed as an interval [a,b], meaning the actual degree of truth lies somewhere between a and b. Similarly, indeterminacy and falsity values can also be represented as intervals.

Secondary k-Range: This refers to a secondary level of granularity or partitioning in the matrix. The parameter k-range could indicate a specific division or range within the matrix that is used to classify or analyze different sections of data. It helps in categorizing and analyzing more complex scenarios where single-range partitioning is not enough.

Symmetric Matrices: A matrix is called symmetric if it is equal to its transpose, i.e., $S = S^T$. In Interval-Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices, symmetry ensures that the relationship between the elements in the matrix is mirrored, which might be useful in modeling relationships like similarity, mutual influence, or mutual dependence.

Quadri Partitioned: The term "quadri partitioned" refers to the division of the matrix into four quadrants or partitions, where each partition could represent a different aspect or dimension of the problem being analyzed. These partitions allow for more detailed analysis and representation of multi-faceted data.

Matrices play a fundamental role in various scientific and engineering disciplines. However, traditional matrix theory often struggles to address problems that involve diverse forms of uncertainty. To address this limitation, Zadeh [1] introduced fuzzy sets (FS), which are characterized by membership values. However, assigning accurate membership values to a fuzzy set can sometimes pose challenges. To tackle this issue, Atanassov [2] developed intuitionistic fuzzy sets (IFS), which incorporate both membership and non-membership values to better represent uncertainty. Building on this, Smarandache [3] introduced neutrosophic sets (NSs), a framework designed to handle indeterminate information and address situations involving imprecision, uncertainty, and inconsistency.

Fuzzy matrices have been applied to address specific types of problems, and numerous researchers have expanded on these applications. However, traditional fuzzy matrices only consider membership values and are unable to handle non-membership values. Khan, Shyamal, and Pal [4] were the first to investigate intuitionistic fuzzy matrices (IFMs). Atanassov [5,6] explored both intuitionistic fuzzy sets (IFS) and operations on interval-valued IFS (IV IFS). Hashimoto [7] worked on the canonical form of a transitive matrix, while Kim and Roush [8] examined generalized fuzzy matrices. Lee [9] delved into secondary skew-symmetric and secondary orthogonal matrices, and Hill and Waters [10] focused on k-real and k-Hermitian matrices. Meenakshi [11] contributed to the understanding of fuzzy matrix theory and its applications.

Measuring membership or non-membership as precise values in real-world scenarios often poses challenges. To address these, Anandhkumar [12,13] investigated pseudo-similarity in neutrosophic fuzzy matrices and explored various inverses of neutrosophic fuzzy matrices. Additionally, Punithavalli and Anandhkumar [14] studied reverse sharp and left-Tand right-T partial ordering on intuitionistic fuzzy matrices. Pal and Susanta Kha [15] have explored interval-valued intuitionistic fuzzy matrices, contributing to the understanding of how these matrices can handle uncertainty more effectively. Vidhya and Irene Hepzibah [16] have focused their research on interval-valued neutrosophic fuzzy matrices, further extending the concept by incorporating neutrosophic logic to address indeterminacy along with membership and non-membership values. Anandhkumar et al. [17] have examined reverse sharp and left-T and right-T partial ordering on neutrosophic fuzzy matrices, which enhances the mathematical framework for handling neutrosophic fuzzy relations and their applications.

Anandhkumar et al. [18] have examined reverse tilde T and minus partial ordering on intuitionistic fuzzy matrices, extending the theoretical framework for fuzzy systems. Additionally, Anandhkumar et al. [19] explored partial orderings, characterizations, and generalizations of k-idempotent neutrosophic fuzzy matrices, further enriching the study of neutrosophic fuzzy matrices in handling uncertainties.Building upon these foundational studies, we propose considering membership values as intervals, rather than fixed points, and similarly treating non-membership values as intervals. This approach accommodates a broader range of uncertainty. In this context, we introduce Interval-Valued (IV) Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices (IVSNRNFMs). Furthermore, we define basic operators on these interval-valued neutrosophic fuzzy matrices (IVNFMs), establishing a groundwork for future research and applications in fields that require handling of imprecise and indeterminate information.

The structure of the article is organized as follows:

In section 1, we present introduction,

In section 2 and 3, we introduce notation and some elementary definitions.

In section 4, we provide graphical representations of RS, CS, and KS adjacency matrices.

In section 5, we discuss Algorithm and application of adjacency Neutrosophic fuzzy matrix of a graph in decision making.

In section 6, we introduce Interval valued Secondary k-RS IVQPNFM

In section 7, we discuss IVQP s – k-RS regular Neutrosophic fuzzy matrices.

1.1 Literature Review

Meenakshi and Jaya Shree [20] have explored the concept of **k**-kernel symmetric matrices, while Meenakshi and Krishnamoorthy [21] characterized secondary **k**-Hermitian matrices. Additionally, Meenakshi and Jaya Shree [22] investigated **k**-range symmetric matrices, and Jaya Shree [23] delved into secondary **k-**kernel symmetric fuzzy matrices. Shyamal and Pal [24] contributed to the study of interval-valued fuzzy matrices, while Meenakshi and Kalliraja [25] examined regular interval-valued fuzzy matrices. Anandhkumar [26] explored kernel and **k**-kernel intuitionistic fuzzy matrices, and Jaya Shree [27] discussed secondary k**-**range symmetric fuzzy matrices. Anandhkumar et al. [28] worked on generalized symmetric neutrosophic fuzzy matrices, and Kaliraja and Bhavani [29] researched interval-valued secondary **k**-range symmetric fuzzy matrices. Let S be any fuzzy matrix. If S occurs, it coincides with the transpose of the matrix, denoted S^T. A fuzzy matrix S, belonging to Fn, is referred to as a kernel symmetric matrix if it satisfies $N(S) = N(S^T)$, where N(S) represents the null space of S. However, this does not imply that the range spaces of S and S^T are equal, i.e., $R(S) \neq R(S^T)$. Nevertheless, the converse holds true, meaning that if $R(S) = R(S^T)$, then $N(S) = N(S^T)$, indicating kernel symmetry.

Symmetric matrices play an essential role in the theory of complex matrix entries, particularly in the study of k-Hermitian matrices. This concept has led to the development of k-EP matrices, which generalize both k-Hermitian matrices and EP (equal projection) matrices. Hill and Waters [30] initiated the exploration of k-Hermitian matrices. Following this, Baskett and Katz [31] introduced theorems on the products of EPr matrices, which further expanded the understanding of matrix product theory. Meenakshi and Krishnamoorthy [32] contributed to this area by studying k-EP matrices.In the context of complex matrices, it is well established that the concepts of range symmetry and kernel symmetry are equivalent. However, this equivalence does not hold for interval-valued fuzzy matrices, where the relationship between range and kernel symmetry breaks down due to the inherent uncertainty in interval-valued membership and non-membership functions. This distinction makes the study of interval-valued fuzzy matrices more nuanced and highlights the need for further research into their unique properties.

The concept of interval-valued s–ks -Hermitian and interval-valued kernel symmetric matrices for fuzzy matrices provides a framework for handling uncertainty in complex systems. These matrices extend classical matrix theory by incorporating interval values to represent both membership and non-membership, offering a more flexible approach to manage fuzzy data.Additionally, we have expanded on several fundamental conclusions regarding these two types of matrices. Specifically, an interval-valued secondary s–ks -kernel symmetric fuzzy matrix is characterized by properties that extend traditional kernel symmetry, and suitable criteria for determining generalized inverses (g-inverses) of these matrices are established. These generalized inverses for interval-valued secondary s–k-kernel symmetric fuzzy matrices serve as essential tools for solving fuzzy matrix equations and other computational problems involving such matrices.

Moreover, we derive the necessary and sufficient conditions for a matrix to qualify as an interval-valued s−k kernel symmetric fuzzy matrix, providing a rigorous foundation for analyzing and utilizing these structures in practical applications. This further enhances the understanding of fuzzy matrix theory in relation to uncertainty and symmetry.Meenakshi, Krishnamoorthy, and Ramesh [33] have also contributed to this area by studying s−k-EP matrices, which play a crucial role in extending matrix theory to more generalized and uncertain environments. These developments highlight the ongoing research in the field of fuzzy matrices and their application in solving real-world problems involving imprecision and indeterminacy.

As a generalization of secondary Hermitian and Hermitian matrices, Meenakshi and Krishnamoorthy [34] introduced the concept of s−k Hermitian matrices, which expand the traditional understanding of matrix symmetry and its applications. Building on this foundation, we extend the notion of s−k Hermitian matrices to include s−k kernel symmetric intuitionistic fuzzy matrices. This extension allows us to incorporate the unique properties of intuitionistic fuzzy sets, such as membership and non-membership values, into the framework of matrix theory.Furthermore, we derive equivalent conditions for various generalized inverses (g-inverses) of secondary k-kernel intuitionistic fuzzy matrices to qualify as secondary k-kernel symmetric intuitionistic fuzzy matrices. This relationship is critical for understanding the interplay between matrix symmetry and the properties of g-inverses in fuzzy systems, thereby enhancing the applicability of these matrices in solving fuzzy matrix equations and related problems.

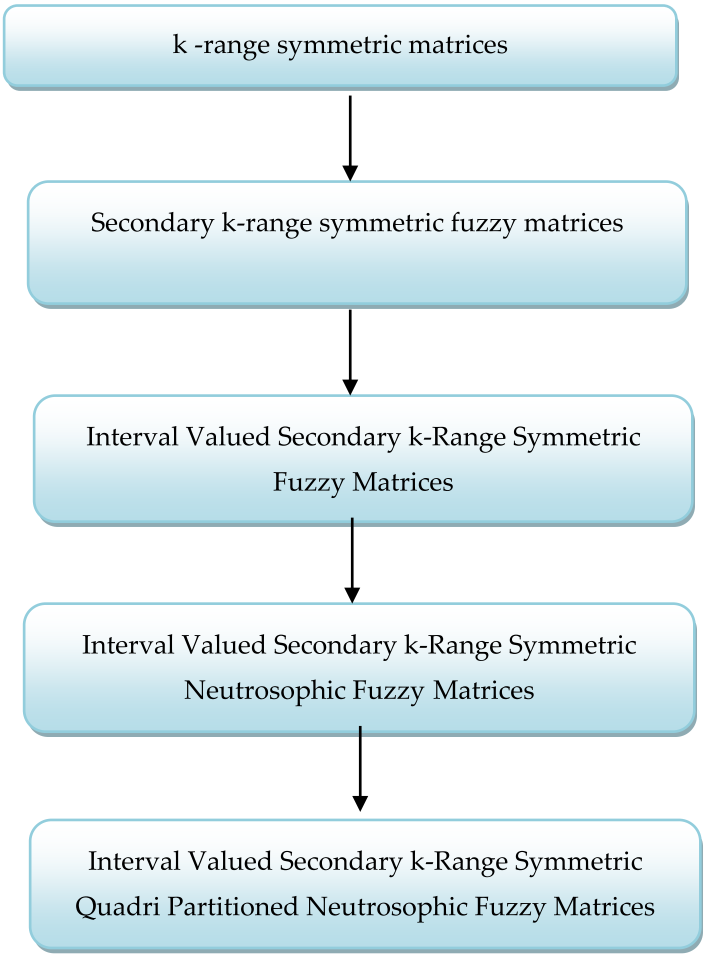
In addition, Meenakshi and Krishnamoorthy [35] have introduced k-EP matrices, which further generalize existing concepts in fuzzy matrix theory. These developments emphasize the ongoing exploration of advanced matrix structures and their significance in handling uncertainty and indeterminacy in various applications.Shyamal and Pal [36] conducted a study on interval-valued fuzzy matrices, expanding the understanding of how fuzzy data can be represented and manipulated using interval values to express uncertainty.The concept of k-symmetric matrices was introduced by various authors, including Ann Lec [37], who explored secondary symmetric and skew-symmetric secondary orthogonal matrices. This work contributes to the broader understanding of matrix symmetry and its implications in different mathematical contexts.

Elumalai and Rajesh Kannan [38] further advanced the study of symmetry in matrices by introducing k-symmetric circulant, sss-symmetric circulant, and s−k -symmetric circulant matrices. These specialized matrices play a vital role in various applications, particularly in fields requiring structured data representations.Additionally, Elumalai and Arthi [39] investigated the properties of k-centrosymmetric and k-skew centrosymmetric matrices, providing valuable insights into how symmetry interacts with the structural properties of matrices.Following this, Gunasekaran and Mohana [40] studied k-symmetric, s-symmetric, and s−k-symmetric double stochastic matrices. Their research emphasizes the importance of symmetry in the context of stochastic processes and reinforces the growing body of knowledge surrounding symmetric matrices in various mathematical and applied domains. Anandhkumar et al. [41] have studied Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. Anandhkumar et al. introduced the concept of secondary k-range symmetric neutrosophic fuzzy matrices, providing a new approach to enhance the matrix's flexibility in representing uncertain information in diverse applications [42]. Expanding on this foundation, Anandhkumar, Bobin, Chithra, and Kamalakannan proposed generalized symmetric Fermatean neutrosophic fuzzy matrices, which incorporate additional dimensions of indeterminacy to enable more comprehensive modeling [43]. Further developments include secondary k-column symmetric neutrosophic fuzzy matrices, as explored by Anandhkumar, Punithavalli, and Janaki, which focus on enhancing matrix structures for specific applications in decision-making and data analysis [44].

Table 1: Extension of Neutrosophic Fuzzy Matrices Based on Previous Works

References	Extension of Neutrosophic Fuzzy Matrices from Fuzzy Matrices	Year
[20]	On k-kernel symmetric matrices	2009
[22]	On k-range symmetric matrices	2009
[23]	Secondary k-kernel symmetric fuzzy matrices	2014
[27]	Secondary k-range symmetric fuzzy matrices	2018
[29]	Interval Valued Secondary k-Range Symmetric Fuzzy Matrices	2022
[41]	Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices	2023
Proposed	Interval Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices with Decision Making	2024

This table summarizes the extensions of Neutrosophic Fuzzy Matrices as derived from previous works. It highlights the evolution of research in this field, culminating in our proposed extension of Interval Valued Secondary k-Range Symmetric Quadri Partitioned Neutrosophic Fuzzy Matrices in 2024.



From Table 1 and the accompanying process flow, it is clear that previous studies have predominantly focused on concepts such as k-kernel, k-range, secondary k-kernel, and secondary k-range matrices within the framework of fuzzy matrices. However, a noticeable research gap exists concerning the exploration of these concepts in a neutrosophic environment, which deals with indeterminate and inconsistent information.

Recognizing this gap, we have made significant strides in establishing results related to kkk-range and secondary k-range matrices specifically within the context of interval-valued neutrosophic fuzzy **matrices**. By addressing this underexplored area, our work contributes to the development of the theoretical foundation necessary for understanding how these matrix types can operate under the complexities of neutrosophic logic. This not only enriches the existing literature but also opens up new avenues for research and applications in dealing with uncertainty and imprecision in various fields.

1.2 Novelties

We have established the concept of Interval-Valued (IV) Secondary k-Range Symmetric (RS) Neutrosophic Fuzzy Matrices. This development is crucial for enhancing the hybrid fuzzy structure, allowing for more sophisticated modeling of uncertainty. We applied this concept within the framework of Neutrosophic Fuzzy Matrices (NFM) and conducted a detailed study of several results stemming from this application. In addition, we present equivalent characterizations of Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. This section includes various examples that illustrate the properties and applications of these matrices, facilitating a deeper understanding of their structure and significance.

Furthermore, we discuss various generalized inverses associated with regular matrices, providing a comprehensive characterization of the set of all inverses through the lens of Secondary s–k-Range Symmetric Neutrosophic Matrices. This analysis contributes to the broader discourse on matrix theory, particularly in the context of fuzzy and neutrosophic environments, enhancing the tools available for dealing with complex systems characterized by uncertainty.

1.3 Notations:

IV =Interval valued,

IVNFM = Interval valued Neutrosophic Fuzzy Matrix,

RS = Range Symmetric

IVQPNFM = Interval valued Quadri Partitioned Neutrosophic Fuzzy Matrices

$[S_T, S_C, S_I, S_F]_L^T$ = Transpose of the IVQPNFM $[S_T, S_C, S_I, S_F]_L$,

$[S_T, S_C, S_I, S_F]_U^T$ = Transpose of the IVQPNFM $[S_T, S_C, S_I, S_F]_U$,

$[S_T, S_C, S_I, S_F]_L^+$ = Moore-Penrose inverse of IVQPNFM $[S_T, S_C, S_I, S_F]_L$,

$[S_T, S_C, S_I, S_F]_U^+$ = Moore-Penrose inverse of IVQPNFM $[S_T, S_C, S_I, S_F]_U$,

$\mathrm{R}\left([S_T, S_C, S_I, S_F]_L\right)$ = Row space of IVQPNFM $[S_T, S_C, S_I, S_F]_L$

$\mathrm{R}\left([S_T, S_C, S_I, S_F]_U\right)$ = Row space of IVQPNFM $[S_T, S_C, S_I, S_F]_U$,

$\mathrm{C}\left([S_T, S_C, S_I, S_F]_L\right)$ = Column space of IVQPNFM $[S_T, S_C, S_I, S_F]_L$,

$\mathrm{C}\left([S_T, S_C, S_I, S_F]_U\right)$ = Column space of IVQPNFM $[S_T, S_C, S_I, S_F]_U$

QPNFM : Quadri Partitioned Neutrosophic fuzzy matrices.

PNFM : Permutation Neutrosophic fuzzy matrices.

2. Preliminaries and Definitions

2.1 Preliminary

If $\kappa(x)=(x_{k[1]}, x_{k[2]}, x_{k[3]},\ldots, x_{k[n]}) \in F_{n\times 1}$ for $(x = x_1, x_2,..,x_n) \in F_{[1\times n]}$, where K is involuntary, The corresponding

Permutation matrix is satisfied using the conditions

(P.2.1) $KK^T = K^T K = I_n$, $K = K^T$, $K^2 = I$

By the definition of V, and $R(x) = Kx$

(P.2.2) $V = V^T$, $VV^T = V^T V = I_n$ and $V^2 = I$

(P.2.3) $R\left([S_T, S_C, S_I, S_F]_L\right) = R\left([S_T, S_C, S_I, S_F]_L\right) V$,

$$R\left([S_T, S_C, S_I, S_F]_L\right) = R\left([S_T, S_C, S_I, S_F]_L\right) K$$

$$R\left([S_T, S_C, S_I, S_F]_U\right) = R\left([S_T, S_C, S_I, S_F]_U\right) V,$$

$$R\left([S_T, S_C, S_I, S_F]_U\right) = R\left([S_T, S_C, S_I, S_F]_U\right) K$$

(P.2.4) $R\left([S_T, S_C, S_I, S_F]_L V\right)^T = R\left(V[S_T, S_C, S_I, S_F]_L^T\right)$,

$$R\left(V[S_T, S_C, S_I, S_F]_L\right)^T = R\left([S_T, S_C, S_I, S_F]_L^T V\right)$$

$$R\left([S_T, S_C, S_I, S_F]_U V\right)^T = R\left(V[S_T, S_C, S_I, S_F]_U^T\right),$$

$$R\left(V[S_T, S_C, S_I, S_F]_U\right)^T = R\left([S_T, S_C, S_I, S_F]_U^T V\right)$$

3.Quadri Partitioned Neutrosophic Soft Set (QPNSS)

Definition 3.1. Let X is an initial universe set and E is a set of parameters. Consider a non-empty set A where A ⊆ E. Let P(X) denote the set of all QPNSS of X. The collection (F, A) is termed the (QPNSS) over X, where F is a mapping given by F : A⟶ P(X).Here,

$A = \{< x, T_A(x), C_A(x), U_A(x), FA(x) >: x \in U\}$ with T_A , F_A, C_A ,$U_A : X \longrightarrow [0,1]$ *and* $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$. In this context

- $T_A(x)$ is the truth membership,
- $C_A(x)$ is contradiction membership,
- $U_A(x)$ is ignorance membership
- $F_A(x)$ is the false membership.

Definition:3.2 A QPNSS (F, A) over the universe X is considered to be an empty QPNSS with respect to the parameter A if $T = 0$, $C = 0$, $U = 1$, $F = 1$.

Definition: 3.3 A QPNSS (F, A) over the universe X is considered to be universe QPNSS with respect to the parameter A if $T = 1$, $C = 1$, $U = 0$, $F = 0$.

Definition: 3.4 Let $S = <[S_T, S_C, S_I, S_F]>$ be a QPNFM, if R $[[S_T, S_C, S_I, S_F]]$ =

R $[[S_T, S_C, S_I, S_F]^T]$ then $S = <[S_T, S_C, S_I, S_F]>$ is called as RS.

Example: 3.1 Consider an QPNFM

$$[S_T,S_C,S_I,S_F]=\begin{bmatrix} <0.4,0,1,0.6> & <1,0,1,1> & <0.2,0.3,0.4,0.7> \\ <1,0,1,1> & <1,0,1,1> & <1,0,1,1> \\ <0.2,0.3,0.4,0.7> & <1,0,1,1> & <1,0,1,1> \end{bmatrix}$$

Here, R $[S_T,S_C,S_I,S_F]$ = R $[[S_T,S_C,S_I,S_F]^T]$

The following matrices does not satisfy the range symmetric condition

$$[S_T,S_C,S_I,S_F]=\begin{bmatrix} <0.4,0,1,0.7> & <1,0,1,1> & <0.2,0.3,0.4,0.6> \\ <1,0,1,1> & <1,0,1,1> & <1,0,1,1> \\ <0.2,0.4,0.4,0.6> & <1,0,1,1> & <1,0,1,1> \end{bmatrix}$$

$$[S_T,S_C,S_I,S_F]^T=\begin{bmatrix} <0.4,0,1,0.7> & <1,0,1,1> & <0.2,0.4,0.4,0.6> \\ <1,0,1,1> & <1,0,1,1> & <1,0,1,1> \\ <0.2,0.3,0.4,0.6> & <1,0,1,1> & <1,0,1,1> \end{bmatrix}$$

$\left[(0.4,0,1,0.7)\ (1,0,1,1)\ (0.2,0.3,0.4,0.6)\right]\in R([S_T,S_C,S_I,S_F])$,

$\left[(0.4,0,1,0.7)\ (1,0,1,1)\ (0.2,0.3,0.4,0.6)\right]\notin R([S_T,S_C,S_I,S_F]^T)$,

$\left[(1,0,1,1)\ (1,0,1,1)\ (1,0,1,1)\right]\in R[S_T,S_C,S_I,S_F]$,

$\left[(1,0,1,1)\ (1,0,1,1)\ (1,0,1,1)\right]\in R([S_T,S_C,S_I,S_F]^T)$,

$\left[(0.2,0.4,0.4,0.6)\ (1,0,1,1)\ (1,0,1,1)\right]\in R([S_T,S_C,S_I,S_F])$,

$\left[(0.2,0.4,0.4,0.6)\ (1,0,1,1)\ (1,0,1,1)\right]\notin R([S_T,S_C,S_I,S_F]^T)$,

$R([S_T,S_C,S_I,S_F])\notin R([S_T,S_C,S_I,S_F]^T)$.

Note:3.1 For QPNFM R with det $[S_T,S_C,S_I,S_F]$ > <0,0,1,1> has non- zero rows and non-columns, hereafter N($[S_T,S_C,S_I,S_F]$) = <0,0,1,1> = N($[S_T,S_C,S_I,S_F]^T$) . Furthermore, a symmetric matrix $[S_T,S_C,S_I,S_F]$ = $[S_T,S_C,S_I,S_F]^T$ that is N($[S_T,S_C,S_I,S_F]$)= N($[S_T,S_C,S_I,S_F]^T$).

Definition :3.5 Let $S=<[S_T,S_C,S_I,S_F]> \in (QPNFM)_n$, if N($[S_T,S_C,S_I,S_F]$) = N($[S_T,S_C,S_I,S_F]^T$) and then R is called KS- QPNFM where N($[S_T,S_C,S_I,S_F]$)= {x/x $[S_T,S_C,S_I,S_F]$} = (0,0,1,1) and x ∈ $F_{1\times n}$}.

Example: 3.2 Consider an QPNFM

$$[S_T,S_C,S_I,S_F]=\begin{bmatrix} <0.3,0.3,0.4,0.7> & <0.5,0.3,0.6,0.3> & <0.2,0.3,0.4,0.5> \\ <0.4,0.7,0.6,0.3> & <0.4,0.8,0.1,0.2> & <0.2,0.5,0.1,0.4> \\ <0.2,0.4,0.4,0.1> & <0.4,0.4,0.5,0.2> & <0.4,0.5,0.6,0.1> \end{bmatrix}$$

$$N([S_T,S_C,S_I,S_F])=N([S_T,S_C,S_I,S_F]^T)=(0,0,1,1).$$

Definition 3.6. Symmetric QPNFM. If $S=<[S_T,S_C,S_I,S_F]> \in (QPNFM)_n$ is said to be symmetric QPNFM if r_{ij} = r_{ji}

Example: 3.3 Consider an QPNFM

$$[S_T,S_C,S_I,S_F]=\begin{bmatrix} <0.4,0,1,0.2> & <1,0,1,1> & <0.2,0.3,0.4,0.7> \\ <1,0,1,1> & <1,0,1,1> & <1,0,1,1> \\ <0.2,0.3,0.4,0.7> & <1,0,1,1> & <1,0,1,1> \end{bmatrix}$$

Here, $[S_T,S_C,S_I,S_F]$= $[S_T,S_C,S_I,S_F]^T$

Every row single (1,1,0,0) with (0,0,1,1) 's everywhere else is called QPPNFM.

Example: 3.4 Consider a QPNFPM, $K=\begin{bmatrix} (1,1,0,0) & (0,0,1,1) & (0,0,1,1) \\ (0,0,1,1) & (1,1,0,0) & (0,0,1,1) \\ (0,0,1,1) & (0,0,1,1) & (1,1,0,0) \end{bmatrix}$

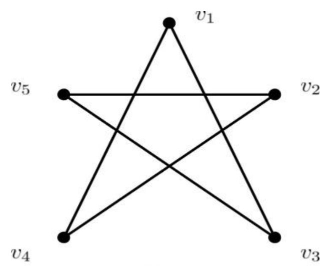
4. Graphical Representation of KS Adjacency IVINFM.

Definition 4.1. Adjacency IVINFM

An QPNM is a square matrix representing a finite graph. The elements of this matrix indicate whether pairs of vertices in the graph are connected. In the case of a finite simple graph, the IVINFM can be represented as a binary matrix, typically denoted as [1,1,0,0] and [0,0,1,1] where the diagonal elements are consistently set to [0,0,1,1]. Let G(V, E) denote a graph with n vertices. The adjacency matrix S = [s_{ij}] is a symmetric matrix defined by $S=[s_{ij}]=\begin{cases}[1,1,0,0] & \text{when } v_i \text{ is adjacent to } v_j \\ [0,0,1,1] & \text{otherwise}\end{cases}$, denoted by S(G) or

Example: 4.1 Consider an IVINM and a equivalent graph

$$S = \begin{matrix} \\ v_1 \\ v_3 \\ v_4 \\ v_2 \\ v_5 \end{matrix} \begin{pmatrix} v_1 & v_3 & v_4 & v_2 & v_5 \\ [0,0,1,1] & [1,1,0,0] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] \\ [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] & [1,1,0,0] \\ [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \\ [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] \\ [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \end{pmatrix}$$



H

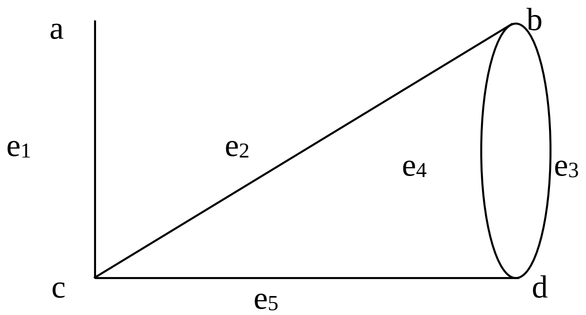
Definition 4.2. Incidence IVINFM

Let G(V, E) represent a simple graph with n vertices. Let V = {V_1, V_2, …, V_n} and E = {e_1, e_2, ..., e_m}. Then, the incidence IVINFM I = [m_{ij}] is a $n \times m$ matrix defined by

$$I = [m_{ij}] = \begin{cases} [1,1,0,0] & \text{when } v_i \text{ is incident to } e_j \\ [0,0,1,1] & \text{otherwise} \end{cases}$$, denoted by S(G) or S_G.

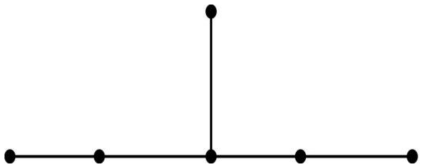
Example:4.2 Consider an incidence IVINFM and a equivalent graph. The incidence IVINFM is

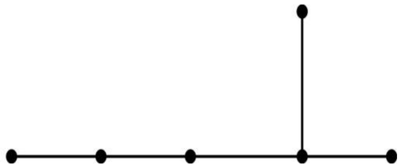
$$A = \begin{matrix} a \\ b \\ c \\ d \end{matrix} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \\ [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] \\ [0,0,1,1] & [1,1,0,0] & [1,1,0,0] & [1,1,0,0] & [0,0,1,1] \\ [1,1,0,0] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [1,1,0,0] \\ [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [1,1,0,0] & [1,1,0,0] \end{bmatrix}$$

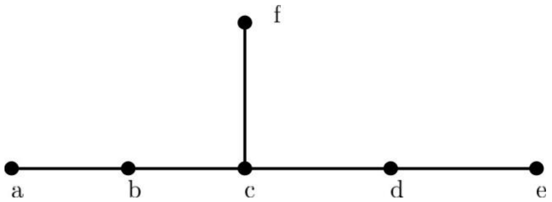


Definition 4.3. **Isomorphic of Graph:** Two graphs are said to be isomorphic if number of vertices, edges, degree sequence and adjacency IVINFM are equal.

Relation between isomorphism, non-isomorphism and KS



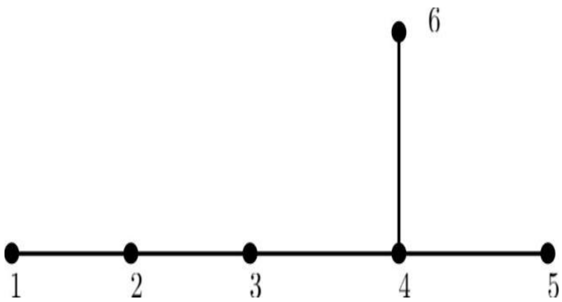




	a	b	c	d	e	f
a	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[0, 0, 1, 1]	[0, 0, 1, 1]	[0, 0, 1, 1]
b	[1, 1, 0, 0]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[0, 0, 1, 1]	[0, 0, 1, 1]
c	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[1, 1, 0, 0]
d	[0, 0, 1, 1]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]
e	[0, 0, 1, 1]	[0, 0, 1, 1]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[0, 0, 1, 1]
f	[0, 0, 1, 1]	[0, 0, 1, 1]	[1, 1, 0, 0]	[0, 0, 1, 1]	[0, 0, 1, 1]	[0, 0, 1, 1]

The adjacency IVINFM of the graph is given by

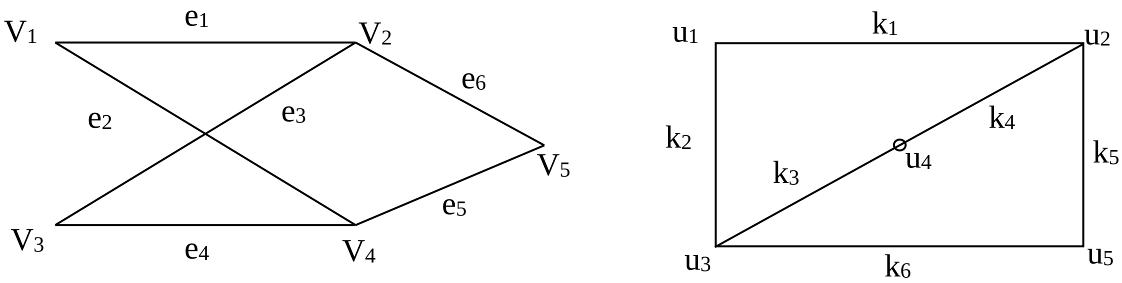
Consider the graph H and name as follows



The adjacency IVINM of the graph is given by

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] \\ [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] & [0,0,1,1] \\ [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] \\ [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [1,1,0,0] \\ [0,0,1,1] & [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] \\ [0,0,1,1] & [0,0,1,1] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [0,0,1,1] \end{bmatrix}$$

The two graphs presented have identical numbers of vertices, edges, and degree sequences, yet their adjacency IVINFMs differ. Therefore the given Graph is not isomorphic but KS.



Let us form the adjacency IVINFM AG and AH

$$A_G = \begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 \\ v_1 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \\ v_2 & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] \\ v_3 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \\ v_4 & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] \\ v_5 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \end{bmatrix}$$

$$A_H = \begin{bmatrix} & u_1 & u_2 & u_3 & u_4 & u_5 \\ u_1 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \\ u_2 & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] \\ u_3 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \\ u_4 & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] \\ u_5 & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] & [1,1,0,0] & [0,0,1,1] \end{bmatrix}$$

The two graphs provided have the same number of vertices, edges, and degree sequences, and their adjacency IVINFMs are also identical. Therefore, the graphs are isomorphic and also KS IVINFM.

Every isomorphic and non-isomorphic graph is KS adjacency IVINM but converse need not be true.

In this section, we introduce an algorithm designed to reduce parameters through the use of an adjacency matrix associated with a soft graph. We then apply this algorithm to a decision-making problem.

5. Algorithm

Consider the product set { M_1, M_2, M_3, …..M_n} with parameters {S_1,S_2,…,S_k}. To select the best product based on these parameters, we propose the following algorithm, which utilizes the adjacency matrix of a soft graph.

• Form a bipartite graph G=(V,E) for the given problem. In this graph:

V represents the set of vertices, which includes the products { M_1, M_2, M_3, …..M_n} and the parameters {S_1,S_2,…,S_k}. If E represents the set of edges, which connect each product M_i to the relevant parameters S_i This bipartite graph effectively illustrates the relationships between the products and their associated parameters.

• To construct a soft graph (F,A) with A={M1,M2,M3,…,Mn} follow these steps:

1. **Define the Set S(x)**: For a given vertex x in the graph, define S(x)={z∈V:d(x,z)≤1} where d(x,z) denotes the distance between vertices x and z.
2. **Define the Set T(x)**: Define T(x)={xu∈E: u∈S(x)} where E represents the set of edges and u is a vertex connected to x.
3. **Construct F(x)**: The soft graph F(x) is then represented as F(x)=(S(x),T(x)), where S(x) is the set of vertices within a distance of (1,1,0,0) from x and T(x) is the set of edges connecting x to these vertices.

• Construct the adjacency matrix of the given soft graph (F,A). In this matrix:

- The rows correspond to the products Mi.
- The columns correspond to the parameters Pj.

Each entry (Mi,Sj) in the matrix represents the relationship or connection between product Mi and parameter Sj. If there is a connection, the entry is (1,1,0,0); otherwise, it is (0,0,1,1).

• If any entry (Mi,Sj) in the adjacency matrix is either (1,1,0,0) or (0,0,1,1) for all i=1,2,…,n, then the parameter Sj should be removed. This indicates that the parameter Sj does not contribute to distinguishing between products and can therefore be excluded from consideration.

• To determine the row totals in the modified adjacency matrix of the soft graph:

Modify the Adjacency Matrix: Make any necessary modifications to the original adjacency matrix based on the specific requirements or criteria provided.

Calculate Row Totals: For each row in the modified adjacency matrix, sum the entries. This sum represents the total number of connections or relationships for each vertex in the graph.

Identify the Last Column: The last column of the matrix should contain these row totals.

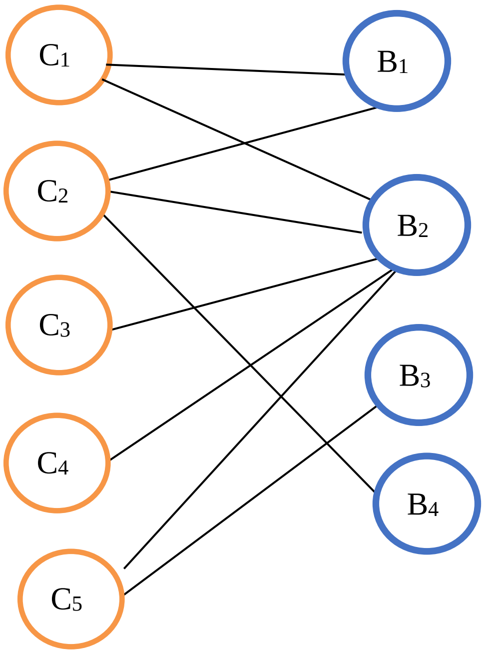
• Determine the product Mi that has the highest row total..

• The product with the highest row total will be the most favorable option.

5.1 Application:

The candidate is selecting a new course to enrol in from five available options: C1, C2, C3, C4, and C5. Each course offers various benefits (B1, B2, B3, and B4).Course C1 provides benefits B1 and B3, Course C2 includes benefits B1, B2, and B4, Course C3 has only benefit B2., Course C4 also offers benefit B2 and

Course C5 features benefits B2 and B3. Which course should the candidate choose to get the most value based on the benefits that align with their learning goals? First, generate a graph based on the given problem as outlined in the description.



Let V={C_1,C_2,C_3,C_4,C_5,B_1,B_2,B_3,B_4}.

Select A={C_1,C_2,C_3,C_4,C_5}.

Define S(x)={ z∈V: d(x,z) ≤ 1},

T(x)={ xu∈E: :u∈S(x)}. Now, let F(x)=(S(x),T(x)), where: F(C1)={C_1,B_1,B_3}, F(C2)={C_2,B_1,B_2,B_4}, F(C3)={C_3,B_2}, F(C4)={C_4,B_2} and F(C5)={C_5,B_2,B_3}.

Thus, (F, A) is a soft graph.

The adjacency matrix for the graph described above is provided as follows.

	B_1	B_2	B_3	B_4	C_1	C_2	C_3	C_4	C_5	Row total =Degree of vertex
C_1	(1,1,0,0)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	2

C_2	(1,1,0,0)	(1,1,0,0)	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	3
C_3	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	1
C_4	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	1
C_5	(0,0,1,1)	(1,1,0,0)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	2
B_1	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	2
B_2	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	5
B_3	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	1
B_4	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	1

The row total in the final column of the adjacency matrix represents the vertex degree in the soft graph (F,A). Given that the entry (M_i,S_2) is (1,1,0,0) for i=1,2,3,4,5 we should remove the second column from the matrix.

	B_1	B_3	B_4	C_1	C_2	C_3	C_4	C_5	Row total = Degree of vertex
C_1	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	1
C_2	(1,1,0,0)	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	2
C_3	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	0
C_4	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	0
C_5	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	1
B_1	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	2
B_2	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	5
B_3	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	1
B_4	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(0,0,1,1)	(1,1,0,0)	(0,0,1,1)	(0,0,1,1)	1

In the final column of the matrix, examine the row totals for new course C_1, C_2, C3, C4, and C_5. The row total for C_2 is the highest among them. Therefore, C_2 is the optimal choice.

6. Interval valued Secondary k-RS IVQPNFM

Definition 6.1. For an IVQPNFM $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \text{IVQPNFM}_{nn}$ is an IVQP s - symmetric IVQPNFM iff $[S_T,S_C,S_I,S_F]_L = V\left([S_T,S_C,S_I,S_F]_L{}^T\right)V$ and $[S_T,S_C,S_I,S_F]_U$

$= V\left([S_T,S_C,S_I,S_F]_U{}^T\right)V.$

Definition 6.2 For an IVQPNFM $S =< [S_T,S_C,S_I,S_F]_L, [S_T,S_C,S_I,S_F]_U > \in \text{IVQPNFM}_{nn}$ is an IVQP s-RS IVQPNFM iff $R\left([S_T,S_C,S_I,S_F]_L\right) = R\left(V[S_T,S_C,S_I,S_F]_L{}^T V\right)$,

$R\left([S_T,S_C,S_I,S_F]_U\right) = R\left(V[S_T,S_C,S_I,S_F]_U{}^T V\right)$.

Definition 6.3. For an IVQPNFM $S =< [S_T,S_C,S_I,S_F]_L, [S_T,S_C,S_I,S_F]_U >$ is an IV s-k- RS IVQPNFM iff $R\left([S_T,S_C,S_I,S_F]_L\right) = R\left(KV[S_T,S_C,S_I,S_F]_L{}^T VK\right)$, $R\left([S_T,S_C,S_I,S_F]_U\right) =$

$R\left(KV[S_T,S_C,S_I,S_F]_U{}^T VK\right)$.

Lemma 6.1. For an IVQPNFM $S =< [S_T,S_C,S_I,S_F]_L, [S_T,S_C,S_I,S_F]_U > \in \text{IVQPNFM}_{nn}$ is an IVQP s- RS IVQPNFM $\Leftrightarrow$ $\text{V}S =< V[S_T,S_C,S_I,S_F]_L, \text{V}[S_T,S_C,S_I,S_F]_U >$ s- RS IVQPNFM.

$\Leftrightarrow S\,\text{V} =< [S_T,S_C,S_I,S_F]_L\,\text{V}, [S_T,S_C,S_I,S_F]_U\,\text{V} >$ is RS IVQPNFM.

Proof. Let an IVQPNFM $S =< [S_T,S_C,S_I,S_F]_L, [S_T,S_C,S_I,S_F]_U > \in \text{IVQPNFM}_{nn}$ is s-RS IVQPNFM

$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_L\right) = R\left(V[S_T,S_C,S_I,S_F]_L{}^T V\right)$

$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_L\,\text{V}\right) = R\left([S_T,S_C,S_I,S_F]_L V\right)^T$

$\Leftrightarrow [S_T,S_C,S_I,S_F]_L\,\text{V}$ is RS. [By P.2.2]

$\Leftrightarrow R\left(\text{V}[S_T,S_C,S_I,S_F]_L\,\text{VV}^T\right) = R\left(VV[S_T,S_C,S_I,S_F]_L{}^T V\right)$

$\Leftrightarrow R\left(\text{V}[S_T,S_C,S_I,S_F]_L\right) = R\left(\text{V}[S_T,S_C,S_I,S_F]_L\right)^T$

$\Leftrightarrow \text{V}[S_T,S_C,S_I,S_F]_L$ is RS.

Similar manner

$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_U\right) = R\left(V[S_T,S_C,S_I,S_F]_U{}^T V\right)$

$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_U\,\text{V}\right) = R\left([S_T,S_C,S_I,S_F]_U V\right)^T$

$\Leftrightarrow [S_T, S_C, S_I, S_F]_U \, \mathrm{V}$ is RS.

$\Leftrightarrow R\left(\mathrm{V}[S_T, S_C, S_I, S_F]_U \, \mathrm{V}\mathrm{V}^T\right) = R\left(VV[S_T, S_C, S_I, S_F]_U{}^T V\right)$

$\Leftrightarrow R\left(\mathrm{V}[S_T, S_C, S_I, S_F]_U\right) = R\left(\mathrm{V}[S_T, S_C, S_I, S_F]_U\right)^T$

$\Leftrightarrow \mathrm{V}[S_T, S_C, S_I, S_F]_U$ is RS.

Therefore, $\mathrm{V}\, S = < V[S_T, S_C, S_I, S_F]_L, \mathrm{V}[S_T, S_C, S_I, S_F]_U >$ is an IV symmetric.

Example 6.1 Let us consider an IVQPNFM

$$S = \begin{bmatrix} <[1,1],[1,1],[0,0],[0,0]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5],[0.3,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5],[0.3,0.5]> & <[1,1],[1,1],[0,0],[0,0]> \end{bmatrix}$$

Lower Limit QPNFM, $[S_T, S_C, S_I, S_F]_L = \begin{bmatrix} <1,1,0,0> & <0.1,0.2,0,2,0.3> \\ <0.1,0.2,0.2,0.3> & <1,1,0,0> \end{bmatrix}$,

Upper Limit QPNFM, $[S_T, S_C, S_I, S_F]_U = \begin{bmatrix} <1,1,0,0> & <0.3,0.4,0.5,0.5> \\ <0.3,0.4,0.5,0.5> & <1,1,0,0> \end{bmatrix}$

$$V = \begin{bmatrix} <0,0,1,1> & <1,1,0,0> \\ <1,1,0,0> & <0,0,1,1> \end{bmatrix}, \ \mathrm{K} = \begin{bmatrix} <1,1,0,0> & <0,0,1,1> \\ <0,0,1,1> & <1,1,0,0> \end{bmatrix},$$

$$KV[S_T, S_C, S_I, S_F]_L{}^T VK = \begin{bmatrix} <1,1,0,0> & <0,0,1,1> \\ <0,0,1,1> & <1,1,0,0> \end{bmatrix} \begin{bmatrix} <0,0,1,1> & <1,1,0,0> \\ <1,1,0,0> & <0,0,1,1> \end{bmatrix}$$

$$\begin{bmatrix} <1,1,0,0> & <0.1,0.2,0,2,0.3> \\ <0.1,0.2,0.2,0.3> & <1,1,0,0> \end{bmatrix} \begin{bmatrix} <0,0,1,1> & <1,1,0,0> \\ <1,1,0,0> & <0,0,1,1> \end{bmatrix} \begin{bmatrix} <1,1,0,0> & <0,0,1,1> \\ <0,0,1,1> & <1,1,0,0> \end{bmatrix}$$

$$KV[S_T, S_C, S_I, S_F]_L{}^T VK = \begin{bmatrix} <1,1,0,0> & <0.1,0.2,0,2,0.3> \\ <0.1,0.2,0.2,0.3> & <1,1,0,0> \end{bmatrix}$$

$$KV[S_T, S_C, S_I, S_F]_L{}^T VK = [S_T, S_C, S_I, S_F]_L$$

Similarly, $KV[S_T, S_C, S_I, S_F]_U{}^T VK = [S_T, S_C, S_I, S_F]_U$

$$[S_T, S_C, S_I, S_F]_L = K[S_T, S_C, S_I, S_F]_L K$$

$$K[S_T,S_C,S_I,S_F]_L K = \begin{bmatrix} <1,1,0,0> & <0,0,1,1> \\ <0,0,1,1> & <1,1,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0,0> & <0.1,0.2,0,2,0.3> \\ <0.1,0.2,0.2,0.3> & <1,1,0,0> \end{bmatrix}$$

$$\begin{bmatrix} <1,1,0,0> & <0,0,1,1> \\ <0,0,1,1> & <1,1,0,0> \end{bmatrix}$$

$$K[S_T,S_C,S_I,S_F]_L K = \begin{bmatrix} <1,1,0,0> & <0.1,0.2,0,2,0.3> \\ <0.1,0.2,0.2,0.3> & <1,1,0,0> \end{bmatrix} = [S_T,S_C,S_I,S_F]_L$$

Similarly, $[S_T,S_C,S_I,S_F]_U = K[S_T,S_C,S_I,S_F]_U K$

Therefore S is symmetric NFM and its satisfies the condition

$$[S_T,S_C,S_I,S_F]_L = [S_T,S_C,S_I,S_F]_L{}^T, [S_T,S_C,S_I,S_F]_U = [S_T,S_C,S_I,S_F]_U{}^T$$

Therefore S is range symmetric QPNFM and its satisfies the condition

$$R[S_T,S_C,S_I,S_F]_L = R[S_T,S_C,S_I,S_F]_L{}^T, R[S_T,S_C,S_I,S_F]_U = R[S_T,S_C,S_I,S_F]_U{}^T$$

Therefore S is kernel symmetric QPNFM and its satisfies the condition

$$N[S_T,S_C,S_I,S_F]_L = N[S_T,S_C,S_I,S_F]_L{}^T, N[S_T,S_C,S_I,S_F]_U = N[S_T,S_C,S_I,S_F]_U{}^T$$

Therefore S is s-k symmetric QPNFM and its satisfies the condition

$$R\left([S_T,S_C,S_I,S_F]_L\right) = R\left(KV[S_T,S_C,S_I,S_F]_L{}^T VK\right),$$

$$R\left([S_T,S_C,S_I,S_F]_U\right) = R\left(KV[S_T,S_C,S_I,S_F]_U{}^T VK\right).$$

not both k –symmetric and s- k - symmetric NFM.
Therefore S is both k –symmetric and and s- k - symmetric QPNFM and also s-k- RS QPNFM.

Theorem 6.1. The following conditions are equivalent for $S \in \mathrm{IVQPNFM}_n$

(i) $S = <[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U> \in \mathrm{IVQPNFM}_{nn}$ is an IVQP s – k RS.

(ii) $\mathrm{KV}\,S = <KV[S_T,S_C,S_I,S_F]_L, \mathrm{KV}[S_T,S_C,S_I,S_F]_U>$ is an IVQP RS.

(iii) $S\,\mathrm{KV} = <[S_T,S_C,S_I,S_F]_L KV, [S_T,S_C,S_I,S_F]_U KV>$ is an IVQP RS.

(iv) $\mathrm{V}\,S = <V[S_T,S_C,S_I,S_F]_L, \mathrm{V}[S_T,S_C,S_I,S_F]_U>$ is an IVQP k- RS.

(v) $S\,\mathrm{K} = < [S_T, S_C, S_I, S_F]_L\,\mathrm{K}, [S_T, S_C, S_I, S_F]_U\,\mathrm{K} >$ is an IVQP s- RS.

(vi) S^T is an IVQP s-k RS.

(vii) $\mathrm{R}([S_T, S_C, S_I, S_F]_L) = \left(\mathrm{R}[S_T, S_C, S_I, S_F]_L{}^T\,\mathrm{VK}\right),$

$\mathrm{R}([S_T, S_C, S_I, S_F]_U) = \left(\mathrm{R}[S_T, S_C, S_I, S_F]_U{}^T\,\mathrm{VK}\right)$

(viii) $\mathrm{R}([S_T, S_C, S_I, S_F]_L{}^T) = \left(\mathrm{R}[S_T, S_C, S_I, S_F]_L\,\mathrm{VK}\right),$

$\mathrm{R}([S_T, S_C, S_I, S_F]_U{}^T) = \left(\mathrm{R}[S_T, S_C, S_I, S_F]_U\,\mathrm{VK}\right)$

(ix) $\mathrm{C}(\mathrm{KV}[S_T, S_C, S_I, S_F]_L) = C\left(\mathrm{KV}[S_T, S_C, S_I, S_F]_L{}^T\right)^T,$

$\mathrm{C}(\mathrm{KV}[S_T, S_C, S_I, S_F]_U) = C\left(\mathrm{KV}[S_T, S_C, S_I, S_F]_U{}^T\right)^T$

(x) $[S_T, S_C, S_I, S_F]_L = \mathrm{VK}[S_T, S_C, S_I, S_F]_L{}^T\,\mathrm{VKH}_1,$

$[S_T, S_C, S_I, S_F]_U = \mathrm{VK}[S_T, S_C, S_I, S_F]_U{}^T\,\mathrm{VKH}_1$ for $\mathrm{H}_1 \in IVQPNFM$

(xi) $[S_T, S_C, S_I, S_F]_L = \mathrm{H}_1\mathrm{KV}[S_T, S_C, S_I, S_F]_L{}^T\,\mathrm{KV},$

$[S_T, S_C, S_I, S_F]_U = \mathrm{H}_1\mathrm{KV}[S_T, S_C, S_I, S_F]_U{}^T\,\mathrm{VK}$ for $\mathrm{H}_1 \in IVQPNFM$

(xii) $[S_T, S_C, S_I, S_F]_L{}^T = \mathrm{KV}[S_T, S_C, S_I, S_F]_L\,\mathrm{VKH}_1,$

$[S_T, S_C, S_I, S_F]_U{}^T = \mathrm{KV}[S_T, S_C, S_I, S_F]_U\,\mathrm{VKH}_1$ for $\mathrm{H}_1 \in IVQPNFM$

(xiii) $[S_T, S_C, S_I, S_F]_L{}^T = \mathrm{H}_1\mathrm{KV}[S_T, S_C, S_I, S_F]_L\,\mathrm{KV},$

$[S_T, S_C, S_I, S_F]_U = \mathrm{H}_1\mathrm{KV}[S_T, S_C, S_I, S_F]_U\,\mathrm{VK}$ for $\mathrm{H}_1 \in IVQPNFM$

Proof: (i) iff (ii) iff (iv)

Let $S = < [S_T, S_C, S_I, S_F]_L, [S_T, S_C, S_I, S_F]_U > \in \mathrm{IVQPNFM}_{nn}$ is an IVQP s – κ RS

Let $[S_T, S_C, S_I, S_F]_L$ is an IVQP s – κ RS.

$$\Leftrightarrow R([S_T, S_C, S_I, S_F]_L) = R(KV[S_T, S_C, S_I, S_F]_L{}^T VK),$$

$$\mathrm{R}([S_T, S_C, S_I, S_F]_U) = R(KV[S_T, S_C, S_I, S_F]_U{}^T VK),$$

$\Leftrightarrow R(KV[S_T,S_C,S_I,S_F]_L)=R(KV[S_T,S_C,S_I,S_F]_L)^T,$

$R([S_T,S_C,S_I,S_F]_U)=R(K[S_T,S_C,S_I,S_F]_U)^T$ By (P.2.3)

$\Leftrightarrow$ KV $S=< KV[S_T,S_C,S_I,S_F]_L, KV[S_T,S_C,S_I,S_F]_U >$ is an IVQP RS

$\Leftrightarrow$ V $S=< V[S_T,S_C,S_I,S_F]_L, V[S_T,S_C,S_I,S_F]_U >$ is an IVQP k- RS

As a conclusion (i) iff (ii) iff (iv) is true

(i) iff (ii) iff (v)

Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \text{IVQPNFM}_{nn}$ is an IVQP s – κ RS

$\Leftrightarrow R(KV[S_T,S_C,S_I,S_F]_L)=R(KV[S_T,S_C,S_I,S_F]_L)^T,$

$R(KV[S_T,S_C,S_I,S_F]_U)=R(KV[S_T,S_C,S_I,S_F]_U)^T,$,

$\Leftrightarrow R(VK(KV[S_T,S_C,S_I,S_F]_L))=R((VK)[S_T,S_C,S_I,S_F]_L{}^T VK(VK)^T)$

$R(VK(KV[S_T,S_C,S_I,S_F]_U))=R((VK)[S_T,S_C,S_I,S_F]_U{}^T VK(VK)^T)$

$\Leftrightarrow SKV=\left[[S_T,S_C,S_I,S_F]_L KV,[S_T,S_C,S_I,S_F]_U KV\right]$ is an IVQP RS

$\Leftrightarrow SK=\left[[S_T,S_C,S_I,S_F]_L K,[S_T,S_C,S_I,S_F]_U K\right]$ is an IVQP s- RS

As a conclusion (i) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (v) is true. (ii) $\Leftrightarrow$ (ix)

$KVS=\left[KV[S_T,S_C,S_I,S_F]_L, KV[S_T,S_C,S_I,S_F]_U\right]$ is an IVQP RS

$\Leftrightarrow R\left(KV[S_T,S_C,S_I,S_F]_L\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right),$

$R\left(KV[S_T,S_C,S_I,S_F]_U\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$

(ii) $\Leftrightarrow$ (ix) is true. (ii) $\Leftrightarrow$ (vii)

$KVS=\left[KV[S_T,S_C,S_I,S_F]_L, KV[S_T,S_C,S_I,S_F]_U\right]$ is an IVQP RS.

$\Leftrightarrow R\left(KV[S_T,S_C,S_I,S_F]_L\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right),$

$R\left(KV[S_T,S_C,S_I,S_F]_U\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$

$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_L\right)=R\left([S_T,S_C,S_I,S_F]_L{}^T VK\right),$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)=R\left([S_T,S_C,S_I,S_F]_U{}^T VK\right)$$

As a conclusion (ii) $\Leftrightarrow$ (vii) is true. (iii) $\Leftrightarrow$ (viii)

$$SVK=\left[[S_T,S_C,S_I,S_F]_L VK,[S_T,S_C,S_I,S_F]_U VK\right]$$

$$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_L VK\right)=R\left(\left([S_T,S_C,S_I,S_F]_L VK\right)^T\right),$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U VK\right)=R\left(\left([S_T,S_C,S_I,S_F]_U VK\right)^T\right)$$

$$\Leftrightarrow R\left([S_T,S_C,S_I,S_F]_L VK\right)=R\left([S_T,S_C,S_I,S_F]_L\right)^T,$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U VK\right)=R\left([S_T,S_C,S_I,S_F]_U\right)^T$$

As a conclusion (iii) $\Leftrightarrow$ (viii) is true.

(i) $\Leftrightarrow$ (vi)

Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP s – κ RS

$$\Leftrightarrow R([S_T,S_C,S_I,S_F]_L)=R(KV[S_T,S_C,S_I,S_F]_L{}^T VK),$$

$$\mathrm{R}([S_T,S_C,S_I,S_F]_U)=R(KV[S_T,S_C,S_I,S_F]_U{}^T VK),$$

$\Leftrightarrow\quad (KVS)^T=(KV[S_T,S_C,S_I,S_F]_L,KV[S_T,S_C,S_I,S_F]_U)^T$ is an IVQP RS

$\Leftrightarrow\quad S^T VK=([S_T,S_C,S_I,S_F]_L VK,[S_T,S_C,S_I,S_F]_U VK)$ is an IVQP RS

$\Leftrightarrow\quad S^T=\left([S_T,S_C,S_I,S_F]_L{}^T,[S_T,S_C,S_I,S_F]_U{}^T\right)$ is an IVQP s – κ RS

As a conclusion (i) $\Leftrightarrow$ (vi) is true

(i) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (xi)

Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP s – κ RS

Consider $[S_T,S_C,S_I,S_F]_L$ is a s – κ RS

$$\Leftrightarrow C([S_T,S_C,S_I,S_F]_L{}^T)=C(\mathrm{KV}[S_T,S_C,S_I,S_F]_L VK),$$

$\mathrm{C}([S_T,S_C,S_I,S_F]_U{}^T)=C(\mathrm{KV}[S_T,S_C,S_I,S_F]_U VK)$ By (P.2.3)

$$\Leftrightarrow [S_T,S_C,S_I,S_F]_L=H_1\,\mathrm{KV}[S_T,S_C,S_I,S_F]_L{}^T VK,$$

$[S_T,S_C,S_I,S_F]_U{}^T=H_1\,\mathrm{KV}[S_T,S_C,S_I,S_F]_U VK$ for $\mathrm{H}_1\in IVNFM$.

As a result (i) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (xi) true.

(ii) $\Leftrightarrow$ (xiii) $\Leftrightarrow$ (x)

$\Leftrightarrow SVK = \left[[S_T, S_C, S_I, S_F]_L VK, [S_T, S_C, S_I, S_F]_U VK\right]$ is an IVQP RS

$\Leftrightarrow SV = \left[[S_T, S_C, S_I, S_F]_L V, [S_T, S_C, S_I, S_F]_U V\right]$ is an IVQP κ- RS

$$\Leftrightarrow R(\mathrm{V}[S_T, S_C, S_I, S_F]_L) = R(\mathrm{K}\left(\mathrm{V}[S_T, S_C, S_I, S_F]_L\right)^T K),$$

$$\mathrm{R}(\mathrm{V}[S_T, S_C, S_I, S_F]_U) = R(K\left(V[S_T, S_C, S_I, S_F]_U\right)^T K),$$

$$\Leftrightarrow R([S_T, S_C, S_I, S_F]_L) = R\left([S_T, S_C, S_I, S_F]_L\right)^T VK),$$

$$\mathrm{R}([S_T, S_C, S_I, S_F]_U) = R\left([S_T, S_C, S_I, S_F]_U\right)^T VK),$$

$$\Leftrightarrow C([S_T, S_C, S_I, S_F]_L{}^T) = C\left(\mathrm{KV}[S_T, S_C, S_I, S_F]_L\right)^T K),$$

$$\mathrm{C}([S_T, S_C, S_I, S_F]_U{}^T) = C\left(KV[S_T, S_C, S_I, S_F]_U\right)$$

$$[S_T, S_C, S_I, S_F]_L = VK[S_T, S_C, S_I, S_F]_L{}^T \mathrm{VK}\, H_1,$$

$$[S_T, S_C, S_I, S_F]_U = VK[S_T, S_C, S_I, S_F]_U \mathrm{VK} \; for\, H_1 \in IVNFM$$

As a conclusion (ii) $\Leftrightarrow$ (xiii) $\Leftrightarrow$ (x) is true

The above statement can be reduced to the equivalent requirement that a matrix be an IVQP s- RS for K = I in particular.

Corollary:6.1 The following statements are equivalent for $S \in \mathrm{IVQPNFM}_{nn}$

(i) $S = < [S_T, S_C, S_I, S_F]_L, [S_T, S_C, S_I, S_F]_U > \in \mathrm{IVQPNFM}_{nn}$ is an IV s-RS.

(ii) $VS = < V[S_T, S_C, S_I, S_F]_L, \mathrm{V}[S_T, S_C, S_I, S_F]_U >$ is an IVQP RS.

(iii) $SV = < [S_T, S_C, S_I, S_F]_L \mathrm{V}, [S_T, S_C, S_I, S_F]_U \mathrm{V} >$ is an IVQP RS.

(iv) $S^T = < [S_T, S_C, S_I, S_F]_L{}^T, [S_T, S_C, S_I, S_F]_U{}^T >$ is an IVQP s – RS.

(v) $R\left([S_T, S_C, S_I, S_F]_L\right) = R\left([S_T, S_C, S_I, S_F]_L{}^T V\right)$,

$\mathrm{R}\left([S_T, S_C, S_I, S_F]_U\right) = R\left([S_T, S_C, S_I, S_F]_U{}^T V\right)$

(vi) $R\left([S_T,S_C,S_I,S_F]_L{}^T\right)=R\left([S_T,S_C,S_I,S_F]_L V\right)$,

$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U{}^T\right)=R\left([S_T,S_C,S_I,S_F]_U \mathrm{V}\right)$

(vii) $C\left(\mathrm{KV}[S_T,S_C,S_I,S_F]_L\right)=C\left(\mathrm{V}[S_T,S_C,S_I,S_F]_L\right)^T$,

$\mathrm{C}\left(\mathrm{KV}[S_T,S_C,S_I,S_F]_U\right)=C\left(\mathrm{V}[S_T,S_C,S_I,S_F]_U\right)^T$

(viii) $[S_T,S_C,S_I,S_F]_L=\mathrm{V}[S_T,S_C,S_I,S_F]_L{}^T\ \mathrm{VH}_1$,

$[S_T,S_C,S_I,S_F]_U=\mathrm{V}[S_T,S_C,S_I,S_F]_U{}^T\ \mathrm{VH}_1$ for $\mathrm{H}_1\in IVQPNFM$

(ix) $[S_T,S_C,S_I,S_F]_L=\mathrm{H}_1\mathrm{V}[S_T,S_C,S_I,S_F]_L{}^T\ \mathrm{V}$,

$[S_T,S_C,S_I,S_F]_U=\mathrm{H}_1\mathrm{V}[S_T,S_C,S_I,S_F]_U{}^T\ \mathrm{V}$ for $\mathrm{H}_1\in IVQPNFM$

(x) $[S_T,S_C,S_I,S_F]_L{}^T=\mathrm{V}[S_T,S_C,S_I,S_F]_L\ \mathrm{VH}_1$,

$[S_T,S_C,S_I,S_F]_U{}^T=\mathrm{V}[S_T,S_C,S_I,S_F]_U\ \mathrm{VH}_1$ for $\mathrm{H}_1\in IVQPNFM$

(xi) $[S_T,S_C,S_I,S_F]_L{}^T=\mathrm{H}_1\mathrm{V}[S_T,S_C,S_I,S_F]_L\ \mathrm{V}$,

$[S_T,S_C,S_I,S_F]_U=\mathrm{H}_1\mathrm{V}[S_T,S_C,S_I,S_F]_U\ \mathrm{V}$ for $\mathrm{H}_1\in IVQPNFM$

Theorem 6.2. For $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ then any two of the conditions below imply the other

(i) $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP κ- RS.

(ii) $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP s-κ- RS.

(iii) $R\left([S_T,S_C,S_I,S_F]_L\right)^T=R\left(\mathrm{VK}[S_T,S_C,S_I,S_F]_L\right)^T$,

$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)^T=R\left(\mathrm{VK}[S_T,S_C,S_I,S_F]_U{}^T\right)$

Proof: (i) and (ii) implies (iii)

Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP s – k RS

$$\Rightarrow R\left([S_T,S_C,S_I,S_F]_L\right)=R\left([S_T,S_C,S_I,S_F]_L{}^T VK\right),$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)=R\left([S_T,S_C,S_I,S_F]_U{}^T VK\right)$$

$$\Rightarrow R\left(K[S_T,S_C,S_I,S_F]_L K\right)=R\left(K[S_T,S_C,S_I,S_F]_L{}^T K\right),$$

$$\mathrm{R}\left(K[S_T,S_C,S_I,S_F]_U K\right)=R\left(K[S_T,S_C,S_I,S_F]_U{}^T K\right)$$

$$\Rightarrow R[S_T,S_C,S_I,S_F]_L{}^T=R\left(\left(VK[S_T,S_C,S_I,S_F]_L\right)^T\right),$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)^T=R\left(\left(VK[S_T,S_C,S_I,S_F]_U\right)^T\right)$$

(i) & (ii) implies (iii) is true

(i)& (iii) implies (ii)

$S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP κ- RS

$$\Rightarrow R\left(K[S_T,S_C,S_I,S_F]_L K\right)=R\left(\left([S_T,S_C,S_I,S_F]_L\right)^T\right),$$

$$\mathrm{R}\left(K[S_T,S_C,S_I,S_F]_U K\right)=R\left(\left([S_T,S_C,S_I,S_F]_U\right)^T\right)$$

Therefore, (i) & (iii)

$$\Rightarrow R\left([S_T,S_C,S_I,S_F]\right)=R\left([S_T,S_C,S_I,S_F]_L{}^T VK\right),$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)=R\left([S_T,S_C,S_I,S_F]_U{}^T VK\right)$$

$$\Rightarrow R\left([S_T,S_C,S_I,S_F]_L\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right),$$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$$

$S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IV s-k-RS

$\Rightarrow$ (ii) is true

(ii) & (iii) implies (i)

$S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP s- κ- RS

$$\Rightarrow R\left(K[S_T,S_C,S_I,S_F]_L K\right)=R\left(K[S_T,S_C,S_I,S_F]_L{}^T K\right),$$

$$\mathrm{R}\left(K[S_T,S_C,S_I,S_F]_U K\right)=R\left(K[S_T,S_C,S_I,S_F]_U{}^T K\right)$$

Therefore,(ii) and (iii) $\Rightarrow R\left([S_T,S_C,S_I,S_F]_L\right)=R\left(K[S_T,S_C,S_I,S_F]_L{}^T K\right),$

$$\mathrm{R}\left([S_T,S_C,S_I,S_F]_U\right)=R\left(K[S_T,S_C,S_I,S_F]_U{}^T K\right)$$

$\mathrm{S}=<[S_T,S_C,S_I,S_F],[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$ is an IVQP κ- RS .

Therefore, (i) is true, Hence the theorem.

7. IVQP s – k RS regular Neutrosophic fuzzy matrices

In this section, we explore the various generalized inverses of matrices in the context of Interval-Valued Quasi-Periodic Neutrosophic Fuzzy Matrices (IVQPNFM). We establish the necessary criteria for different generalized inverses of an IVQP s−ks-ks−k Range Symmetric (RS) Neutrosophic Fuzzy Matrix to also qualify as IVQP s−ks-ks−k RS matrices. Additionally, we characterize the generalized inverses of an IVQP s−k RS matrix corresponding to the sets S={1,2}, S={1,2,3}, and S={1,2,4}. This characterization enhances our understanding of the structural properties of these matrices and their inverses.

Theorem 7.1: Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$, Z belongs to S{1,2} and SZ, ZS are an IV s- κ- RS. Then S is an IVQP s- κ - RS iff $Z=<[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L,[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U>$ is an IVQP s- κ – RS.

Proof: Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \mathrm{IVQPNFM}_{nn}$

$$R\left(KV[S_T,S_C,S_I,S_F]_L\right)=R\left(KV[S_T,S_C,S_I,S_F]_L\,\mathrm{Z}[S_T,S_C,S_I,S_F]\right)\subseteq R\left(\mathrm{Z}[S_T,S_C,S_I,S_F]_L\right)$$

$$=R\left(ZVV[S_T,S_C,S_I,S_F]_L\right)\subseteq R\left(ZVKKV[S_T,S_C,S_I,S_F]_L\right)\subseteq R\left(KV[S_T,S_C,S_I,S_F]_L\right)$$

Hence, $\mathrm{R}\left(KV[S_T,S_C,S_I,S_F]_L\right)=R\left(\mathrm{Z}[S_T,S_C,S_I,S_F]_L\right)$

$$=\mathrm{R}\left(KV\left(\mathrm{Z}[S_T,S_C,S_I,S_F]_L\right)^T VK\right)$$

$$=\mathrm{R}\left([S_T,S_C,S_I,S_F]_L{}^T[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L{}^T VK\right)$$

$$=\mathrm{R}\left([\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L{}^T VK\right)=\mathrm{R}\left(\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)^T\right)$$

$$R\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right)=R\left([S_T,S_C,S_I,S_F]_L{}^T VK\right)$$

$$=\mathrm{R}\left(KV[S_T,S_C,S_I,S_F]_L[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)$$

$$=\mathrm{R}\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)$$

Similarly,

Hence, $\mathrm{R}\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$

$\Leftrightarrow R\left(KV[S_T,S_C,S_I,S_F]_L\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]\right)^T\right),$

$R\left(KV[S_T,S_C,S_I,S_F]_U\right)=R\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$

$\Leftrightarrow R\left(KV[Z_T,Z_C,Z_I,Z_F]_L\right)=R\left(\left(KV[Z_T,Z_C,Z_I,Z_F]_L\right)^T\right),$

$R\left(KV[Z_T,Z_C,Z_I,Z_F]_U\right)=R\left(\left(KV[Z_T,Z_C,Z_I,Z_F]_U\right)^T\right)$

$\Leftrightarrow KVX=\left[KV[Z_T,Z_C,Z_I,Z_F]_L,KV[Z_T,Z_C,Z_I,Z_F]_U\right]$ is an IVQP RS

$Z=<[Z_T,Z_C,Z_I,Z_F]_L,[Z_T,Z_C,Z_I,Z_F]>$ is an IVQP RS.

Theorem 7.2:, Let $S=<[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U>\in \text{IVQPNFM}_{nn}$, $Z=<[Z_T,Z_C,Z_I,Z_F]_L$ $,[Z_T,Z_C,Z_I,Z_F]_U>\in$S{1,2,3 $\text{R}(\text{KV}[S_T,S_C,S_I,S_F]_L)=R\left(\text{KV}[X_{\mu L},X_{\lambda L},X_{\nu L}]\right)^T$ $,\text{R}(\text{KV}[S_T,S_C,S_I,S_F]_U)=R\left(\text{KV}[Z_{\mu U},Z_{\lambda U},Z_{\nu U}]\right)^T$.Then $S=<[S_T,S_C,S_I,S_F]_L,$ $[S_T,S_C,S_I,S_F]_U>\in \text{IVQPNFM}_{nn}$ is IVQP s-κ- RS $\Leftrightarrow$ $Z=<[Z_T,Z_C,Z_I,Z_F]_L,[Z_T,Z_C,Z_I,Z_F]_U>$ is IVQP s- k – RS.

Proof: Given P{1,2,3}, Hence ,

$[S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L[S_T,S_C,S_I,S_F]_L=[S_T,S_C,S_I,S_F]_L,$

$[Z_T,Z_C,Z_I,Z_F]_L[S_T,S_C,S_I,S_F][Z_T,Z_C,Z_I,Z_F]_L=[Z_T,Z_C,Z_I,Z_F]_L,$

$\left([S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L\right)^T=[S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L$

Consider, $\text{R}\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right)$ = $\text{R}\left([Z_T,Z_C,Z_I,Z_F]_L{}^T[S_T,S_C,S_I,S_F]_L{}^T VK\right)$

= $\text{R}\left(KV\left([S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L\right)^T\right)$

= $\text{R}\left(\left([S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L\right)^T\right)$ $\left[By\ P._{2.3}\right]$

= $\text{R}\left([S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L\right)$

= $\text{R}\left([Z_T,Z_C,Z_I,Z_F]_L\right)$

$\left[\text{By using } [Z_T,Z_C,Z_I,Z_F]_L=[Z_T,Z_C,Z_I,Z_F]_L[S_T,S_C,S_I,S_F]_L[Z_T,Z_C,Z_I,Z_F]_L\right]$

= $\text{R}\left(KV[Z_T,Z_C,Z_I,Z_F]_L\right)$ $\left[By\ P_{2.3}\right]$

Similarly, we can consider, $\mathrm{R}\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right) = \mathrm{R}\left([\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_v]_U{}^T[S_T,S_C,S_I,S_F]_U{}^T VK\right)$

$= \mathrm{R}\left(KV\left([S_T,S_C,S_I,S_F]_U[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)^T\right) = \mathrm{R}\left(\left([S_T,S_C,S_I,S_F]_U[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)^T\right)$

$= \mathrm{R}\left([S_T,S_C,S_I,S_F]_U[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)$ $\left[(PZ)^T = \mathrm{PZ}\right]$

$= \mathrm{R}\left([\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)$ $\left[\text{By using Z} = \mathrm{Z}PZ\right]$

$= \mathrm{R}\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)$ $\left[\text{By } P_{2.3}\right]$

If KVA is an IVQP RS

$\Leftrightarrow R\left(KV[S_T,S_C,S_I,S_F]_L\right) = \mathrm{R}\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right)$,

$R\left(KV[S_T,S_C,S_I,S_F]_U\right) = \mathrm{R}\left(\left(KV[S_T,S_C,S_I,S_F]_U\right)^T\right)$

$\Leftrightarrow R\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right) = \mathrm{R}\left(\left(KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)^T\right)$,

$KVX = [K[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L, KV[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U]$ is an IVQP RS.

$Z = <[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L,[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U>$ is an IVQP s-k RS.

Theorem 7.3: Let $S = <[S_T,S_C,S_I,S_F]_L,[S_T,S_C,S_I,S_F]_U> \in \mathrm{IVQPNFM}_{nn}$, Z ∈ S {1, 2, 4},

$\mathrm{R(KV}[S_T,S_C,S_I,S_F]_L)^T = R\left(\mathrm{KV}[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right), \mathrm{R(KV}[S_T,S_C,S_I,S_F]_U)^T$

$= R\left(\mathrm{KV}[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U\right)$. Then KVP is an IV s- κ-Ks iff $Z = <[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L$,

$[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_U >$ is an IVQP s- κ- RS.

Proof: Given, S {1, 2, 4}, Hence $[S_T,S_C,S_I,S_F]_L[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L[S_T,S_C,S_I,S_F]_L = [S_T,S_C,S_I,S_F]_L$,

$[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L[\mathrm{P}_\mu,\mathrm{P}_\lambda,\mathrm{P}_v]_L[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L = [\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L$,

$\left([\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L[S_T,S_C,S_I,S_F]_L\right)^T = [\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L[S_T,S_C,S_I,S_F]_L$

Consider, $\mathrm{R}\left(\left(KV[S_T,S_C,S_I,S_F]_L\right)^T\right) = \mathrm{R}\left([\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L{}^T[S_T,S_C,S_I,S_F]_L{}^T VK\right)$

$= R\left(KV\left([S_T,S_C,S_I,S_F]_L[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)^T\right)$

$= R\left(\left([S_T,S_C,S_I,S_F]_L[\mathrm{Z}_T,\mathrm{Z}_C,\mathrm{Z}_I,\mathrm{Z}_F]_L\right)^T\right)$ $\left[By\ P_{2.3}\right]$

$$= R\left([S_T, S_C, S_I, S_F]_L [Z_T, Z_C, Z_I, Z_F]_L\right) = R\left([Z_T, Z_C, Z_I, Z_F]_L\right)$$

$$= R\left(KV[Z_T, Z_C, Z_I, Z_F]_L\right) \qquad \left[By\ P_{2.3}\right]$$

$$R\left(\left(KV[S_T, S_C, S_I, S_F]_U\right)^T\right) = R\left([Z_T, Z_C, Z_I, Z_F]_U{}^T [S_T, S_C, S_I, S_F]_U{}^T VK\right)$$

$$= R\left(KV\left([S_T, S_C, S_I, S_F]_U [Z_T, Z_C, Z_I, Z_F]_U\right)^T\right)$$

$$= R\left(\left([S_T, S_C, S_I, S_F]_U [Z_T, Z_C, Z_I, Z_F]_U\right)^T\right) \qquad \left[By\ P_{2.3}\right]$$

$$= R\left([S_T, S_C, S_I, S_F]_U [Z_T, Z_C, Z_I, Z_F]_U\right) \qquad \left[\ (S\,Z)^T = SZ\ \right]$$

$$= N\left([X_{\mu U}, X_{\lambda U}, X_{\nu U}]\right) = R\left(KV[Z_T, Z_C, Z_I, Z_F]_U\right) \qquad \left[By\ P_{2.3}\right]$$

If KVP is an IVQP RS

$$\Leftrightarrow R\left(KV[S_T, S_C, S_I, S_F]_L\right) = R\left(\left(KV[S_T, S_C, S_I, S_F]_L\right)^T\right),$$

$$R\left(KV[S_T, S_C, S_I, S_F]\right) = R\left(\left(KV[S_T, S_C, S_I, S_F]\right)^T\right)$$

$$\Leftrightarrow R\left(KV[Z_T, Z_C, Z_I, Z_F]_L\right) = R\left(\left(KV[Z_T, Z_C, Z_I, Z_F]_L\right)^T\right),$$

$KVX = [KV[Z_T, Z_C, Z_I, Z_F]_L, KV[Z_T, Z_C, Z_I, Z_F]_U]$ is an IVQP RS.

$Z = <[Z_T, Z_C, Z_I, Z_F],[Z_T, Z_C, Z_I, Z_F]>$ is an IVQP s-k RS.

The aforementioned Theorems reduce to comparable criteria, in particular for K = I, for different g-inverses of interval valued s- RS to be IV secondary RS.

Corollary 7.1: For $S = <[S_T, S_C, S_I, S_F]_L, [S_T, S_C, S_I, S_F]_U> \in$ IVQPNFM$_{nn}$, Z∈ P {1, 2} and

$SZ = <[S_T, S_C, S_I, S_F]_L [Z_T, Z_C, Z_I, Z_F], [S_T, S_C, S_I, S_F]_U [Z_T, Z_C, Z_I, Z_F]_U>$,

$ZS = <[Z_T, Z_C, Z_I, Z_F]_L [S_T, S_C, S_I, S_F]_L, [Z_T, Z_C, Z_I, Z_F]_U [S_T, S_C, S_I, S_F]_U>$, are is an IVQP s-RS. Then S is an IVQP s- RS iff $Z = <[Z_T, Z_C, Z_I, Z_F]_L, [Z_T, Z_C, Z_I, Z_F]_U>$ is an IVQP s- RS.

Corollary 7.2: For $S = <[S_T, S_C, S_I, S_F]_L, [S_T, S_C, S_I, S_F]_U> \in$ IVQPNFM$_{nn}$ Z∈ S {1, 2, 3},

$R(KV[S_T, S_C, S_I, S_F]_L) = R\left(V[Z_T, Z_C, Z_I, Z_F]_L\right)^T, R(KV[S_T, S_C, S_I, S_F])$

$= R\left(V[Z_T, Z_C, Z_I, Z_F]_U\right)^T$. Then S is an IVQP s- RS iff $Z = <[Z_T, Z_C, Z_I, Z_F]_L, [Z_T, Z_C, Z_I, Z_F]_U>$ is an IVQP s- RS.

Corollary 7.3: For $S =< [S_T, S_C, S_I, S_F]_L, [S_T, S_C, S_I, S_F]_U > \in$ IVQPNFM$_{nn}$, Z∈ P {1, 2, 4}

, $R(V[S_T, S_C, S_I, S_F]_L)^T = R\left(V[Z_T, Z_C, Z_I, Z_F]_L\right)^T$, $R(V[S_T, S_C, S_I, S_F]_U)^T$

$= R\left(V[Z_T, Z_C, Z_I, Z_F]_U\right)$. Then S is an IVQP s- RS iff Z is an IVQP s-RS.

8. Conclusion:

Firstly, we present equivalent characterizations for IVQP k-Range Symmetric (RS), IVQP RS, Interval-Valued s-Range Symmetric, and IVQP s−k Range Symmetric Neutrosophic Fuzzy Matrices. Additionally, we provide an example demonstrating that while an s−k-symmetric fuzzy matrix can be considered an s−k-RS Neutrosophic fuzzy matrix, the reverse does not necessarily hold true. Secondly, we examine various generalized inverses associated with regular matrices and establish a characterization of the complete set of all inverses. This exploration aids in understanding the intricate relationships between different types of matrix inverses. Thirdly, we determine equivalent conditions for various generalized inverses of IVQPNFM s−k-Range Symmetric and s-Range Symmetric Neutrosophic Fuzzy Matrices. This analysis provides crucial insights into the structural properties and potential applications of these matrices. Additionally, we provide a graphical representation of RS, CS, and KS adjacency and incidence QPNFM. While every adjacency QPNFM is symmetric, RS, CS, and KS, the incidence matrix satisfies only KS conditions. Every RS adjacency QPNFM is a KS adjacency QPNFM, but a KS adjacency QPNFM does not necessarily imply RS QPNFM. Soft graphs represent a novel area of research in mathematics. In this paper, we explore their application in decision-making by utilizing the adjacency matrix of a soft graph and developing a corresponding algorithm. Finally, we conclude by introducing the concept of IVQP Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices. Looking ahead, our future work will focus on investigating IVQP Secondary k-Kernel Symmetric Neutrosophic Fuzzy Matrices, further enriching the field of neutrosophic fuzzy matrix theory.

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