NSS

Neutrosophic Sets and Systems, Vol. 78, 2025

University of New Mexico

A Neutrosophic Fixed Charge Multi-Objective 4 Dimensional Multi-item Transportation Problem with Carbon Emission and Budget Constraints

A.N. Revathi[1] and S. Mohanaselvi[1,*]

[1]Department of Mathematics, SRM Valliammai Engineering College,

[1,*] Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chennai– 603 203, Tamil Nadu, India.

anrevathi 77@gmail.com

[*]Correspondence: mohanass@srmist.edu.in; Tel.:+91 − 9940087667

Abstract. This paper seeks to address 4-dimensional transportation challenges by integrating budget constraints, vehicle speed, carbon emissions, and optimizing for maximum profit and minimum time. The rising number of vehicles, varying road conditions, and diverse vehicle types used to meet daily needs have exacerbated issues like global warming and greenhouse gas (GHG) emissions. Our goal is to reduce carbon emissions for a cleaner environment. Single-objective, single-item transportation systems are often insufficient for scenarios with multiple criteria. Thus, we adopt a multi-objective, multi-item decision-making approach to tackle real-life transportation problems. Practical challenges frequently involve uncertainties stemming from time pressures, insufficient data, information gaps, or inaccuracies in measurement. Recognizing this, we incorporate indeterminacy in our problem design. To address the uncertainties and ambiguities found in fuzzy and intuitionistic fuzzy settings, we utilize a neutrosophic set, which is well-suited to handle such complexities. Consequently, we employ neutrosophic linear programming to derive a compromise solution for the proposed neutrosophic fixed charge multi-objective, four-dimensional, multi-item transportation problem, considering carbon emissions and budget constraints. The model's effectiveness is showcased using a numerical example, with the results displayed and analyzed. Finally, sensitivity analysis and conclusions, along with future research directions, are presented.

Keywords: Neutrosophic linear programming; Carbon emission;Fixed charge transportation problem; Decision making on multi-objective.

1. Introduction

In recent decades, the traditional transportation system has primarily focused on a single objective function. However, transportation problems in the real world are inherently multi-dimensional, involving criteria such as transit cost, average delivery time, and fixed charges (FC) for routes. These factors become critical when transporting a homogeneous product to various destinations within a competitive economic landscape. Due to these complexities, real-world decision-making cannot be solved effectively by single-objective transportation problem (TP) models.

To address this, traditional TPs have evolved to incorporate multi-objective decision-making models, which handle competing objectives such as cost, time, and carbon emissions. Hitchcock [1] first defined the TP, combining it with linear programming (Hitchcock-Koopmans). Numerous studies on multi-objective TP (MOTP) in both precise and imprecise environments have emerged since then. Maity et al. [2] investigated the application of artificial intelligence to multi-modal TP, while Ebrahimnejad [3] introduced an approach for solving TP using generalized trapezoidal fuzzy numbers. Li and Lai [4] explored MOTP in a fuzzy environment, and Maity et al. [5] designed a framework for uncertain environments, emphasizing cost reliability. These studies underline the increasing complexity of transportation issues and the need for sophisticated models.

Motivation for this research arises from the limitations of existing methods in handling real-world complexities, particularly the combination of fixed costs, multiple objectives, carbon emissions, and budget constraints. The fixed-charge transportation problem (FCTP), for example, extends the basic TP by introducing a 0-1 decision variable and fixed charges, including vehicle rental, highway tolls, and machine establishment costs. First introduced by Hirsch and Dantzig [6] , FCTP plays a crucial role in optimizing transportation costs by accounting for both fixed and direct costs. Recent research by Midya and Roy [7, 8] applied interval and rough interval programming to FCTP in stochastic environments, while Roy et al. [9] addressed random rough variables.

Tools such as interval programming and neutrosophic logic have emerged as powerful methods for addressing uncertainty in transportation problems. The Solid Transportation Problem (STP), originally defined by Haley [10], adds conveyance constraints along with source and destination constraints, while Roy and Midya [11] tackled the Fixed Charge Solid Transportation Problem (FCSTP) using intuitionistic fuzzy logic. Once a single-type conveyance is utilized in FCTP, TP is redefined and renamed as the fixed charge solid transportation problem (FC-STP). FCSTP was resolved in intuitionistic fuzzy (IF) by Roy and Midya [11] using concepts of product blending and multi-objective setting. The STP and facility location with carbon policies were represented by Das et al. [12]. A MOSTP with IF-type variables and parameters was solved by Ghosh et al. [13]. Using two-fold uncertainty, Roy et al. [14] examined FCSTP with multi-item and multi-objective.

In the context of an uncertain interval programming environment, Sifaouil and Aider [15] examined safety precautions and budgetary restrictions in a FCSTP with multiple objectives and items. The transportation industry primarily uses trucks, cars, ships, planes, buses, trains, and other modes of transportation to move goods and people. The transportation system is a major contributor to CO2 and other greenhouse gas emissions due to the combustion processes in engines. Light-duty vehicles, including passenger cars and minibuses, account for half of these emissions, while heavy-duty vehicles such as trucks, ships, and freight transport are responsible for the remaining half.

Air and environmental pollution are major risks associated with greenhouse gas emissions. The type of fuel, engine, traffic laws, road conditions, driving regulations, etc. all affect the carbon emission(CE). A range of researchers have explored CE across various contexts. Here are some key studies that focus on the impact of CE within transportation-related challenges These studies demonstrate the growing complexity of transportation problems, particularly when considering environmental factors like carbon emissions (CE). The transportation industry is a major source of CE, with light-duty vehicles and freight transport contributing heavily to greenhouse gas emissions.

Given the growing environmental concerns, many researchers have focused on the impact of CE in transportation. Ding et al. [16] examined potential reductions in China's transport sector, while Sengupta et al. [17] used the Gamma type-2 defuzzification approach to address CE within STPs. Song and Leng [18] explored CE policies for single-period problems. Strategies for reducing CO2 emissions in smart transport were studied by Tarulescu et al. [19]. FCSTP with multi-stage, multi-objective in a green supply chain was presented by Midya et al. [20]. These studies highlight the need for transportation models that effectively balance cost, time, and environmental impact. Ali and Muthuswamy [21] emphasize the need for sustainable evaluation of power production systems to achieve a cleaner energy future, incorporating environmental impacts, resource efficiency, and economic viability through a multi-criteria decision-making (MCDM) approach using the VIKOR method integrated with neutrosophic sets.

Novelty in this research lies in the introduction of a Neutrosophic Fixed Charge Multi-Objective 4-Dimensional Multi-Item Transportation Problem (NFCMO4DMITP) model, which integrates fixed charges, carbon emissions, and budget constraints. The neutrosophic theory provides a powerful framework to handle uncertainty, allowing for the formulation of models that account for both objective and subjective uncertainties in transportation scenarios. This model is particularly useful in multi-dimensional problems where multiple products, routes, and vehicles are involved.

The environment in which the model is applied involves uncertainty in both transportation costs and emissions, as well as the presence of budget constraints, making it suitable for real-world scenarios with complex logistics challenges. The contribution of this research is the development of a comprehensive model that addresses not only the traditional transportation objectives but also the emerging concerns related to carbon emissions and environmental sustainability.

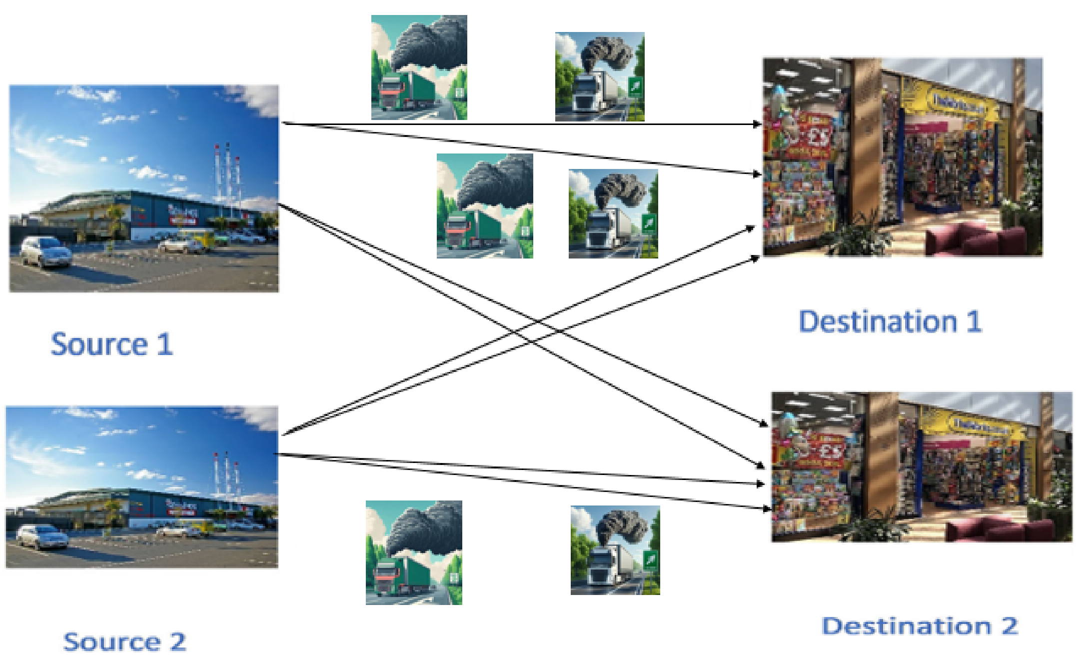

Figure 1. Pictorial representation of 4DTP

The rest of the paper is structured as follows: Section 2 provides fundamental definitions and theorems related to neutrosophic theory. Section 3 outlines the mathematical formulation

Table 1. Existing work on TP

References	Dimension	Objective	FC	TC	CE	Time	Environment	Solution method
Thamaraiselvi and Santhi [22]	2	S	-	✓	-	-	N	NLGP
Sengupta et al. [23]	3	M	-	✓	-	✓	T2Z	IFP
Roy et al. [24]	2	M	-	✓	✓	-	IF	Interval-valued IFP
Gessesse et al. [25]	2	M	-	✓	✓	-	Burr distribution	GA based FP
Paul et al. [26]	3	S	-	✓	-	-	N	LINGO 17.0 (ranking function)
Midya et al. [27]	3	M	✓	✓	✓	✓	IF	Min-max GP
Aktar et al. [28]	4	S	-	✓	_		T2F	GRGT
Adhami and Ahmad [29]	2	M	-	✓	-	-	PFN	PHFP
Malik and Gupta [30]	2	M	-	✓	-	-	Interval-valued IF	GP
Samanta et al. [31]	4	M	-	-	✓	-	LR-type IF	Convex combination method
Mohammed and Wang [32]	3	M	-	✓	✓	✓	F	LP-matrix, GP
Rizk-Allah et al. [33]	2	M	-	✓	✓	-	C	NCPA
Kumar et al. [34]	2	S	-	✓	-	-	PF	PF
Pratihar et al. [35]	2	S	✓	-	-	-	T2F	Modified VAM
Khalifa et al. [36]	2	S	-	✓	-	-	N	KKM approach
This paper	4	M	✓	✓	✓	✓	N	Interval-valued NP

S-Single, M-Multi, F-Fuzzy, N-Neutrosophic IF-Intuitionistic fuzzy, T2F-Type 2 Fuzzy, FP-Fuzzy Programming, PF-Pythagorean Fuzzy, PHFP-Pythagorean hesitant Fuzzy programming, T2Z-Type 2 zigzag, GP- Goal Programming, NP-Neutrosophic Programming, GA-Genetic Algorithm, GRGT- generalised reduced gradient technique, PFN- parabolic Fuzzy Number C-Crisp

of the proposed NFCMO4DMITP model. Section 4 details the solution methodology, while Section 5 discusses the benefits of the study. Finally, Sections 6, 7, and 8 present a numerical example, sensitivity analysis, and conclusions with future research directions.

2. **Preliminaries**

In this section, we begin by presenting some fundamental definitions of neutrosophic set theory, followed by a discussion on the neutrosophic compromise programming approach.

Definition 2.1. A neutrosophic set(NS) ${}^{n}\tilde{N}$ defined in the universal set Y is characterized by truth membership function (TMF) $T_{{}^{n}\tilde{N}}(y)$ indeterminacy membership function(IMF) $I_{{}^{n}\tilde{N}}(y)$

and a falsity membership function (FMF) $F_{^n\tilde{N}}(y)$ and is denoted by

$$^n\tilde{N} = \{\langle y, T_{^n\tilde{N}}(y), I_{^n\tilde{N}}(y), F_{^n\tilde{N}}(y)\rangle \,|y \in Y\} \tag{1}$$

where $T_{^n\tilde{N}}(y), I_{^n\tilde{N}}(y), F_{^n\tilde{N}}(y)$ are real standard or non standard subsets belonging to]0- ,1+[. Also TMF, IMF and FMF are the functions from Y to]0- ,1+[. Also we have $0^- \leq \sup T_{^n\tilde{N}}(y) + \sup I_{^n\tilde{N}}(y) + \sup F_{^n\tilde{N}}(y) \leq 3^+$.
Wang et al. [37] introduced Single valued Neutrosophic set (SVNS) in engineering problem as it is computationally more comfortable.

Definition 2.2. A SVNS $^n\tilde{N}$ defined on Y is expressed as
$^n\tilde{N} = \{\langle y, T_{^n\tilde{N}}(y), I_{^n\tilde{N}}(y), F_{^n\tilde{N}}(y)\rangle \,|y \in Y\}$ where
$T_{^n\tilde{N}}(y), I_{^n\tilde{N}}(y), F_{^n\tilde{N}}(y) \in [0,1], \forall y \in Y$ and $0 \leq T_{^n\tilde{N}}(y), I_{^n\tilde{N}}(y), F_{^n\tilde{N}}(y) \leq 3$.
Clearly, SVNS is subset of NS.

Definition 2.3. The complement of NS $^n\tilde{N}$ is denoted by $c^n\tilde{N}$ and is defined by $T_{c^n\tilde{N}}(y) = F_{^n\tilde{N}}(y), I_{c^n\tilde{N}}(y) = 1 - I_{^n\tilde{N}}(y)$, $F_{c^n\tilde{N}}(y) = T_{^n\tilde{N}}$, $\forall y \in Y$

Definition 2.4. The standard formulation of an optimization problem with i objectives, i variables and K constraints is as follows
Minimize $Z(Y) = (Z_1(y), Z_2(y), Z_3(y), \cdots, Z_i(y))$
subject to
$h_k(y) \leq 0, (k = 1, 2, \cdots, K)$
$y_u \geq 0, (u = 1, 2, \cdots, i), y_u \in Y \subset R^n$.

By individually solving the n objective functions subject to the constraints, we obtained i solutions $Y_1, Y_2, \cdots, Y_i$.
To find the bounds for each objective function, we obtain the values of each objective at each of these n solutions.

$$\begin{aligned} &(i.e.)\, U_i = \max\{Z_i(Y_1), Z_i(Y_2), \cdots, Z_i(Y_i)\} \\ &\text{and } L_i = \min\{Z_i(Y_1), Z_i(Y_2), \cdots, Z_i(Y_i)\} \end{aligned} \tag{2}$$

The TMF $T_{^n\tilde{N}}(y)$, IMF $I_{^n\tilde{N}}(y)$ and FMF $F_{^n\tilde{N}}(y)$ functions for the considered objective functions $Z_i(y)$ are respectively defined. Hence, the upper and lower bounds for TMF, IMF and FMF are given by

$$\left.\begin{aligned} &U_i^T = U_i, L_i^T = L_i \\ &U_i^F = U_i^T, L_i^F = L_i^T + t_i(U_i^T - L_i^T) \\ &U_i^I = L_i^T + s_i(U_i^T - L_i^T), L_i^I = L_i^T \end{aligned}\right\} \tag{3}$$

where t_i, s_i are predetermined real numbers in (0,1).

Using the above upper and lower bounds, TMF, IMF and FMF are interpreted as follows:

$$T_i(Z_i^*(y)) = \begin{cases} 1 & \text{if } Z_i^*(y) < L_i^T \\ \dfrac{U_i^T - Z_i^*(y)}{U_i^T - L_i^T} & \text{if } L_i^T \leq Z_i^*(y) \leq U_i^T \\ 0 & \text{if } Z_i^*(y) > U_i^T \end{cases} \quad (4)$$

$$I_i(Z_i^*(y)) = \begin{cases} 1 & \text{if } Z_i^*(y) < L_i^I \\ \dfrac{U_i^I - Z_i^*(y)}{U_i^I - L_i^I} & \text{if } L_i^I \leq Z_i^*(y) \leq U_i^I \\ 0 & \text{if } Z_i^*(y) > U_i^I \end{cases} \quad (5)$$

$$F_i(Z_i^*(y)) = \begin{cases} 1 & \text{if } Z_i^*(y) > U_i^F \\ \dfrac{Z_i^*(y) - L_i^F}{U_i^F - L_i^F} & \text{if } L_i^F \leq Z_i^*(y) \leq U_i^F \\ 0 & \text{if } Z_i^*(y) < L_i^F \end{cases} \quad (6)$$

where $U_i^{(.)} \neq L_i^{(.)}$ for all objectives.

Definition 2.5. Interval valued neutrosophic number: Samarandache [38] An neutrosophic number(NN) is represented by ${}^n\tilde{b} = c + dI$, where c,d $\in R$ and I $\subset [0,1]$ is the indeterminacy. Now $I = [I^L, I^U]$, and therefore the interval form of ${}^n\tilde{b} = [c + dI^L, c + dI^U] = [b^L, b^U]$.

Definition 2.6. Neutrosophic linear programming problem(NLPP): Ye [39] A NLPP is defined as the general optimization problem if the following conditions are met;

(1) The objective function in the neutrosophic framework is linear.
(2) Every decision variable is non-negative.
(3) All structural constraints are either " $\leq$ " or " $\geq$ " types.

Definition 2.7. Ya [39] In general, a constrained optimization problem in n decision variable with neutrosophic number(NN) is formulated as below:

$$\text{minimize/maximize}\,{}^nZ(y, I)$$
$$\text{subject to}$$
$$f_i(y, I) \leq 0, i = 1, 2, \cdots, m$$
$$p_k(y, I) = 0, k = 1, 2, \cdots, K$$

$y \in Y$ When indeterminacy I is treated as a potential interval range, the optimal solutions across all feasible intervals define the feasible region or feasible set for y and $I = [I^L, I^U]$. In this scenario, the value of the NN objective function represents an optimal possible interval NN for ${}^nZ(y, I)$. For instance, consider the following optimization problem with $I = [0, 1]$.

$$\text{maximize }{}^nZ(y, I) = (4 + 5I)y_1 + (6 + I)y_2$$

$$= [4y_1 + 6y_2, 9y_1 + 7y_2]$$
subject to
$$(2 + 2I)y_1 + (3 + I)y_2 \leq (8 + 2I)$$
$$= [2y_1 + 3y_2, 4y_1 + 4y_2] \leq [8, 10]$$
$$(5 + 2I)y_1 + (8 + I)y_2 \leq (20 + 2I)$$
$$= [5y_1 + 8y_2, 7y_1 + 9y_2] \leq (20 + 2I)$$

This problem can now be novelly transformed into two equivalent crisp sub-problems to determine the worst (lower bound) and best (upper bound) solutions as follows:

Sub-Problem(i)

maximize $4y_1 + 6y_2$

subject to

$2y_1 + 3y_2 \leq 8$

$4y_1 + 4y_2 \leq 10$

Sub-Problem(ii)

maximize $9y_1 + 7y_2$

subject to

$2y_1 + 3y_2 \leq 8$

$4y_1 + 4y_2 \leq 10$

By addressing the two subproblems, we derive the optimal solution for the original problem, which is represented as the NN interval: $Z = [15, 22.5]$.

Nomenclature

m	index for origins
t	index for destinations
v	indexed by the type of transport
w	index for the routes
g	index for the goods
${}^{n}(\tilde{Z}_i)$	objective function with NN (i=1,2)
${}^{\mathrm{In}}\widetilde{\mathbf{Z}}_{\mathbf{i}}$	the interval valued objective function (i=1,2)
${}^{\mathrm{L}}\mathbf{Z}_{\mathbf{i}}$	deterministic form of the lower-level objective functions(i=1,2)
${}^{\mathrm{U}}\mathbf{Z}_{\mathbf{i}}$	deterministic form of the upper-level objective functions(i=1,2)
${}^{\mathrm{n}}\widetilde{\mathbf{P}}_{\mathrm{mg}}$	unit purchasing price of the g^{th} product at m^{th} origin

Nomenclature

${}^{\mathbf{n}}\widetilde{\mathbf{S}}_{\mathrm{tg}}$	unit selling price of g^{th} product at t^{th} destination
$\mathbf{y}_{\mathrm{mtvwg}}$	quantity of g^{th} goods that to be transit from m^{th} origin to t^{th} destination with v^{th} vehicle via w^{th} route
${}^{\mathbf{n}}\widetilde{\mathbf{C}}_{\mathrm{mtvwg}}$	the unit transportation cost of the product from m^{th} origin to t^{th} destination via v^{th} mode of conveyance per unit distance
${}^{\mathrm{n}}\widetilde{\mathbf{f}}_{\mathrm{mtvwg}}$	the FC, during the transit activity happens from m^{th} origin to t^{th} destination by v^{th} conveyance via w^{th} route
${}^{\mathbf{n}}\widetilde{\mathbf{d}}_{\mathrm{mtvwg}}$	distance from from m^{th} origin to t^{th} destination via w^{th} route
V_v	speed of the v^{th} transport
γ_{mtw}	disturbance rate of the speed due to w^{th} route from m^{th} origin to the t^{th} destination
${}^{\mathrm{n}}\widetilde{\mathbf{L}}_{\mathrm{mtvg}}$	time for loading and unloading the g^{th} goods during transit from the m^{th} origin to the t^{th} destination using the v^{th} mode of transport
${}^{\mathrm{n}}\widetilde{\mathbf{a}}_{\mathrm{mg}}$	g^{th} good's availability at m^{th} origin
${}^{\mathrm{n}}\widetilde{\mathbf{b}}_{\mathrm{tg}}$	g^{th} good's demand at the t^{th} destination
${}^{\mathbf{n}}\widetilde{\mathbf{e}}_{\mathbf{v}}$	v^{th} transport's capacity
$h(\mathbf{y}_{\mathrm{mtvwg}})$	binary variable takes the value 1 if $\mathbf{y}_{\mathrm{mtvwg}} \neq \mathbf{0}$ **and 0 if** $\mathbf{y}_{\mathrm{mtvwg}} = \mathbf{0}$
${}^{\mathrm{n}}\widetilde{\mathrm{Bud}}_{\mathbf{t}}$	total budget at the t^{th} destination
${}^{\mathrm{n}}\widetilde{\alpha}$	carbon tax at per unit of its CE
${}^{\mathbf{n}}\widetilde{\mathbf{e}}_{\mathrm{mtv}}$	charge of CE per unit

3. Mathematical Formation

This section contains the mathematical formulation of the proposed NFCMO4DMITPCEBC.

An NFCMO4DMITPCEBC with vehicle speed is formulated as follows. Let there be M origins O_m $(m = 1, 2, \ldots M)$, T-demands D_t $(t = 1, 2, \ldots, T)$, W-roads Q_w $(w = 1, 2 \ldots W)$, G- goods P_g $(g = 1, 2, \ldots G)$, V-conveyances E_v $(v = 1, 2, \ldots, V)$. The transportation model seeks to maximize profits and minimize transit times, but accurately estimating parameters is challenging due to uncertainties in practical applications. Decision-makers face unknowns such as raw material availability, fluctuating shipping costs, and demand variations. The novelty of this research is its approach to managing these uncertainties, improving decision-making in complex transportation systems.

Maximizing the total profit is the first objective and minimizing the transportation time is the second objective of the model which are given below.

$$\text{Maximum profit} = \text{Total revenue} - \text{Total purchase price}$$
$$- \text{Total transportation cost}$$
$$\text{Minimum time} = \text{Transporting time} + \text{Loading/Unloading time}$$

The transporting time will be taken into consideration if the goods are transported from m^{th} origin to t^{th} destination by v^{th} transport via w^{th} road. First constraint require that all goods available (a_m) at each origin m be delivered,

Second constraint force, delivery (b_t) units of goods to each destination,

Third constraint force capacity of the vehicle (e_v),

Fourth constraint force, budget of the destination, Fifth constraints represent the fixed carbon capacity of the v^{th} transport mode.

Assume that H is a function of y_{mtvwg} takes values 0 and 1 to describe the transportation activity from source m to destination t through v^{th} transport mode. Finally, sixth and seventh constraints set the ranges of y_{umv} and H(x_{umv}) respectively.

The mathematical formulation of NFCMO4DMITPCEBC is given as below Model 1:

Model 1:

$$\text{Max } {}^{n}\widetilde{\mathbf{Z}}_{\mathbf{1}} = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}[{}^{n}\widetilde{S}_{\text{tg}} - {}^{n}\widetilde{P}_{\text{mg}} - {}^{n}\,\widetilde{\alpha}\;{}^{n}\widetilde{e_{\text{mtv}}} - {}^{n}\widetilde{C}_{\text{mtvwg}}d_{\text{mtw}}]y_{\text{mtvwg}}$$
$$-\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}{}^{n}\widetilde{f}_{\text{mtvg}}\;h_{\text{mtvwg}})$$
$$\text{Min } {}^{n}\widetilde{\mathbf{Z}}_{\mathbf{2}} = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\frac{d_{\text{mtw}}\text{x}_{\text{mtvw}}}{V_{\text{v}}\,(1-\gamma_{\text{mtw}})} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}({}^{n}\widetilde{\delta}_{\text{mtvg}}\;y_{\text{mtvwg}})$$

where

$$\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}y_{\text{mtvwg}} \le {}^{n}\widetilde{a}_{\text{mg}}\;,\; m=1,\cdots,M,\; g=1,\cdots,G$$
$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}y_{\text{mtvwg}} \ge {}^{n}\widetilde{b}_{\text{tg}}\;,\; t=1,\cdots,T,\; g=1,\cdots,G$$
$$\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{g=1}^{G}y_{\text{mtvwg}} \le {}^{n}\widetilde{e}_{v}\;\;,\; v=1,\cdots,V$$
$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}\left({}^{n}\widetilde{P}_{\text{mg}} + {}^{n}\widetilde{C}_{\text{mtvwg}} + {}^{n}\,\widetilde{\alpha}\;\;{}^{n}\widetilde{e_{\text{mtv}}}\right)y_{\text{mtvwg}}$$
$$+\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}{}^{n}\widetilde{f}_{\text{mtvw}}\text{x}_{\text{mtvw}} \le {}^{n}\widetilde{\text{Bud}}_{t}\;,\;\forall\; t=1,\cdots,T.$$

$$y_{\text{mtvwg}}\;\ge 0,\;\forall\; m,t,v,w,g.$$
$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{N}}\sum_{w=1}^{W}\sum_{g=1}^{G}{}^{n}\widetilde{e}_{\text{mtv}}y_{\text{mtvwg}} \le {}^{n}\widetilde{E}_{\text{v}}\;\;,v=1,\cdots,V.$$

$$x_{\mathrm{mtvw}} = \begin{cases} 1\ if\ \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}}\ = 0 \end{cases}$$

$$h_{\mathrm{mtvwg}} = \begin{cases} 1\ if \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}} = 0 \end{cases}$$

The condition for feasibility of the TP is as below

$$\sum_{m=1}^{M}\sum_{g=1}^{G} {}^{n}\tilde{a}_{mg} \geq \sum_{t=1}^{T}\sum_{g=1}^{G} {}^{n}\tilde{b}_{tg}; \sum_{v=1}^{V} {}^{n}\tilde{e}_{v} \geq \sum_{t=1}^{T}\sum_{g=1}^{G} {}^{n}\tilde{b}_{tg}. \tag{7}$$

The parameters considered in the above model are NN, leading to Model 1 being identified as NFCMO4DMITPCEBC. To derive the Pareto optimal solution, we treat minimizing ${}^{In}\tilde{Z}_1$ is equivalent to maximizing$(-{}^{n}\tilde{Z}_1\ )$ and minimizing ${}^{In}\tilde{Z}_2$ is equivalent to minimizing ${}^{n}\tilde{Z}_2$.

Without losing generality, we specify the NN in Model 1 as ${}^{n}\tilde{a} = q + rI$, with I representing indeterminacy. As a result, Model 1 is reformulated into Model 2, as outlined below:

Model 2:

$$\begin{aligned} \text{Min } {}^{In}\widetilde{\mathbf{Z}}_{\mathbf{1}} = & \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} [-(S^{1}_{\mathrm{tg}} + I_{\mathrm{s}} S^{2}_{\mathrm{tg}}) + (P^{1}{}_{\mathrm{mg}} + I_{\mathrm{p}} P^{2}{}_{\mathrm{mg}}) + (\alpha^{\ 1} \\ & + I_{\ \alpha}\alpha^{\ 2})({e_{\mathrm{mtv}}}^{1} + I_{\mathrm{e}}{e_{\mathrm{mtv}}}^{2}) + (C^{1}{}_{\mathrm{mtvwg}} + I_{\mathrm{c}} C^{2}{}_{\mathrm{mtvwg}}) d_{\mathrm{mtw}}] y_{\mathrm{mtvwg}} \\ & - \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} (f^{1}{}_{\mathrm{mtvg}} + (I_{\mathrm{f}} f^{2}_{\mathrm{mtvg}}\ h_{\mathrm{mtvwg}}) \end{aligned}$$

$$\text{Min } {}^{In}\widetilde{\mathbf{Z}}_{\mathbf{2}} = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W} \frac{d_{\mathrm{mtw}} \mathrm{x}_{\mathrm{mtvw}}}{V_{\mathrm{v}}\ (1 - \gamma_{\mathrm{mtw}})} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} (\delta^{1}{}_{\mathrm{mtvg}}\ + I_{\mathrm{mtvg}}\ \delta^{2}{}_{\mathrm{mtvg}}\ ) y_{\mathrm{mtvwg}}$$

where

$$x_{\mathrm{mtvw}} = \begin{cases} 1\ if\ \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}}\ = 0 \end{cases}$$

$$h_{\mathrm{mtvwg}} = \begin{cases} 1\ if \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}} = 0 \end{cases}$$

$$\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\mathrm{mtvwg}}\ \leq (a^{1}{}_{\mathrm{mg}} + I_{\mathrm{a}} a^{2}{}_{\mathrm{mg}})\ ,\ m = 1, \cdots, M,\ g = 1, \cdots, G$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\mathrm{mtvwg}} \geq (b^{1}{}_{\mathrm{tg}} + I_{\mathrm{b}} b^{2}{}_{\mathrm{tg}})\ ,\ t = 1,\cdots,T,\ g = 1,\cdots,G$$

$$\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{g=1}^{G} y_{\mathrm{mtvwg}} \leq (e^{1}{}_{v} \quad + I_{v} \quad e^{2}{}_{v} \quad ,)\ v = 1,\cdots,V$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} \left((P^{1}{}_{\mathrm{mg}} + I_{\mathrm{p}} P^{2}{}_{\mathrm{mg}}) + (C^{1}{}_{\mathrm{mtvwg}} + I_{\mathrm{C}} C^{2}{}_{\mathrm{mtvwg}}) + (\alpha^{\ 1} \ + I^{1}\alpha^{\ 2})\ (e^{1}_{\mathrm{mtv}} + I_{\mathrm{e}} e^{2}_{\mathrm{mtv}})\right) y_{\mathrm{mtvwg}}$$

$$+ \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{1}{}_{\mathrm{mtvw}} + I_{\mathrm{f}} f^{2}{}_{\mathrm{mtvw}} \mathrm{x}_{\mathrm{mtvw}} \leq (\mathrm{Bud}^{1}{}_{t} + \mathrm{I}_{\mathrm{Bud}} \mathrm{Bud}^{2}{}_{t})\ ,\ \forall\ t = 1,\cdots,T.$$

$$y_{\mathrm{mtvwg}} \ \geq 0,\ \forall\ m,t,v,w,g.$$

$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{T}}\sum_{w=1}^{W}\sum_{g=1}^{G} (e^{1}{}_{\mathrm{mtv}} + I_{\mathrm{e}} e^{2}{}_{\mathrm{mtv}} y_{\mathrm{mtvwg})} \leq (E^{1}{}_{\mathrm{v}} \quad + I_{\mathrm{E}} \quad E^{2}{}_{\mathrm{v}} \quad ,v = 1,\cdots,V.)$$

Now a NN can be represented as ${}^{n}\tilde{a} = [q+rI^{L}, q+rI^{U}] = [a^{L}, a^{U}]$, which constitutes an interval number with defined lower and upper bounds.This leads to the need to decompose Model 2 into two distinct crisp problems, namely Model 3 and Model 4. Here, Model 3 is recognized as the lower-level problem, whereas Model 4 is identified as the upper-level problem. By applying inequalities, the interval constraints are converted into a deterministic format. Therefore, the deterministic versions of these models are as follows:

Model 3:

$$\mathrm{Min}\ {}^{IL}\mathbf{Z_1} = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} [-S^{L}{}_{\mathrm{tg}} + P^{L}{}_{\mathrm{mg}} + \alpha^{L}\ e^{L}_{\mathrm{mtv}} + C^{L}{}_{\mathrm{mtvwg}} d_{\mathrm{mtw}}] y_{\mathrm{mtvwg}}$$

$$+ \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{L}{}_{\mathrm{mtvg}}\ h_{\mathrm{mtvwg}})$$

$$\mathrm{Min}\ {}^{IL}\mathbf{Z_2} = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W} \frac{d_{\mathrm{mtw}} \mathrm{x}_{\mathrm{mtvw}}}{V_{\mathrm{v}}\ (1-\gamma_{\mathrm{mtw}})} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} (\delta^{L}{}_{\mathrm{mtvg}}\ y_{\mathrm{mtvwg}})$$

where

$$x_{\mathrm{mtvw}} = \begin{cases} 1\ if\ \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}} \ = 0 \end{cases}$$

$$h_{\mathrm{mtvwg}} = \begin{cases} 1\ if \sum_{g} y_{\mathrm{mtvwg}} > 0 \\ 0\ if \sum_{g} y_{\mathrm{mtvwg}} = 0 \end{cases}$$

$$\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\mathrm{mtvwg}} \ \leq a^{U}{}_{\mathrm{mg}}\ ,\ m = 1,\cdots M,\ g = 1,\cdots,G$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\mathrm{mnvwg}} \geq b^{L}{}_{\mathrm{ng}}\ ,\ t=1,\cdots,T,\ g=1,\cdots,G$$

$$\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{g=1}^{G} y_{\mathrm{mtvwg}} \leq e^{U}{}_{v}\ ,\ v=1,\cdots,V$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}\left(P^{L}{}_{\mathrm{mg}}+C^{L}{}_{\mathrm{mtvwg}}+\alpha^{\ L}\ e^{L}_{\mathrm{mtv}}\right) y_{\mathrm{mtvwg}} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{L}{}_{\mathrm{mtvw}}\mathrm{x}_{\mathrm{mtvw}} \leq \mathrm{Bud}^{L}{}_{t}\ ,\ \forall\ t=1,\cdots,T.$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}\left(P^{U}{}_{\mathrm{mg}}+C^{U}{}_{\mathrm{mtvwg}}+\alpha^{\ U}\ e^{U}_{\mathrm{mtv}}\right) y_{\mathrm{mtvwg}} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{U}{}_{\mathrm{mtvw}}\mathrm{x}_{\mathrm{mtvw}} \leq \mathrm{Bud}^{U}{}_{t}\ ,\ \forall\ t=1 \text{ to T.}$$

$$y_{\mathrm{mtvwg}} \geq 0,\ \forall\ m,t,v,w,g.$$

$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{T}}\sum_{w=1}^{W}\sum_{g=1}^{G} e^{L}{}_{\mathrm{mtv}} y_{\mathrm{mtvwg}} \leq E^{L}{}_{\mathrm{v}}\ , v=1,\cdots,V.$$

$$y_{\mathrm{mtvwg}} \geq 0,\ \forall\ m,t,v,w,g.$$

$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{T}}\sum_{w=1}^{W}\sum_{g=1}^{G} e^{U}{}_{\mathrm{mtv}} y_{\mathrm{mtvwg}} \leq E^{U}{}_{\mathrm{v}}\ , v=1,\cdots,V.$$

Model 4:

$$\text{Min }{}^{IU}\mathbf{Z_1}=\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}[-S^{U}{}_{\mathrm{tg}}+P^{U}{}_{\mathrm{mg}}+\alpha^{U}\ e^{U}_{\mathrm{mtv}}+C^{U}{}_{\mathrm{mtvwg}} d_{\mathrm{mtw}}] y_{\mathrm{mtvwg}} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{U}{}_{\mathrm{mtvg}}\ h_{\mathrm{mtvwg}})$$

$$\text{Min }{}^{IU}\mathbf{Z_2}=\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\frac{d_{\mathrm{mtw}}\mathrm{x}_{\mathrm{mtvw}}}{V_{\mathrm{v}}\left(1-\gamma_{\mathrm{mtw}}\right)}+\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}(\delta^{U}{}_{\mathrm{mtvg}}\ y_{\mathrm{mtvwg}})$$

where

$$x_{\mathrm{mtvw}}=\begin{cases}1\ if\ \sum_{g} y_{\mathrm{mtvwg}}>0\\ 0\ if \sum_{g} y_{\mathrm{mtvwg}}=0\end{cases}$$

$$h_{\mathrm{mtvwg}}=\begin{cases}1\ if \sum_{g} y_{\mathrm{mtvwg}}>0\\ 0\ if \sum_{g} y_{\mathrm{mtvwg}}=0\end{cases}$$

$$\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\text{mtvwg}} \ \le a^{U}{}_{\text{mg}}\ ,\ m=1,\cdots,M,\ g=1,\cdots,G$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W} y_{\text{mtvwg}} \ \ge b^{L}{}_{\text{tg}}\ ,\ t=1\ldots\ldots T,\ g=1,\cdots,G$$

$$\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{g=1}^{G} y_{\text{mtvwg}} \le e^{U}{}_{v}\ \ ,\ v=1,\cdots,V.$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}\left(P^{L}{}_{\text{mg}}+C^{L}{}_{\text{mtvwg}}+\alpha^{\ L}\ e^{L}_{\text{mtv}}\right) y_{\text{mtvwg}}$$
$$+\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{L}{}_{\text{mtvw}}\text{x}_{\text{mtvw}} \le \text{Bud}^{L}{}_{t}\ ,\ \forall\ t=1,\cdots,T.$$

$$\sum_{m=1}^{M}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G}\left(P^{U}{}_{\text{mg}}+C^{U}{}_{\text{mtvwg}}+\alpha^{\ U}\ e^{U}_{\text{mtv}}\right) y_{\text{mtvwg}}$$
$$+\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{w=1}^{W}\sum_{g=1}^{G} f^{U}{}_{\text{mtvw}}\text{x}_{\text{mtvw}} \le \text{Bud}^{U}{}_{t}\ ,\ \forall\ t=1,\cdots,T.$$

$$y_{\text{mtvwg}} \ \ge 0,\ \forall\ m,t,v,w,g.$$
$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{T}}\sum_{w=1}^{W}\sum_{g=1}^{G} e^{L}{}_{\text{mtv}} y_{\text{mtvwg}} \le E^{L}{}_{\text{v}}\ \ ,v=1 \text{ to V}$$
$$y_{\text{mtvwg}} \ \ge 0,\ \forall\ m,t,v,w,g.$$
$$\sum_{\mathbf{m=1}}^{\mathbf{M}}\sum_{\mathbf{t=1}}^{\mathbf{T}}\sum_{w=1}^{W}\sum_{g=1}^{G} e^{U}{}_{\text{mtv}} y_{\text{mtvwg}} \le E^{U}{}_{\text{v}}\ \ ,v=1 \text{ to V}$$

Definition 3.1. A Pareto-optimal solution, also known as a compromise solution, for Model 3 and Model 4 is a feasible solution $y^* = (y^*_{mtvwg})$ such that there does not exist any other feasible solution $y = (y_{mtvwg})$ with ${}^{In}Z_i(y) \le^{In} Z_i(y^)$ for $i = 1, 2$ and ${}^{In}Z_i(y) \le^{In} Z_i(y^*)$ for at least one i.

Theorem 3.2. *The union of the Pareto-optimal solutions from Model 3 and Model 4 constitutes the Pareto-optimal solution for Model 2, and by extension, for Model 1 when expressed as an interval NN.*

Proof: *Theorem 3.2 follows directly from Definition 2.5.*

4. **Solution Methodology**

Models 3 and 4 offer the lower and upper bounds for the objective functions, respectively; however, they do not present a comprehensive compromise solution for Model 2. To address

TABLE 2. Pay-off matrix

	Z_1	Z_2
Y_1^*	$Z_1(Y_1^*)$	$Z_2(Y_1^*)$
Y_2^*	$Z_1(Y_2^*)$	$Z_2(Y_2^*)$

this, we employ NLPP techniques to derive an optimal compromise solution for the multi-objective decision-making problem. The steps to solve the proposed model using NLPP are outlined below.

(1) Convert the neutrosophic optimization problem into a crisp format, resulting in two distinct sub-problems.
(2) Address each sub-problem separately while adhering to all constraints.
(3) Designate the upper bound as the Positive Ideal Solution (PIS) and the lower bound as the Negative Ideal Solution (NIS) for each objective function within the pay-off matrix, as illustrated in Table 2. The PIS and NIS are defined as follows:
PIS=Z_i^* =Max $(Z_i(Y_1^*), Z_i(Y_2^*)\ )$(i=1,2) and
NIS=Z_i =Min $(Z_i(Y_1^*), Z_i^*(Y_2^*)\ )$(i=1,2) respectively.
(4) Construct the TMF and the IMF with the highest degree, and the FMF with the lowest degree.
(5) Define the tolerance levels and create the membership functions in accordance with the limits specified in Equations (4), (5), and (6). Here,

$$\left.\begin{aligned} &U_i^T = U_i, L_i^T = L_i \\ &U_i^F = U_i^T, L_i^F = L_i^T + r_i(U_i^T - L_i^T) \\ &U_i^I = L_i^T + s_i(U_i^T - L_i^T), L_i^I = L_i^T \end{aligned}\right\} \quad (8)$$

where r_i, s_i are predetermined real numbers in (0,1).

(6) Choose the values of $\alpha_T, \alpha_I, \text{and}\alpha_F$ within the interval $[0, 1]$ for each NN as the truth, indeterminacy and falsity degrees respectively.

(7) Construct NLPP that represents in Model 5

Model 5

$$\text{Maximize}\{T_i({}^{In}Z_i^*(Y))\} : i = 1, 2.$$

$$\text{Minimize}\{F_i({}^{In}Z_i^*(Y))\} : i = 1, 2.$$

$$\text{Maximize}\{I_i({}^{In}Z_i^*(Y))\} : i = 1, 2.$$

subject to the constraints given in model 3.

Model 5 can be transformed into Model 6 as outlined below:

Model 6

$$\begin{aligned}
&\text{Max}\,\alpha_T \\
&\text{Max}\,\alpha_I \\
&\text{Min}\,\alpha_F \\
&\text{subject to} \\
&T_i({}^{In}Z_i(y)) \geq \alpha_T,\ I_i({}^{In}Z_i)(y) \geq \alpha_I,\ F_i({}^{In}z_i)(y) \leq \alpha_F \\
&\alpha_T \geq \alpha_I, \alpha_T \geq \alpha_F, \alpha_T + \alpha_I + \alpha_F \leq 3, \alpha_T + \alpha_I + \alpha_F \geq 0, \alpha_T, \alpha_I, \alpha_F \in [0,1]
\end{aligned} \tag{9}$$

also with constraints given in model 3. The simplified NLPP model (Model 6) that yields the compromise solution for the MOTP (i.e., Model 7) is as follows:

Model 7

$$\begin{aligned}
&\text{Max}\,\alpha_T - \alpha_F + \alpha_I \\
&\text{subject to} \\
&{}^{In}Z_i^*(y) + (U_i^T - L_i^T)\alpha_T \leq U_i^T \\
&{}^{In}Z_i^*(y) + (U_i^I - L_i^I)\alpha_I \leq U_i^I \\
&{}^{In}Z_k^*(y) - (U_i^F - L_i^F)\alpha_F \leq L_i^F \\
&\alpha_T \geq \alpha_I, \alpha_T \geq \alpha_F, \alpha_T + \alpha_I + \alpha_F \leq 3, \\
&\alpha_T, \alpha_I, \alpha_F \in [0,1], i = 1,2.
\end{aligned} \tag{10}$$

with the constraints given in model 3.

5. Advantages of the Proposed Model

(1) NS stand out due to their three independent membership degrees: TMF (T), IMF (I), and F (F), enabling them to effectively handle imprecise parameters. Unlike Intuitionistic Fuzzy Sets (IFS), which are restricted to managing incomplete information, NS can cope with both incomplete and indeterminate information. Consequently, NS are more proficient at addressing complete uncertainty compared to IFSs.

(2) In a NS, decision makers aim to maximize the TMF and the IMF while minimizing the FMF, reflecting more realistic scenarios in real-life problems.

(3) In the context of transportation systems, CE contribute significantly to air pollution, increasing CE charges and subsequently reducing the system's profit. Therefore, reducing CE is crucial for maximizing profit, a factor addressed in this paper. Additionally,

the inclusion of vehicle speed and disturbance rate enhances system reliability and effectively reduces transportation time.

(4) To enhance profit maximization, budget constraints and carbon capacity are also factored into this problem. These extra limitations contribute to reducing the overall time required to solve the proposed problem.

Despite these advantages, it is essential to consider the following potential drawbacks of the model to ensure well-rounded decision-making.

5.1. *Limitations of the Model*

While the model proves highly effective for real-world decision-making, it also presents certain limitations, as outlined below:

- Sensitivity to Input Variability: Minor fluctuations in key input data, such as changes in fuel prices or emission rates, can significantly influence the results. This heightened sensitivity can diminish the reliability of the model's recommendations.

- Regulatory and Compliance Challenges: Carbon emission regulations differ across regions, making it difficult to adapt the transportation model uniformly. This can lead to reduced flexibility and operational constraints in certain regions.

6. **Numerical Example**

To demonstrate the applicability of the suggested approach with maximum profit and minimum time in the transportation system, we include a real-world example in this section. Taking into account the two Orissa export hubs for two different kinds of sea fish (like Jharkhand and Chhattisgarh) in the transit network. CE charges for heavy-duty vehicles in transportation systems are in addition to other expenses like selling price, purchasing cost, transportation cost, and FC. In addition, the time factor needs to be included when sending data over long distances. NN are defined as supply, demand, and conveyance based on a number of complex factors. All NN, which are provided in Tables 3, 4, 5, 6 , 7 and 8, are the transportation cost per unit item from source to destination, FC, distance between the origin and destination, rate of disturbances of different vehicles, loading and unloading timings and the rate of CE. The cost of transport and CE are expressed in dollars. each unit and time, measured in hours. Additionally, NN are selected here, along with the selling price, purchasing cost, carbon tax,

Table 3. Unit transportation cost

m	t	v	$\tilde{C}_{mtv11}$	$\tilde{C}_{mtv12}$	$\tilde{C}_{mtv21}$	$\tilde{C}_{mtv22}$
1	1	1	4+3I	3+2I	3+2I	3+3I
		2	4+4I	5-I	8+I	8-5I
	2	1	2+I	6+3I	7+I	2+5I
		2	5+I	3+I	2+2I	2+3I
2	1	1	2+3I	4+2I	4+2I	5+3I
		2	2+3I	5+2I	2+2I	5+3I
	2	1	1+2I	1+2I	2+2I	2+2I
		2	1+2I	1+2I	6+2I	2+2I

source, demand, conveyance, budget, loading and unloading schedules, and carbon capacity. The goal is to maximize selling price while minimizing fixed costs, transportation costs, CE charges, and purchasing costs to achieve maximum profit. Additionally, time is limited, necessitating a boundary for the budget and carbon capacity. As a result, the issue is now NFCMO4DMITPCEBC due to the description of such conditions with the objective functions that contradict each other. The problem can be expressed mathematically by using the models 3 and 4. The following provides an illustration of the problem's solution. Furthermore, the following presumptions apply to the planned study.

Selling price $={}^{\mathrm{n}}\tilde{S}_{\mathrm{tg}}$:$\{\ {}^{\mathrm{n}}\tilde{S}_{11} = 100 + 175I;\ {}^{\mathrm{n}}\tilde{S}_{12} = 119 + 135I,^{n}\ \tilde{S}_{21} = 221 + 52I;\ \ {}^{\mathrm{n}}\tilde{S}_{22} = 203 + 50I\ \}$

Purchase price $={}^{\mathrm{n}}\tilde{P}_{\mathrm{mg}}$:$\{\ {}^{\mathrm{n}}\tilde{P}_{11} = 6 + 12I;\ {}^{\mathrm{n}}\tilde{P}_{12} = 11 + 2I,^{n}\ \tilde{P}_{21} = 12 + 2I;\ \ {}^{\mathrm{n}}\tilde{P}_{22} = 11 + I\}$

Carbon tax= ${}^{\mathrm{n}}\ \tilde{\alpha}$:{ α=2+I}

Source $=({}^{\mathrm{n}}\tilde{a}_{\mathrm{mg}})$: $\{\ {}^{\mathrm{n}}\tilde{a}_{11} = 380 + 8I;\ {}^{\mathrm{n}}\tilde{a}_{12} = 290 + 10I;^{n}\ \tilde{a}_{21} = 380 + 8I;\ \ ;^{n}\ \tilde{a}_{22} = 380 + 8I\}$

Demand $=({}^{\mathrm{n}}\tilde{b}_{\mathrm{tg}})$: $\{\ {}^{\mathrm{n}}\tilde{b}_{11} = 150 + 7I;\ {}^{\mathrm{n}}\tilde{b}_{12} = 130 + 10I;\ {}^{n}\tilde{b}_{21} = 90 + 7I;^{n}\ \tilde{b}_{22} = 120 + 8I\}$

Conveyance$=({}^{\mathrm{n}}\tilde{e}_{v})$: $\{\ {}^{\mathrm{n}}\tilde{e}_{1} = 425 + 12I;\ {}^{\mathrm{n}}\tilde{e}_{2} = 430 + 19I\ \}$

Budget $=({}^{\mathrm{n}}\tilde{\mathrm{Bud}}_{t})$: $\{\ {}^{\mathrm{n}}\tilde{B}_{1} = 10800 + 1200I;\ {}^{\mathrm{n}}\tilde{B}_{2} = 7400 + 1800I\}$

Carbon Capacity $=({}^{\mathrm{n}}\tilde{c}_{v})$: $\{\ {}^{\mathrm{n}}\tilde{c}_{1} = 2900 + 100I;\ {}^{\mathrm{n}}\tilde{c}_{2} = 3400 + 100I\}$

Here, we define indeterminacy as I=[0 1].

Vehicle Speed $=(V_v)$: (V_1= 35; $V_2 = 25$)

Solve the models which are obtained from model 3 and model 4 individually By employing the LINGO18 iterative scheme to solve the models which are obtained from model 3 and model 4 individually , we derive their respective solutions and compute the values of the objective functions, as detailed in Tables 9 and 10. Due to the contradictory nature of these solutions, we seek an overall compromise solution by solving Model 7 using the proposed NLPP. The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS), denoted as IU and IL respectively, are

Table 4. Fixed charge

m	t	v	$\tilde{f}_{mtv11}$	$\tilde{f}_{mtv12}$	$\tilde{f}_{mtv21}$	$\tilde{f}_{mtv22}$
1	1	1	12+2I	10+2I	14+2I	12+2I
		2	20+5I	5-I	25+I	5-2I
	2	1	4+2I	1+I	4+I	20+2I
		2	4+2I	12+2I	5+2I	10+5I
2	1	1	10+2I	8+2I	7+7I	6+6I
		2	4+20I	2+3I	1+24I	4+2I
	2	1	4+I	18+2I	3+1I	17+4I
		2	4+I	12+I	4+2I	12+2I

Table 5. Distance between the origins and destinations

m	D_{m11}	D_{m12}	D_{m21}	D_{m22}
1	43	45	45	35
2	43	40	56	45

Table 6. Rate of disturbances of different vehicles

m	γ_{m11}	γ_{m12}	γ_{m21}	γ_{m22}
1	0.03	0.04	0.02	0.03
2	0.04	0.034	0.022	0.05

Table 7. Time for loading and unloading the goods

m	t	$\tilde{\delta}_{mt11}$	$\tilde{\delta}_{mt12}$	$\tilde{\delta}_{mt21}$	$\tilde{\delta}_{mt22}$
1	1	6+13I	4+10I	3+6I	5+I
	2	5+4I	3+I	4+11I	6+I
2	1	6+I	3+3I	4+I	8+I
	2	7+I	4+5I	3+6I	7+I

obtained from Tables 9 and 10. Ultimately, we develop Model 8 using NLPP to find the compromise solution for Model 2.

TABLE 8. The rate of carbon emission

m	$\tilde{e}_{m11}$	$\tilde{e}_{m12}$	$\tilde{e}_{m21}$	$\tilde{e}_{m22}$
1	2+4I	3+3I	3+2I	2+1I
2	2+3I	2+1I	2+2I	3+2I

TABLE 9. Pay off matrix for ${}^{IL}Z_i$

${}^{IL}Y$	${}^{IL}Z_1$	${}^{IL}Z_1$
${}^{IL}Y_1$	-99440.4	1947.75
${}^{IL}Y_1$	-31437.4	1578.64

TABLE 10. Pay off matrix for ${}^{IU}Z_i$

${}^{IU}Y$	${}^{IU}Z_1$	${}^{IU}Z_1$
${}^{IU}Y_1$	-80110.53	4334.3
${}^{IU}Y_1$	-261.27	2927.89

Model 8

$$\begin{aligned}
&\text{Max}\, \alpha_T - \alpha_F + \alpha_I \\
&\text{subject to} \\
&{}^{IL}Z_i^*(y) + (U_i^L - L_i^L)\alpha_T \leq U_i^L \\
&{}^{IL}Z_i^*(y) + (U_i^L - L_i^L)(\alpha_I - 1)s_1 i \leq L_i^L \\
&{}^{IL}Z_I^*(y) - (U_i^L - L_i^L)\alpha_F(1 - r_1 i) \leq U_i^L \\
&{}^{IU}Z_i^*(y) + (U_i^U - L_i^U)\alpha_T \leq U_i^U \\
&{}^{IU}Z_i^*(y) + (U_i^U - L_i^U)(\alpha_I - 1)s_2 i \leq L_i^U \\
&{}^{IU}Z_I^*(y) - (U_i^U - L_i^U)\alpha_F(1 - r_2 i) \leq U_i^U \\
&\alpha_T \geq \alpha_I, \alpha_T \geq \alpha_F, \alpha_T + \alpha_I + \alpha_F \leq 3, \\
&\alpha_T, \alpha_I, \alpha_F \in [0, 1], i = 1, 2.
\end{aligned} \tag{11}$$

Where U_i^L = PIS of ${}^{IL}Z_i$, L_i^L = NIS of ${}^{IL}Z_i$; U_i^U = PIS of ${}^{IU}Z_i$, L_i^U = NIS of ${}^{IU}Z_i$. The solutions for Model 8 are determined as follows;$\alpha_T = 0.7437, \alpha_I = 0.7437, \alpha_F = 0$,$x_{11211} = 63.23$, $x_{11221} = 2.379$, $x_{12112} = 119.4603$, $x_{12122} = 8.5396$, $x_{21112} = 139.495$, $x_{21122} = 0.5049$, $x_{21211} = 50.611$, $x_{21221} = 40.778$, $x_{22211} = 97$. ${}^{IL}Z_1 = 10379.72$, ${}^{IL}Z_2 = 1673.21$, ${}^{IU}Z_1 = 20332.83$ ${}^{IU}Z_2 = 3288.25$. Therefore the compromise solutions of proposed Model 1 is $Z_1 = [10379.72, 20332.83]$ and $Z_2 = [1673.21, 3288.25]$.

6.1. *Results and Discussion*

By employing NLPP to solve Model 2, the solutions for Model 1 are illustrated in Section 6. NLPP effectively addresses the MOTP. The analysis highlights the crucial need to incorporate additional constraints on CE and vehicle speed in our formulated problem. This approach not only safeguards the environment but also maximizes profitability in the transportation system by minimizing overall transportation costs and time.

7. Sensitivity analysis

To elucidate and interpret the impact of changes in the coefficients of objective functions, sensitivity analysis is a critical and meticulous procedure in optimization problems. Analyzing the range of all parametric values and their slight changes, while ensuring the optimal value remains constant, is challenging. There are several research papers on sensitivity analysis in TP using linear programming, such as Ebrahimnejad [40]. However, when there are significant changes in variables and constraints, complexity arises, leading to changes in the values of basic variables. To manage these complexities, we introduce sensitivity analysis for the NFCMO4DMITPCEBC with NLPP, ensuring that all basic variables remain fixed. The steps to determine the parameter ranges in NFCMO4DMITPCEBC are as follows:

(1) Keep all basic variables fixed for the optimal solution of NFCMO4DMITPCEBC.
(2) Vary each parameter's value one at a time, keeping the others fixed, and solve the NFCMO4DMITPCEBC using the LINGO 18 iterative scheme.
(3) Repeat Step 2 until either a basic variable changes or no feasible solution arises.
(4) Determine the range of each parameter from Step 3.

For supply, demand, and conveyance parameters, sensitivity analysis is conducted as follows:

- Let ${}^n\tilde{a}_{mg}$ change to ${}^n\tilde{a}^*_{mg}$ as ${}^n\tilde{a}^*_{mg} =^n \tilde{a}_{mg} + \theta_m g$
- Let ${}^n\tilde{b}_{tg}$ change to ${}^n\tilde{b}^*_{tg}$ as ${}^n\tilde{b}^*_{tg} =^n \tilde{b}_{tg} + \phi_t g$
- Let ${}^n\tilde{e}_v$ change to ${}^n\tilde{e}^*_v$ as ${}^n\tilde{e}^*_v =^n \tilde{e}_v + \omega_v$

Using this procedure, we derive the values of ${}^n\tilde{a}^*_m$,${}^n\tilde{b}^*_n$ and ${}^n\tilde{e}^*_v$, which are presented in Table 11.

8. Conclusion and Future Work

The transportation system is influenced by a variety of factors, including time, profit, budget, deterioration, CE, purchasing cost, selling price, and FC. Given that all data are sourced from real-world scenarios, complexities, restrictions, and uncertainties inevitably arise. Our model aims to address these complexities using neutrosophic data to find a compromise solution. By representing all data as NN and applying NLPP, we maximize the TMF and IMF while minimizing the FMF. Additionally, incorporating constraints on CE, vehicle speed, and

TABLE 11. Ranges of availability, demand and conveyance

Values of ${}^{n}\tilde{a}_{mg}, {}^{n}\tilde{b}_{ng} and {}^{n}\tilde{e}_{v}$	Changes values of ${}^{n}\tilde{a}_{mg}, {}^{n}\tilde{b}_{ng} and {}^{n}\tilde{e}_{v}$
${}^{n}\tilde{a}_{11}= 380$	$100 \leq^{n} \tilde{a}^{*}_{11} \leq \infty$
${}^{n}\tilde{a}_{12}= 290$	$125 \leq^{n} \tilde{a}^{*}_{12} \leq \infty$
${}^{n}\tilde{a}_{21}= 250$	$90 \leq^{n} \tilde{a}^{*}_{21} \leq \infty$
${}^{n}\tilde{a}_{22}= 260$	$80.5 \leq^{n} \tilde{a}^{*}_{22} \leq \infty$
${}^{n}\tilde{b}_{11}= 150$	$105 \leq^{n} \tilde{b}^{*}_{11} \leq 170$
${}^{n}\tilde{b}_{12}= 130$	$60 \leq^{n} \tilde{b}^{*}_{12} \leq 180$
${}^{n}\tilde{b}_{21}= 90$	$10 \leq^{n} \tilde{b}^{*}_{21} \leq 150$
${}^{n}\tilde{b}_{22}= 120$	$35 \leq^{n} \tilde{b}^{*}_{22} \leq 130$
${}^{n}\tilde{e}_{1}= 425$	$230 \leq^{n} \tilde{e}^{*}_{1} \leq \infty$
${}^{n}\tilde{e}_{2}= 430$	$225 \leq^{n} \tilde{e}^{*}_{2} \leq \infty$

disturbance rates improves profitability and reduces air pollution. NLPP has proven to be the most effective and straightforward method for multi-objective decision-making problems, yielding a compromise solution for our proposed model.

Unlike fuzzy sets and IFS, which only address uncertainty through membership and non-membership values, the neutrosophic system includes an indeterminacy membership value, providing a more reliable framework for handling incomplete and indeterminate information. The practicality of our approach has been demonstrated through a real-life example, with sensitivity analysis revealing stable parameter ranges. Furthermore, we have discussed decisions regarding budget and CE that can assist organizations in addressing economic and environmental concerns.

Future research could evolve the neutrosophic system into a bipolar-neutrosophic system for both linear and non-linear challenges. Additionally, the CE system could utilize a cap-and-trade policy with neutrosophic numbers. Neutrosophic programming using these numbers could also address fractional problems or multi-item transportation issues. Our model could be expanded to include various uncertainties, such as type-2 neutrosophic, type-2 uncertain variables, uncertain-random, and type-2 intuitionistic fuzzy variables. Furthermore, numerous heuristics, meta-heuristics, and hybrid approaches could be devised to solve larger instances of our proposed problem.

References

1. Hitchcock, F.L. The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics 1941, 20(1-4), 224–230.
2. Maity, G.; Roy, S.K.; Verdegay, J.L. Analyzing multimodal transportation problem and its application to artificial intelligence. Neural Computing and Applications 2019, 32(7), 2243–2256.

3. Ebrahimnejad, A. A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. Applied Soft Computing 2014, 19, 171–176.
4. Li, L.; Lai, K.K. A fuzzy approach to the multiobjective transportation problem. Computers and Operations Research 2000, 27(1), 43–57.
5. Maity, G.; Roy, S.K.; Verdegay, J.L. Multi-objective transportation problem with cost reliability under uncertain environment. International Journal of Computational Intelligence Systems 2016, 9(5), 839–849.
6. Hirsch, W.M.; Dantzig G.B. The Fixed Charge Problem. Naval Research Logistics 1968, 15(3), 413-424.
7. Midya, S.; Roy, S.K. Analysis of interval programming in different environments and its application to fixed charge transportation problem. Discrete Mathematics, Algorithms and Applications 2017, 9(03), 1750040.
8. Midya, S.; Roy, S.K. Solving single-sink, fixed charge, multi-objective, multi-index stochastic transportation problem. American Journal of Mathematical and Management Sciences 2004, 33(4), 300–314.
9. Roy, S.K.; Midya, S.; Yu, V.F. Multi-objective fixed charge transportation problem with random rough variables. International Journal Uncertainty Fuzziness and Knowledge-Based Systems 2008, 26(06), 971–996.
10. Haley, K. The solid transportation problem, Operations Research 1962, 10(4), 448–463.
11. Roy, S.K.; Midya, S. Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. Applied Intelligence 2019, 49(10), 3524–3538.
12. Das, S.K.; Roy, S.K.; Weber, G.W. Application of type-2 fuzzy logic to a multi-objective green solid transportation– location problem with dwell time under carbon tax, cap, and offset policy: Fuzzy versus nonfuzzy techniques. IEEE Transactions on Fuzzy Systems 2020, 28(11), 2711–2725.
13. Ghosh, S.; Roy, S.K. Fuzzy-rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station. Rairo Operations Research 2021, 55, S2923–S29527.
14. Maity, G.; Roy, S.K.; Verdegay, J.L. Analyzing multi-modal transportation problem and its application to artificial intelligence. Neural Computing and Applications 2019, 32, 2243–2256.
15. Sifaouil, T.; Aider, M. Uncertain interval programming model for multi-objective multi-item fixed charge solid transportation problem with budget constraint and safety measure. Soft Computing 2020, 24, 10123–10147.
16. Ding, J.; Jin, F.; Li, Y.; Wang, J. Analysis of transportation carbon emissions and its potential for reduction in china. Chinese Journal of Population Resources and Environment 2013, 11(1), 17–25.
17. Sengupta, D.; Das, A.; Bera, U.K. A gamma type- 2 defuzzification method for solving a solid transportation problem considering carbon emission. Applied Intelligence 2018, 48(11), 3995–4022.
18. Song, J.; Leng, M. Analysis of the single-period problem under carbon emissions policies. In Handbook of news vendor problems, Springer 2012, 297–313.
19. Tarulescu, S.; Tarulescu, R.; Soica, A.; Leahu, C.I. Smart transportation CO2 emission reduction strategies. In IOP Conference Series: Material Science and Engineering, IOP Publishing: 2017, 252, 012-051.
20. Midya, S.; Roy, S.K.; Vincent, F.Y. Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. International Journal of Machine Learning Cybernetics 2021, 12(3), 699–717.
21. M.Ali, A.; Muthuswamy, M. Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources, Sustainable Machine Intelligence Journal 2023, 4(3), 1–10.
22. Thamaraiselvi, A.; Santhi, R. A new approach for optimization of real-life transportation problem in neutrosophic environment. Mathematical Problems in Engineering 2016, Article ID: 5950747. https://doi.org/10.1155/2016/5950747
23. Sengupta, D.; Das, A.; Dutta, A.; Bera, U.K. A carbon emission optimization model with reduction method of type-2 zigzag uncertain variable. Neural Computing and Applications 2018, 32, 10895–10914.

24. Roy, S.K.; Ebrahimnejad, A.; Verdegay, J.L.; Das, S. New approach for solving intuitionistic fuzzy multi-objective transportation problem. Sadhana 2018, 43(1), 3. https://doi.org/10.1007/s12046-017-0777-7
25. Gessesse, A.A.; Mishra, R.; Acharya, M.M.; Das, K.N. Genetic algorithm based fuzzy programming approach for multi-objective linear fractional stochastic transportation problem involving four parameter burr distribution. International Journal of Systems Assurance Engineering Management 2020, 11(1), 93–109.
26. Paul, N.; Sarma, D.; Bera, U.K. A neutrosophic solid transportation model with insufficient supply. In: 2019 IEEE region 10 symposium (TENSYMP). IEEE, 2019, 379–384. https://doi.org/10. 1109/TENSYMP46218.2019.8971130
27. Midya, S.; Roy, S.K.; Vincent, F.Y. Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. International Journal Machine Learning Cybernetics 2021, 12(3), 699–717.
28. Aktar, M.S.; De, M.; Maity, S.; Mazumder, S.K.; Maiti, M. (2020) Green 4D transportation problems with breakable incompatible items under type-2 fuzzy-random environment. Journal of Cleaner Production 2020, 275, 122376.
29. Adhami, A.Y.; Ahmad, F. Interactive Pythagorean-hesitant fuzzy computational algorithm for multi-objective transportation problem under uncertainty. International Journal of Management Science and Engineering Management 2020, 15(4), 288–297.
30. Malik, M.; Gupta, S. Goal programming technique for solving fully interval-valued intuitionistic fuzzy multiple objective transportation problems. Soft Computing 2020, 24(18), 13955–13977.
31. Samanta, S.; Jana, D.K.; Panigrahi, G.; Maiti, M. Novel multi-objective, multi-item and four-dimensional transportation problem with vehicle speed in LR-type intuitionistic fuzzy environment. Neural Computing and Applications 2020, 32, 11937–11955.
32. Mohammed, A.; Wang, Q. The fuzzy multi-objective distribution planner for a green meat supply chain. International Journal Production Economics 2017, 184, 47–58.
33. Rizk-Allah, R.M.; Hassanien, A.E.; Elhoseny, M. A multi-objective transportation model under neutrosophic environment. Computers and Electrical Engineering 2018, 69, 705–719.
34. Kumar, R.; Edalatpanah, S.A.; Jha, S.; Singh, R. A Pythagorean fuzzy approach to the transportation problem. Complex and Intelligence Systems 2019, 5, 255–263.
35. Pratihar, J.; Kumar, R.; Edalatpanah, S.A.; Dey, A. Modified Vogel's approximation method for transportation problem under uncertain environment. Complex and Intelligence Systems 2021, 7, 29–40.
36. Khalifa HAEW.; Kumar, P.; Mirjalili, S. A KKM approach for inverse capacitated transportation problem in neutrosophic environment. Sadhana 2021, 46, 166. https://doi.org/10.1007/s12046- 021-01682-5
37. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic set, Multi-space and multi-structure, 2010, 4, 410-413.
38. Smarandache, F. Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability, Education Publishing: Columbus, OH, USA, 2013;
39. Ye, J. Neutrosophic number linear programming method and its application under neutrosophic number environments. Soft computing 2018, 22(14), 4639–4646.
40. Ebrahimnejad, A. A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. Applied Soft Computing 2014, 19, 171–176.

Received: Sep 6, 2024. Accepted: Dec 4, 2024