Neutrosophic Sets and Systems, Vol. 79, 2025
University of New Mexico

Neutrosophic DUS Exponential Distribution

Megha.C.M [1,4], Divya.P.R [2], and Sajesh.T.A [3,*]

[1] Department of Statistics, St Thomas College (Autonomous), Thrissur; meghacm@gmail.com
[2] Department of Statistics, Vimala College (Autonomous), Thrissur; divyastat@vimalacollege.edu.in
[3] Department of Statistics, St Thomas College (Autonomous), Thrissur; sajesh.t.abraham@gmail.com
[4] University of Calicut

* Correspondence: sajesh.t.abraham@gmail.com; Tel.: (+919747936823)

Abstract: The exponential distribution is a widely used lifetime model with applications spanning numerous disciplines. The DUS transformation extends the applicability of the exponential distribution by improving its flexibility for diverse real-world scenarios. This study proposes the Neutrosophic DUS Exponential Distribution (NDUS-ED), an extension of the DUS Exponential Distribution, designed to effectively address and quantify uncertainty, inconsistency, and indeterminacy in data. Key statistical properties of NDUS-ED, including quantiles, moments, moment-generating functions, and order statistics, are derived under neutrosophic conditions. The performance of estimated parameters is evaluated through simulation, revealing superior results with larger sample sizes, particularly in managing imprecise and indeterminate data. Finally, the proposed distribution is applied to an actual dataset, and the results are compared with those of the Neutrosophic Exponential Distribution and the DUS Exponential Distribution, demonstrating superior performance.

Keywords: Neutrosophy; DUS transformation; Probability distribution; Exponential distribution

1. Introduction

The idea of vagueness or ambiguity is often used nowadays to solve a variety of real-world issues. Classical statistics is the term used to describe statistics, which primarily works with numerical data. We can't always obtain numerical data in its precise form in the real world because of the indeterminacy of data. Numerous advancements have been made recently to model such imprecise situations by taking fuzzy logic and neutrosophy into consideration [1-4]. In order to address indeterminacy or partially ambiguous aspects in the data, Smarandache [5] proposed neutrosophic statistics. In 1995, he presented the idea of neutrosophic logic by denoting the components as T, I, and F, which stand for truth, indeterminate, and falsehood, respectively. Neutrosophic is a multiple-valued logic that defines imprecise probability, fuzzy logic and classical logic. This logic has been applied in various domains of science and engineering.

Probability distributions have become an integral part of all scientific study. These probabilistic frameworks describe many real-world unforeseen occurrences [6]. Many studies have been conducted using probability distributions that incorporates neutrosophic logic [7]. Patro et al. [8] developed the neutrosophic probability distributions, such as the normal and binomial distributions and presented related case studies. Alhabib et al. [9] introduced neutrosophic Poisson, uniform, and exponential distributions by generalizing the respective classical distributions. Alhasan et al. [10] proposed and studied the neutrosophic Weibull distribution and the neutrosophic version of its related family of distributions. Aslam [11] introduced the neutrosophic Rayleigh distribution under the neutrosophic statistics and discussed its various properties.

Sherwani et al. [12] extended the classical beta distribution to a neutrosophic environment and proposed neutrosophic beta distribution. They have established many statistical characteristics of the distribution and the results were validated by applying the distribution on two real life datasets. Khan et al. [13] established gamma distribution under indeterminacy with applications to complex data analysis. Khan et al. [14] derived neutrosophic lognormal distribution and data from Nitrogen oxide emissions of Denmark was used to validate the established characteristics of the distribution. Ahsan-ul-Haq [15] proposed the neutrosophic Kumaraswamy distribution to analyze bounded data sets under an indeterminacy environment.

In 2023, more distributions pertaining to imprecise data were introduced. Norouzirad et al. [16] introduced neutrosophic generalized Rayleigh distribution and shown that the distribution is ideal to model skewed lifetime data. Rao [17] established neutrosophic log-logistic distribution model in complex alloy metal melting point applications. Alanaz et al. [18] proposed neutrosophic exponentiated inverse Rayleigh distribution to describe diverse survival data with indeterminacies.

Neutrosohic beta-Lindley distribution was established by Algamal et al. [18] and modeled bladder cancer data. Neutrosophic Birnbaum-Saunders distribution for imprecise data was introduced by Hassan et al. [19] and the results were validated through datasets based on alloy melting points and lifetime of batteries. Neutrosophic Topp-Leone distribution for interval-valued data analysis [20] was proposed by Ahsan-ul-Haq [20]. Under indeterminacy, discrete geometric distribution was introduced by Khan et al. [21] with emphasis to reliability functions and relevant case studies. Jamal et al. [22] proposed neutrosophic BURR III to model COVID 19 data. Recently, Aslam [23] established the neutrosophic negative binomial distribution and developed algorithms for generating data based on this distribution.

A common lifetime distribution among the classical distributions is the exponential distribution, which is simple in terms of the closeness of its cumulative density function (cdf) and other statistics. Since exponential distribution have fixed failure rate constraint, transformations are applied to arrive at more robust and meaningful distribution. One such transformation is DUS transformation [24]. The DUS transformation yields a parsimonious distribution since it has no additional parameters, making it simple to understand. When the exponential distribution undergoes the DUS transformation, it leads to the DUS-exponential distribution (DUS-ED) which is a more robust distribution. Considering the exponential distribution using neutrosophic reasoning Duan et al. [25] presented neutrosophic exponential distribution. Later, in 2022 Khan et al. [26] developed the neutrosophic design of the exponential model with application to lifetime failures (in hours) of air conditioning instrument used in 720-Boeing planes.

This study proposes a new lifetime distribution the neutrosophic DUS-exponential distribution (NDUS-ED), an extension of DUS-exponential distribution intended to efficiently capture and quantify uncertainty, inconsistency, and indeterminacy. The primary purpose of the proposed distribution is to incorporate the indeterminate information about the variables under study into the existing classical distribution. It is necessary to consider indeterminacy of the study parameters in practical situations and integrate it with the proposed model to present a system.

This paper is structured as follows: Section 2 introduces the proposed model and corresponding plots. Statistical properties are derived in section 3. The parametric estimation is done in section 4. Simulation study is explained in section 5. Real data set study is included in section 6. Conclusions are given in section 7.

2. Proposed Model

Neutrosophic DUS-Exponential Distribution (NDUS-ED)

Let $f(x)$ and $F(x)$ be the pdf and cdf of the baseline distribution respectively, then the pdf $g(x)$ of the distribution obtained by DUS transformation of the baseline distribution is given by

$$g(x) = \frac{1}{(e^{(1)} - 1)} f(x) e^{F(x)} \tag{1}$$

Now, the corresponding cdf and hazard function are given by

$$G(x) = \frac{1}{(e^{(1)} - 1)} \left[e^{F(x)} - 1\right] \tag{2}$$

and

$$h(x) = \frac{1}{\left(e^{(1)} - e^{F(x)}\right)} f(x) e^{F(x)} \tag{3}$$

respectively.

The neutrosophic random variable X, which equals the distance between successive events in a Poisson process, follows the neutrosophic exponential distribution (NED) model with the following neutrosophic density function

$$f_N(x) = \theta_N \, e^{-\theta_N x}, x > 0, \theta_N \in [\theta_L, \theta_U], \theta_N > 0 \tag{4}$$

The corresponding neutrosophic cumulative distribution function

$$F_N(x) = 1 - e^{-\theta_N x}, x > 0, \theta_N \in [\theta_L, \theta_U], \theta_N > 0 \tag{5}$$

Now, using (1), (2), and (3), neutrosophic- DUS exponential model is proposed. The neutrosophic density function (npdf) and neutrosophic cumulative distribution function (ncdf) of the proposed distribution respectively are

$$g_N(x) = \frac{1}{\left(e^{(1)} - 1\right)} \theta_N \, e^{-\theta_N x} \, e^{\left(1 - e^{-\theta_N x}\right)}, x > 0, \theta_N \in [\theta_L, \theta_U], \theta_N > 0 \tag{6}$$

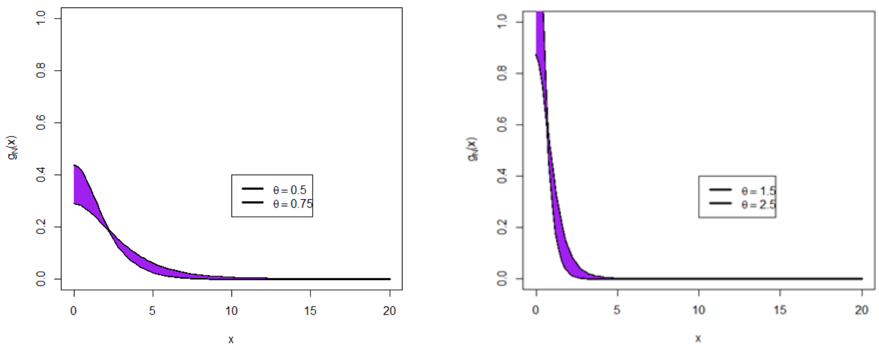
and

$$G_N(x) = \frac{1}{e^{(1)} - 1} \left[e^{\left(1 - e^{-\theta_N x}\right)} - 1 \right], \; x > 0, \theta_N \in [\theta_L, \theta_U], \theta_N > 0 \tag{7}$$

The hazard function is given by

$$h_N(x) = \frac{1}{e^{(1)} - e^{\left(1 - e^{-\theta_N x}\right)}} \theta_N \, e^{-\theta_N x} \, e^{\left(1 - e^{-\theta_N x}\right)}, x > 0, \theta_N \in [\theta_L, \theta_U], \theta_N > 0 \tag{8}$$

The plots of npdf are given below.

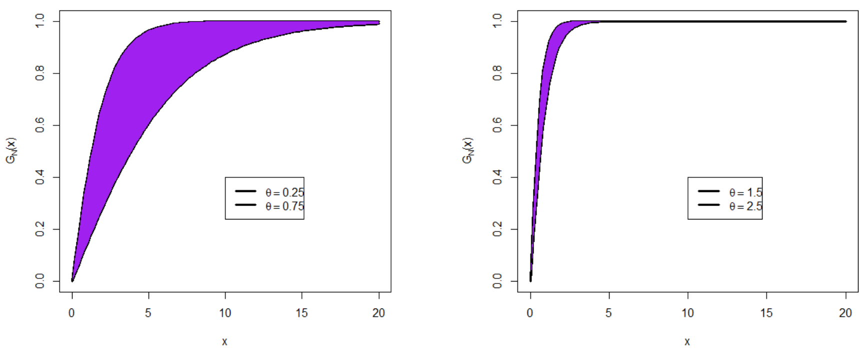


(a) (b)

.

Figure 1 Density graph of neutrosophic DUS-exponential
(a) $\theta_N \epsilon$ [1.5,2.5] (b) $\theta_N \epsilon$ [3.5,4.5].

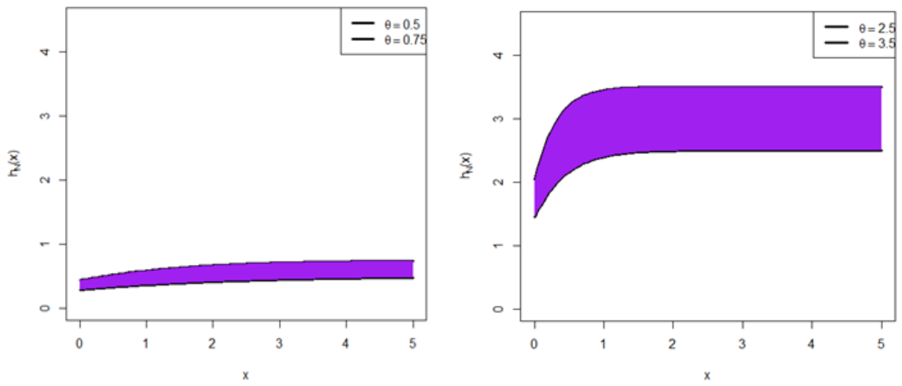
The plots of ncdf are given below.



(a) (b)

Figure 2 Cumulative Density graph of neutrosophic DUS-exponential
(a) $\theta_N \epsilon$ [0.25,0.75] (b) $\theta_N \epsilon$ [1.5,2.5].

The plots of hazard function are given below.



(a) (b)

Figure 3. Hazard function graph of neutrosophic DUS-exponential (a) $\theta_N \epsilon$ [0.5,0.75] (b) $\theta_N \epsilon$ [2.5,3.5]

3. Statistical Properties

The main properties like moments, moment generating function, cumulant generating function, characteristic function, quantiles and order statistics are studied below.

3.1. Moments

The r^{th} moment of the distribution, μ_r' is given by

$$\mu_r' = \int_0^\infty x^r \frac{1}{(e^{(1)}-1)} \theta_N\, e^{-\theta_N x}\, e^{(1-e^{-\theta_N x})}\, dx \tag{9}$$

To solve (9) we use exponential expansion $e^z = \sum_{j=0}^{\infty} \frac{z^j}{j!}$ and binomial series of expansion $(1-z)^a = \sum_{j=0}^{\infty}(-1)(aC_j)z^j$. Now Equation (9) takes the form

$$\mu_r' = \frac{1}{\theta_N^r(e^{(1)}-1)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^n}{m!}\,(mC_n)\frac{\Gamma(r+1)}{(n+1)^{r+1}} \tag{10}$$

Put $r = 1$, then we get

$$E(X) = \frac{1}{\theta_N(e^{(1)}-1)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^n}{m!}\,(mC_n)\frac{1}{(n+1)^{2}} \tag{11}$$

The expression for V(X) is given in equation (12)

$$V(X) = \frac{1}{\theta_N^2(e^{(1)}-1)}\left[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!}\,(mC_n)\frac{\Gamma(3)}{(n+1)^3} - \frac{1}{(e^{(1)}-1)}\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!}\,(mC_n)\frac{1}{(n+1)^2}\right\}^2\right] \qquad (12)$$

3.2 Moment Generating Function

If X ~ NDUS-ED, the moment generating function is derived as

$$M_X(\mathrm{t}) = E(e^{tX}) = \frac{\theta_N}{e^{(1)}-1}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!}\,(mC_n)\frac{\Gamma(1)}{(\theta_N(n+1)-t)} \qquad (13)$$

3.3 Cumulant Generating Function

The cumulant generating function of X, where X ~NDUS-ED is given as

$$K_X(\mathrm{t}) = \log M_X(\mathrm{t}) = \log\left[\frac{\theta_N}{e^{(1)}-1}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!}\,(mC_n)\frac{\Gamma(1)}{(\theta_N(n+1)-t)}\right] \qquad (14)$$

3.4 Characteristic Function

The characteristic function of X, where X ~NDUS-ED is as follows

$$\varphi_X(t) = E\left(e^{itX}\right) = \frac{\theta_N}{e^{(1)}-1}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!}\,(mC_n)\frac{\Gamma(1)}{(\theta_N(n+1)-it)} \qquad (15)$$

3.5 Quantile Function

The p^{th} quantile function, denoted by $Q(p)$ of NDUS-ED is obtained by solving

$$G_N\big(Q(p)\big) = p, 0 < p < 1.$$

The quantile function is given by

$$Q(p) = \frac{-\ln\left(1-\ln\left(p\left(e^{(1)}-1\right)+1\right)\right)}{\theta_N} \tag{16}$$

While setting $p = \frac{1}{4}$, we get the first quartile Q_1, setting $p = \frac{1}{2}$, we get the second quartile/Median Q_2 and setting $p = \frac{3}{4}$, we get the third quartile Q_3.

3.6 Order Statistics

The hazard rate function plays a vital role in the domains of survival analysis and reliability theory. This makes the relevance of order statistics in the domains. Let $X_1, X_2, \dots, X_n$ be n independently and identically random variables with the corresponding order statistics $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ from NDUS-ED with the density function $g_N(x)(6)$ and the distribution function $G_N(x)(7)$. The pdf and cdf of the r^th^ order statistics are given respectively by $g_r(x)$ and $G_r(x)$

$$g_r(x) = \frac{n!}{(r-1)!\,(n-r)!} G_N{}^{(r-1)}(x)[1-G_N(x)]^{(n-r)}\, g_N(x)$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{e^{(1)}-1}\left[e^{\left(1-e^{-\theta_N x}\right)}-1\right]\right]^{(r-1)}$$

$$\left[1-\frac{1}{e^{(1)}-1}\left[e^{\left(1-e^{-\theta_N x}\right)}-1\right]\right]^{(n-r)} \frac{1}{\left(e^{(1)}-1\right)}\theta_N\, e^{-\theta_N x}\, e^{\left(1-e^{-\theta_N x}\right)} \tag{17}$$

$$G_r(x) = \sum_{k=r}^{N} nC_k\,[G_N(x)]^k[1-G_N(x)]^{(n-k)}$$

$$= \sum_{k=r}^{N} nC_k \left[\frac{1}{e^{(1)}-1}\left[e^{\left(1-e^{-\theta_N x}\right)}-1\right]\right]^k \left[1-\frac{1}{e^{(1)}-1}\left[e^{\left(1-e^{-\theta_N x}\right)}-1\right]\right]^{(n-k)} \tag{18}$$

4. Method of Estimation

Here, the parameters are estimated using the method of maximum likelihood is discussed. Consider $x_1, x_2, \dots, x_n$ are n observations taken from NDUS-ED. Then, the likelihood function is given by

$$L(x:\theta_N) = \prod_{i=1}^{n} g_N\,(x;\theta_N)$$

$$= \prod_{i=1}^{n} \frac{1}{\left(e^{(1)}-1\right)}\theta_N\, e^{-\theta_N x}\, e^{\left(1-e^{-\theta_N x}\right)}$$

$$= \left[\frac{e^{(1)}}{e^{(1)}-1}\right]^n \theta_N{}^n e^{-\,(\theta_N \sum_{i=1}^{n} x_i)} e^{-e^{-(\theta_N \sum_{i=1}^{n} x_i)}}$$

Now, the log-likelihood function is given by

$$\log L = nlog\left(e^{(1)}\right) - n\log\left(e^{(1)} - 1\right) + n\log\theta_N - \theta_N\left(\sum_{i=1}^{n} x_i\right) - e^{-\left(\theta_N \sum_{i=1}^{n} x_i\right)}$$

The maximum likelihood estimators are obtained by maximizing the log-likelihood function with respect to the unknown parameter θ_N

$$\frac{\partial logL}{\partial\theta_N} = \frac{n}{\theta_N} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i\, e^{-\left(\theta_N \sum_{i=1}^{n} x_i\right)}$$

This equation has a non-linear form. Therefore, it can be numerically solved using the statistical software R by choosing arbitrary initial values.

5. Simulation study

A comprehensive evaluation has been conducted in order to assess the MLE performance of the NDUS-ED. The neutrosophic root mean square error ($RMSE_N$) shows how well a model predicts the data; lower $RMSE_N$ values are indicative of higher model performance. The following definition of neutrosophic root mean square error have been used to evaluate the effectiveness of the neutrosophic maximum likelihood estimator:

$$RMSE_N = \sqrt{\frac{\sum_{i=1}^{N}\left(\hat{\theta}_{N_i} - \theta_N\right)^2}{N}}$$

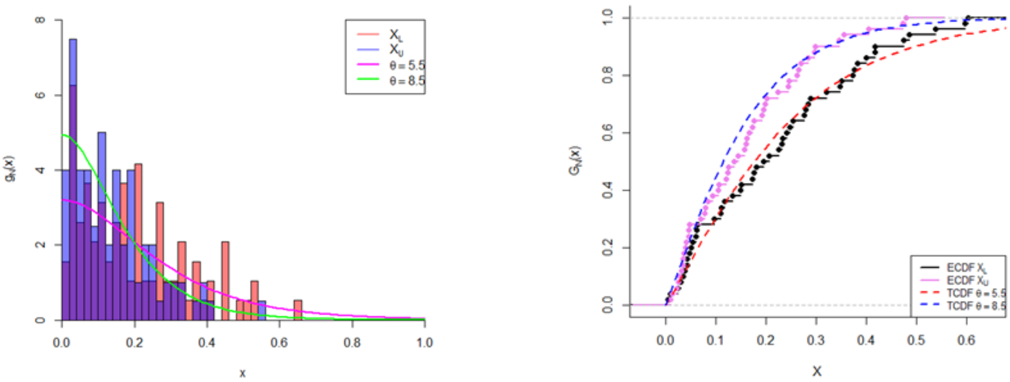
The inversion approach is used to obtain the samples for the two interval measures $\theta_N \epsilon$ [1,2] and $\theta_N \epsilon$ [3,4] and the analysis is repeated for N=10,000 times. The analysis is carried out using R software. For sample sizes n= 10, 25, 40, 55, 70, $RMSE_N$ is computed. Table 1 displays the results of a study on the performance of measurements of the neutrosophic maximum likelihood (NML) estimator.

Table 1 Performance of NML estimate of the NDUS-ED for simulated neutrosophic data

Sample size	$RMSE_N$ for $\theta \epsilon$ [1,2]	$RMSE_N$ $\theta \epsilon$ [3,4]
10	[0.2912116, 0.5996537]	[0.8736341, 1.199315]
25	[0.1740633, 0.4682503]	[0.5221849, 0.936499]
40	[0.1463233, 0.4443628]	[0.4389749, 0.888727]
55	[0.134346, 0.4342111]	[0.4030462, 0.868416]
70	[0.127095, 0.4286541]	[0.3812866, 0.857305]

From the table, it can be seen that, as the sample size (n) increases $RMSE_N$ decreases for both θ ϵ [1,2] and θ ϵ [3,4]. The study concludes that the NML estimator offers reliable and accurate parameter estimates by showing a steady increase in estimation accuracy with increasing sample size.

The pdf and cdf plots of the simulated values are given below



(a) (b)

Figure 4. Graphical overview of simulated values (a) pdf plot (b) cdf plot

6. Real-World application

In this section, a real dataset is used to illustrate the application of the proposed model. The data is taken from [27]. We use two statistical criteria, Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) to test the performance of our proposed distribution. We have extracted the values of AIC and BIC of neutrosophic exponential distribution (NED) from [28] and DUS-exponential distribution (DUS-ED) from [24] and presented their values.

Table 2. AIC and BIC of NDUS-ED, NED and DUS-ED (DUS-exponential) distributions

Distributions	AIC	BIC
NDUS-ED	(823.8715, 827.376)	(826.7235, 830.2281)
NED	(825.8715, 829.3760)	(841.2796, 844.7842)
DUS-ED	834.044	836.896

Table 2 shows the comparison between the proposed distribution with that of the other distributions. Here, we can observe that the AIC (823.8715, 827.376) and BIC (826.7235, 830.2281) values of the proposed distribution are lower than those of the other distributions. Lower AIC and BIC values indicate that the neutrosophic framework is producing accurate and trustworthy parameter estimates by effectively interpreting data uncertainty. The suggested model (6) is more accurate and adaptable in terms of imprecision and uncertainty when compared to the other distributions.

7. Conclusions

In contrast to classical probability distributions, neutrosophic probability distributions take an intrusive approach to explaining and resolving a variety of real-world scenarios. A wide range of disciplines, including engineering, actuarial sciences, health sciences, etc. heavily rely on the exponential distribution. A neutrosophic DUS-exponential distribution has been developed to better account for the imprecisions that are frequently present in real-world data, especially when it comes to interval form. The generating functions, order statistics, and hazard rate function are among the properties of the suggested neutrosophic DUS-exponential distribution that have been established. Using the neutrosophic maximum likelihood estimation method, parameters were estimated, and a simulation study was used to assess the distribution's efficiency. This model's adaptability was further illustrated by using it on a real-world dataset and contrasting its results with those of the neutrosophic DUS-exponential distribution, neutrosophic exponential distribution and DUS-exponential distribution. Results show that the suggested NDUS-ED yields trustworthy outcomes while managing data uncertainty and indeterminacy.

The NDUS-ED distribution could be extended or modified to accommodate different types of data distributions by introducing additional flexibility through parameterization or hybridization with other distributions. Adding shape parameters to account for skewness or heavy tails could make the distribution more adaptable to asymmetric or leptokurtic data. Developing multivariate versions of the NDUS-ED distribution could enable modeling of relationships and dependencies between multiple variables.

Funding: "This research received no external funding"

Acknowledgments: The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality of the paper.

Conflicts of Interest: "The authors declare no conflict of interest."

References

1. Atanassov, K.T., Intuitionistic fuzzy sets. *Fuzzy Sets and Syst.* **1986**, 20(1), pp. 87-96.
2. Ashbacher, C. Introduction to Neutrosophic logic. *Infinite Study* **2014.**

3. Smarandache, F. A unifying field in Logics: Neutrosophic Logic. In *Philosophy* **1999**, pp.1-141.
4. Smarandache, Florentin.; Stefan, Vladutescu.; Miihaela, Colhon.; Wadei, Al-Omeri.; Saeid, Jafari.; Muhammad, Zahir, Khan.; Muhammad, Farid, Khan.; Muhammad, Aslam.; Abdur, Razzaque, Mughal. *New Challenges in Neutrosophic Theory and Applications*. MDPI, Basel, Switzerland,**2020.**
5. Smarandache, F. Introduction to Neutrosophic Statistics. *Sitech & Education Publishing* **2014.**
6. Dimitrov, B. N.; Chukova, S.; Green Jr, D. Probability distributions in periodic random environment. *SIAM J. Appl. Math* **1994.**
7. Zeina, M. B.; Hatip, A. Neutrosophic random variables. *Neutrosophic Sets and Syst.* **2021,** 39, pp.44-52.
8. Patro, S. K.; Smarandache, F. The neutrosophic statistical distribution, more problems, more solutions. *Infinite Study* **2016.**
9. Alhabib, R.; Ranna, M. M.; Farah, H., Salama, A. A. Some neutrosophic probability distributions. *Neutrosophic Sets and Syst*. **2018,** 22, pp.30-38.
10. Alhasan, K. F. H.; Smarandache, F. Neutrosophic Weibull distribution and neutrosophic family Weibull distribution. *Infinite Study* **2019**.
11. Aslam, M.A. Neutrosophic Rayleigh distribution with some basic properties and application, *Neutrosophic Sets in Decision Analysis and Operations Research, IGI Global* **2020**, pp. 119-128.
12. Sherwani, RAK.; Naeem, M.; Aslam, M.; Raza, M.A.; Abid, M.; Abbas, S. Neutrosophic beta distribution with properties and applications. *Neutrosophic Sets Syst* **2021**,41, pp.209–214
13. Khan, Z.; Al-Bossly, A.; Almazah, M. M.; Alduais, F. S. On statistical development of neutrosophic gamma distribution with applications to complex data analysis. Complexity, **2021**(1), 3701236.
14. Khan, Z.; Amin, A.; Khan, S. A.; Gulistan, M. Statistical development of the neutrosophic lognormal model with application to environmental data. *Neutrosophic Sets and Syst.* **2021**, 47(1), p.1.
15. Ahsan-ul-Haq, M. J. N. S. S. Neutrosophic Kumaraswamy distribution with engineering application. *Neutrosophic Sets and Syst.* **2022**, 49(1), pp. 269-276.
16. Norouzirad, M.; Rao, G. S.; Mazarei, D. Neutrosophic Generalized Rayleigh Distribution with Application. *Neutrosophic Sets and Syst.* **2023,***58*(1), p. 15.
17. Rao, G. S. Neutrosophic log-logistic distribution model in complex alloy metal melting point applications. *Int. J. Comput. Intell. Syst.* **2023,** *16*(1), p. 48.
18. Algamal, Z. Y.; Alobaidi, N. N.; Hamad, A. A.; Alanaz, M. M.; Mustafa, M. Y. Neutrosophic Beta-Lindley distribution: mathematical properties and modeling bladder Cancer data. *Int. J. Neutrosophic Sci.* **2024**, *23*(2), pp.186-194.

19. Hassan, M.K.; Aslam, M. Birnbaum Saunders distribution for imprecise data: statistical properties, estimation methods, and real life applications. *Sci Rep* **2024**,14, p.6955.
20. Ahsan-ul-Haq, M.; Zafar, J.; Aslam, M.; Tariq, S. Neutrosophic Topp-Leone Distribution for Interval-Valued Data Analysis. *J. Stat. Theory Appl.* **2024**, pp.1-10.
21. Khan.; Rehan.; Iqbal.; Sadia.; Abbas.; Shumaila.; Saleem.; Muhammad, Aslam., Muhammad.; Smarandache, Florentin. Neutrosophic discrete Geometric distribution. *Adv. Appl. Stat.* **2024**, 91, pp. 855-877.
22. Jamal, F.; Shafiq, S.; Aslam, M.; Khan, S.; Hussain, Z.; Abbas, Q. Modeling COVID-19 data with a novel neutrosophic Burr-III distribution. *Sci. Rep.* **2024,**14(1), p. 10810.
23. Aslam, M. The neutrosophic negative binomial distribution: algorithms and practical application, *Revstat Stat. J.* accepted-August **2024**.
24. Kumar, D.; Singh, U.; Singh S. K. A method of proposing new distribution and its application to Bladder cancer patients data. *J Stat Appl Pro Lett* **2015,** 2(3), pp.235–245
25. Duan, W. Q.; Khan, Z.; Gulistan, M.; Khurshid, A. Neutrosophic exponential distribution: modeling and applications for complex data analysis. *Complexity* **2021,** p. 5970613.
26. Khan, Z.; Gulistan, M. Neutrosophic Design of the Exponential Model with Applications. *Neutrosophic Sets and Syst.* **2022,** *48*, pp.291-305.
27. Lee, E. T.; Wang, J. *Statistical methods for survival data analysis,* 3rd ed.; John Wiley & Sons, Inc.: Hoboken, New Jersey. **2003**; p. 231.
28. Rao, G. S.; Norouzirad, M.; Mazarei, D. Neutrosophic generalized exponential distribution with application. *Neutrosophic Sets and Syst.* **2023**,*55*(1), p. 28.

Received: Sep 27, 2024. Accepted: Dec 13, 2024