The First Resolution of the Travelling Salesman Problem under Neutrosophic Octagonal Fuzzy Environment

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Abstract: The Travelling Salesman Problem (TSP) is a challenging combinatorial optimization problem classified as NP-hard. Its objective is to identify the shortest cycle that visits every city precisely once before returning to the initial city. To the best of our knowledge there is no one in the literature who solved the TSP under the neutrosophic octagonal fuzzy environment. That's why, in this paper the novel heuristic namely Dhouib-Matrix-TSP1 (DM-TSP1) is exploited to optimize the TSP under the neutrosophic octagonal fuzzy domain. So, this research work represents the first application of DM-TSP1 on this mentioned environment. A defuzzification function is used to convert neutrosophic octagonal fuzzy numbers to crisp ones then the four simple steps of DM-TSP1 are launched. A numerical example illustrating a step-by-step application of DM-TSP1 on novel created benchmark instances is provided to prove its performance and efficiency in solving the neutrosophic octagonal fuzzy TSP.

Keywords: Urban development, Infrastructure governance, Artificial Intelligence, Intelligent Computing and Algorithms, Data science applications, Neutrosophic Set.

1. Introduction

The Travelling Salesman Problem (TSP) is one of the most important problems in the operational research and it is extensively used in the real-world. It aims to generate the shortest cycle between all nodes (cities, customers, suppliers, … etc.) namely the Hamiltonian cycle: each node is visited only once excepting the starting node which will be also the last visited node.

The mathematical formulation of the TSP is as follows:

Minimize:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} x_{ij} \tag{1}$$

Subject to:

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,...,n \\ &\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,...,n \\ &x_{ij} = 0 \text{ or } 1, \ i = 1,...,n \ , \quad \mathrm{j} = 1,...,n \end{aligned} \tag{2}$$

x_{ij} is the binary variable used to specify if city i is linked to city j (so x_{ij} = 1) or if cities i and j are not linked (x_{ij} = 0). d is the parameter that represents the distance (cost, time, … etc.) while d_{ij} denotes the distance between cities i and j.

In industrial companies, data are usually uncertain and imprecise that's why fuzzy, intuitionistic and neutrosophic numbers are considered in almost of optimization problems (instead of crisp numbers). The neutrosophic concept is firstly proposed by Smarandache [1]. The neutrosophic components are: the Truth (T), the Indeterminacy (I) and the Falsity (F) with T, I and F are subsets of [-0,1+].

Several research works are interested in solving the TSP under the neutrosophic environment. The neutrosophic TSP using hexagonal fuzzy numbers is optimized in [2] via the nearest neighbor technique which is a local approximative method that uses the ancient values closed to the current one to estimate its trajectory by the means of the trajectories of its neighbors. The neutrosophic TSP with trapezoidal fuzzy numbers is solved in [3] by the use of the branch and bound method that divides the problem into several sub-problems and uses a function to remove sub-problems that cannot have the optimal solution. A formulation and a genetic algorithm are proposed in [4] to optimize the neutrosophic TSP with the neutrosophic edge weight and a new heuristic crossover as well as a heuristic mutation for the proposed GA are designed. A novel heuristic for a new pentagonal neutrosophic TSP is introduced in [5]. In fact, the novel heuristic Dhouib-Matrix-TSP1 (DM-TSP1) is applied using a ranking function in order to transform the fuzzy numbers to crisp data and a range function to select cities. The Pascal's graded mean method and the ones assignment approach are used to unravel the neutrosophic TSP in [6]. The novel column-row heuristic namely Dhouib-Matrix-TSP1 is enhanced by the means of the center of gravity ranking function and the standard deviation metric to optimize the neutrosophic TSP in [7]. The Yager's ranking function is applied to convert the neutrosophic triangular fuzzy numbers to crisp numbers then the neutrosophic TSP is solved by the means of the Dhouib-Matrix-TSP1 heuristic in [8].

Other research papers are focused on the optimization of the Transportation Problem (TP) using neutrosophic numbers. The neutrosophic TP is modelled as a parametric Linear Programming Problem (LPP) then it is solved by a dynamic optimal solution approach in [9]. The constraints containing pentagonal neutrosophic quantities are converted into crisp constraints with the help of the score function then the pentagonal neutrosophic TP with an interval cost is solved in [10]. A modeling study of the TP using the neutrosophic logic and neutrosophic mathematical formulas for the TP are presented in [11]. A neutrosophic approach for the TP by the use of single-valued trapezoidal neutrosophic numbers is introduced in [12] and the obtained optimal solution is compared with solutions given by other approaches. The bilevel TP in the neutrosophic environment is optimized in [13] using a goal programming strategy to arrive at a satisfactory solution. The neutrosophic multi-objective green four-dimensional fixed-charge TP is optimized in [14] by the means of two new approaches based on the neutrosophic programming and the pythagorean hesitant fuzzy programming. The single-valued trapezoidal neutrosophic TP is solved in [15] by enriching the Dhouib-Matrix-TP1 heuristic with two functions namely the score and the metric functions.

Concerning the resolution of the Assignment Problem (AP) under the neutrosophic domain, many methods are proposed. The neutrosophic AP is optimized by the means of interval-valued trapezoidal neutrosophic numbers and the use of the order relations in [16]. The AP with neutrosophic costs is studied in [17] and its solution methodology is presented where two real problems are solved to prove the efficiency of the proposed method. An approach to optimize the multi-objective AP with neutrosophic numbers is introduced in [18]. This approach is used by the help of a weighting Tchebycheff problem which is applied by defining relative weights and ideal targets. A new method to rank pentagonal neutrosophic numbers using its magnitude and its applications to solve the AP is proposed in [19]. The neutrosophic set theory is applied on the generalized AP and after evaluating the score function matrix the problem is solved by the use of the Extremum Difference Method (EDM) to get the optimal assignment in [20]. A solution for the multi-criteria AP is obtained through the neutrosophic set theory and by applying two different methods in [21].

To optimize the Shortest Path Problem (SPP) using neutrosophic numbers, few techniques are introduced. The neutrosophic SPP is solved in [22] by applying the Dijkstra algorithm which is redesigned to handle the case where the most of parameters of a network are given as neutrosophic numbers. An overview about the SPP in fuzzy, intuitionistic and neutrosophic environments and advantages as well as limitations of different types of sets are given in [23]. A mathematical model of the SPP in the neutrosophic environment and three different approaches for solving the neutrosophic SPP are presented in [24]. The neutrosophic SPP is optimized in [25] by the means of the Bellman algorithm and a comparative analysis is carried out with other existing methods which proves the performance of the new technique. The basic concepts about neutrosophic sets and interval valued neutrosophic graphs are reviewed then an algorithm is suggested in [26] to find the shortest path and distance in an interval valued neutrosophic graph. A novel linear programming approach for solving the gaussian valued neutrosophic SPP without using any ranking method is proposed in [27]. An algorithm is developed in [28] to find the shortest path on a network in which the weights of the edges are represented as bipolar neutrosophic numbers.

In this research work, the TSP under the neutrosophic octagonal fuzzy environment is optimized by the means of the novel heuristic namely Dhouib-Matrix-TSP1. To the best of our knowledge, this research work marks the inaugural resolution of the TSP under the neutrosophic octagonal fuzzy domain and that's why novel benchmark instances are created. Additionally, this paper presents the first application of DM-TSP1 on this mentioned environment. The remaining of this paper is structured as follows. Section 2 contains all details about the neutrosophic octagonal fuzzy numbers. Section 3 describes how to solve the TSP under the neutrosophic octagonal fuzzy environment using the novel heuristic namely DM-TSP1. Section 4 summarizes a numerical example. Finally, section 5 gives a conclusion and further research perspectives.

2. The neutrosophic octagonal fuzzy number

In this section, some definitions concerning the neutrosophic octagonal fuzzy numbers are presented.

A neutrosophic octagonal fuzzy number $\tilde{S}_N$ is described as follows:

$$\tilde{S}_N = < [(a_1; b_1; c_1; d_1; e_1; f_1; g_1; h_1) ; \rho] ;$$
$$[(a_2; b_2; c_2; d_2; e_2; f_2; g_2; h_2) ;\sigma] ;$$
$$[(a_3; b_3; c_3; d_3; e_3; f_3; g_3; h_3 ; \omega] >$$

Where $\rho, \sigma, \omega \in [0,1]$.

Three membership functions are defined for $\tilde{S}_N$:

1) The Truth $T_{\tilde{S}_N} : R \to [0, \rho]$ (see Figure 1.a),

2) The Indeterminacy membership function $I_{\tilde{S}} : R \to [\sigma,1]$ (see Figure 1.b),

3) The Falsity membership function $F_{\tilde{S}} : R \to [\omega,1]$ (see Figure 1.c).

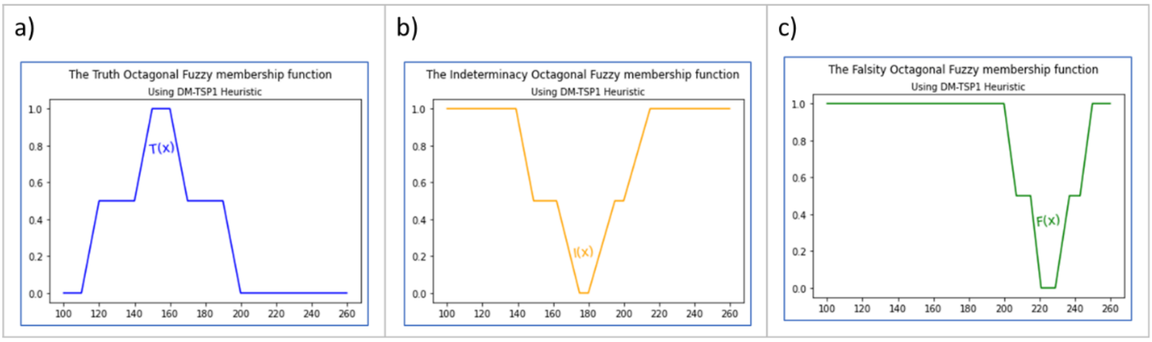

Figure 1. A graphical representation of: a) The thruth membership function b) The Indeterminacy membership function c) The Falsity membership function

These membership functions are defined as follows:

$$T_{\tilde{S}_N}(x)=\begin{cases}\dfrac{x-a_1}{b_1-a_1}, & a_1\le x\le b_1\\ \dfrac{x-b_1}{c_1-b_1}, & b_1\le x\le c_1\\ \dfrac{x-c_1}{d_1-c_1}, & c_1\le x\le d_1\\ 1, & x=d_1\\ 1, & x=e_1\\ \dfrac{f_1-x}{f_1-e_1}, & e_1\le x\le f_1\\ \dfrac{g_1-x}{g_1-f_1}, & f_1\le x\le g_1\\ \dfrac{h_1-x}{h_1-g_1}, & g_1\le x\le h_1\\ 0, & otherwise\end{cases}$$

$$I_{\tilde{S}_N}(x)=\begin{cases}\dfrac{b_2-x}{b_2-a_2}, & a_2\le x\le b_2\\ \dfrac{c_2-x}{c_2-b_2}, & b_2\le x\le c_2\\ \dfrac{d_2-x}{d_2-c_2}, & c_2\le x\le d_2\\ 0, & x=d_2\\ 0, & x=e_2\\ \dfrac{x-e_2}{f_2-e_2}, & e_2\le x\le f_2\\ \dfrac{x-f_2}{g_2-f_2}, & f_2\le x\le g_2\\ \dfrac{x-g_2}{h_2-g_2}, & g_2\le x\le h_2\\ 1, & otherwise\end{cases}$$

$$F_{\tilde{S}_N}(x)=\begin{cases}\dfrac{b_3-x}{b_3-a_3}, & a_3\le x\le b_3\\ \dfrac{c_3-x}{c_3-b_3}, & b_3\le x\le c_3\\ \dfrac{d_3-x}{d_3-c_3}, & c_3\le x\le d_3\\ 0, & x=d_3\\ 0, & x=e_3\\ \dfrac{x-e_3}{f_3-e_3}, & e_3\le x\le f_3\\ \dfrac{x-f_3}{g_3-f_3}, & f_3\le x\le g_3\\ \dfrac{x-g_3}{h_3-g_3}, & g_3\le x\le h_3\\ 1, & otherwise\end{cases}$$

Figure 2 represents a graphical representation of a neutrosophic octagonal fuzzy number where $-0\le T_{\tilde{S}_N}(x)+I_{\tilde{S}_N}(x)+F_{\tilde{S}_N}(x)\le 3+$.

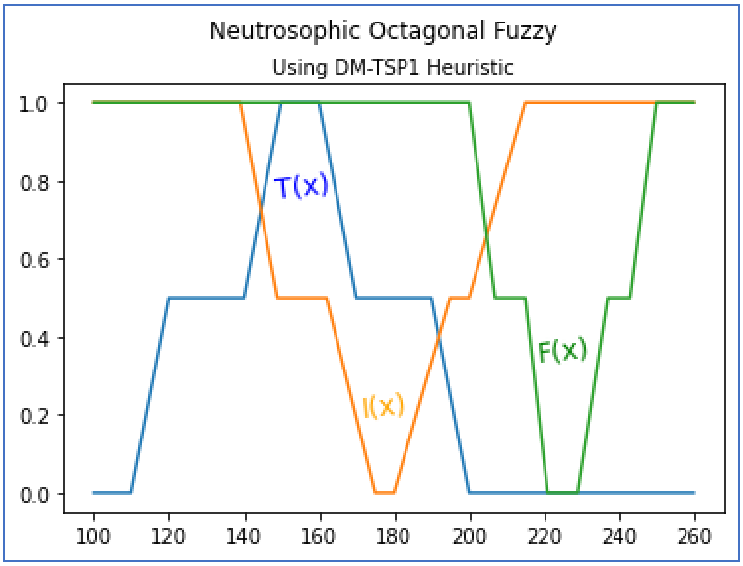

Figure 2. A graphical representation of a neutrosophic octagonal fuzzy number

3. Solving the neutrosophic octagonal fuzzy TSP using Dhouib-Matrix-TSP1

Recently and more precisely in 2021, the novel theory of optimization methods named Dhouib-Matrix (DM) has been invented. DM encompasses multiple optimization methods (see Figure 3). Regarding the approximative techniques, a new multi-start method entitled Dhouib-Matrix-4 (DM4) using statistical metrics is proposed in [29, 30, 31, 32, 33]. Furthermore, a new iterated stochastic metaheuristic named Dhouib-Matrix-3 (DM3) is introduced in [34, 35, 36, 37]. In addition, a new local search algorithm entitled Far-to-Near (FtN) using a guided search procedure is presented in [38]. Hence, concerning the optimization of the Assignment Problem two heuristics namely Dhouib-Matrix-AP1 (DM-AP1) and Dhouib-Matrix-AP2 (DM-AP2) are proposed in [39, 40, 41, 42]. Besides, the technique entitled Dhouib-Matrix-TP1 (DM-TP1) is designed in [43, 44] to optimize the Transportation Problem. The methods named Dhouib-Matrix-TSP1 (DM-TSP1) and Dhouib-Matrix-TSP2 (DM-TSP2) are developed in [45, 46, 47, 48, 49] to solve the Travelling Salesman Problem. Concerning the optimal techniques, two novel methods are developed: Dhouib-Matrix-SPP (DM-SPP) which aims to optimize the standard shortest path problem in [50] as well as the autonomous mobile robot shortest path planning problem in [51, 52, 53]; and Dhouib-Matrix-ALL-SPP (DM-ALL-SPP) to solve the all pairs (Multiple pair) shortest path problem [54]. Thus, a new technique named Dhouib-Matrix-MSTP (DM-MSTP) is proposed to unravel the Minimum Spanning Tree problem in [55].

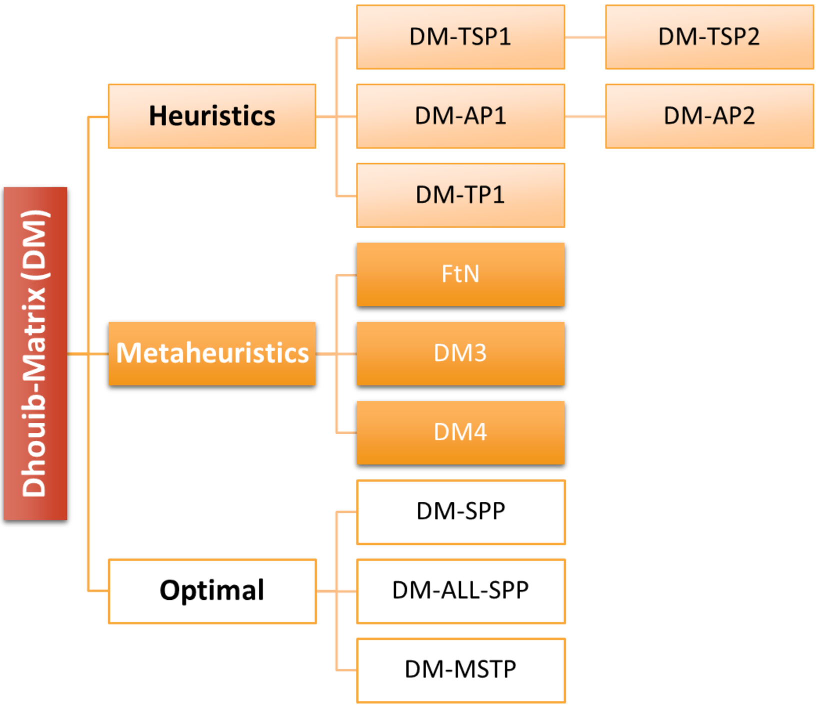

Figure 3. The Dhouib-Matrix techniques

At first, the heuristic DM-TSP1 is proposed to optimize the generalized TSP. Then, DM-TSP1 is adapted to solve the trapezoidal fuzzy TSP [56] as well as the octagonal fuzzy TSP [57] and the intuitionistic fuzzy TSP [58]. Furthermore, new versions of DM-TSP1 are introduced in [59, 60] to minimize the total distance.

As far as we know, this research work presents the first resolution of the TSP under the neutrosophic octagonal fuzzy environment as well as the first application of the novel heuristic namely DM-TSP1 on this mentioned environment. DM-TSP1 starts by applying the defuzzification function (see equation 1) used by [61].

$$D_{\tilde{S}_N} = \frac{D_{T_{\tilde{S}_N}} + D_{I_{\tilde{S}_N}} + D_{F_{\tilde{S}_N}}}{3} \quad (1)$$

Where:

$$D_{T_{\tilde{S}_N}} = \frac{1}{6}(a_1 + h_1) + k(b_1 + c_1 + f_1 + g_1) + (d_1 + e_1)$$

$$D_{I_{\tilde{S}_N}} = \frac{1}{6}(a_2 + h_2) + k(b_2 + c_2 + f_2 + g_2) + (d_2 + e_2)$$

$$D_{F_{\tilde{S}_N}} = \frac{1}{6}(a_3 + h_3) + k(b_3 + c_3 + f_3 + g_3) + (d_3 + e_3)$$

DM-TSP1 is composed of four steps where step 2 and step 3 are iterated n times (with n is the number of cities). Whereas steps 1 and 4 are executed only once. Actually, DM-TSP1 is characterized by its flexibility to use different descriptive statistical metrics. In this research work, the range metric is used. DM-TSP1 starts by converting the neutrosophic octagonal fuzzy numbers to crisp ones using the defuzzification function then the four steps of DM-TSP1 are executed (see Figure 4).

Step1: Compute the range function (max-min) for each row and write it on the right-hand side of the matrix. Next, find the minimal range and select its row. Then, select the smallest element in this row which will specify the two first cities x and y to be inserted in the list *List-cities* $\{x, y\}$. Finally, discard the respected columns of cities x and y.

Step 2: Find the minimal element for city x and for city y and select the smallest distance which will indicate city z.

Step 3: Add city z to the list *List-cities* and discard its column. Next, if there is no column go to Step 4 otherwise go to Step 2.

Step 4: Modify the realizable solution in *List-cities* in order to generate a cycle (the starting city in the cycle has to be also the last one). First, to ensure that the starting city will be at the first position, translate all the cities (one by one) before the starting one at the end of the list. Second, duplicate the starting city at the last position.

Figure 4. Steps of DM-TSP1

DM-TSP1 is a heuristic and not a metaheuristic so it is dedicated to solve only the TSP and not other problems such as the Transportation Problem. That's why, another heuristic is developed for the Transportation Problem namely DM-TP1 inspired from DM-TSP1. In addition, DM-TSP1 is used to generate a starting solution for different metaheuristics such as the Artificial Bee Colony in [49], the novel Dhouib-Matrix-3 in [36, 37] and Dhouib-Matrix-4 in in [29, 30].

4. Numerical example

A salesman is looking for the Hamiltonian cycle joining six cities to visit. It is required to visit each city only once and generating the shortest cycle. The distances between the six cities are represented as neutrosophic octagonal fuzzy numbers as represented in Figure 5.

$$\begin{bmatrix} \infty & d_{01} & d_{02} & d_{03} & d_{04} & d_{05} \\ d_{10} & \infty & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{20} & d_{21} & \infty & d_{23} & d_{24} & d_{25} \\ d_{30} & d_{31} & d_{32} & \infty & d_{34} & d_{35} \\ d_{40} & d_{41} & d_{42} & d_{43} & \infty & d_{45} \\ d_{50} & d_{51} & d_{52} & d_{53} & d_{54} & \infty \end{bmatrix}$$

Figure 5. Neutrosophic octagonal fuzzy distance matrix

Where:

$$d_{01} = \langle 4,8,10,15,16,16.5,18,22; 8.5,9.5,10.5,12,14,16.5,17.5,18.5; 11,12,15,16,19,21,22,23 \rangle$$

$$d_{02} = \langle 5,6,7,8,9,10,11,12; 3.5,4.5,5.5,6.5,7.5,8.5,9.5,10.5; 7,8,9,10,11,12,13,14 \rangle$$

$$d_{03} = \langle 3,4,5,6,7,8,9,10; 4.5,5.5,6.5,7.5,8.5,9.5,10.5,11.5; 9,11,12,13,14,15,16,17 \rangle$$

$$d_{03}=\langle 0,1,2,3,5,6,7,8;2.5,2.5,4.5,5.5,6.5,7.5,8.5,9.5;5,7,9,11,12,13,14,15\rangle$$

$$d_{04}=\langle 4,5,6,7,8,9,10,11;5.5,6.5,7.5,8.5,9.5,11.5,13.5,14.5;9,11,12,13,14,15,16,17\rangle$$

$$d_{05}=\langle 5,6,7,8,10,12,13,14;6.5,7.5,8.5,9.5,11.5,12,13,14;11,12,13,14,15,16,17,18\rangle$$

$$d_{10}=\langle 4,8,10,15,16,16.5,18,22;8.5,9.5,10.5,12,14,16.5,17.5,18.5;11,12,15,16,19,21,22,23\rangle$$

$$d_{12}=\langle 12,13,15,19,20,21,22,24;13,15,16,17,19,21,22,23;14,15,16,17,18,19,21,22\rangle$$

$$d_{13}=\langle 7,8,11,12,13,14,15,16;8.5,9.5,10.5,11.5,12.5,13.5,15.5,17.5;13,14,15,16,17,18,19,20\rangle$$

$$d_{14}=\langle 11,12,13,14,15,16,17,18;7.5,8.5,9.5,10.5,11.5,12.5,13.5,14.5;12,13,14,15,16,17,18,19\rangle$$

$$d_{15}=\langle 10,11,12,13,14,15,16,17;9.5,10.5,11.5,12.5,13.5,14.5,15.5,17.5;12,13,14,15,16,17,18,19\rangle$$

$$d_{20}=\langle 5,6,7,8,9,10,11,12;3.5,4.5,5.5,6.5,7.5,8.5,9.5,10.5;7,8,9,10,11,12,13,14\rangle$$

$$d_{21}=\langle 12,13,15,19,20,21,22,24;13,15,16,17,19,21,22,23;14,15,16,17,18,19,21,22\rangle$$

$$d_{23}=\langle 7,8,11,15,16,17,18,20;10.5,11.5,12.5,13.5,15.5,17.5,18.5,19.5;14,15,16,17,18,20,21,22\rangle$$

$$d_{24}=\langle 1,2,3,4,5,6,7,8;2.5,3.5,4.5,5.5,6.5,7.5,8.5,9.5;4,5,7,9,11,12,13,14\rangle$$

$$d_{25}=\langle 6,8,11,15,16,17,18,20;7.5,10.5,12.5,14.5,16.5,17.5,18.5,19.5;13,14,15,17,18,20,21,23\rangle$$

$$d_{30}=\langle 3,4,5,6,7,8,9,10;4.5,5.5,6.5,7.5,8.5,9.5,10.5,11.5;9,11,12,13,14,15,16,17\rangle$$

$$d_{31}=\langle 7,8,11,12,13,14,15,16;8.5,9.5,10.5,11.5,12.5,13.5,15.5,17.5;13,14,15,16,17,18,19,20\rangle$$

$$d_{32}=\langle 7,8,11,15,16,17,18,20;10.5,11.5,12.5,13.5,15.5,17.5,18.5,19.5;14,15,16,17,18,20,21,22\rangle$$

$$d_{34}=\langle 9,10,14,17,19,20,21,22;8,9,11,12,13,15,16,17;11,12,13,14,15,16,17,18\rangle$$

$$d_{35}=\langle 11,13,16,17,18,20,21,22;11,12,13,14,15,16,17,18;12,13,14,15,16,17,18,19\rangle$$

$$d_{40}=\langle 4,5,6,7,8,9,10,11;5.5,6.5,7.5,8.5,9.5,11.5,13.5,14.5;9,11,12,13,14,15,16,17\rangle$$

$$d_{41}=\langle 11,12,13,14,15,16,17,18;7.5,8.5,9.5,10.5,11.5,12.5,13.5,14.5;12,13,14,15,16,17,18,19\rangle$$

$$d_{42}=\langle 1,2,3,4,5,6,7,8;2.5,3.5,4.5,5.5,6.5,7.5,8.5,9.5;4,5,7,9,11,12,13,14\rangle$$

$$d_{43}=\langle 9,10,14,17,19,20,21,22;8,9,11,12,13,15,16,17;11,12,13,14,15,16,17,18\rangle$$

$$d_{45}=\langle 5,6,7,8,11,12,13,14;6.5,7.5,8.5,9.5,10.5,11.5,12.5,13.5;10,11,12,13,14,15,16,17\rangle$$

$$d_{50}=\langle 5,6,7,8,10,12,13,14;6.5,7.5,8.5,9.5,11.5,12,13,14;11,12,13,14,15,16,17,18\rangle$$

$$d_{51}=\langle 10,11,12,13,14,15,16,17;9.5,10.5,11.5,12.5,13.5,14.5,15.5,17.5;12,13,14,15,16,17,18,19\rangle$$

$$d_{52}=\langle 6,8,11,15,16,17,18,20;7.5,10.5,12.5,14.5,16.5,17.5,18.5,19.5;13,14,15,17,18,20,21,23\rangle$$

$$d_{53}=\langle 11,13,16,17,18,20,21,22;11,12,13,14,15,16,17,18;12,13,14,15,16,17,18,19\rangle$$

$$d_{54}=\langle 5,6,7,8,11,12,13,14;6.5,7.5,8.5,9.5,10.5,11.5,12.5,13.5;10,11,12,13,14,15,16,17\rangle$$

The first step is to convert all neutrosophic octagonal fuzzy numbers to crisp numbers using the defuzzification function. The generated crisp distance matrix is represented in Figure 6.

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 6. The crisp distance matrix

Now, DM-TSP1 is launched. The range for each row is computed and the smallest element (4.45) is selected and its corresponding element is designated at position d_{13} (see Figure 7).

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & \boxed{13.63} & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix} \begin{matrix} 6.23 \\ \textcircled{4.45} \\ 11.45 \\ 7.17 \\ 7.91 \\ 4.75 \end{matrix}$$

Figure 7. The smallest range is selected

Next, cities 1 and 3 are inserted in the list *List-cities* {1-3} and their corresponding columns are discarded. Subsequently, the smallest elements at row 1 (which is 13.67) and at row 3 (which is 8.58) are selected and their smallest element (8.58 at position d_{03}) is nominated for the next step (see Figure 8).

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 8. The smallest elements of rows 1 and 3 are selected

Hence, city 0 is inserted in the list *List-cities* {1-3-0} and its column is discarded and the smallest elements at row 1 (13.67) and at row 0 (8.67) are selected (see Figure 9) and their minimal element (8.67 at position d_{02}) is nominated for the next step.

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 9. The smallest elements of rows 0 and 1 are selected

Following, city 2 is inserted in the list *List-cities* {1-3-0-2} and its column is discarded and the smallest elements at row 1 (13.67) and at row 2 (6.63) are selected (see Figure 10) and their minimal element (6.63 at position d_{24}) is nominated for the next step.

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 10. The smallest elements of row 1 and 2 are selected

Subsequently, city 4 is inserted in the list *List-cities* {1-3-0-2-4} and its column is discarded and the smallest elements at row 1 (14.04) and at row 4 (11) are selected (see Figure 11) and their minimal element (11 at position d_{45}) is nominated for the next step.

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 11. The smallest elements of row 1 and 2 are selected

Hence, city 5 is inserted in the list *List-cities* {1-3-0-2-4-5} and its column is discarded (see Figure 12).

$$\begin{bmatrix} \infty & 14.81 & 8.67 & 8.58 & 10.17 & 11.40 \\ 14.81 & \infty & 18.08 & 13.63 & 13.67 & 14.04 \\ 8.67 & 18.08 & \infty & 15.58 & 6.63 & 15.42 \\ 8.58 & 13.63 & 15.58 & \infty & 14.54 & 15.75 \\ 10.17 & 13.67 & 6.63 & 14.54 & \infty & 11 \\ 11.40 & 14.04 & 15.42 & 15.75 & 11 & \infty \end{bmatrix}$$

Figure 12. All the columns are discarded

Now, all the columns are discarded and step 4 is launched in order to create a cycle from *List-cities* {1-3-0-2-4-5}. Thus, all cities before city 0 are translated at the end of *List-cities*: At first, city 1 is added at the last position of *List-cities* {3-0-2-4-5-1} and at second city 3 is added at the end of *List-cities* {0-2-4-5-3-1}. Finally, the starting city is added at the last position of *List-cities* {0-2-4-5-1-3-0}. Accordingly, the optimal solution generated by DM-TSP1 is $x_{02} = 1;\ x_{24} = 1;\ x_{45} = 1;\ x_{51} = 1;\ x_{13} = 1; x_{30} = 1$ with the path crisp total cost $z = 8.67 + 6.63 + 11 + 14.04 + 13.63 + 8.58 = 62.55$.

Consequently, the neutrosophic octagonal fuzzy solution is:

$z^N = < 31, 37, 45, 51, 59, 65, 71, 77; 35, 41, 47, 53, 59, 65, 72, 80; 55, 62, 69, 76, 83, 89, 95, 101 >$ and

its graphical representation is illustrated in Figure 13.

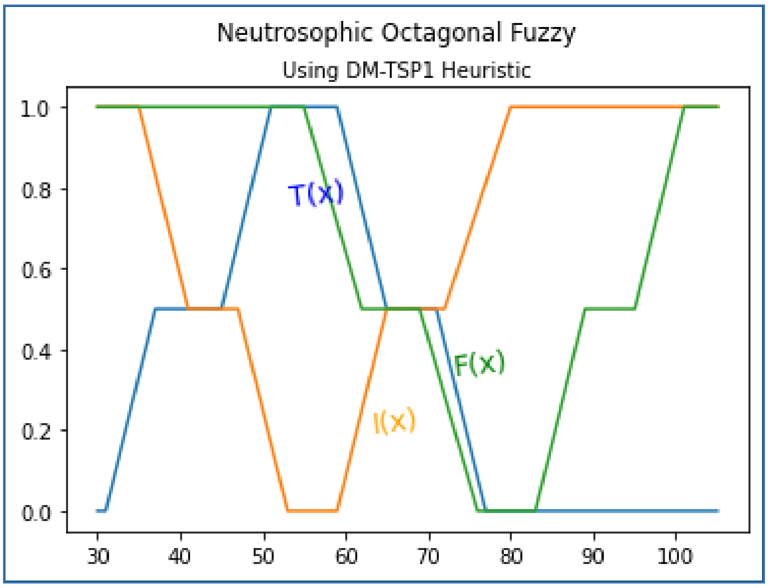

Figure 13. The neutrosophic octagonal fuzzy optimal solution generated by DM-TSP1

The corresponding truth membership function is presented by Equation 5.

$$T_{\tilde{S}_N}(x)=\begin{cases}\dfrac{x-31}{37-31}, & 31\le x\le 37\\ \dfrac{x-37}{45-37}, & 37\le x\le 45\\ \dfrac{x-45}{51-45}, & 45\le x\le 51\\ 1, & x=51\\ 1, & x=59\\ \dfrac{65-x}{65-59}, & 59\le x\le 65\\ \dfrac{71-x}{71-65}, & 65\le x\le 71\\ \dfrac{77-x}{77-71}, & 71\le x\le 77\\ 0, & otherwise\end{cases} \quad (5)$$

In the same way, the indeterminacy membership function is denoted by Equation 6.

$$I_{\tilde{S}_N}(x)=\begin{cases}\dfrac{41-x}{41-35}, & 35\le x\le 41\\ \dfrac{47-x}{47-41}, & 41\le x\le 47\\ \dfrac{53-x}{53-47}, & 47\le x\le 53\\ 0, & x=53\\ 0, & x=59\\ \dfrac{x-59}{65-59}, & 59\le x\le 65\\ \dfrac{x-65}{72-65}, & 65\le x\le 72\\ \dfrac{x-72}{80-72}, & 72\le x\le 80\\ 1, & otherwise\end{cases} \quad (6)$$

And the falsity membership function is offered by Equation 7.

$$F_{\tilde{S}_N}(x)=\begin{cases}\dfrac{62-x}{62-55}, & 55\le x\le 62\\ \dfrac{69-x}{69-62}, & 62\le x\le 69\\ \dfrac{76-x}{76-69}, & 69\le x\le 76\\ 0, & x=76\\ 0, & x=83\\ \dfrac{x-83}{89-83}, & 83\le x\le 89\\ \dfrac{x-89}{95-89}, & 89\le x\le 95\\ \dfrac{x-95}{101-95}, & 95\le x\le 101\\ 1, & otherwise\end{cases} \quad (7)$$

5. Conclusion

The Travelling Salesman Problem (TSP) stands as a challenging problem in the field of the operational research with widespread real-world applications. Its objective is to determine the shortest Hamiltonian cycle that connects all nodes which can represent cities, customers, suppliers and more. This cycle ensures that each node is visited exactly once excepting the starting node which also serves as the final point. In this current research work, a new variant of the TSP under the neutrosophic octagonal fuzzy domain is optimized using the novel heuristic DM-TSP1. As far as we know, this paper represents the first resolution of the neutrosophic octagonal fuzzy TSP in addition to the inaugural application of the new heuristic DM-TSP1 on this new variant of the TSP. That's why new benchmark instances are created. All neutrosophic octagonal fuzzy numbers are converted to crisp ones using a defuzzification function. Then, the four simple steps of DM-TSP1 are launched. A numerical example shows the rapidity and the efficiency of DM-TSP1 in optimizing the neutrosophic octagonal fuzzy TSP. Further research works aim to apply DM-TSP1 on other variants of the TSP: the generalized TSP, the multi-objective TSP and the TSP under a dynamic environment.

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