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Neutrosophic Soft Sets in One And Two-Dimensions Using Iteration Method

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Abstract

This paper introduces a different perspective of *Neutrosophic Fractals* and *Neutrosophic Soft Fractals*, merging the principles of Neutrosophic Logic, Soft set theory, and Fractal Geometry to address indeterminacy in complex, self-similar structures specifically the Von Koch curve and the Sierpinski triangle. It sightsees the complex qualities of Neutrosophic soft sets by incorporating attributes of falsification, indefiniteness, and truth into union and intersection operations. The research elucidates the interplay between Neutrosophic Logic and fractal geometry, leading to more precise modeling of complex systems. Proving theorems and providing examples examine the intricate interactions between membership characteristics in these fractal structures, demonstrating self-similarity. Fractal geometry is applied innovatively to improve the representation of uncertainty, indeterminacy, and falsity in Neutrosophic Logic, enhancing mathematical modeling techniques. Results show that the Sierpinski triangle provides a better representation than the Koch curve.

Keywords: Fuzzy set, Neutrosophic soft set, Fractals, Von Koch curve, Sierpinski triangle.

1. Introduction

Lotfi Zadeh is obtainable to us with a fuzzy set. The membership value of fuzzy sets can occasionally be difficult to ascertain [4, 13]. Interval-valued fuzzy sets were therefore proposed as a way to take into consideration the degree of uncertainty in membership values. From the point of view of philosophy, real standard or unusual subsets of] −0, 1 + [define the amount for the Neutrosophic set [1, 6]. Neutrosophic soft sets have three categories truth, indeterminacy, and falsity [2, 8]. The truth and the falsity of memberships must be considered in certain real-world scenarios, such as expert systems, belief systems, information fusion, and so on, for it to be appropriately defined in an unclear, ambiguous environment [3].

Florentin Smarandache introduced Neutrosophic logic in 1995 as an extension of Neutrosophic logic, encompassing fuzzy logic, intuitionistic logic, and three other logics incorporating an indeterminate value. A Neutrosophic set of logical variables (x) is represented in triple order as $X= (t, i, f)$, where t denotes the truth value of the membership function, i represents the indefiniteness value of membership operation, and f signifies falsification value for the function defining the membership [1].

The maps in Fractals are likeness transforms for n = m, and the dimension of R is log r/log n. But if n is greater than m, then a new strategy is needed, mainly because under the iteration of the mappingsf_i, squares are stretched into narrow rectangles [4, 11]. This frame of mind has evolved in the past few decades [9]. The mathematics of non-smooth things is a subject about which a lot is said, and worth saying. Additionally, irregular sets offer a far superior depiction of several natural events compared to conventional geometry figures [10]. Studying such irregular collections may be done broadly with the help of fractal geometry. One of the most well-known and simple-to-make fractals is the middle third Cantor set; yet, it exhibits many typical fractal characteristics [12].

Such recursive processes may be used to generate many more sets. For instance, equilateral triangles of unit side lengths can be repeatedly removed from an original equilateral triangle to get the Sierpinski triangle or gasket [2]. It is preferable to conceptualize this process as repeatedly substituting three half-height triangles for an equilateral triangle for numerous purposes. A Cantor dust, a plane equivalent of the Cantor set. Every square that is left gets split into 16 smaller squares at each step [5].

Section 2 defines the Neutrosophic soft set with an example, Neutrosophic value class, and Neutrosophic two sets of pairs for union and intersection. Section 3 explains the methods, detailing the Neutrosophic soft set and its examples, followed by the Neutrosophic union and intersection of two pairs with a theorem. Neutrosophic logic is used in the Von Koch curve and Sierpinski triangle explained with iterations, and demonstrated using Neutrosophic logic with figures.

2. Preliminaries

2.1 Neutrosophic Soft Set

Consider the scenario wherever G is a set, a subset of G is A, and P (U) is said to be a collection of all Neutrosophic sets defined by the universal set U. In this context, pairs (F, A) are termed soft Neutrosophic sets, where F represents a mapping of the subset A to the set P (U) [8].

Example

Let U be the usual cars below process and G be the set. Every parameter consists of a word or phrase that uses a Neutrosophic term. Examine G = {expensive, repair, non-repair, models, types of engines, water wash}. In this instance, defining a Neutrosophic soft set entails highlighting expensive, repairs, models, types of engines, and so on. Let's state that there are five cars in the universe U, denoted by U = {c1, c2, c3, c4, c5} and that the set of parameters A = {g1, g2, g3, g4}, where g1 denotes the parameter *costly*, g2 denotes *repair*, g3 denotes *models*, and g4 denotes *types of engines [8]*.

Table 1-Neutrosophicset of Car Engines

Universe U	C1	C2	C3	C4	C5
COSTLY	0.43,0.74,0.89	0.57,0.64,0.47	0.15,0.51,0.42	0.65,0.25,0.41	0.67,0.51,0.42
REPAIR	0.54,0.22,0.47	0.86,0.23,0.42	0.71,0.17,0.15	0.63,0.14,0.28	0.78,0.12,0.95
MODELS	0.77,0.38,0.12	0.65,0.46,0.91	0.56,0.71,0.42	0.43,0.11,0.25	0.46,0.52,0.61
TYPES OF MACHINES	0.67,0.75,0.83	0.96,0.18,0.55	0.16,0.75,0.83	0.85,0.17,0.98	0.28,0.54,0.66

2.2 Neutrosophic Set in Value Class

Neutrosophic soft sets (F, G) are characterized by the numerical value class denoted by H (F, G), encompassing a set of all values corresponding to the Neutrosophic set. For instance, to provided example (2.1), H (F, G) is defined as $\{v_1, v_2, \cdots v_{10}\}$. H (F, G) is a subset of P (U).

2.3 Neutrosophic Soft Set in Two Pairs

Consider two pairs of Neutrosophic soft sets (NSs) F and B defined across the unified Universe. If $A \subset B$ for all $x \in U, g \in A$, then the following conditions hold: TF (g)(x) ≤ TE(g)(x), similarly IF ≤ IG and FF ≥ FE, then the pair (F, A) termed as of (E, B) denoted by(F, A) ⊆ (E, B). Conversely, if (E, B) is an NS of (F, A).

2.4 Neutrosophic Union with Two Sets

Let (C, A) and (E, B) be the two pairs of neutrosophic Soft sets in the same Universe. The (C, A) ∪ (E, B) can be expressed (K, H), where H= A ∪ B.

a) TK(g)(n) = TC(g)(n) if g ∈ A − B, TE(g)(n) if g ∈ B − A, and max (TC(g)(n), TE(g)(n)) if g ∈ A ∩ B.
b) IK(g)(n) = IC(g)(n) if g ∈ A - B, IE(g)(n) if g ∈ B - A, and IC(g)(n) + IE(g)(n)/2 if g ∈ A ∩ B.
c) FK(g)(n) = FC(g)(n) for g ∈ A − B, FE(g)(n) for g ∈ B − A, and min (FC(g)(n), FE(g)(n)) for g ∈ A ∩ B.

2.5 Neutrosophic Intersection of Two Sets

Let us consider the two pairs (C, A) and (E, B) to be the Neutrosophic sets spanning identical universes [1, 2]. The intersection of the two pairs is indicated as (C, A)∩ (E, B) and it is also defined by (K, H), where H=A intersection B. The three membership functions of (K, H) are as follows:

a) Truth membership of K(g)(n) = min (TH(g)(n), TE(g)(n))
b) Indefiniteness membership of K(g)(n) = IC(g)(m) + IE(g)(n)/2
c) Falsification membership of K (g) (n) = max (F C (g)(n), FE(g)(n)), for g ∈ H.

3. Methods

The methods section demonstrates the Neutrosophic soft set of union and intersection with theorems and how the Sierpinski triangle can represent Neutrosophic soft sets. The union of parings with T, I, and F values is a Neutrosophic set with a numerical value given by H (F, G). These variables are divided into subcomponents like $(T_1, T_2, \dots T_A)$ for truth, $(I_1, I_2, \dots I_B)$ for indefiniteness, and $(F_1, F_2, \dots F_C)$ for falsification, where [A+B+C = n ≥1]. Iterations 1, 2, 3, and 4 of this quadratic Sierpinski triangle are discussed. Koch curve and Sierpinski triangle for these iterations, as shown in the figure and tables.

Theorem 3.1

a) *For any two neutrosophic soft sets (C, A) and (E, B), the distributive property between union and intersection is satisfied [6],*

i. $(C, A) \cup (C, A) = (C, A)$.

ii. $(C, A) \cup (E, B) = (E, B) \cup (C, A)$

iii. $(C, A) \cap (C, A) = C, A$.

iv. $(C, A) \cap (E, B) = (E, B) \cap (C, A)$.

v. $(C, A) \cup \Phi = (C, A)$.

vi. $(C, A) \cap \Phi = \Phi. 2r$

vii. $[(C, A)^c]^c = (C, A)$

b) *For any neutrosophic soft sets (E, B), (K, H), and (C, A), the following properties hold:*

i. Property of distribution of a union across an intersection

$$[(E, B) \cup (K, H)]\, U\, (C, A) = [(K, H) \cup (E, B)] \cup (C, A).$$

ii. An intersection's associative attribute:

$$[(E, B) \cap (K, H)] \cap (C, A) = [(K, H) \cap [(C, A) \cap (E, B)].$$

iii. The union's associative property:

$$[(E, B) \cap (K, H)] \cap (C, A) = [(C, A) \cup (E, B)] \cap [(K, H) \cup (C, A)].$$

iv. The property of the distribution of intersection over union

$$[(E, B) \cup (K, H)] \cap (C, A) = [(C, A) \cap (E, B)]\, U\, [(C, A) \cap (K, H)].$$

Proof

A pair of sets unions gives the same set; the intersection of the two pairs has a variable in pair one; one pair of sets union of the null set gives the pair of the same set; one pair of set interests with null set gives null set; and the complement of one pair of sets having another complement gives the same set. These examples illustrate the apparent nature of the proof. Hypotheses 1 through 4 are evident in part b, as well as the distributive property of the union across the intersection, an intersection between two pairs, property with the associative union, and distributive property of intersection over the union.

3.1von Koch Curve

The techniques of standard geometry and calculus are not adequate for humans to investigate fractals; instead, they require alternative approaches. In fractals, geometry has to form a dimension, which is a fundamental tool. The idea is that a (smooth) curve has one dimension while a surface has two. The Cantor set should have the size log 2⁄log 3 = 0.63 and the von Koch curve has a dimension log 4/ log 3 = 1.26. If the Koch curve is *higher than 1-dimensional* and *smaller than 2-dimensional* or has an unlimited length, then this latter estimate is at least consistent with that. An analysis of the sense that the *dimensions* validate and stand for self-similarity clambering properties.

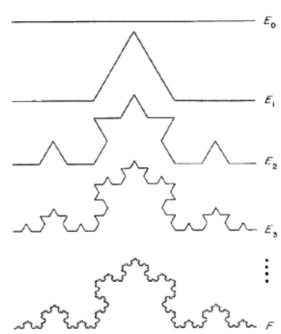

Figure 1 Von Koch curve

The von Koch curve has features in many ways similar to those listed for the middle third cantor set. It is made up of four quarters each similar to the whole, but scaled by a factor 1/3. The fine structure is reflected in the irregularities at all scales nevertheless, this intricate structure stems from a simple construction. Whilst it is reasonable to call F a curve, it is much too irregular to have tangents in the classical sense. A simple calculation shows that E_K is of length $(\frac{4}{3})^k$ letting k tend to infinity implies that F has infinite length. On the other hand, F occupies zero area in the plane, so neither length nor area provides a handy description of the size of F [4].

$$dim_H F = dim_B F = \frac{log4}{log3}$$

3.1.1 Iteration for Koch Curve

The von Koch curve method to explore the fractals in the Sierpinski triangle with the baseline was eliminated.

i) Let an equilateral triangle, a generate (zig-zag curve) be created by replacing every line segment with four copies of the original line portion, each of which is shortened to one-third of its original length. D is equal to minus log N/log S. The fractal size for any strictly identical fractal may be found using this method.
T= 0.6= $1/3^6$=0.001

I=0.5= $1/3^5$=0.004

F=0.2=$1/3^2$=0.111

$$0 \leq T + I + F \leq 3 = 0.001 + 0.004 + 0.111 = 0.116$$

$$= \frac{\log 4}{\log 0.001} + \frac{\log 4}{\log 0.004} + \frac{\log 4}{\log 0.111}$$

$$= \frac{0.602059}{3} - \frac{0.602059}{2.3979} - \frac{0.602059}{0.9546}$$

= 0.2006+0.2510+0.2510=0.7026

3.2 Neutrosophic Logic

In Neutrosophic sets the indeterminate value Iinto Paradox (true and false) and Uncertainty (real or untrue), Belnap's four-valued logic expands. Furthermore, dividing Iinto inconsistency, unpredictability, and ambiguity yields logic. In advanced Neutrosophic logic, truth is subdivided into sub-components $(T_1, T_2, \ldots T_A)$, indefiniteness$(I_1, I_2, \ldots I_B)$, and falsification$(F_1, F_2, \ldots F_C)$, where [A+B+C = n ≥1]. Their subcomponents T_A, I_B, F_Ccan comprise numerous sets, possibly countable or unbounded. In the context of implementing software engineering concepts, the conventional unit interval *[0, 1]* may be employed. For instance, an assertion can range from *[0.4, 0.6]* true, *[0.5]* indeterminate, or anywhere from (0.15, 0.25) ambiguous, corresponding to 0.4 or 0.6 false.

3.3 Sierpinski Triangle

The Sierpinski triangle or gasket is obtained by repeatedly removing (inverted) equilateral triangles from an initial equilateral triangle of unit side length. For many purposes, this procedure is better understood as repeatedly replacing an equilateral triangle with three triangles of half the height.

Example

The Sierpinski triangle or gasket F is constructed from an equilateral triangle by repeatedly removing inverted equilateral triangles. Then

$$dim_H F = dim_B F = \frac{log3}{log2}$$

Solution: The set F is the attractor of the three obvious similarities of ratios ½, which maps from the triangle E_0 onto the triangles of E_1. The open set condition holds, taking V as the interior of E_0. Thus, the open set condition $V \supset \bigcup_{i=1}^{m} S_i(V)$ holds for the similarities $S_i$on $\mathbb{R}^n$ with ratios $0 < V_i < 1$ for$1 \leq i \leq m$. If F is the attractor of the IFS$\{S_1, \dots, S_m\}$, that is $F = \bigcup_{i=1}^{m} S_i(F)$, then $dim_H F = dim_B F = S$, where S is given by $\sum_{i=1}^{m} r_i^s = 1$. Moreover, for this value of S, $0 < \mathcal{H}^s(F) < \infty$.

3.3.1 Iteration 1

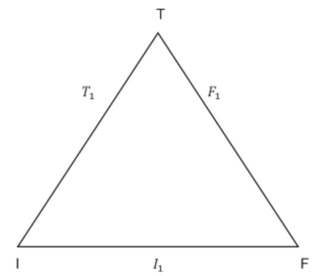

Figure 2 **Sierpinski equilateral Triangle with membership functions**

Assume that the triangle has 1 ratio with truth (T) = 0.6, indefiniteness (I) = 0.5, and falsification (F) = 0.2, which is 0.6+0.5+0.2 = 1.3≤ 3, satisfying the condition of the Neutrosophic set value of T+I+F≤3. This iteration 1 states that the triangle contains three sides of the same length, width, and height identified in the NSs as truth, indeterminacy, and falsity. It could be represented by a membership function between*]-0 to 1+ [*.

3.3.2 Iteration 2

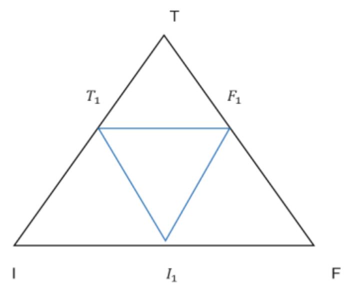

Figure 3 **Sierpinski Triangle iteration 2 with membership functions**

In this second iteration, the triangle's three sides with the same length, blue color, width, and height are referred to as truth, truth, indefiniteness, and falsification in the Neutrosophic set. Its membership function may be represented by a range from*]-0 to 1+ [*. Assuming truth (T_1) = 0.8, indeterminacy (I_1) = 0.5, and falsehood (F_1) = 0.1, the iteration is satisfied by 0.8+0.5+0.1= 1.4≤3.

3.3.3 Iteration 3

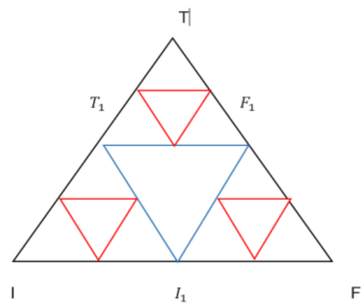

Figure 4 **Sierpinski Triangle iteration 3 with membership functions**

The triangle in iteration 3 indicates that there is a difference between the first and second iterations with a 3 sides red color triangle. This difference is found in iteration 2, where another quadrilateral triangle with the same width and height separates the two iterations. Indicates a Neutrosophic membership value in a similar manner, provided that the requirement is satisfied [5].

3.3.4 Iteration 4

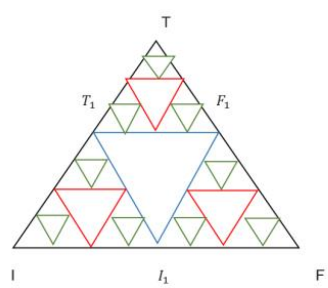

Figure 5 **Sierpinski Triangle iteration 4 with membership functions**

And so on. Eventually, it will be possible to state that the Neutrosophic Sierpinski triangle, which is the result of the combination of the self-similarity in the Sierpinski triangle and the criteria in the Neutrosophic set, fulfills the Sierpinski triangle. The Sierpinski triangle has an equilateral triangle and it is also dented by the Neutrosophic set notations like the truth of $T_A(x)$, the indeterminacy of $I_B(x)$, the falsity of$F_C(x)$.

4. Results

The Von Koch curve proposes the elimination of both self-similarity and adjacent lines when applied, offering unique characteristics for various applications. In the antenna response, there are sudden spikes at Iteration 1 =1 and Figure *1*, Iteration 2 =1 /2 and Figure *2*, Iteration 3 =1 /4 and *Figure 3*, Iteration 4 =1/8 and *Figure 4*, and goes on this frequency. Sierpinski triangle must be represented as a Neutrosophic soft set with three membership values using an example like five cars in the universe, denoted by U = {c1, c2, c3, c4, c5} and that parameters A = {g1, g2, g3, g4}, where g1 denotes the parameter *costly*, g2 denotes *repair*, g3 denotes *models*, and g4 denotes *types of engines.*

5. Conclusion

The *Neutrosophic Fractals* and *Neutrosophic Soft Fractals* cover the boundaries of Fractal analysis by incorporating the dimensions of truth, indeterminacy, and falsity to the Von Koch curve and Sierpiński triangle demonstrate that memberships representing truth, indefiniteness, and falsification alignment can effectively the geometric patterns observed. This approach bridges mathematical concepts from different domains, underscoring the relevance of Neutrosophic soft sets in fractal analysis, supported by theorems and empirical evidence. The study ranks the Sierpiński Triangle higher than the Koch Curve, with relative closeness scores of 0.625 and 0.375, respectively, indicating that the Sierpiński Triangle better satisfies the combined fractal dimension, length, and area criteria. It highlights the potential of Neutrosophic soft sets for innovative applications in mathematics and computational sciences, demonstrating the interconnectedness between mathematical frameworks and geometric patterns.

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