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Pseudo Functions in N-Cylindrical Fuzzy Neutrosophic Topological Spaces

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Abstract: An interpretations supplied the study investigators the current research investigation is presented coupled with an exploration subsequent to trends to whatever individuals have provided. Let's start through a beginning to defined a new set in a space $nCy\mathbb{F}$ called an $nCy\bar{\bar{\alpha}}\wr$-incorporates the ideas of closed and open sets and examine the map of a function Δ from $nCy\mathbb{F}^1$ to $nCy\mathbb{F}^2$ has been stated to be $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP, $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL and decisively $nCy\bar{\bar{\alpha}}\text{-}OpFn$ and offer an understanding of the achieved outcomes by introducing the notion of the spaces in a $nCyFNTS's$ is referred to as ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space, ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space and ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space. In addition to the examples, assumptions, and theorems, the distance between two $n\text{-}CyFNS$, in conjunction with their attributes and fundamental operations, was defined.

Keywords: nCy-pseudo alpha open functions, nCy-pseudo alpha closed functions, nCy-delta spaces, nCy- decisively alpha functions, nCy αδ0-space, nCy αδ1-space, n-CyFNS.

1. Introduction

Topological spaces are a key notion in mathematics, notably in topology, which investigates the features of space that remain constant during Continuous distortions such as twisting and bending but not ripping either glueing. In layman's words, a topological space is a set that contains a collection of subsets known as "open sets" that meet certain qualities, such as proximity and continuity. These open sets determine the topology of the space and allow us to define concepts such as convergence, continuity, compactness, and connectedness, which are important in many fields of mathematics, including analysis, geometry, and algebraic topology. One of the distinguishing characteristics of topological spaces is their generality. Topological spaces, as opposed to more rigid geometric structures such as Euclidean spaces, may be very flexible and abstract, allowing mathematicians to examine a diverse variety of forms and structures, from common lines and planes to more exotic and complicated ones. Overall, topological spaces provide a rich framework for exploring the shape and structure of mathematical objects, and they act as a link between different branches of mathematics, allowing ideas and techniques from one to be applied to another.

FTS build on the notion of classical TS by integrating fuzziness or uncertainty into the definitions of open sets and other topological features. This addition is especially beneficial when dealing with circumstances where specific limits or distinctions are not clearly specified. In a fuzzy topological space, rather of having crisp, well-defined open sets, we have "fuzzy" open sets that assign a level of being a member to each point. The level of being a member specifies how much point belongs to the set. The higher the degree of membership, the closer the point is to being completely contained within the set. The formal definition of a fuzzy topological space entails replacing the traditional concept of open sets with fuzzy sets, which are distinguished by membership functions. These membership functions assign a value between 0 and 1 to each point in the space, reflecting its degree of belonging to the set. Fuzzy topological spaces are used in a variety of domains, including fuzzy logic, approximation reasoning, pattern recognition, and decision making, when ambiguity and imprecision exist in the data or issue formulation. They provide an effective framework for dealing with ambiguous or partial information, as well as a more flexible and nuanced approach to modelling complex systems.

Intuitionistic fuzzy topological spaces are an extension of classical topological spaces that account for the inherent ambiguity and vagueness seen in real-world data and decision-making processes. This expansion provides intuitionistic fuzzy sets, which provide a more expressive approach to manage uncertainty than classical fuzzy sets. Unlike classical FS assign a single level of being a member to each element reflecting its degree of membership, a set, IFS include the ideas of being a member and none being a member degrees. They also provide a third parameter, the hesitation degree, which expresses the level of ambiguity or indecision about an element's membership status. Intuitionistic Fuzzy Topology, like classical topology, relies heavily on the idea of open sets. However, in intuitionistic fuzzy topology, open sets are defined as IFS. These sets have distinguished being a member, non-membership, hesitation functions, allowing for a more sophisticated portrayal of the thought of openness in space. IFTS have applications in a variety of disciplines where uncertainty and ambiguity play important roles, including decision making, pattern recognition, image processing, and expert systems. By giving a more detailed representation of uncertainty, they provide a strong foundation for modelling and analysing complex systems in a more realistic and flexible way.

Neutrosophic fuzzy topological spaces combine two important theories in mathematical modelling: neutrosophic set theory and fuzzy topology. These spaces handle both uncertainty and indeterminacy, making them very adaptable to real-world settings that include ambiguity. Neutrosophic theory of sets is a refinement of traditional set theory that includes aspects of uncertainty. In a neutrosophic the set, every component has one of the following values: truth, indeterminacy, or false being a member. Neutrosophic fuzzy topological spaces are a combination of two significant mathematical modelling theories: neutrosophic set theory and fuzzy topology. These spaces can manage both uncertainty and indeterminacy, making them well-suited to ambiguous actual events situations. Neutrosophic theory of sets is an adaptation on classical theory of sets that incorporates uncertainty. In a neutrosophic established every element is assigned one of three attributes: truth, indeterminacy, or false membership.

Zadeh [43] created the groundwork for the topic of uncertainty known as fuzzy sets. Topology was the primary discipline of mathematics where fuzzy set notions and ideas formed parallels. Chang [14] used Zadeh's notion to breathe new life into the concept of fuzzy topological spaces. Since then, several concepts from classical topology have been applied to fuzzy topological spaces. In the latter part of 1970 and the early part of 1980, numerous authors made significant contributions to this emerging discipline. Later, Atanassov [2], [3] created a new set known as the Intuitionistic Fuzzy Set (IFS), in which the total of the acceptance and rejection severity grades does not exceed one. Coker [15] later got IFTS via IFSs, while Lee [23] established the features of continuous, open, and closed maps.

In 2013, Yager [42] introduced the Pythagorean Fuzzy Set (PyFS) as an extension of IFS that assures that the square sum of its individual degrees is less than or equal to one. Olgun et al. and Parimala M et al. established the notion of a PNTS [24, 26, 27, 28]. Cuong [16] originated the concept of the Picture FS (PFS). He used three indices in PFS: membership degree P (x), neutral being a member degree I (x), and non-membership degree N (x), with the requirement that $0 \leq P(x) + I(x) + N(x) \leq 1$. Obviously, PFSs are better suited than IFS and PyFS for dealing with fuzziness and ambiguity. Razaq et al. [30] introduced the concept of image FTS. Later, Kahraman and Gündogdu introduced spherical fuzzy sets (SFS) [18, 19]. SFS should meet the requirement that the squared total of being a member degree, none being a member degree, and hesitancy degree be equal to or less than one. Princy and Mohana [29] proposed spherical FTS.

Smarandache [33, 34] introduced the neutrosophic set, which is a generalisation of the IFS. Salama and Alblowi [36] developed the idea of NTS. They defined NTS as a generalisation of IFTS, as well as a neutrosophic set, in addition to each element's degree of membership, indeterminacy, and nonmembership. Smarandache [37,41] originally established the dependency degree (also known as the independence degree) of the fuzzy and neutrosophic components in 2006. P. Basker and Broumi Said [4,5,6,7,8,9,10,11,12,13,17,25,32,35,38,39] studied neutrosophic sets in the Applications of Sets and Functions by using an open set in Fuzzy Neutrosophic Topological Spaces, Neutrosophic Homeomorphisms, $(\beta_{\rho n})$ - OS in Pythagorean Neutrosophic Topological Spaces, $N\psi_{\alpha}^{\#\,0}$-spaces and $N\psi_{\alpha}^{\#\,1}$-spaces. $I^{(T)}\alpha$-open and $I^{(T)}\beta$–open sets. Neighbourhoods. $\mathcal{M}_X\alpha\delta\langle\widetilde{\mathcal{H}}\rangle$ in $\mathcal{M}$-structures. Single-valued neutrosophic graphs, Correlation coefficient of interval neutrosophic set, Neutrosophic soft matrices.

Arockiarani and Jency [1] proposed the concept of a fuzzy neutrosophic set in which the total of all three membership characteristics does not exceed 3. Veereswari presented a FNTS with fundamental operations [40]. Sarannya Kumari et al. [30, 31, 20, 21, 22] recently proposed n-Cylindrical Fuzzy Neutrosophic Sets, with T and F regarded as dependent portions and I as independent components. Except for fuzzy neutrosophic sets, the n-CyFNS is the most extensive extension of fuzzy sets. In this situation, the level to which any of the neutral, positive, and negative members meet the criteria, $0 \le \beta A(x) < 1$ and $0 \le \alpha An(x) + \gamma An(x) \le 1$, $n > 1$, is an integer. They also specified the distance between two n-CyFNS, as well as their attributes and fundamental operations. In this study, we define topological space in the n-CyFNS environment. This is a novel form of fuzzy neutrosophic set, with dependent components T and F and independent components I. We defined n-CyFN topological space and n-CyFN open sets. We also started the n-CyFN base, n-CyFN subbase, and various related experiments. In this paper the following abbreviation are used OS-Open Set, CS-Closed Set, $nCyFNTS$-nCylindrical Fuzzy Neutrosophic Topological Spaces, $n\text{-}CyFNS$-nCylindrical Fuzzy Neutrosophic Sets. The following are the preliminary definitions.

Definition 1.1. An n-CyFNS N on S is an entity of the type $N = \{< i, \alpha N(i), \beta N(i), \gamma N(i) > \mid i \in S\}$ where $\alpha N(i) \in [0,1]$, termed the degree of positive membership. of i in N, $\beta N(i) \in [0,1]$, Known as the degree of neutral membership of i in N and $\gamma N(i) \in [0,1]$, termed the degree of negative membership. of i in N, which satisfies the condition, $(\forall\, i \in S)$ $(0 \le \beta N(i) \le 1$ and $0 \le \alpha Nn(i) + \gamma Nn(i) \le 1, n > 1$, is an integer. Here T and F are dependent neutrosophic components and I is 100% independent.
For the convenience, $< i, \alpha N(i), \beta N(i), \gamma N(i) >$ is called as n-Cylindrical Fuzzy Neutrosophic Number (n-CyFNN) and is denoted as $A =< i, \alpha N(i), \beta N(i), \gamma N(i) >$.

Definition 1.2. Inclusion: $\forall$ two $N, O \in C_n(S), N \subseteq O \Leftrightarrow (\forall\, i \in S, \alpha N(i) \le \alpha O(i)$ and $\beta N(i) \le \beta O(i)$ and $\gamma N(i) \ge \gamma O(i))$ and $N = O \Leftrightarrow$ $(N \subseteq O$ and $O \subseteq N)$.

Definition 1.3. Union: $\forall$ two $N, O \in C_n(S)$, the union of two n-CyFNSs N and O is $N \cup O(i) = \{\langle i, max\,(\alpha N(i), \alpha O(i)), max\,(\beta N(i), \beta O(i)), min\,(\gamma N(i), \gamma O(i))\rangle \mid i \in S\}$.

Definition 1.4. Intersection: $\forall$ two $N, O \in C_n(S)$,, the intersection of two n-CyFNSs N and O is $N \cap O(i) = \{\langle i, min\,(\alpha N(i), \alpha O(i)), min\,(\beta N(i), \beta O(i)), max\,(\gamma N(i), \gamma O(i))\rangle \mid i \in S\}$.

Definition 1.5. Complementation: For every $N \in C_n(S)$, the complement of an n-CyFNS N is $N^C = \{< i,\ \gamma A(i), \beta A(i), \alpha A(i) > \mid i \in\ S\}$

2. $nCy\sim\bar{\bar{\alpha}} < on$-Pseudo functions

Definition 2.1. An n-Cylindrical subset Y of a space $nCy\mathbb{F}$ is called an $nCy\bar{\bar{\alpha}}\wr$-open set ($nCy\bar{\bar{\alpha}}\wr OS$) if Y is a subsets of $nCyI\left(nCyC\left(nCyI(Y)\right)\right)$ and a $nCy\bar{\bar{\alpha}}\wr$-closed set if $nCyC\left(nCyI\left(nCyC(Y)\right)\right)$ is a subset of Y.

Example 2.2. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \dots 8$, where,
$Y_i = \{< n_1; \mathfrak{R}_j >, < n_2; \mathfrak{H}_k >, < n_2; \mathfrak{I}_l >\};\ j, k, l = 1,2,3, \dots 8$

Y_i	$\mathfrak{R}_j$	$\mathfrak{H}_j$	$\mathfrak{I}_l$
Y_1	$\mathfrak{R}_1 = \frac{49}{400}, \frac{58}{400}, \frac{175}{400}$	$\mathfrak{H}_1 = \frac{125}{400}, \frac{74}{400}, \frac{82}{400}$	$\mathfrak{I}_1 = \frac{285}{400}, \frac{122}{400}, \frac{115}{400}$
Y_2	$\mathfrak{R}_2 = \frac{38}{400}, \frac{49}{400}, \frac{49}{400}$	$\mathfrak{H}_2 = \frac{174}{400}, \frac{95}{400}, \frac{112}{400}$	$\mathfrak{I}_2 = \frac{177}{400}, \frac{98}{400}, \frac{79}{400}$
Y_3	$\mathfrak{R}_3 = \frac{82}{400}, \frac{192}{400}, \frac{74}{400}$	$\mathfrak{H}_3 = \frac{325}{400}, \frac{118}{400}, \frac{98}{400}$	$\mathfrak{I}_3 = \frac{192}{400}, \frac{57}{400}, \frac{48}{400}$
Y_4	$\mathfrak{R}_4 = \frac{58}{400}, \frac{125}{400}, \frac{82}{400}$	$\mathfrak{H}_4 = \frac{311}{400}, \frac{277}{400}, \frac{97}{400}$	$\mathfrak{I}_4 = \frac{115}{400}, \frac{78}{400}, \frac{74}{400}$
Y_5	$\mathfrak{R}_5 = \frac{192}{400}, \frac{134}{400}, \frac{115}{400}$	$\mathfrak{H}_5 = \frac{49}{400}, \frac{301}{400}, \frac{88}{400}$	$\mathfrak{I}_5 = \frac{49}{400}, \frac{48}{400}, \frac{58}{400}$
Y_6	$\mathfrak{R}_6 = \frac{74}{400}, \frac{57}{400}, \frac{285}{400}$	$\mathfrak{H}_6 = \frac{54}{400}, \frac{157}{400}, \frac{99}{400}$	$\mathfrak{I}_6 = \frac{54}{400}, \frac{59}{400}, \frac{95}{400}$
Y_7	$\mathfrak{R}_7 = \frac{82}{400}, \frac{95}{400}, \frac{74}{400}$	$\mathfrak{H}_7 = \frac{68}{400}, \frac{168}{400}, \frac{108}{400}$	$\mathfrak{I}_7 = \frac{134}{400}, \frac{221}{400}, \frac{95}{400}$
Y_8	$\mathfrak{R}_8 = \frac{125}{400}, \frac{124}{400}, \frac{49}{400}$	$\mathfrak{H}_8 = \frac{115}{400}, \frac{198}{400}, \frac{49}{400}$	$\mathfrak{I}_8 = \frac{82}{400}, \frac{117}{400}, \frac{74}{400}$

Here $\left\{< n_1; \frac{58}{400}, \frac{125}{400}, \frac{82}{400} >, < n_2; \frac{311}{400}, \frac{277}{400}, \frac{97}{400} >, < n_2; \frac{115}{400}, \frac{78}{400}, \frac{74}{400} >\right\}$ is a subsets of $nCyI\left(nCyC\left(nCyI\left(\left\{< n_1; \frac{58}{400}, \frac{125}{400}, \frac{82}{400} >, < n_2; \frac{311}{400}, \frac{277}{400}, \frac{97}{400} >, < n_2; \frac{115}{400}, \frac{78}{400}, \frac{74}{400} >\right\}\right)\right)\right)$. Thus $\left\{< n_1; \frac{58}{400}, \frac{125}{400}, \frac{82}{400} >, < n_2; \frac{311}{400}, \frac{277}{400}, \frac{97}{400} >, < n_2; \frac{115}{400}, \frac{78}{400}, \frac{74}{400} >\right\}$ is a $nCy\bar{\bar{\alpha}}\wr OS$.

Definition 2.3. A map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ It has been stated to be $nCy\sim Pse(\bar{\bar{\alpha}})$-OP if every instance of $nCy\bar{\bar{\alpha}}\wr OS$ in $nCy\mathbb{F}^1$ is OS in $nCy\mathbb{F}^2$. It is noticeable the concepts exist $nCy\sim Pse(\bar{\bar{\alpha}})$-OP and $nCy\bar{\bar{\alpha}}$- enduring correspond if the bijection function is used.

Example 2.4. Let $C_y = \{c1, c2\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \dots 8$, where,
$Y_i = \{< n_1; \mathfrak{R}_j >, < n_2; \mathfrak{H}_k >, < n_2; \mathfrak{I}_l >\};\ j, k, l = 1,2,3, \dots 8$

Y_i	$\mathfrak{R}_j$	$\mathfrak{H}_j$
Y_1	$\mathfrak{R}_1 = \frac{5}{8}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_1 = \frac{17}{40}, \frac{9}{40}, \frac{1}{8}$
Y_2	$\mathfrak{R}_2 = \frac{20}{40}, \frac{27}{40}, \frac{15}{40}$	$\mathfrak{H}_2 = \frac{10}{40}, \frac{8}{40}, \frac{3}{8}$
Y_3	$\mathfrak{R}_3 = \frac{15}{40}, \frac{14}{40}, \frac{11}{40}$	$\mathfrak{H}_3 = \frac{22}{40}, \frac{11}{40}, \frac{5}{40}$

Y_4	$\mathfrak{R}_4 = \frac{13}{40}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_4 = \frac{13}{40}, \frac{7}{40}, \frac{25}{40}$
Y_5	$\mathfrak{R}_5 = \frac{25}{40}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_5 = \frac{19}{40}, \frac{5}{40}, \frac{5}{40}$
Y_6	$\mathfrak{R}_6 = \frac{22}{40}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_6 = \frac{10}{40}, \frac{18}{40}, \frac{15}{40}$
Y_7	$\mathfrak{R}_7 = \frac{11}{40}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_7 = \frac{12}{40}, \frac{19}{40}, \frac{5}{40}$
Y_8	$\mathfrak{R}_8 = \frac{21}{40}, \frac{17}{40}, \frac{13}{40}$	$\mathfrak{H}_8 = \frac{1}{40}, \frac{13}{40}, \frac{7}{40}$

Clearly $(nC_y, nCy\mathbb{F}^1)$ is an $nCy\bar{\bar{\alpha}} \wr$ open set and $(nC_y, nCy\mathbb{F}^2)$ is an open set. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_2, \Delta(c2) = n_1$, then Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP.

Theorem 2.5. A map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP $\Leftrightarrow$ $\forall$ portion $L^{\#}$ of $nCy\mathbb{F}^1$, $\Delta(nCy\bar{\bar{\alpha}}_{In}(L^{\#})) \subset nCyIN(\Delta(L^{\#}))$.

Proof: Here Δ be a $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP. As of presently, we've $nCyIN(L^{\#}) \subset L^{\#}$ and $nCy\bar{\bar{\alpha}}_{In}(L^{\#})$ is a $nCy\bar{\bar{\alpha}} \wr OS$. Thus, we emerge at that $\Delta(nCy\bar{\bar{\alpha}}_{In}(L^{\#})) \subset \Delta(L^{\#})$. As $\Delta(nCy\bar{\bar{\alpha}}_{In}(L^{\#}))$ is OS, $\Delta(nCy\bar{\bar{\alpha}}_{In}(L^{\#})) \subset nCyIN(\Delta(L^{\#}))$. On the other hand, suppose that $L^{\#}$ is a $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Then, $\Delta$of $L^{\#}$is equal to Δ of $nCy\bar{\bar{\alpha}}_{In}$ of $L^{\#}$ $\subset nCyIN(\Delta(L^{\#}))$ but $nCyIN$ of $\Delta(L^{\#})$ is a subset of $\Delta(L^{\#})$ Thereby, Δ of $L^{\#}$ is equal to $nCyIN$ of $\Delta(L^{\#})$ and consequently Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP.

Theorem 2.6. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$- OP, then $nCy\bar{\bar{\alpha}}_{In}(\Delta^{-1}(L^{\#})) \subset \Delta^{-1}(nCyIN(L^{\#}))$ $\forall$ portion $L^{\#}$ of $nCy\mathbb{F}^2$.

Proof. Here $L^{\#}$ be any unpredictability subset within $nCy\mathbb{F}^2$. Then, $nCy\bar{\bar{\alpha}}_{In}(\Delta^{-1}(L^{\#}))$ is a $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$ and Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$ -OP, then $\Delta\left(nCy\bar{\bar{\alpha}}_{In}(\Delta^{-1}(L^{\#}))\right) \subset nCyIN\left(\Delta(\Delta^{-1}(L^{\#}))\right) \subset nCyIN(L^{\#})$. Thereby, $nCy\bar{\bar{\alpha}}_{In}(\Delta^{-1}(L^{\#})) \subset \Delta^{-1}(nCyIN(L^{\#}))$. Remember that a subset $K_!$ is referred to as a point l of $nCy\mathbb{F}^1$ is $nCy\bar{\bar{\alpha}} \wr$-neighborhood, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$ L_* the kind that $l \in L_* \subset K_!$.

Theorem 2.7. Concerning a map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$, Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP $\Leftrightarrow$ $\forall$ subset $L_1^{\#}$ of $nCy\mathbb{F}^1$, $\Delta(nCy\bar{\bar{\alpha}}_{In}) \subset nCyIN\left(\Delta(L_1^{\#})\right)$ $\Leftrightarrow$ $\forall$ $l \in nCy\mathbb{F}^1$ and $\forall$ $nCy\bar{\bar{\alpha}} \wr$-neighbourhood $L_1^{\#}$ of l in $nCy\mathbb{F}^1$, $\exists$ a neighbourhood $\Delta(L_1^{\#})$ of $\Delta(l)$ in $nCy\mathbb{F}^2$ such that $L^{\#}{}_2 \subset \Delta(L_1^{\#})$.

Proof: Let $l \in nCy\mathbb{F}^1$ and $L_1^{\#}$ be an arbitrary $nCy\bar{\bar{\alpha}} \wr$-neighbourhood $L_1^{\#}$ of l in $nCy\mathbb{F}^1$. Then $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$, $L^{\#}{}_2$ in $nCy\mathbb{F}^1$ such that $l \in L^{\#}{}_2$ in $L_1^{\#}$, we've Δ of $L_2^{\#}$ is a $\Delta(nCy\bar{\bar{\alpha}}_{In}) \subset nCyIN\left(\Delta(L^{\#}{}_2)\right)$ and hence $\Delta(L^{\#}{}_2) = nCyIN\left(\Delta(L^{\#}{}_2)\right)$. Therefore, it follows that $\Delta(L^{\#}{}_2)$ is OS in $nCy\mathbb{F}^2$ such that $\Delta(l) \in \Delta(L^{\#}{}_2) \subset \Delta(L_1^{\#})$.

Let $L_1^{\#}$ be an arbitrary $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Then $\forall$ $m \in \Delta(L_1^{\#})$, $\exists$ a neighbourhood $L^{\#}{}_{2_m}$ of m in $nCy\mathbb{F}^2$ such that $L^{\#}{}_{2_m} \subset \Delta(L_1^{\#})$. As $L^{\#}{}_{2_m}$ is a neighbourhood of m, $\exists$ an OS $L^{\#}{}_{3_m}$ in $nCy\mathbb{F}^2$ such that $m \in L^{\#}{}_{3_m}$ is in $L^{\#}{}_{2_m}$. Thus Δ of $L_1^{\#}$ is a $\cup$ of $\{L^{\#}{}_{3_m} : m \in \Delta \text{ of } L_1^{\#}\}$ which is an OS in $nCy\mathbb{F}^2$.This implies that Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP.

Theorem 2.8. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP $\Leftrightarrow$ $\forall$ subset $D_{\#}$ of $nCy\mathbb{F}^2$ and $\forall$ $nCy\bar{\bar{\alpha}} \wr$ CS, $P^{(1}$ of $nCy\mathbb{F}^1$ containing $\Delta^{-1}(D_{\#})$, $\exists$ a CS $P^{(2}$ of $nCy\mathbb{F}^2$ containing $D_{\#}$ $\mid$ $\Delta^{-1}(P^{(2)}) \subset P^{(1}$.

Proof: In case Δ is $nCy\bar{\bar{\alpha}} \wr OS$. Let $D_\# \subset nCy\mathbb{F}^2$ and $P^{(1}$ be a $nCy\bar{\bar{\alpha}} \wr CS$ of $nCy\mathbb{F}^1$ containing $\Delta^{-1}(D_\#)$. Now place $P^{(2} = nCy\mathbb{F}^2 - \Delta(nCy\mathbb{F}^1 - P^{(1})$. It is obvious that Δ^{-1} of $P^{(2}$ is in $P^{(1} \Rightarrow D_\#$ is in $P^{(2}$. Since Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP, We acquire $P^{(2}$ as a CS of $nCy\mathbb{F}^2$. Furthermore, we've $\Delta^{-1}(P^{(2}) \subset P^{(1}$.

In contrast, let $L^\#$ be a $nCy\bar{\bar{\alpha}} \wr OS$ of $nCy\mathbb{F}^1$ and place $D_\# = nCy\mathbb{F}^2 \backslash \Delta(L^\#)$. Then $nCy\mathbb{F}^1 \backslash L^\#$ is a $nCy\bar{\bar{\alpha}} \wr CS$ in set in $nCy\mathbb{F}^1$ containing $\Delta^{-1}(D_\#)$. By supposition, $\exists$ a CS, $P^{(1}$ of $nCy\mathbb{F}^2 \mid D_\# \subset P^{(1}$ and Δ^{-1} of $P^{(1} \subset nCy\mathbb{F}^1 \backslash L^\#$. Thus, we acquire $\Delta(L^\#) \subset nCy\mathbb{F}^2 \backslash P^{(1}$. Conversely, nevertheless, it implies that $D_\# \subset P^{(1}$, $nCy\mathbb{F}^2 \backslash P^{(1} \subset nCy\mathbb{F}^2 \backslash D_\# = \Delta(L^\#)$. Hence, we get $\Delta(L^\#) = nCy\mathbb{F}^2 \backslash P^{(1}$ which is OS and hence Δ is a $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP function.

Theorem 2.9. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP $\Leftrightarrow$ $\Delta^{-1}\big(nCyCL(D_\#)\big) \subset nCy\bar{\bar{\alpha}}_{Cl}\big(\Delta^{-1}(D_\#)\big)$ $\forall$ subset $D_\#$ of $nCy\mathbb{F}^2$.

Proof: Suppose that Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$ -OP. For any subset $D_\#$ of $nCy\mathbb{F}^2$, $\Delta^{-1}(D_\#) \subset nCy\bar{\bar{\alpha}}_{Cl}\big(\Delta^{-1}(D_\#)\big)$. Therefore, $\exists$ a CS $P^{(1}$ in $nCy\mathbb{F}^2 \mid D_\# \subset P^{(1}$ and Δ^{-1} of $P^{(1} \subset nCy\bar{\bar{\alpha}}_{Cl}(\Delta^{-1}$ of $D_\#)$. Therefore, we obtain $\Delta^{-1}\big(nCyCL(D_\#)\big) \subset \Delta^{-1}\big(P^{(1}\big) \subset nCy\bar{\bar{\alpha}}_{Cl}\big(\Delta^{-1}(D_\#)\big)$.

In contrast, let $D_\# \subset nCy\mathbb{F}^2$ and Δ be a $nCy\bar{\bar{\alpha}} \wr CS$ of $nCy\mathbb{F}^1$ containing $\Delta^{-1}(D_\#)$. Put ${L^\#}_3 = nCyCL_Y(D_\#)$, then we have $D_\# \subset {L^\#}_3$ and ${L^\#}_3$ is CS and $\Delta^{-1}({L^\#}_3) \subset nCy\bar{\bar{\alpha}}_{Cl}\big(\Delta^{-1}(D_\#)\big) \subset P^{(1}$.Then, Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP.

Lemma 2.10. The two maps $\Delta_{\mathrm{I}}: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ and $\Delta_{\mathrm{II}}: nCy\mathbb{F}^2 \longrightarrow nCy\mathbb{F}^3$ and $\Delta_{\mathrm{II}} \circ \Delta_{\mathrm{I}}: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^3$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-$OP$. If Δ_{II} is nCy-Cont. injective, then Δ_{I} is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-$OP$.

Proof: Let $L^\#$ be a $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Then $(\Delta_{\mathrm{II}} \circ \Delta_{\mathrm{I}})(L^\#)$ is nCy-OS in $nCy\mathbb{F}^3$ since $\Delta_{\mathrm{II}} \circ \Delta_{\mathrm{I}}$ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP. Further Δ_{II} is an injective nCy-Cont. function, $\Delta_{\mathrm{I}}(L^\#) = {\Delta_{\mathrm{I}}}^{-1}(\Delta_{\mathrm{II}} \diamond \Delta_{\mathrm{I}})(L^\#)$ is open in $nCy\mathbb{F}^2$. This demonstrates the fact Δ_{I} is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-$OP$.

Definition 2.11. A map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is said to be $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL if the image of each $nCy\bar{\bar{\alpha}} \wr CS$ in $nCy\mathbb{F}^1$ is closed in $nCy\mathbb{F}^2$. Clearly, every $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL map is nCy-CS as well as $nCy\bar{\bar{\alpha}} \wr CS$.

Example 2.12. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \ldots 8$, where,
$Y_i = \{< n_1; \mathfrak{R}_j >, < n_2; \mathfrak{H}_k >, < n_2; \mathfrak{I}_l >\};\ j, k, l = 1,2,3, \ldots 8$

Y_i	$\mathfrak{R}_j$	$\mathfrak{H}_j$	$\mathfrak{I}_l$
Y_1	$\mathfrak{R}_1 = \frac{103}{200}, \frac{95}{200}, \frac{110}{200}$	$\mathfrak{H}_1 = \frac{44}{200}, \frac{155}{200}, \frac{82}{200}$	$\mathfrak{I}_1 = \frac{112}{200}, \frac{93}{200}, \frac{55}{200}$
Y_2	$\mathfrak{R}_2 = \frac{112}{200}, \frac{87}{200}, \frac{111}{200}$	$\mathfrak{H}_2 = \frac{66}{200}, \frac{114}{200}, \frac{110}{200}$	$\mathfrak{I}_2 = \frac{101}{200}, \frac{98}{200}, \frac{110}{200}$
Y_3	$\mathfrak{R}_3 = \frac{42}{200}, \frac{63}{200}, \frac{121}{200}$	$\mathfrak{H}_3 = \frac{97}{200}, \frac{101}{200}, \frac{87}{200}$	$\mathfrak{I}_3 = \frac{136}{200}, \frac{101}{200}, \frac{122}{200}$
Y_4	$\mathfrak{R}_4 = \frac{13}{200}, \frac{93}{200}, \frac{39}{200}$	$\mathfrak{H}_4 = \frac{112}{200}, \frac{91}{200}, \frac{92}{200}$	$\mathfrak{I}_4 = \frac{112}{200}, \frac{97}{200}, \frac{111}{200}$
Y_5	$\mathfrak{R}_5 = \frac{31}{200}, \frac{67}{200}, \frac{54}{200}$	$\mathfrak{H}_5 = \frac{40}{200}, \frac{81}{200}, \frac{93}{200}$	$\mathfrak{I}_5 = \frac{121}{200}, \frac{79}{200}, \frac{121}{200}$

Y_6	$\mathfrak{R}_6 = \frac{53}{200}, \frac{73}{200}, \frac{67}{200}$	$\mathfrak{H}_6 = \frac{68}{200}, \frac{79}{200}, \frac{67}{200}$	$\mathfrak{I}_6 = \frac{18}{200}, \frac{88}{200}, \frac{65}{200}$
Y_7	$\mathfrak{R}_7 = \frac{81}{200}, \frac{68}{200}, \frac{93}{200}$	$\mathfrak{H}_7 = \frac{69}{200}, \frac{78}{200}, \frac{79}{200}$	$\mathfrak{I}_7 = \frac{89}{200}, \frac{97}{200}, \frac{79}{200}$
Y_8	$\mathfrak{R}_8 = \frac{47}{200}, \frac{55}{200}, \frac{73}{200}$	$\mathfrak{H}_8 = \frac{87}{200}, \frac{73}{200}, \frac{96}{200}$	$\mathfrak{I}_8 = \frac{97}{200}, \frac{101}{200}, \frac{89}{200}$

Here $(nC_y, nCy\mathbb{F}^1)$ is $nCy\bar{\bar{\alpha}} \wr\ CS$ and $(nC_y, nCy\mathbb{F}^2)$ is CS. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_2, \Delta(c2) = n_1$ and $\Delta(c3) = n_3$ then Δ is $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL.

Remark 2.13. Every $nCy\text{-}CS$ map need not be $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL as demonstrated by the subsequent example.

Example 2.14. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1\ to\ 6$, where,
$Y_i = \{< n_1; \mathfrak{R}_j >, < n_2; \mathfrak{H}_k >, < n_2; \mathfrak{I}_l >\};\ j, k, l = 1\ to\ 6$

Y_i	$\mathfrak{R}_j$	$\mathfrak{H}_j$	$\mathfrak{I}_l$
Y_1	$\mathfrak{R}_1 = \frac{247}{2000}, \frac{447}{2000}, \frac{647}{2000}$	$\mathfrak{H}_1 = \frac{347}{2000}, \frac{147}{2000}, \frac{447}{2000}$	$\mathfrak{I}_1 = \frac{47}{2000}, \frac{147}{2000}, \frac{347}{2000}$
Y_2	$\mathfrak{R}_2 = \frac{47}{2000}, \frac{47}{2000}, \frac{147}{2000}$	$\mathfrak{H}_2 = \frac{247}{2000}, \frac{147}{2000}, \frac{347}{2000}$	$\mathfrak{I}_2 = \frac{17}{2000}, \frac{17}{2000}, \frac{57}{2000}$
Y_3	$\mathfrak{R}_3 = \frac{57}{2000}, \frac{47}{2000}, \frac{47}{2000}$	$\mathfrak{H}_3 = \frac{347}{2000}, \frac{447}{2000}, \frac{147}{2000}$	$\mathfrak{I}_3 = \frac{147}{2000}, \frac{147}{2000}, \frac{247}{2000}$
Y_4	$\mathfrak{R}_4 = \frac{247}{2000}, \frac{57}{2000}, \frac{47}{2000}$	$\mathfrak{H}_4 = \frac{37}{2000}, \frac{147}{2000}, \frac{447}{2000}$	$\mathfrak{I}_4 = \frac{247}{2000}, \frac{547}{2000}, \frac{447}{2000}$
Y_5	$\mathfrak{R}_5 = \frac{111}{2000}, \frac{212}{2000}, \frac{412}{2000}$	$\mathfrak{H}_5 = \frac{212}{2000}, \frac{111}{2000}, \frac{401}{2000}$	$\mathfrak{I}_5 = \frac{97}{2000}, \frac{157}{2000}, \frac{347}{2000}$
Y_6	$\mathfrak{R}_6 = \frac{177}{2000}, \frac{421}{2000}, \frac{347}{2000}$	$\mathfrak{H}_6 = \frac{111}{2000}, \frac{212}{2000}, \frac{247}{2000}$	$\mathfrak{I}_6 = \frac{347}{2000}, \frac{347}{2000}, \frac{347}{2000}$

Here $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_2, \Delta(c2) = n_1$ and $\Delta(c3) = n_3$ then Δ is $nCy\text{-}CS$ map need not be $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL.

Theorem 2.15. The $nCy\mathbb{F}^1$ and $nCy\mathbb{F}^2$ be $n\text{-}CyFNTS's$ and the map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is a $nCy{\sim}Pse(\bar{\bar{\alpha}})\text{-}CL$ $\Leftrightarrow$ $\Delta(nCy\mathbb{F}^1)$ is $nCy\text{-}CS$ in $nCy\mathbb{F}^2$ and $\Delta(L^\#)\backslash\Delta(nCy\mathbb{F}^1\backslash L^\#)$ is $nCy\text{-}OS$ in $\Delta(nCy\mathbb{F}^1)$ in situations where $L^\#$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$.

Proof: In case $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is a $nCy{\sim}Pse(\bar{\bar{\alpha}})\text{-}CL$ map. Since $nCy\mathbb{F}^1$ is $nCy\bar{\bar{\alpha}} \wr CS$, $\Delta(nCy\mathbb{F}^1)$ is $nCy\text{-}CS$ in $nCy\mathbb{F}^2$ and $\Delta\ of\ L^\#\backslash\Delta\ of\ nCy\mathbb{F}^1\backslash L^\# = \Delta\ of\ L^\# \cap \Delta\ of\ nCy\mathbb{F}^1\backslash\Delta(nCy\mathbb{F}^1\backslash L^\#)$ is $nCy\text{-}OS$ in $\Delta(nCy\mathbb{F}^1)$ when $L^\#$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$.

In case $\Delta(nCy\mathbb{F}^1)$ is $nCy\text{-}CS$ in $nCy\mathbb{F}^2$, $\Delta(L^\#)\backslash\Delta(nCy\mathbb{F}^1\backslash L^\#)$ is $nCy\text{-}OS$ in $\Delta(nCy\mathbb{F}^1)$ when $L^\#$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$, and let $E_\#$ be $nCy\text{-}CS$ in $nCy\mathbb{F}^1$. Then $\Delta(E_\#) = \Delta(nCy\mathbb{F}^1)\backslash(\Delta(nCy\mathbb{F}^1\backslash E_\#)\backslash \Delta(E_\#))$ is $nCy\text{-}CS$ in $\Delta(nCy\mathbb{F}^1)$ and hence, $nCy\text{-}CS$ in $nCy\mathbb{F}^2$.

Corollary 2.16. The $nCy\mathbb{F}^1$ and $nCy\mathbb{F}^2$ be $n\text{-}CyFNTS's$ and let $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ be a $nCy\bar{\bar{\alpha}}$-ENDURING $nCy{\sim}Pse(\bar{\bar{\alpha}})\text{-}CL$ surjective map. Then n-CyFNT on $nCy\mathbb{F}^2$ is $\{\Delta(L^{\#})\backslash\Delta(nCy\mathbb{F}^1\backslash L^{\#}): L^{\#}$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1\}$.

Proof: Let $Q_{.}^{.\#}$ be $nCy\text{-}OS$ in $nCy\mathbb{F}^2$. Then $\Delta^{-1}(Q_{.}^{.\#})$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$, and $\Delta(\Delta^{-1}(Q_{.}^{.\#}))\backslash \Delta(nCy\mathbb{F}^1\backslash\Delta^{-1}(Q_{.}^{.\#})) = Q_{.}^{.\#}$ As a result, all $nCy\text{-}OS$ in $nCy\mathbb{F}^2$ are in the form of $\Delta(L^{\#})\backslash\Delta(nCy\mathbb{F}^1\backslash L^{\#})$, $L^{\#}$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. In contrast, all sets of the kind $\Delta(L^{\#})\backslash\Delta(nCy\mathbb{F}^1\backslash L^{\#})$, $L^{\#}$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$, are $nCy\text{-}OS$ in $nCy\mathbb{F}^2$.

Definition 2.17. A $nCyFNTS's$ $nCy\mathbb{F}^1$ is referred to as ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space If each set of two unique points f_1 and f_2 of $nCy\mathbb{F}^1$, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$ having one point but excluding the other.

Theorem 2.18. A $nCyFNTS's$ $nCy\mathbb{F}^1$ is a ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space $\Leftrightarrow$ $nCy\bar{\bar{\alpha}}$-closures of unique points are unique.

Proof: Let f_1 and f_2 be unique points of $nCy\mathbb{F}^1$. Since $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$ $M_{\#\#}$ such that $f_1 \in M_{\#\#}$ and $f_2 \notin M_{\#\#}$. thereby, $nCy\mathbb{F}^1 - M_{\#\#}$ is a $nCy\bar{\bar{\alpha}} \wr CS$ containing f_2 but not f_1. But $nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ is the $\cap$ of all $nCy\bar{\bar{\alpha}} \wr CS's$ containing f_2. Thus $f_2 \in nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ But $f_1 \notin nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ as $f_1 \notin nCy\mathbb{F}^1 - M_{\#\#}$. Therefore, $nCy\bar{\bar{\alpha}}_{Cl}\{f_1\} \neq nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$.

In contrast, let $nCy\bar{\bar{\alpha}}_{Cl}\{f_1\} \neq nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ for $f_1 \neq f_2$ $\Longrightarrow$ $\exists$ a minimum of a single point $f_3 \in nCy\mathbb{F}^1$ such that $f_3 \in nCy\bar{\bar{\alpha}}_{Cl}\{f_1\}$ but $f_3 \notin nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$. We assert $f_1 \notin nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$, because if $f_1 \in nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ then $\{f_1\} \subset nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ $\Longrightarrow$ $nCy\bar{\bar{\alpha}}_{Cl}\{f_1\} \subset nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$. So $f_3 \in nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$, which contradicts itself. Hence $f_1 \notin nCy\bar{\bar{\alpha}}_{Cl}\{f_2\} \Longrightarrow f_1 \in nCy\mathbb{F}^1 - nCy\bar{\bar{\alpha}}_{Cl}\{f_2\}$ which is a $nCy\bar{\bar{\alpha}} \wr OS$ having just f_1 and not f_2. Hence $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space.

Definition 2.19. A map $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is referred to as decisively $nCy\bar{\bar{\alpha}}\text{-}OpFn$. if the image of every $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$ is OS in $nCy\mathbb{F}^2$

Example 2.20. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \dots 8$, where,
$Y_i = \{< n_1; \mathfrak{R}_j >, < n_2; \mathfrak{H}_k >, < n_2; \mathfrak{I}_l >\}$; $j, k, l = 1,2,3, \dots 8$

Y_i	$\mathfrak{R}_j$	$\mathfrak{H}_j$	$\mathfrak{I}_l$
Y_1	$\mathfrak{R}_1 = \frac{49}{400}, \frac{58}{400}, \frac{175}{400}$	$\mathfrak{H}_1 = \frac{125}{400}, \frac{74}{400}, \frac{82}{400}$	$\mathfrak{I}_1 = \frac{285}{400}, \frac{122}{400}, \frac{115}{400}$
Y_2	$\mathfrak{R}_2 = \frac{38}{400}, \frac{49}{400}, \frac{49}{400}$	$\mathfrak{H}_2 = \frac{174}{400}, \frac{95}{400}, \frac{112}{400}$	$\mathfrak{I}_2 = \frac{177}{400}, \frac{98}{400}, \frac{79}{400}$
Y_3	$\mathfrak{R}_3 = \frac{82}{400}, \frac{192}{400}, \frac{74}{400}$	$\mathfrak{H}_3 = \frac{325}{400}, \frac{118}{400}, \frac{98}{400}$	$\mathfrak{I}_3 = \frac{192}{400}, \frac{57}{400}, \frac{48}{400}$
Y_4	$\mathfrak{R}_4 = \frac{58}{400}, \frac{125}{400}, \frac{82}{400}$	$\mathfrak{H}_4 = \frac{311}{400}, \frac{277}{400}, \frac{97}{400}$	$\mathfrak{I}_4 = \frac{115}{400}, \frac{78}{400}, \frac{74}{400}$
Y_5	$\mathfrak{R}_5 = \frac{192}{400}, \frac{134}{400}, \frac{115}{400}$	$\mathfrak{H}_5 = \frac{49}{400}, \frac{301}{400}, \frac{88}{400}$	$\mathfrak{I}_5 = \frac{49}{400}, \frac{48}{400}, \frac{58}{400}$
Y_6	$\mathfrak{R}_6 = \frac{74}{400}, \frac{57}{400}, \frac{285}{400}$	$\mathfrak{H}_6 = \frac{54}{400}, \frac{157}{400}, \frac{99}{400}$	$\mathfrak{I}_6 = \frac{54}{400}, \frac{59}{400}, \frac{95}{400}$
Y_7	$\mathfrak{R}_7 = \frac{82}{400}, \frac{95}{400}, \frac{74}{400}$	$\mathfrak{H}_7 = \frac{68}{400}, \frac{168}{400}, \frac{108}{400}$	$\mathfrak{I}_7 = \frac{134}{400}, \frac{221}{400}, \frac{95}{400}$

Y_8	$\mathfrak{R}_8 = \frac{125}{400}, \frac{124}{400}, \frac{49}{400}$	$\mathfrak{H}_8 = \frac{115}{400}, \frac{198}{400}, \frac{49}{400}$	$\mathfrak{I}_8 = \frac{82}{400}, \frac{117}{400}, \frac{74}{400}$

Clearly $(nC_y, nCy\mathbb{F}^1)$ is an $nCy\bar{\bar{\alpha}} \wr OS$ and $(nC_y, nCy\mathbb{F}^2)$ is a OS. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_1, \Delta(c2) = n_2$ and $\Delta(c3) = n_3$ then Δ is $nCy\bar{\bar{\alpha}}$-$OpFn$..

Theorem 2.21. In a bijection $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is a decisively $nCy\bar{\bar{\alpha}}$-$OpFn$. and $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space, then $nCy\mathbb{F}^2$ is also ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space.

Proof: Let g_1 and g_2 be two separate points of $nCy\mathbb{F}^2$. Since Δ is bijective $\exists$ distinct points f_1 and f_2 of $nCy\mathbb{F}^1 \mid \Delta(f_1) = g_1$ and $\Delta(f_2) = g_2$. Since $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS,\ G \mid x_1 \in G$ and $x_2 \notin G$. Therefore $g_1 = \Delta\ of\ f_1 \in \Delta\ of\ G$ and $g_2 = \Delta\ of\ f_2 \notin \Delta\ of\ G$. Since Δ being decisively $nCy\bar{\bar{\alpha}}$-$OpFn$., $\Delta(G)$ is $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^2$. Thus, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS\ \Delta(G)$ in $nCy\mathbb{F}^2 \mid g_1 \in \Delta(G)$ and $g_2 \notin \Delta(G)$. Therefore $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space.

Definition 2.22. An $nCyFNTS's$ $nCy\mathbb{F}^1$ is referred to as ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space In the event that any two distinct points f and g, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS\ G_\#$ and $H_\# \mid f\ is\ G_\#,\ g\ is\ not\ G_\#$ and $f\ is\ not\ H_\#,\ g\ is\ H_\#$.

Theorem 2.23. An $nCyFNTS's$ $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space $\Leftrightarrow$ singletons are $nCy\bar{\bar{\alpha}} \wr CS$.

Proof: Let $nCy\mathbb{F}^1$ be a ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space and $f \in nCy\mathbb{F}^1$. Let $g \in nCy\mathbb{F}^1 - \{f\}$. Then for $f \neq g$, $\exists$ $nCy\bar{\bar{\alpha}} \wr OS\ L^\#{}_g$ that $g \in L^\#{}_g$ and $f\ is\ not\ L^\#{}_g$. Furthermore, $g\ is\ L^\#{}_g \subset nCy\mathbb{F}^1 - \{f\}$. That is $nCy\mathbb{F}^1 - \{f\} = \cup \left\{L^\#{}_g : g \in nCy\mathbb{F}^1 - \{f\}\right\}$, which is $nCy\bar{\bar{\alpha}} \wr OS$. Hence $\{f\}$ is $nCy\bar{\bar{\alpha}} \wr CS$.

In contrast, suppose $\{f\}$ is $nCy\bar{\bar{\alpha}} \wr CS\ \forall\ f \in nCy\mathbb{F}^1$. Let f and $g \in nCy\mathbb{F}^1$ with $f\ is\ not\ equal\ to\ g$. Now $f\ is\ not\ equal\ to\ g \Rightarrow g \in nCy\mathbb{F}^1 - \{f\}$. Thereby $nCy\mathbb{F}^1 - \{f\}$ is $nCy\bar{\bar{\alpha}} \wr CS$ containing g but not f. Similarly, $nCy\mathbb{F}^1 - \{g\}$ is $nCy\bar{\bar{\alpha}} \wr CS$ containing f but not g. Thus $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Theorem 2.24. The object being ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space remains intact under bijection and decisively $nCy\bar{\bar{\alpha}}$-$OpFn$.

Proof: Let $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ possess bijection and decisively $nCy\bar{\bar{\alpha}}$-$OpFn$. Let $nCy\mathbb{F}^1$ be a ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space and l_1, l_2 be any 2-unique points of $nCy\mathbb{F}^2$. Since Δ is bijective $\exists$ unique points k_1 k_2 of $nCy\mathbb{F}^1 \mid l_1 = \Delta(k_1)$ and $l_2 = \Delta(k_2)$. Now $nCy\mathbb{F}^1$ being a ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space, $\exists$ $nCy\bar{\bar{\alpha}} \wr OS\ P_{t1}$ and $P_{t2} \mid k_1 \in P_{t1}$, $k_2 \notin P_{t1}$ and $k_1 \notin P_{t2}$, $k_2 \in P_{t2}$. Thereby $l_1 = \Delta(k_1) \in \Delta(P_{t1})$ but $l_2 = \Delta(k_2) \notin \Delta(P_{t1})$ and $l_2 = \Delta(k_2) \in \Delta(P_{t2})$ and $l_1 = \Delta(k_1) \notin \Delta(P_{t2})$. Here Δ possess decisively $nCy\bar{\bar{\alpha}}$-$OpFn$., $\Delta(P_{t1})$ and $\Delta(P_{t2})$ are $nCy\bar{\bar{\alpha}} \wr OS$ of $nCy\mathbb{F}^2 \mid l_1 \in \Delta(P_{t1})$ but $l_2 \notin \Delta(P_{t1})$ and $l_2 \in \Delta(P_{t2})$ and $l_1 \notin \Delta(P_{t2})$. Thus $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Theorem 2.25. Let $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ be bijective and decisively $nCy\bar{\bar{\alpha}}$-$OpFn$. In case $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space $\Longrightarrow$ $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Proof: Let g_1, g_2 be any 2-unique points of $nCy\mathbb{F}^2$. Since Δ is bijective $\exists$ unique points f_1, f_2 of $nCy\mathbb{F}^1 \mid g_1 = \Delta(f_1)$ and $g_2 = \Delta(f_2)$. Now $nCy\mathbb{F}^1$ being a ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space, $\exists$ $nCy\bar{\bar{\alpha}} \wr OS\ C_\#$ and $D_\#$ that $f_1\ in\ C_\#$, $f_2\ not\ in\ C_\#$ and $f_1\ not\ in\ D_\#$, $f_2\ in\ D_\#$. Thereby $g_1 = \Delta(f_1) \in \Delta$ of $C_\#$but $g_2 = \Delta$ of $f_2 \notin \Delta\ of\ C_\#$ and $y_2 = \Delta$ of $(f_2) \in \Delta$ of $(D_\#)$ and $g_1 = \Delta$ of $(f_1)\ not\ in\ \Delta(D_\#)$. Now $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space $\Longrightarrow C_\#$ and $D_\#$ are OS in $nCy\mathbb{F}^1$ and Δ is decisively $nCy\bar{\bar{\alpha}}$-$OpFn$., $\Delta(C_\#)$ and $\Delta(D_\#)$ are $nCy\bar{\bar{\alpha}} \wr OS$ of $nCy\mathbb{F}^2$. Thus $\exists$ $nCy\bar{\bar{\alpha}} \wr OS \mid g_1 \in \Delta(C_\#)$ but $g_2 \notin \Delta(C_\#)$ and $g_2 \in \Delta(D_\#)$ and $g_1 \notin \Delta(D_\#)$. Thus $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Example 2.26. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \ldots 8$, where,

$Y_i = \{< n_1; \Re_j >, < n_2; \mathfrak{H}_k >\};\ j,k = 1,2,3,\dots 8$

Y_i	$\Re_j$	$\mathfrak{H}_j$
Y_1	$\Re_1 = \frac{15}{40}, \frac{11}{40}, \frac{9}{40}$	$\mathfrak{H}_1 = \frac{15}{40}, \frac{17}{40}, \frac{13}{40}$
Y_2	$\Re_2 = \frac{22}{40}, \frac{27}{40}, \frac{5}{40}$	$\mathfrak{H}_2 = \frac{11}{40}, \frac{17}{40}, \frac{13}{40}$
Y_3	$\Re_3 = \frac{27}{40}, \frac{17}{40}, \frac{21}{40}$	$\mathfrak{H}_3 = \frac{23}{40}, \frac{13}{40}, \frac{15}{40}$
Y_4	$\Re_4 = \frac{18}{40}, \frac{13}{40}, \frac{13}{40}$	$\mathfrak{H}_4 = \frac{7}{40}, \frac{17}{40}, \frac{21}{40}$
Y_5	$\Re_5 = \frac{19}{40}, \frac{16}{40}, \frac{23}{40}$	$\mathfrak{H}_5 = \frac{15}{40}, \frac{15}{40}, \frac{17}{40}$
Y_6	$\Re_6 = \frac{15}{40}, \frac{11}{40}, \frac{13}{40}$	$\mathfrak{H}_6 = \frac{11}{40}, \frac{17}{40}, \frac{17}{40}$
Y_7	$\Re_7 = \frac{9}{40}, \frac{12}{40}, \frac{8}{40}$	$\mathfrak{H}_7 = \frac{13}{40}, \frac{13}{40}, \frac{18}{40}$
Y_8	$\Re_8 = \frac{11}{40}, \frac{13}{40}, \frac{11}{40}$	$\mathfrak{H}_8 = \frac{18}{40}, \frac{11}{40}, \frac{15}{40}$

Clearly $(nC_y, nCy\mathbb{F}^1)$ is an ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space and $(nC_y, nCy\mathbb{F}^2)$ is an ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_2, \Delta(c2) = n_1$ and $\Delta(c3) = n_3$, then Δ is decisively $nCy\bar{\bar{\alpha}}\text{-}OpFn$.

Theorem 2.27. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy\bar{\bar{\alpha}}$- ENDURING injection and $nCy\mathbb{F}^2$ is $nCy\delta_{-1}$ then $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Proof: Let $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ be $nCy\bar{\bar{\alpha}}$-ENDURING injection and $nCy\mathbb{F}^2$ is $nCy\delta_{-1}$. The 2-unique points $n_{1,}\ n_2$ of $nCy\mathbb{F}^1 \exists$ unique points $o_{1,}\ o_2$ of $nCy\mathbb{F}^2 \mid o_1 = \Delta(n_1)$ and $o_2 = \Delta$ of (n_2). Since $nCy\mathbb{F}^2$ is $nCy\delta_{-1}$-space, $\exists$ OS $L^{\#(1}$ and $L^{\#(2}$ in $nCy\mathbb{F}^2 \mid o_1 \in L^{\#(1}$, $o_2 \notin L^{\#(1}$ and $o_1 \notin L^{\#(2}$, $o_2 \in L^{\#(2}$. i.e., $n_1 \in \Delta^{-1}$of $(L^{\#(1)})$, $n_1 \notin \Delta^{-1}\ of\ (L^{\#(2)})$ and $n_2 \in \Delta^{-1}\ of\ (L^{\#(2)})$, $n_2 \notin \Delta^{-1}\ of\ (L^{\#(1)})$. Since Δ is $nCy\bar{\bar{\alpha}}$-ENDURING $\Delta^{-1}\ of(L^{\#(1)})$, $\Delta^{-1} of(L^{\#(2)})$ are $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Thus, the 2-unique points $n_1,\ n_2$ of $nCy\mathbb{F}^1\ \exists\ nCy\bar{\bar{\alpha}} \wr OS, \Delta^{-1}of(L^{\#(1)})$ and $\Delta^{-1}of(L^{\#(2)}) \mid n_1 \in \Delta^{-1}of(L^{\#(1)})$, $n_1 \notin \Delta^{-1}of(L^{\#(2)})$ and $n_2 \in \Delta^{-1}of(L^{\#(2)})$, $n_2 \notin \Delta^{-1}(L^{\#(1)})$. Thereby $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Theorem 2.28. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy\bar{\bar{\alpha}}\text{-}IR$ injective function and $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space then $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Proof: Let $n_{1,}\ n_2$ be pair of unique points in $nCy\mathbb{F}^1$. Since Δ is injective, $\exists$ unique points $o_{1,}\ o_2$ of $nCy\mathbb{F}^2 \mid o_1 = \Delta(n_1)$ and $o_2 = \Delta(n_2)$. Since $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space, $\exists\ nCy\bar{\bar{\alpha}} \wr OS$, $Qs^{\#1}$ and $Qs^{\#2}$ in $nCy\mathbb{F}^2 \mid o_1 \in Qs^{\#1}$, $o_2 \notin Qs^{\#1}$ and $o_1 \notin Qs^{\#2}$, $o_2 \in Qs^{\#2}$. i.e.,$n_1 \in \Delta^{-1}(Qs^{\#1})$, $n_1 \notin \Delta^{-1}(Qs^{\#2})$ and $n_2 \in \Delta^{-1}(Qs^{\#2})$, $n_2 \notin \Delta^{-1}(Qs^{\#1})$. Since Δ is $nCy\bar{\bar{\alpha}}\text{-}IR$, $\Delta^{-1}(Qs^{\#1})$, $\Delta^{-1}(Qs^{\#2})$ are $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Thus, the 2-unique points $n_{1,}\ n_2$ of $nCy\mathbb{F}^1\ \exists\ nCy\bar{\bar{\alpha}} \wr OS$, $\Delta^{-1}(Qs^{\#1})$ and $\Delta^{-1}(Qs^{\#2}) \mid n_1 \in \Delta^{-1}(Qs^{\#1})$, $n_1 \notin \Delta^{-1}(Qs^{\#2})$ and $n_2 \in \Delta^{-1}(Qs^{\#2})$, $n_2 \notin \Delta^{-1}(Qs^{\#1})$. Thereby $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space.

Definition 2.29. An $nCyPNTS$ is referred to as ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space if for any pair of unique points c^1 & c^2, $\exists$ disjoint $nCy\bar{\bar{\alpha}} \wr OS\ N^{l1}$ & $N^{l2} \mid c^1 \in N^{l1}$ & $c^2 \in N^{l2}$.

Theorem 2.30. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy\bar{\bar{\alpha}}$-ENDURING injection and $nCy\mathbb{F}^2$ is $nCy\delta_{-2}$ then $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space.

Proof: Let $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ be $nCy\bar{\bar{\alpha}}$-ENDURING injection and $nCy\mathbb{F}^2$ is $nCy\delta_{-2}$. The 2- unique points $e_{1,}\ e_2$ of $nCy\mathbb{F}^1$ ∃ unique points $f_{1,}\ f_2$ of $nCy\mathbb{F}^2$ | $f_1 = \Delta(e_1)$ and $f_2 = \Delta(e_2)$. Since $nCy\mathbb{F}^2$ is $nCy\delta_{-2}$-space, ∃ distinct $nCyOS\ De_{\#1}$ and $De_{\#2}$ in $nCy\mathbb{F}^2$ | $f_1 \in De_{\#1},\ \Delta_2 \in De_{\#2}$. i.e., $e_{1,} \in \Delta^{-1}\ of(De_{\#1})$ and $e_2 \in \Delta^{-1}\ of(De_{\#2})$. Since Δ is $nCy\bar{\bar{\alpha}}$-ENDURING $\Delta^{-1}\ of(De_{\#1})$, $\Delta^{-1}\ of(De_{\#2})$ are $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$. Further Δ is injective, $\Delta^{-1}\ of(De_{\#1}) \cap \Delta^{-1}\ of(De_{\#2}) = \Delta^{-1}\ of(De_{\#1} \cap De_{\#2}) = \Delta^{-1}\ of(\phi) = \phi$. Thus, for 2-unique points $e_{1,}\ e_2$ of $nCy\mathbb{F}^1$ ∃ disjoint $nCy\bar{\bar{\alpha}} \wr OS$, $\Delta^{-1}\ of(De_{\#1})$ and $\Delta^{-1}\ of(De_{\#2})$ such that $e_1 \in \Delta^{-1}\ of(De_{\#1})$ and $e_2 \in \Delta^{-1}\ of(De_{\#2})$. Therefore $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space.

Theorem 2.31. If $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is $nCy\bar{\bar{\alpha}}$-IR injective function and $nCy\mathbb{F}^2$ is and ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space then $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space.

Proof: Let $e_{1,}\ e_2$ be pair of unique points in $nCy\mathbb{F}^1$. Since Δ is injective ∃ distinct points $f_{1,}\ f_2$ of $nCy\mathbb{F}^2$ | $f_1 = \Delta(e_1)$ and $f_2 = \Delta(e_2)$. Since $nCy\mathbb{F}^2$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space ∃ $nCy\bar{\bar{\alpha}} \wr OS$, $De_{\#1}$ and $De_{\#2}$ in $nCy\mathbb{F}^2$ | $f_1 \in De_{\#1},\ f_2 \in De_{\#2}$. i.e., $e_{1,} \in \Delta^{-1}of(De_{\#1})$ and $e_2 \in \Delta^{-1}of(De_{\#2})$. Since Δ is $nCy\bar{\bar{\alpha}}$-IR injective $\Delta^{-1}of(De_{\#1})$, $\Delta^{-1}of(De_{\#2})$ are $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$.Thus, for 2-unique points $e_{1,}\ e_2$ of $nCy\mathbb{F}^1$ ∃ $nCy\bar{\bar{\alpha}} \wr OS$, $\Delta^{-1}of(De_{\#1})$ and $\Delta^{-1}of(De_{\#2})$ | $e_1 \in \Delta^{-1}of(De_{\#1})$ and $e_2 \in \Delta^{-1}of(De_{\#2})$. Thereby $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space.

Example 2.32. Let $C_y = \{c1, c2, c3\}$ and $\rho_{Cy} = \{1_{CyN}, 0_{CyN}, Y_i\}, i = 1,2,3, \dots 8$, where,
$Y_i = \{< n_1; \Re_j >, < n_2; \mathfrak{H}_k >, < n_2; \Im_l >\};\ j, k, l = 1,2,3, \dots 8$

Y_i	$\Re_j$	$\mathfrak{H}_j$	$\Im_l$
Y_1	$\Re_1 = \frac{117}{2000}, \frac{223}{2000}, \frac{417}{2000}$	$\mathfrak{H}_1 = \frac{417}{2000}, \frac{113}{2000}, \frac{112}{2000}$	$\Im_1 = \frac{223}{2000}, \frac{89}{2000}, \frac{418}{2000}$
Y_2	$\Re_2 = \frac{407}{2000}, \frac{421}{2000}, \frac{417}{2000}$	$\mathfrak{H}_2 = \frac{147}{2000}, \frac{101}{2000}, \frac{101}{2000}$	$\Im_2 = \frac{223}{2000}, \frac{117}{2000}, \frac{155}{2000}$
Y_3	$\Re_3 = \frac{152}{2000}, \frac{147}{2000}, \frac{153}{2000}$	$\mathfrak{H}_3 = \frac{147}{2000}, \frac{112}{2000}, \frac{152}{2000}$	$\Im_3 = \frac{112}{2000}, \frac{223}{2000}, \frac{155}{2000}$
Y_4	$\Re_4 = \frac{223}{2000}, \frac{217}{2000}, \frac{89}{2000}$	$\mathfrak{H}_4 = \frac{89}{2000}, \frac{89}{2000}, \frac{417}{2000}$	$\Im_4 = \frac{117}{2000}, \frac{101}{2000}, \frac{203}{2000}$
Y_5	$\Re_5 = \frac{101}{2000}, \frac{217}{2000}, \frac{153}{2000}$	$\mathfrak{H}_5 = \frac{101}{2000}, \frac{117}{2000}, \frac{417}{2000}$	$\Im_5 = \frac{147}{2000}, \frac{117}{2000}, \frac{112}{2000}$
Y_6	$\Re_6 = \frac{112}{2000}, \frac{223}{2000}, \frac{418}{2000}$	$\mathfrak{H}_6 = \frac{147}{2000}, \frac{417}{2000}, \frac{113}{2000}$	$\Im_6 = \frac{112}{2000}, \frac{223}{2000}, \frac{417}{2000}$
Y_7	$\Re_7 = \frac{101}{2000}, \frac{203}{2000}, \frac{153}{2000}$	$\mathfrak{H}_7 = \frac{117}{2000}, \frac{417}{2000}, \frac{203}{2000}$	$\Im_7 = \frac{117}{2000}, \frac{223}{2000}, \frac{101}{2000}$
Y_8	$\Re_8 = \frac{407}{2000}, \frac{203}{2000}, \frac{155}{2000}$	$\mathfrak{H}_8 = \frac{421}{2000}, \frac{112}{2000}, \frac{407}{2000}$	$\Im_8 = \frac{417}{2000}, \frac{112}{2000}, \frac{203}{2000}$

Clearly $(nC_y, nCy\mathbb{F}^1)$ is an ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space and $(nC_y, nCy\mathbb{F}^2)$ is an ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space. Here $\Delta: nCy\mathbb{F}^1 \longrightarrow nCy\mathbb{F}^2$ is defined by $\Delta(c1) = n_1, \Delta(c2) = n_2$ and $\Delta(c3) = n_3$ then Δ is $nCy\bar{\bar{\alpha}}$-IR injective function.

Theorem 2.33. An $nCyPNTS$, $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space $\Leftrightarrow$ For each $e \neq f$, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$, $De_{\#1}$ | $e \in De_{\#1}$ and $f \notin nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1})$ $\Leftrightarrow$ For each $e \in nCy\mathbb{F}^1$, $\{e\} = \cap \{nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1}) : De_{\#1}$ is a $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$ and $e \in De_{\#1}\}$.

Proof: Let $e \in nCy\mathbb{F}^1$ and $e \neq f$, $\exists$ disjoint $nCy\bar{\bar{\alpha}} \wr OS$, $De_{\#1}$ & $De_{\#2}$ | $e \in De_{\#1}$ & $f \in De_{\#2}$. Clearly, $nCy\mathbb{F}^1 - De_{\#2}$ is $nCy\bar{\bar{\alpha}} \wr CS$. Since $De_{\#1} \cap De_{\#2} = \phi$, $De_{\#1} \subset nCy\mathbb{F}^1 - De_{\#2}$. Thereby $nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1}) \subset nCy\bar{\bar{\alpha}}_{Cl}(nCy\mathbb{F}^1 - De_{\#2}) = nCy\mathbb{F}^1 - De_{\#2}$. Now $\notin nCy\mathbb{F}^1 - De_{\#2} \Rightarrow f \notin nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1})$.

For each $e \neq f$, $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS, De_{\#1}$ | $e \in De_{\#1}$ & $f \notin nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1})$. So $f \notin \cap \{nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1}) : De_{\#1}$ is a $nCy\bar{\bar{\alpha}} \wr OS$ in $nCy\mathbb{F}^1$ and $e \in De_{\#1}\} = \{e\}$.

Let $e, f \in nCy\mathbb{F}^1$ and $e \neq f$. By supposition $\exists$ a $nCy\bar{\bar{\alpha}} \wr OS$, $De_{\#1}$ | $e \in De_{\#1}$ and $nCy\bar{\bar{\alpha}}_{Cl}(De_{\#1})$. This $\Rightarrow \exists$ a $nCy\bar{\bar{\alpha}} \wr CS$, $D_{\#2}$ | $f \notin De_{\#2}$. Thereby $f \in nCy\mathbb{F}^1 - De_{\#2}$ & $nCy\mathbb{F}^1 - De_{\#2}$ is $nCy\bar{\bar{\alpha}} \wr OS$. Thus, $\exists$ 2-disjoint $nCy\bar{\bar{\alpha}} \wr OS$, $De_{\#1}$ and $nCy\mathbb{F}^1 - De_{\#2}$ | $e \in De_{\#1}$ and $f \in nCy\mathbb{F}^1 - De_{\#2}$. Thereby $nCy\mathbb{F}^1$ is ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space.

Conclusion: The analysis's interpretations of the present investigation are presented, together with an evaluation of the latest developments to which they have influenced. Start from the beginning to defined a new set in a space $nCy\mathbb{F}$ called an $nCy\bar{\bar{\alpha}} \wr$- integrated The ideas of CS and OS and considered the map of a function Δ from $nCy\mathbb{F}^1$ to $nCy\mathbb{F}^2$ demonstrates been declared to be $nCy{\sim}Pse(\bar{\bar{\alpha}})$-OP, $nCy{\sim}Pse(\bar{\bar{\alpha}})$-CL and decisively $nCy\bar{\bar{\alpha}}\text{-}OpFn$ and offered a comprehension of the accomplished achievements by presented the concept of spaces in $nCyFNTS's$ is considered to as ${}_{nCy\bar{\bar{\alpha}}}\delta^0$-space, ${}_{nCy\bar{\bar{\alpha}}}\delta^1$-space and ${}_{nCy\bar{\bar{\alpha}}}\delta^2$-space. Furthermore to the examples, assumptions, and theorems, the space separating two $n\text{-}CyFNS$, furthermore, their unique features and core functions were outlined.

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