Neutrosophic Approach to Solving First-Order Differential Equations: Applications to Heat Convection with Uncertain Initial Conditions

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Abstract: This study explores the solution of first-order differential equations (DE) using trapezoidal neutrosophic numbers ($TrapN_{number}$) as initial conditions. It examines various forms of $TrapN_{number}$ based on the dependencies of truth (T), indeterminacy (I), and falsity (F). The application first order DE is illustrated through heat conduction problems in fluids. The temperature distribution $T_1(x, \alpha)$, $T_2(x, \alpha)$, $T_1'(x, \beta)$, $T_1''(x, \gamma)$, $T_2'(x, \beta)$ and $T_2''(x, \gamma)$ are analyzed through tables and graphs. A solution procedure for the system of first-order ODEs is developed and demonstrated with numerical examples.

Keywords: difference equation; trapezoidal neutrosophic number, trapezoidal single valued neutrosophic number

1. Introduction

F_{set} theory, significantly developed by Zadeh [1], is defined by the membership function $\mu_{\tilde{A}}(x)$. Atanassov [2, 3] extended this concept by introducing a non-membership function $\vartheta_{\tilde{A}}(x)$, leading to IF_{set}. Liu and Yuan [4] then combined triangular F_{sets} with IF_{set}. to create triangular IF_{sets}. Later, Ye [5] replaced triangular $F_{numbers}$ with trapezoidal $F_{numbers}$ in both truth and falsity membership functions, resulting in trapezoidal IF_{sets}. However, traditional fuzzy and intuitionistic fuzzy logic lack the concept of indeterminacy. Smarandache [6-7] introduced indeterminacy term, enhancing the framework beyond fuzzy logic. Thus, neutrosophic logic consists of three components: truth value ($T_{\tilde{A}_{N_{set}}}$), indeterminacy value ($I_{\tilde{A}_{N_{set}}}$), and false value($F_{\tilde{A}_{N_{set}}}$), with these membership values defined within the non-standard interval $]-0, 1+[$. Smarandache [8] introduced the concept of N_{sets} as a generalization of IF_{sets} expanding the framework of fuzzy logic by incorporating indeterminacy. This structure allows for the representation of uncertainty, imprecision, and inconsistency.

First-order systems of ordinary differential equations have a wide range of applications, including population dynamics and disease spread, electrical circuits and fluid dynamics, motion and heat transfer, economic growth and market analysis, pollutant dispersion and ecosystem dynamics, and drug dosage and physiological processes. There are several approaches to solve first-order ODEs. Several researchers have utilized neutrosophic logic, including $SVN_{numbers}$, $TriN_{numbers}$, and $TrapN_{numbers}$, to solve DEs and explore their applications. Sumathi and Mohana Priya [9] presented a novel perspective on neutrosophic DEs, exploring how neutrosophic concepts can be applied to solve and interpret DEs. Sumathi and Crispin Sweety [10] proposed a new method for solving DEs using $TrapN_{numbers}$, showcasing a specific application of neutrosophic logic in mathematical modeling. Alamin et al. [11] explored solutions and interpretations of neutrosophic homogeneous DEs, enhancing the understanding of discrete systems in the context of neutrosophic logic. Mondal et al. [12] address the solution of a system of differential equations within an intuitionistic fuzzy environment. Chakraborty et al. [13] have explored various linear and non-linear forms of $TrapN_{numbers}$ and discuss de-neutrosophication techniques. Shanmugapriya et al. [14] explored the solution of system of first-order simultaneous DEs within a neutrosophic environment, which allows handling indeterminacy, truth, and falsity values in the context of uncertainty. Their research emphasizes applications in time-cost optimization and sequencing problems.

1.1 Research gap

The research gap in this study emphasizes the limited exploration of $TrapN_{numbers}$ as initial conditions in solving first-order DEs. It highlights the need for systematic analysis of their relationships among truth, indeterminacy, and falsity, as well as their application to heat conduction problems in fluids. Addressing these gaps could improve the use of neutrosophic logic in modeling uncertainty.

1.2 Objective

The aim of this study is to investigate solutions for first-order DEs using $TrapN_{numbers}$ as initial conditions. It seeks to explore various forms of $TrapN_{numbers}$ based on the relationships among truth, indeterminacy, and falsity. Additionally, the study aims to apply these concepts to heat conduction problems in fluids, analyzing temperature distributions through specific functions and presenting findings through tables and graphs.

1.3 Novelty

The novelty of this present study lies in the use of $TrapN_{numbers}$ as initial conditions for solving first-order differential equations (DEs). This innovative approach explores various forms of $TrapN_{numbers}$, capturing the relationships between truth, indeterminacy, and falsity values, which extends traditional methods of solving DEs by incorporating uncertainty. Additionally, applying these neutrosophic concepts to heat conduction problems in fluids, particularly in analyzing temperature distributions, offers a fresh perspective on addressing complex physical systems with uncertain parameters.

1.4 Applications

Neutrosophic logic significantly enhances modeling in various fields by addressing uncertainties inherent in complex systems. In population dynamics, it enables more accurate modeling of species interactions and growth rates. For electrical circuits, this approach facilitates the analysis of circuit behaviors under uncertain conditions. In fluid dynamics, neutrosophic logic aids in understanding fluid behaviors across different states of uncertainty. It also plays a crucial role in economic growth by evaluating models that may involve incomplete or uncertain data. Furthermore, it assists in pollutant dispersion by predicting the spread of pollutants with varying degrees of certainty. Lastly, in drug dosage optimization, neutrosophic logic allows for the tailoring of drug delivery methods based on uncertain patient responses, ultimately improving decision-making across these diverse disciplines.

2. Preliminaries

In this section, the fundamental concepts of neutrosophic logic utilized in this article are presented as follows:

Definition 2.1. N_{set}: [1] Let X be a universe set. An N_{set} $\tilde{A}_{N_{set}}$ on X is defined as:

$\tilde{A}_{N_{set}} = \left\{\langle x, T_{\tilde{A}_{N_{set}}}(x), I_{\tilde{A}_{N_{set}}}(x), F_{\tilde{A}_{N_{set}}}(x)\rangle : x \in X\right\}$,

where

i. $T_{\tilde{A}_{N_{set}}}(x): X \to ^{-}]0,1[^{+}$ is called the truth membership (T_{mem}), representing the degree of confidence in the membership of $x \in X$ in $\tilde{A}_{N_{set}}$

ii. $I_{\tilde{A}_{N_{set}}}(x): X \to ^{-}]0,1[^{+}$ is called the indeterminacy membership (I_{mem}), representing the degree of uncertainty in the membership of $x \in X$ in $\tilde{A}_{N_{set}}$

iii. $F_{\tilde{A}_{N_{set}}}(x): X \to ^{-}]0,1[^{+}$ is called the falsity membership (F_{mem})representing the degree of skepticism in the membership of $x \in X$ in $\tilde{A}_{N_{set}}$.

These membership functions satisfy the condition: that $^{-}0 \leq T_{\tilde{A}_{N_{set}}}(x) + I_{\tilde{A}_{N_{set}}}(x) + F_{\tilde{A}_{N_{set}}}(x) \leq 3^{+}$.

Definition 2.2. SVN_{set}: [2] An N_{set} $\tilde{A}_{N_{set}}$ on X is said to be SVN_{set} $\left(\widetilde{SA}_{N_{set}}\right)$ if x is a single-valued independent variable. The set $\widetilde{SA}_{N_{set}}$is defined as:

$\widetilde{SA}_{N_{set}} = \left\{\langle x, T_{\widetilde{SA}_{N_{set}}}(x), I_{\widetilde{SA}_{N_{set}}}(x), F_{\widetilde{SA}_{N_{set}}}(x)\rangle : x \in X\right\}$,

where the functions $T_{\widetilde{SA}_{N_{set}}}(x), I_{\widetilde{SA}_{N_{set}}}(x), F_{\widetilde{SA}_{N_{set}}}(x): X \to ^{-}]0,1[^{+}$ represents the $T_{mem}, I_{mem,}$ and F_{mem} functions, respectively, for the element $x \in X$ in $\widetilde{SA}_{N_{set}}$, such that:

$^{-}0 \leq T_{\widetilde{SA}_{N_{set}}}(x) + I_{\widetilde{SA}_{N_{set}}}(x) + F_{\widetilde{SA}_{N_{set}}}(x) \leq 3^{+}$.

Definition 2.3. (α, β, γ)-cut: [3] The (α, β, γ)-cut N_{set} is $\tilde{A}_{N_{set}}$ defined as:

$\tilde{A}_{N_{set}}^{(\alpha,\beta,\gamma)} = \left\{\langle T_{\tilde{A}_{N_{set}}}(x), I_{\tilde{A}_{N_{set}}}(x), F_{\tilde{A}_{N_{set}}}(x)\rangle : x \in X, T_{\tilde{A}_{N_{set}}}(x) \geq \alpha, I_{\tilde{A}_{N_{set}}}(x) \leq \beta, F_{\tilde{A}_{N_{set}}}(x) \leq \gamma\right\}$,

where $\alpha, \beta, \gamma \in [0,1]$ are fixed values, and they satisfy the condition:

$\alpha + \beta + \gamma \leq 3$.

Definition 2.4. N_{number}: [3] A N_{set} $\tilde{A}_{N_{set}}$ over the set of real numbers $\mathbb{R}$ is said to be N_{number} if it satisfies the following properties:

i. $\tilde{A}_{N_{set}}$ is normal: if there exist $x_0 \in \mathbb{R}$ such that $T_{\tilde{A}_{N_{set}}}(x_0) = 1$. $\left(I_{\tilde{A}_{N_{set}}}(x_0) = F_{\tilde{A}_{N_{set}}}(x_0) = 0\right)$.

ii. $\tilde{A}_{N_{set}}$ is convex for the $T_{\tilde{A}_{N_{set}}}(x)$ function.

(ie) $T_{\tilde{A}_{N_{set}}}(\mu x_1 + (1-\mu)x_2) \geq \min\left(T_{\tilde{A}_{N_{set}}}(x_1), T_{\tilde{A}_{N_{set}}}(x_2)\right)$, for all $x_1, x_2 \in \mathbb{R}$ & $\mu \in [0,1]$.

iii. $\tilde{A}_{N_{set}}$ is concave set for the $I_{\tilde{A}_{N_{set}}}(x)$ and $F_{\tilde{A}_{N_{set}}}(x)$ functions.

(ie) $I_{\tilde{A}_{N_{set}}}(\mu x_1 + (1-\mu)x_2) \geq \max\left(I_{\tilde{A}_{N_{set}}}(x_1), I_{\tilde{A}_{N_{set}}}(x_2)\right)$ and $F_{\tilde{A}_{N_{set}}}(\mu x_1 + (1-\mu)x_2) \geq \max\left(F_{\tilde{A}_{N_{set}}}(x_1), F_{\tilde{A}_{N_{set}}}(x_2)\right)$, for all $x_1, x_2 \in \mathbb{R}$ & $\mu \in [0,1]$.

Definition 2.5. $TrapN_{number}$: [3] A $TrapN_{number}$, $\tilde{A}_{N_{set}}$ is a subset of N_{number}, $\tilde{A}_{N_{set}}$in $\mathbb{R}$ with the following $T, I, and\ F$ functions which is given by the following:

$$T_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{x-a_1}{a_2-a_1}\right)u_{\tilde{A}_{N_{set}}} & for\ a_1\le x\le a_2\\ u_{\tilde{A}_{N_{set}}} & for\ a_2\le x\le a_3\\ \left(\frac{a_4-x}{a_4-a_3}\right)u_{\tilde{A}_{N_{set}}} & for a_3\le x\le a_4\\ 0 & otherwise\end{cases};\ I_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{a_2-x}{a_2-a_1}\right)v_{\tilde{A}_{N_{set}}} & for\ a_1\le x\le a_2\\ v_{\tilde{A}_{N_{set}}} & for\ a_2\le x\le a_3\\ \left(\frac{a_4-x}{a_4-a_3}\right)u_{v_{N_{set}}} & for\ a_3\le x\le a_4\\ 1 & otherwise\end{cases}$$

$$F_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{a_2-x}{a_2-a_1}\right)w_{\tilde{A}_{N_{set}}} & for\ a_1\le x\le a_2\\ w_{\tilde{A}_{N_{set}}} & for\ a_2\le x\le a_3\\ \left(\frac{a_4-x}{a_4-a_3}\right)w_{\tilde{A}_{N_{set}}} & for a_3\le x\le a_4\\ 1 & otherwise\end{cases}$$

where $^{-}0\le T_{\tilde{A}_{N_{set}}}(x)+I_{\tilde{A}_{N_{set}}}(x)+F_{\tilde{A}_{N_{set}}}(x)\le 3^{+}, x\in \tilde{A}_{N_{set}}$.

Definition 2.6. (α,β,γ) - cut of a $TrapN_{number}$, $\tilde{A}_{N_{set}}$: The (α,β,γ)-cut of a $TrapN_{number}$ $\tilde{A}_{N_{set}}$, where: $\tilde{A}_{N_{set}}=\langle(a_1,a_2,a_3,a_4);u_{\tilde{A}_{N_{set}}},v_{\tilde{A}_{N_{set}}},w_{\tilde{A}_{N_{set}}}\rangle$ is defined as:

$$\left(\tilde{A}_{N_{set}}\right)_{\alpha,\beta,\gamma}=\left[T_{\tilde{A}_{N1_{set}}}(\alpha),T_{\tilde{A}_{N2_{set}}}(\alpha);I_{\tilde{A}_{N1_{set}}}(\beta),I_{\tilde{A}_{N2_{set}}}(\beta);F_{\tilde{A}_{N1_{set}}}(\gamma),F_{\tilde{A}_{N2_{set}}}(\gamma)\right],$$

where

i. T_{mem}: $T_{\tilde{A}_{N1_{set}}}(\alpha)=[a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}, T_{\tilde{A}_{N2_{set}}}(\alpha)=[a_4-\alpha(a_4-a_3)]u_{\tilde{A}_{N_{set}}}$.

ii. I_{mem}: $I_{\tilde{A}_{N1_{set}}}(\beta)=[a_2-\beta(a_2-a_1)]v_{\tilde{A}_{N_{set}}}, I_{\tilde{A}_{N2_{set}}}(\beta)=[a_3+\beta(a_4-a_3)]v_{\tilde{A}_{N_{set}}}$.

iii. F_{mem}: $F_{\tilde{A}_{N1_{set}}}(\gamma)=[a_2-\gamma(a_2-a_1)]w_{\tilde{A}_{N_{set}}}, F_{\tilde{A}_{N2_{set}}}(\gamma)=[a_3+\gamma(a_4-a_3)]w_{\tilde{A}_{N_{set}}}$.

here $\alpha,\beta,\gamma\in(0,1]$ *and* $^{-}0<\alpha+\beta+\gamma\le 3^{+}$.

Linear Generalized N_{number}:

Definition: 2.7. The quantity of the $T, I, and\ F$ are not dependent then the $TrapSVN_{number}\ \tilde{A}_{N_{set}}=\langle(a_1,a_2,a_3,a_4);(b_1,b_2,b_3,b_4);\ (c_1,c_2,c_3,c_4)\rangle$ whose $T_{mem}, I_{mem,}$ and F_{mem} functions are defined as follows:

$$T_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1\le x<a_2\\ 1 & for\ a_2<x\le a_3\\ \left(\frac{a_4-x}{a_4-a_3}\right) & for a_3<x\le a_4\\ 0 & otherwise\end{cases};I_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{b_2-x}{b_2-b_1}\right) & for\ b_1\le x<b_2\\ 0 & for\ b_2<x\le b_3\\ \left(\frac{x-b_3}{b_4-b_3}\right) & for\ b_3<x\le b_4\\ 1 & otherwise\end{cases}$$

$$F_{\tilde{A}_{N_{set}}}(x)=\begin{cases}\left(\frac{c_2-x}{c_2-c_1}\right) & for\ c_1\le x<c_2\\ 0 & for\ c_2<x\le c_3\\ \left(\frac{x-c_3}{c_4-c_3}\right) & for\ c_3<x\le c_4\\ 1 & otherwise\end{cases}$$

where $^{-}0\le T_{\tilde{A}_{N_{set}}}(x)+I_{\tilde{A}_{N_{set}}}(x)+F_{\tilde{A}_{N_{set}}}(x)\le 3^{+}, x\in \tilde{A}_{N_{set}}$.

Definition: 2.8 The (α,β,γ)-cut of a $TrapSVN_{number}$ $\tilde{A}_{N_{set}}=\langle(a_1,a_2,a_3,a_4);(b_1,b_2,b_3,b_4);\ (c_1,c_2,c_3,c_4)\rangle$ of definition 2.7 is defined as follows:

$\left(\tilde{A}_{N_{set}}\right)_{\alpha,\beta,\gamma} = \left[T_{\tilde{A}_{N1_{set}}}(\alpha), T_{\tilde{A}_{N2_{set}}}(\alpha); I_{\tilde{A}_{N1_{set}}}(\beta), I_{\tilde{A}_{N2_{set}}}(\beta); F_{\tilde{A}_{N1_{set}}}(\gamma), F_{\tilde{A}_{N2_{set}}}(\gamma)\right]$,

where $T_{\tilde{A}_{N1_{set}}}(\alpha) = [a_1 + \alpha(a_2 - a_1)], T_{\tilde{A}_{N2_{set}}}(\alpha) = [a_4 - \alpha(a_4 - a_3)]; I_{\tilde{A}_{N1_{set}}}(\alpha) = [b_2 - \beta(b_2 - b_1)], I_{\tilde{A}_{N2_{set}}}(\alpha) = [b_3 + \beta(b_4 - b_3)]; F_{\tilde{A}_{N1_{set}}}(\alpha) = [c_2 - \gamma(c_2 - c_1)], F_{\tilde{A}_{N2_{set}}}(\alpha) = [c_3 + \gamma(c_4 - c_3)]$,
here $\alpha, \beta, \gamma \in (0,1]\ and\ \ {}^{-}0 < \alpha + \beta + \gamma \leq 3^{+}$.

Definition: 2.9. The quantity of $I, and\ F$ are dependent then the $\tilde{A}_{N_{set}} = \langle (a_1, a_2, a_3, a_4); (b_1, b_2, b_3, b_4);\ v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ whose $T_{mem}, I_{mem,}$ and F_{mem} functions are defined as follows: $TrapSVN_{number}$

$$T_{\tilde{A}_{N_{set}}}(x) = \begin{cases} \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\ a_1 \leq x < a_2 \\ 1 & for\ a_2 < x \leq a_3 \\ \left(\dfrac{a_4 - x}{a_4 - a_3}\right) & for a_3 < x \leq a_4 \\ 0 & otherwise \end{cases} ; I_{\tilde{A}_{N_{set}}}(x) = \begin{cases} \left(\dfrac{b_2 - x + v_{\tilde{A}_{N_{set}}}(x - b_1)}{b_2 - b_1}\right) & for\ b_1 \leq x < b_2 \\ v_{\tilde{A}_{N_{set}}} & for\ b_2 < x \leq b_3 \\ \left(\dfrac{x - b_3 + v_{\tilde{A}_{N_{set}}}(b_4 - x)}{b_4 - b_3}\right) & for\ b_3 < x \leq b_4 \\ 1 & otherwise \end{cases}$$

$$F_{\tilde{A}_{N_{set}}}(x) = \begin{cases} \left(\dfrac{b_2 - x + w_{\tilde{A}_{N_{set}}}(x - b_1)}{b_2 - b_1}\right) & for\ b_1 \leq x < b_2 \\ w_{\tilde{A}_{N_{set}}} & for\ b_2 < x \leq b_3 \\ \left(\dfrac{x - b_2 + w_{\tilde{A}_{N_{set}}}(b_3 - x)}{b_3 - b_2}\right) & for\ b_3 < x \leq b_3 \\ 1 & otherwise \end{cases}$$

where ${}^{-}0 \leq T_{\tilde{A}_{N_{set}}}(x) + I_{\tilde{A}_{N_{set}}}(x) + F_{\tilde{A}_{N_{set}}}(x) \leq 2^{+}, x \in \tilde{A}_{N_{set}}$.

Definition: 2.10 The (α, β, γ)-cut of a $TrapSVN_{number}$ $A_{TNe} = \langle (a_1, a_2, a_3, a_4); (b_1, b_2, b_3, b_4);\ v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ of definition 2.9 is defined as follows:
$(A_{TNe})_{\alpha,\beta,\gamma} = \left[T_{A_{TNe1}}(\alpha), T_{A_{TNe2}}(\alpha); I_{A_{TNe1}}(\beta), I_{A_{TNe2}}(\beta); F_{A_{TNe1}}(\gamma), f_{A_{TNe2}}(\gamma)\right]$, where $T_{A_{TNe1}}(\alpha) = [a_1 + \alpha(a_2 - a_1)], T_{A_{TNe2}}(\alpha) = [a_4 - \alpha(a_4 - a_3)]; I_{A_{TNe1}}(\beta) = \left[\frac{b_2 - v_{\tilde{A}_{N_{set}}} b_1 - \beta(b_2 - b_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right], I_{A_{TNe2}}(\beta) = \left[\frac{b_3 - v_{\tilde{A}_{N_{set}}} b_4 + \beta(b_4 - b_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right]; F_{A_{TNe1}}(\gamma) = \left[\frac{b_2 - w_{\tilde{A}_{N_{set}}} b_1 - \gamma(b_2 - b_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right], F_{A_{TNe2}}(\gamma) = \left[\frac{b_3 - w_{\tilde{A}_{N_{set}}} b_4 + \gamma(b_4 - b_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right]$,
here $0 < \alpha \leq 1, v_{\tilde{A}_{N_{set}}} < \beta \leq 1, w_{\tilde{A}_{N_{set}}} < \gamma \leq 1\ and - 0 < \beta + \gamma \leq 1 +\ and\ - 0 < \alpha + \beta + \gamma \leq 2 +$.

Definition: 2.11 The quantity of the $T, I, and\ F$ are dependent then the $TrapSVN_{number}\ of\ T_{mem}, I_{mem,}$ and F_{mem} function defined as

$$T_{A_{TNe}}(x) = \begin{cases} \left(\dfrac{x - a_1}{a_2 - a_1}\right) u_{\tilde{A}_{N_{set}}} & for\ a_1 \leq x < a_2 \\ 1 & for\ a_2 < x \leq a_3 \\ \left(\dfrac{a_4 - x}{a_4 - a_3}\right) u_{\tilde{A}_{N_{set}}} & for a_3 < x \leq a_4 \\ 0 & otherwise \end{cases} ;$$

$$I_{A_{TNe}}(x) = \begin{cases} \left(\dfrac{a_2 - x + u_{\tilde{A}_{N_{set}}}(x - a_1)}{a_2 - a_1}\right) & for\ a_1 \le x < a_2 \\ u_{\tilde{A}_{N_{set}}} & for\ a_2 < x \le a_3 \\ \left(\dfrac{x - a_3 + u_{\tilde{A}_{N_{set}}}(a_4 - x)}{a_4 - a_3}\right) & for\ a_3 < x \le a_4 \\ 1 & otherwise \end{cases};$$

$$F_{A_{TNe}}(x) = \begin{cases} \left(\dfrac{a_2 - x + w_{\tilde{A}_{N_{set}}} x - a_1)}{a_2 - a_1}\right) & for\ a_1 \le x < a_2 \\ w_{\tilde{A}_{N_{set}}} & for\ a_2 < x \le a_3 \\ \left(\dfrac{x - a_2 + w_{\tilde{A}_{N_{set}}}(a_3 - x)}{a_3 - a_2}\right) & for\ a_3 < x \le a_3 \\ 1 & otherwise \end{cases}$$

where $-0 \le T_{A_{TNe}}(x) + I_{A_{TNe}}(x) + F_{A_{TNe}}(x) \le 1+, x \in A_{TNe}$.

Definition: 2.12 (α, β, γ)-cut of a $TrapSVN_{number}$ $A_{TNe} = \langle (a_1, a_2, a_3, a_4); u_{\tilde{A}_{N_{set}}}, v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ of definition 2.11 is defined as follows:

$(A_{TNe})_{\alpha,\beta,\gamma} = [T_{A_{TNe1}}(\alpha), T_{A_{TNe2}}(\alpha); I_{A_{TNe1}}(\beta), I_{A_{TNe2}}(\beta); F_{A_{TNe1}}(\gamma), f_{A_{TNe2}}(\gamma)]$, where, $T_{A_{TNe1}}(\alpha) = \left[a_1 + \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_2 - a_1)\right]$, $T_{A_{TNe2}}(\alpha) = \left[a_4 - \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_4 - a_3)\right]$; $I_{A_{TNe1}}(\beta) = \left[\frac{a_2 - v_{\tilde{A}_{N_{set}}} a_1 - \beta(a_2 - a_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right]$, $I_{A_{TNe2}}(\beta) = \left[\frac{a_3 - v_{\tilde{A}_{N_{set}}} a_4 + \beta(a_4 - a_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right]$; $F_{A_{TNe1}}(\gamma) = \left[\frac{a_2 - w_{\tilde{A}_{N_{set}}} a_1 - \gamma(a_2 - a_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right]$, $F_{A_{TNe2}}(\gamma) = \left[\frac{a_3 - w_{\tilde{A}_{N_{set}}} a_4 + \gamma(a_4 - a_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right]$,
here, $0 < \alpha \le u_{\tilde{A}_{N_{set}}}, v_{\tilde{A}_{N_{set}}} < \beta \le 1, w_{\tilde{A}_{N_{set}}} < \gamma \le 1$ and $-0 < \alpha + \beta + \gamma \le 1+$.

3. Solution of Neutrosophic boundary value problem

In this section, we discuss the solution of first-order ODE with a $TrapN_{numbers}$ and a different form of linear $TrapN_{numbers}$ as initial conditions.

3.1 Solution of First-Order ODE using $\boldsymbol{TrapN_{number}}$

Let us consider the first order differential equation $\frac{dy}{dx} = ky$ (1)

with boundary condition $y(x_0) = \tilde{a}$, where $\tilde{a} = \langle (a_1, a_2, a_3, a_4); u_{\tilde{A}_{N_{set}}}, v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ is a $TrapN_{numbers}$.

Case $(\boldsymbol{i})$ when k is a positive constant $(ie) k > 0$

The (α, β, γ)-cut of Eq. (1) is

$$\frac{d}{dx}[y_1(x,\alpha), y_2(x,\alpha); y_1'(x,\beta), y_2'(x,\beta); y_1''(x,\gamma), y_2''(x,\gamma)]$$
$$= k[y_1(x,\alpha), y_2(x,\alpha); y_1'(x,\beta), y_2'(x,\beta); y_1''(x,\gamma), y_2''(x,\gamma)] \quad (2)$$

with the initial condition

$$y(x_0;\alpha,\beta,\gamma) = \langle [a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}, [a_4-\alpha(a_4-a_3)]u_{\tilde{A}_{N_{set}}};\ [a_2-\beta(a_2-a_1)]v_{\tilde{A}_{N_{set}}}, [a_3 + \beta(a_4-a_3)]v_{\tilde{A}_{N_{set}}}; [a_2-\gamma(a_2-a_1)]w_{\tilde{A}_{N_{set}}}, [a_3 + \gamma(a_4-a_3)]w_{\tilde{A}_{N_{set}}}\rangle \quad (3)$$

Solution: The general solution of Eq.(1) is $y_1(x,\alpha) = e^{kx+c_1}$ (4)

Applying boundary condition in Eq.(4) we get

$$c_1 = \log[a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}\ e^{-kx_0} \quad (5)$$

substituting Eq.(5) in Eq.(4), we get

$$y_1(x,\alpha) = [a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}e^{k(x-x_0)},\ y_2(x,\alpha) = [a_4-\alpha(a_4-a_3)]u_{\tilde{A}_{N_{set}}}e^{k(x-x_0)}$$

similarly,

$$y_1'(x,\beta) = [a_2-\beta(a_2-a_1)]v_{\tilde{A}_{N_{set}}}e^{k(x-x_0)}, y_2'(x,\beta) = [a_3+\beta(a_4-a_3)]v_{\tilde{A}_{N_{set}}}e^{k(x-x_0)}$$

$$y_1''(x,\gamma) = [a_2-\gamma(a_2-a_1)]w_{\tilde{A}_{N_{set}}}e^{k(x-x_0)}, y_2''(x,\gamma) = [a_3+\gamma(a_4-a_3)]w_{\tilde{A}_{N_{set}}}e^{k(x-x_0)}.$$

Case $(\boldsymbol{ii})$ when k is a negative constant $(ie)k < 0$, let us take $k = -p$ and $p > 0$

The (α,β,γ)-cut of Eq. (1) is

$$\frac{d}{dx}[y_1(x,\alpha), y_2(x,\alpha); y_1'(x,\beta), y_2'(x,\beta); y_1''(x,\gamma), y_2''(x,\gamma)] = -p[y_1(x,\alpha), y_2(x,\alpha); y_1'(x,\beta), y_2'(x,\beta); y_1''(x,\gamma), y_2''(x,\gamma)] \quad (6)$$

with the initial condition

$$y(x_0;\alpha,\beta,\gamma) = \langle [a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}, [a_4-\alpha(a_4-a_3)]u_{\tilde{A}_{N_{set}}};\ [a_2-\beta(a_2-a_1)]v_{\tilde{A}_{N_{set}}}, [a_3 + \beta(a_4-a_3)]v_{\tilde{A}_{N_{set}}}; [a_2-\gamma(a_2-a_1)]w_{\tilde{A}_{N_{set}}}, [a_3 + \gamma(a_4-a_3)]w_{\tilde{A}_{N_{set}}}\rangle \quad (7)$$

then the general solution of the above Eqs.(6) and (7) are as follows:

$$y_1(x,\alpha) = [a_1+\alpha(a_2-a_1)]u_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}, y_2(x,\alpha) = [a_4-\alpha(a_4-a_3)]u_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}$$

$$y_1'(x,\beta) = [a_2-\beta(a_2-a_1)]v_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}, y_2'(x,\beta) = [a_3+\beta(a_4-a_3)]v_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}$$

$$y_1''(x,\gamma) = [a_2-\gamma(a_2-a_1)]w_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}, y_2''(x,\gamma) = [a_3+\gamma(a_4-a_3)]w_{\tilde{A}_{N_{set}}}e^{-k(x-x_0)}$$

3.2 Solutions of First-Order ODE using Linear Generalized $\boldsymbol{N_{number}}$

Category $(\boldsymbol{i})$: ***when the quantity of the truth, indeterminacy and falsity are not dependent***

The general solution of the DE $\frac{dy}{dx} = ky$, with boundary condition $y(x_0) = \tilde{a}$, where $\tilde{a} = \langle (a_1,a_2,a_3,a_4); (b_1,b_2,b_3,b_4);\ (c_1,c_2,c_3,c_4)\rangle$ is a $TrapSVN_{number}$ is as follows:

Case$(\boldsymbol{i})$ when $k > 0$

$$y_1(x,\alpha) = [a_1+\alpha(a_2-a_1)]e^{k(x-x_0)}, y_2(x,\alpha) = [a_4-\alpha(a_4-a_3)]e^{k(x-x_0)}$$

$$y_1'(x,\beta) = [b_2-\beta(b_2-b_1)]e^{k(x-x_0)}, y_2'(x,\beta) = [b_3+\beta(b_4-b_3)]e^{k(x-x_0)}$$

$$y_1''(x,\gamma) = [c_2-\gamma(c_2-c_1)]e^{k(x-x_0)}, y_2''(x,\gamma) = [c_3+\gamma(c_4-c_3)]e^{k(x-x_0)}$$

Case$(\boldsymbol{ii})$ when $k < 0$

$$y_1(x,\alpha) = [a_1+\alpha(a_2-a_1)]e^{-k(x-x_0)}, \qquad y_2(x,\alpha) = [a_4-\alpha(a_4-a_3)]e^{-k(x-x_0)}$$

$$y_1'(x,\beta) = [b_2-\beta(b_2-b_1)]e^{-k(x-x_0)}, \qquad y_2'(x,\beta) = [b_3+\beta(b_4-b_3)]e^{-k(x-x_0)}$$

$$y_1''(x,\gamma) = [c_2-\gamma(c_2-c_1)]e^{-k(x-x_0)}, \qquad y_2''(x,\gamma) = [c_3+\gamma(c_4-c_3)]e^{-k(x-x_0)}$$

Category (ii): ***when the quantity of indeterminacy and falsity are dependent***

The general solution of the DE $\frac{dy}{dx} = ky, (k > 0)$ with boundary condition $y(x_0) = \tilde{a}$, where $\tilde{a} = \langle (a_1, a_2, a_3, a_4); (b_1, b_2, b_3, b_4);\ v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ is a $TrapSVN_{number}$ is as follows:

Case(i) when $k > 0$

$$y_1(x, \alpha) = [a_1 + \alpha(a_2 - a_1)]e^{k(x-x_0)}, y_2(x, \alpha) = [a_4 - \alpha(a_4 - a_3)]e^{k(x-x_0)}$$

$$y_1'(x, \beta) = \left[\frac{b_2 - v_{\tilde{A}_{N_{set}}} b_1 - \beta(b_2 - b_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}, y_2'(x, \beta) = \left[\frac{b_3 - v_{\tilde{A}_{N_{set}}} b_4 + \beta(b_4 - b_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}$$

$$y_1''(x, \gamma) = \left[\frac{b_2 - w_{\tilde{A}_{N_{set}}} b_1 - \gamma(b_2 - b_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}, y_2''(x, \gamma) = \left[\frac{b_3 - w_{\tilde{A}_{N_{set}}} b_4 + \gamma(b_4 - b_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}$$

Case(ii) when $k < 0$

$$y_1(x, \alpha) = [a_1 + \alpha(a_2 - a_1)]e^{-k(x-x_0)}, y_2(x, \alpha) = [a_4 - \alpha(a_4 - a_3)]e^{-k(x-x_0)}$$

$$y_1'(x, \beta) = \left[\frac{b_2 - v_{\tilde{A}_{N_{set}}} b_1 - \beta(b_2 - b_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}, y_2'(x, \beta) = \left[\frac{b_3 - v_{\tilde{A}_{N_{set}}} b_4 + \beta(b_4 - b_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}$$

$$y_1''(x, \gamma) = \left[\frac{b_2 - w_{\tilde{A}_{N_{set}}} b_1 - \gamma(b_2 - b_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}, y_2''(x, \gamma) = \left[\frac{b_3 - w_{\tilde{A}_{N_{set}}} b_4 + \gamma(b_4 - b_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}.$$

Category (iii): ***when the quantity of the truth, indeterminacy and falsity are dependent***

The general solution of the DE $\frac{dy}{dx} = ky, (k > 0)$ with boundary condition $y(x_0) = \tilde{a}$, where $\tilde{a} = \langle (a_1, a_2, a_3, a_4); u_{\tilde{A}_{N_{set}}}, v_{\tilde{A}_{N_{set}}}, w_{\tilde{A}_{N_{set}}} \rangle$ is a $TrapSVN_{number}$ is as follows:

Case(i) when $k > 0$

$$y_1(x, \alpha) = \left[a_1 + \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_2 - a_1)\right] e^{k(x-x_0)}, y_2(x, \alpha) = \left[a_4 - \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_4 - a_3)\right] e^{k(x-x_0)}$$

$$y_1'(x, \beta) = \left[\frac{a_2 - v_{\tilde{A}_{N_{set}}} a_1 - \beta(a_2 - a_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}, y_2'(x, \beta) = \left[\frac{a_3 - v_{\tilde{A}_{N_{set}}} a_4 + \beta(a_4 - a_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}$$

$$y_1''(x, \gamma) = \left[\frac{a_2 - w_{\tilde{A}_{N_{set}}} a_1 - \gamma(a_2 - a_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}, y_2''(x, \gamma) = \left[\frac{a_3 - w_{\tilde{A}_{N_{set}}} a_4 + \gamma(a_4 - a_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{k(x-x_0)}$$

Case(ii) when $k < 0$

$$y_1(x, \alpha) = \left[a_1 + \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_2 - a_1)\right] e^{-k(x-x_0)}, y_2(x, \alpha) = \left[a_4 - \frac{\alpha}{u_{\tilde{A}_{N_{set}}}}(a_4 - a_3)\right] e^{-k(x-x_0)}$$

$$y_1'(x, \beta) = \left[\frac{a_2 - v_{\tilde{A}_{N_{set}}} a_1 - \beta(a_2 - a_1)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}, y_2'(x, \beta) = \left[\frac{a_3 - v_{\tilde{A}_{N_{set}}} a_4 + \beta(a_4 - a_3)}{1 - v_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}$$

$$y_1''(x, \gamma) = \left[\frac{a_2 - w_{\tilde{A}_{N_{set}}} a_1 - \gamma(a_2 - a_1)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}, y_2''(x, \gamma) = \left[\frac{a_3 - w_{\tilde{A}_{N_{set}}} a_4 + \gamma(a_4 - a_3)}{1 - w_{\tilde{A}_{N_{set}}}}\right] e^{-k(x-x_0)}.$$

4. Application of the First-Order DE to the Heat Convection in Fluid

4.1 Newtons Law of Cooling

Newtons Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large.
If we let $T(t)$ be the temperature of the object at time t, T_s be the bulk environmental temperature, $\alpha = \frac{h}{\rho c V}$, and A_s the constant surface area then we can formulate Newton's law of Cooling as differential equation:

$$\frac{dT(t)}{dt} = -\alpha A_s[T(t) - T_s] \quad (8)$$

with initial condition $T(t)|_{t=0} = T(0) = T_0$.

4.2 Example: Solution of Heat Convection in Fluid

The initial temperature of an object is 80°C we place it in a refrigerator and maintained it in 5°C. If $\alpha = 0.002m^2s$ and $A_s = 0.2m^2$. What will be the temperature after 8 minutes?

Crisp solution:

We have the initial condition $T(0) = 80$°C, the temperature of the surrounding $T_s = 30$°C, $\alpha = 0.002m^2s$ and $A_s = 0.2m^2$. Using this condition in Eq. (8) then the analytical solutions become $T(t) = 5 + 75e^{-0.0004t}$. At $t = 8$, we get $T(8) = 79.7604$.

Solution:

The standard heat convection equation and the corresponding boundary conditions are:

$$\frac{dT(t)}{dt} = -\alpha A_s[T(t) - T_s]$$

with initial condition $T(t)|_{t=0} = T(0) = T_0$.
After applying neutrosophic parameters the above equation is reformulated to the neutrosophic differential equation.

$$(ie)\frac{d}{dt}[T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)]$$
$$= -\alpha A_s\big[[T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)] - T_s\big]$$

with the initial condition

$$T(t_0;\alpha,\beta,\gamma) = \langle [a_1 + \alpha(a_2 - a_1)]u_{\tilde{A}_{N_{set}}}, [a_4 - \alpha(a_4 - a_3)]u_{\tilde{A}_{N_{set}}};\ [a_2 - \beta(a_2 - a_1)]v_{\tilde{A}_{N_{set}}}, [a_3 + \beta(a_4 - a_3)]v_{\tilde{A}_{N_{set}}}; [a_2 - \gamma(a_2 - a_1)]w_{\tilde{A}_{N_{set}}}, [a_3 + \gamma(a_4 - a_3)]w_{\tilde{A}_{N_{set}}}\rangle$$

(i) Using $TrapN_{number}$

If the initial temperature is a $TrapN_{number}$ $\tilde{a} = \langle(3,4,5,6); 0.7, 0.6, 0.4\rangle$ then the solution of the above example is:

$[T(t)]_{\alpha,\beta,\gamma} = [T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)]$

where

$T_1(t,\alpha) = [3+\alpha]0.7[5 + 75e^{-0.0004t}]$; $T_2(t,\alpha) = [6-\alpha]0.7[5 + 75e^{-0.0004t}]$

$T_1'(t,\beta) = [4-\beta]0.6[5 + 75e^{-0.0004t}]$; $T_2'(t,\beta) = [5+\beta]0.6[5 + 75e^{-0.0004t}]$

$T_1''(t,\gamma) = [4-\gamma]0.4[5 + 75e^{-0.0004t}]$; $T_2''(t,\gamma) = [5+\gamma]0.4[5 + 75e^{-0.0004t}]$

here, $0 < \alpha \le 1, 0 < \beta \le 1, 0 < \gamma \le 1\ and - 0 < \alpha + \beta + \gamma \le 3 +$.

(ii) Using Linear Generalized N_{number}

Category (i):

If the initial temperature is a $TrapSVN_{number}$ $\tilde{a} = \langle(1,2,3,4); (5,6,7,8); (9,10,11,12)\rangle$ then the solution of the above example is:

$[T(t)]_{\alpha,\beta,\gamma} = [T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)]$

where

$T_1(t,\alpha) = [1+\alpha][5+75e^{-0.0004t}]$; $T_2(t,\alpha) = [4-\alpha][5+75e^{-0.0004t}]$

$T_1'(t,\beta) = [5-\beta][5+75e^{-0.0004t}]$; $T_2'(t,\beta) = [7+\beta][5+75e^{-0.0004t}]$

$T_1''(t,\gamma) = [10-\gamma][5+75e^{-0.0004t}]$; $T_2''(t,\gamma) = [11+\gamma][5+75e^{-0.0004t}]$

here, $0 < \alpha \le 1, 0 < \beta \le 1, 0 < \gamma \le 1\ and -0 < \alpha + \beta + \gamma \le 3+$.

Category (ii):

If the initial temperature is a $TrapSVN_{number}$ $\tilde{a} = \langle(1,2,3,4); (5,6,7,8);\ 0.1,0.2\rangle$ then the solution of the above example is:

$[T(t)]_{\alpha,\beta,\gamma} = [T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)]$

where

$T_1(t,\alpha) = [1+\alpha][5+75e^{-0.0004t}]$; $T_2(t,\alpha) = [4-\alpha][5+75e^{-0.0004t}]$

$$T_1'(t,\beta) = \left[\frac{3.5-\beta}{0.9}\right][5+75e^{-0.0004t}];\ T_2'(t,\beta) = \left[\frac{6.2+\beta}{0.9}\right][5+75e^{-0.0004t}]$$

$$T_1''(t,\gamma) = \left[\frac{5-\gamma}{0.8}\right][5+75e^{-0.0004t}];\ T_2''(t,\gamma) = \left[\frac{5.4+\gamma}{0.8}\right][5+75e^{-0.0004t}]$$

here, $0 < \alpha \le 1, v_A < \beta \le 1, w_A < \gamma \le 1\ and -0 < \beta + \gamma \le 1 +\ and\ -0 < \alpha + \beta + \gamma \le 2+$.

Category (iii):

If the initial temperature is a $TrapSVN_{number}$ $\tilde{a} = \langle(1,2,3,4); 0.1,0.2,0.3\rangle$ then the solution of the above example is:

$[T(t)]_{\alpha,\beta,\gamma} = [T_1(t,\alpha), T_2(t,\alpha); T_1'(t,\beta), T_2'(t,\beta); T_1''(t,\gamma), T_2''(t,\gamma)]$

where

$$T_1(t,\alpha) = \left[1+\frac{\alpha}{0.1}\right][5+75e^{-0.0004t}];\ T_2(t,\alpha) = \left[4-\frac{\alpha}{0.1}\right][5+75e^{-0.0004t}]$$

$$T_1'(t,\beta) = \left[\frac{1.8-\beta}{0.8}\right][5+75e^{-0.0004t}];\ T_2'(t,\beta) = \left[\frac{2.2+\beta}{0.8}\right][5+75e^{-0.0004t}]$$

$$T_1''(t,\gamma) = \left[\frac{1.7-\gamma}{0.7}\right][5+75e^{-0.0004t}];\ T_2''(t,\gamma) = \left[\frac{1.8+\gamma}{0.7}\right][5+75e^{-0.0004t}]$$

here, $0 < \alpha \le u_A, v_A < \beta \le 1, w_A < \gamma \le 1$ and $-0 < \alpha + \beta + \gamma \le 1+$.

The solution of the temperature distribution of $T(t) = 5 + 75e^{-0.0004t}$ for various t and $0 < \alpha,\beta,\gamma \le 1$ $TrapN_{number}$ and linear generalized N_{number} with different categories are depicted in Tables 1-4. These tables summarize the behavior of truth, indeterminacy, and falsity under various (α,β,γ) - cut values at $t = 8$. It is observed that the results of $T(t)$ satisfies the condition of strong solution.

Table 1. $TrapN_{number}$ solution for the different values of α,β and γ at $t = 8$

α	$T_1(t,\alpha)$	$T_2(t,\alpha)$	β	$T_1'(t,\beta)$	$T_2'(t,\beta)$	γ	$T_1''(t,\gamma)$	$T_2''(t,\gamma)$
0.1	173.0800	329.4103	0.1	186.6392	244.0667	0.1	124.4261	162.7111
0.2	178.6632	323.8271	0.2	181.8536	248.8523	0.2	121.2357	165.9015
0.3	184.2464	318.2439	0.3	177.0680	253.6380	0.3	118.0453	169.0920
0.4	189.8297	312.6607	0.4	172.2824	258.4236	0.4	114.8549	172.2824
0.5	195.4129	307.0774	0.5	167.4968	263.2092	0.5	111.6645	175.4728
0.6	200.9961	301.4942	0.6	162.7111	267.9948	0.6	108.4741	178.6632
0.7	206.5793	295.9110	0.7	157.9255	272.7805	0.7	105.2837	181.8536
0.8	212.1626	290.3277	0.8	153.1399	277.5661	0.8	102.0932	185.0440
0.9	217.7458	284.7445	0.9	148.3543	282.3517	0.9	98.9028	188.2345
1	223.3290	279.1613	1	143.5686	287.1373	1	95.7124	191.4249

Table 2. $TrapSVN_{number}$ solution for the different values of α, β and γ at $t = 8$ when the quantity of the truth, indeterminacy and falsity are not dependent

α	$T_1(t,\alpha)$	$T_2(t,\alpha)$	β	$T_1'(t,\beta)$	$T_2'(t,\beta)$	γ	$T_1''(t,\gamma)$	$T_2''(t,\gamma)$
0.1	87.7364	311.0654	0.1	470.5862	566.2987	0.1	789.6277	885.3402
0.2	95.7124	303.0894	0.2	462.6102	574.2747	0.2	781.6517	893.3162
0.3	103.6884	295.1134	0.3	454.6341	582.2508	0.3	773.6757	901.2923
0.4	111.6645	287.1373	0.4	446.6581	590.2268	0.4	765.6996	909.2683
0.5	119.6405	279.1613	0.5	438.6821	598.2028	0.5	757.7236	917.2444
0.6	127.6166	271.1853	0.6	430.7060	606.1789	0.6	749.7476	925.2204
0.7	135.5926	263.2092	0.7	422.7300	614.1549	0.7	741.7715	933.1964
0.8	143.5686	255.2332	0.8	414.7539	622.1309	0.8	733.7955	941.1725
0.9	151.5447	247.2571	0.9	406.7779	630.1070	0.9	725.8194	949.1485
1	159.5207	239.2811	1	398.8019	638.0830	1	717.8434	957.1246

Table 3. $TrapSVN_{number}$ solution for the different values of α, β and γ at $t = 8$ when the quantity of indeterminacy and falsity are dependent

α	$T_1(t,\alpha)$	$T_2(t,\alpha)$	β	$T_1'(t,\beta)$	$T_2'(t,\beta)$	γ	$T_1''(t,\gamma)$	$T_2''(t,\gamma)$
0.10	87.7364	311.0654	0.20	469.7000	567.1849	0.300	468.5922	568.292
0.15	91.7244	307.0774	0.25	465.2689	571.6160	0.325	466.0997	570.7852
0.20	95.7124	303.0894	0.30	460.8377	576.0472	0.350	463.6072	573.2777
0.25	99.7004	299.1014	0.35	456.4066	580.4783	0.375	461.1147	575.7702
0.30	103.6884	295.1134	0.40	451.9755	584.9094	0.400	458.6222	578.2627
0.35	107.6765	291.1254	0.45	447.5443	589.3406	0.425	456.1296	580.7552
0.40	111.6645	287.1373	0.50	443.1132	593.7717	0.450	453.6371	583.2478
0.45	115.6525	283.1493	0.55	438.6821	598.2028	0.475	451.1446	585.7403
0.50	119.6405	279.1613	0.60	434.2509	602.6340	0.500	448.6521	588.2328
0.55	123.6285	275.1733	0.65	429.8198	607.0651	0.525	446.1596	590.7253

Table 4. $TrapSVN_{number}$ solution for the different values of α, β and γ at $t = 8$ when the quantity of the truth, indeterminacy and falsity are dependent

α	$T_1(t,\alpha)$	$T_2(t,\alpha)$	β	$T_1'(t,\beta)$	$T_2'(t,\beta)$	γ	$T_1''(t,\gamma)$	$T_2''(t,\gamma)$
0.01	87.7364	311.0654	0.250	154.5357	244.2661	0.325	156.6721	242.1297
0.02	95.7124	303.0894	0.275	152.0432	246.7586	0.350	153.8235	244.9783
0.03	103.6884	295.1134	0.300	149.5507	249.2511	0.375	150.9750	247.8269
0.04	111.6645	287.1373	0.325	147.0582	251.7437	0.400	148.1264	250.6754

0.05	119.6405	279.1613	0.350	144.5656	254.2362	0.425	145.2778	253.5240
0.06	127.6166	271.1853	0.375	142.0731	256.7287	0.450	142.4292	256.3726
0.07	135.5926	263.2092	0.400	139.5806	259.2212	0.475	139.5806	259.2212
0.08	143.5686	255.2332	0.425	137.0881	261.7137	0.500	136.7320	262.0698
0.09	151.5447	247.2571	0.450	134.5956	264.2062	0.525	133.8835	264.9184
0.1	159.5207	239.2811	0.475	132.1031	266.6987	0.550	131.0349	267.7670

5. Results and Discussion

5.1 Analysis of 3D contour plots

The temperature distribution of $T_1(t,\alpha) = [3+\alpha]0.7[5+75e^{-0.0004t}]$; $T_2(t,\alpha) = [6-\alpha]0.7[5+75e^{-0.0004t}]$ for $0 < \alpha \le 1$ and $100 \le t \le 1000$, which is shown if Fig. 1 and 2. The temperature distribution of $T_1(t,\alpha) = [1+\alpha][5+75e^{-0.0004t}], T_2(t,\alpha) = [4-\alpha][5+75e^{-0.0004t}]$ for $0 < \alpha \le 1$ and $100 \le t \le 1000$, which is shown if Fig. 3 and 4. The temperature distribution of $T_1(t,\alpha) = [1+\alpha][5+75e^{-0.0004t}], T_2(t,\alpha) = [4-\alpha][5+75e^{-0.0004t}]$ for $0 < \alpha \le 0.55$ and $100 \le t \le 1000$, which is shown if Fig. 5 and 6. The temperature distribution of $T_1(t,\alpha) = \left[1+\frac{\alpha}{0.1}\right][5+75e^{-0.0004t}], T_2(t,\alpha) = \left[4-\frac{\alpha}{0.1}\right][5+75e^{-0.0004t}]$for $0 < \alpha \le 0.1$ and $100 \le t \le 1000$, which is shown if Fig. 7 and 8. It is noted that, the upsurging values of α,β,γ increase $T_1(t,\alpha)$ and decrease $T_2(t,\alpha)$ where as $T_1'(t,\beta)$ and $T_1''(t,\gamma)$ are diminishing functions and $T_2'(t,\beta)$ and $T_2''(t,\gamma)$ are elevation functions. Therefore, the mentioned heat convection problem satisfies the conditions of the strong solutions of a neutrosophic difference equation [12] $(ie)\, T_1(t,\alpha) \le T_2(t,\alpha), T_1'(t,\beta) \le T_2'(t,\beta), T_1''(t,\gamma) \le T_2''(t,\gamma)$ and $\frac{\partial}{\partial\alpha}[T_1(t,\alpha)] > 0$, $\frac{\partial}{\partial\alpha}[T_2(t,\alpha)] < 0$; $\frac{\partial}{\partial\beta}[T_1'(t,\beta)] < 0, \frac{\partial}{\partial\beta}[T_2'(t,\beta)] > 0$; $\frac{\partial}{\partial\gamma}[T_1''(t,\gamma)] < 0, \frac{\partial}{\partial\gamma}[T_2''(t,\gamma)] > 0$. Hence the obtained solution is a strong solution.

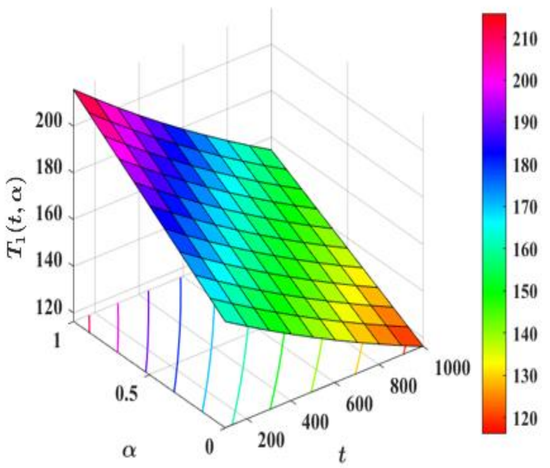


Fig. 1. Impact of *t* in $T_1(t,\alpha)$

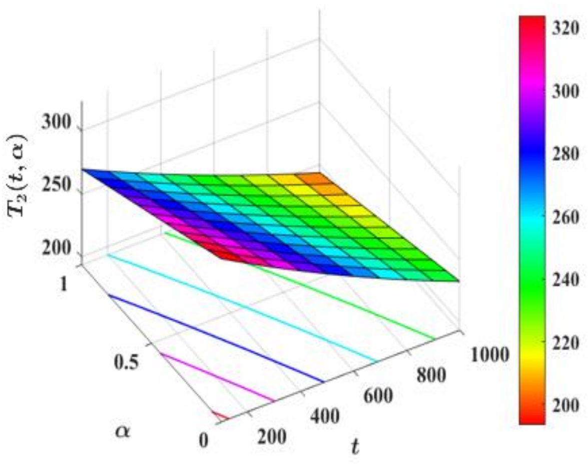


Fig. 2. Impact of *t* in $T_2(t,\alpha)$

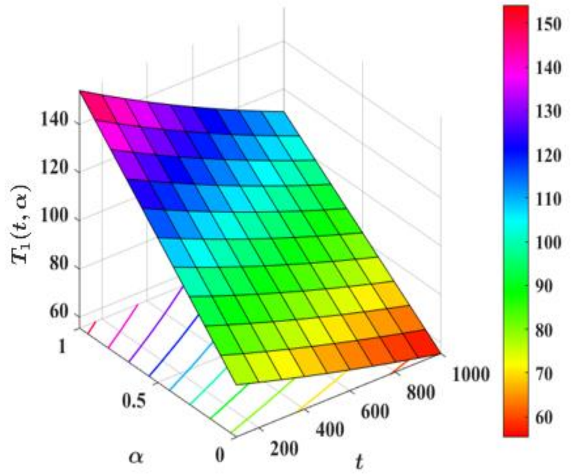


Fig. 3. Impact of t in $T_1(t,\alpha)$

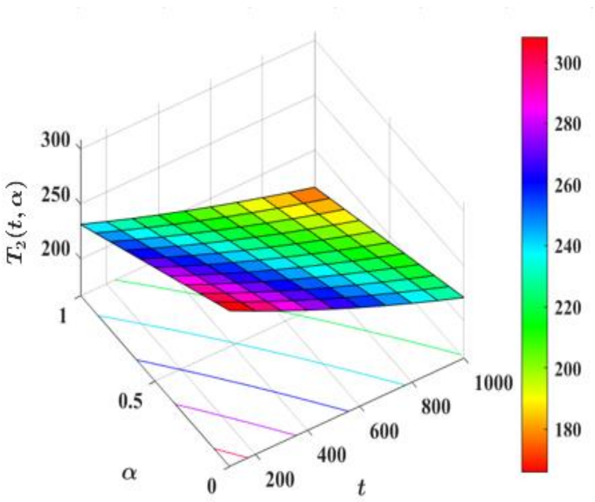


Fig. 4. Impact of t in $T_2(t,\alpha)$

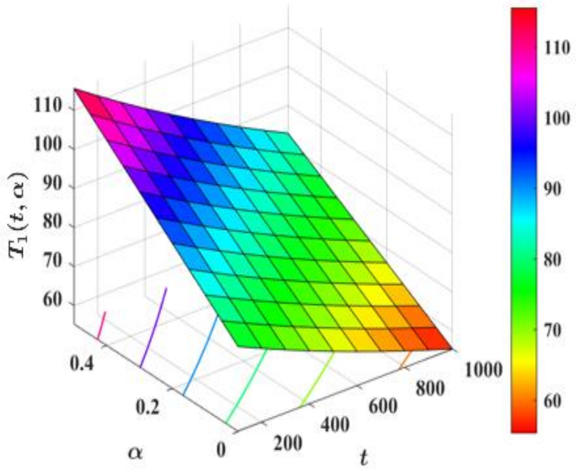


Fig. 5. Impact of t in $T_1(t,\alpha)$

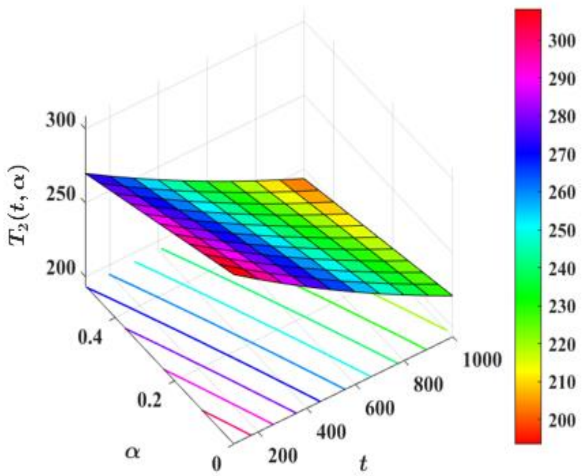


Fig. 6. Impact of t in $T_2(t,\alpha)$

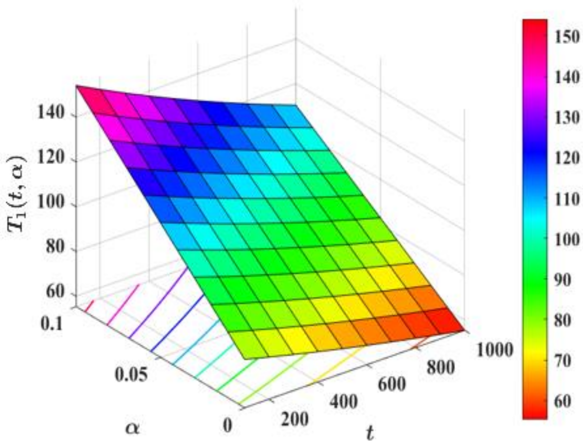


Fig. 7. Impact of t in $T_1(t,\alpha)$

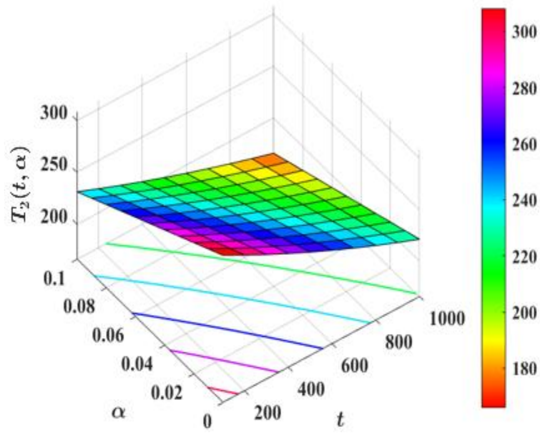


Fig. 8. Impact of t in $T_2(t,\alpha)$

5.2 *Analysis of* $\boldsymbol{TrapN_{number}}$ *and* $\boldsymbol{TrapSVN_{number}}$ *plots*

Figure 9 demonstrates that, as $\alpha - cut$ values increase, the lower bound of truth membership, $T_1(t, \alpha)$ rises, while the upper bound, $T_2(t, \alpha)$ decreases. This trend shows a diminishing uncertainty as the values approach the exact solution. Similarly, Figure 10 shows that for indeterminacy membership, both the lower bound $T_1'(t, \beta)$ and $T_2'(t, \beta)$ upper bound decrease with increasing $\beta - cut$ values, confirming that indeterminacy is minimized as the $\beta -$cut values approach. In figures 11 $(a), (b)$ *and* (c), when the dependency of falsity on $\gamma - cut$ is introduced, the lower bound $T_1''(t, \gamma)$ shows a decreasing trend, while the upper bound $T_2''(t, \gamma)$ rises. This reveals that falsity becomes more important at higher $\gamma - cut$ values. Finally, figures 12 $(a), (b)$ *and* (c) presents the truth, indeterminacy, and falsity are considered dependent. The data suggests that as $\alpha - cuts$ values increase and $\beta - cut$ and higher $\gamma - cut$ values decrease, the lower bound of truth membership rises, and indeterminacy and falsity gradually diminish. This points to a convergence toward the exact solution when the dependencies are balanced.

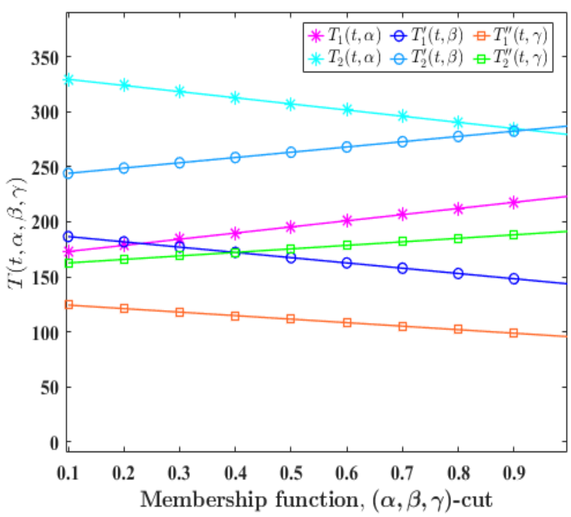


Fig. 9. Impact of t in $T(t, \alpha, \beta, \gamma)$ using $TrapN_{number}$

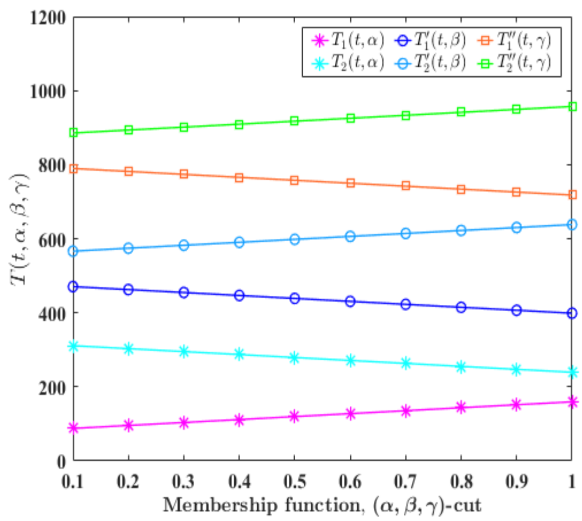
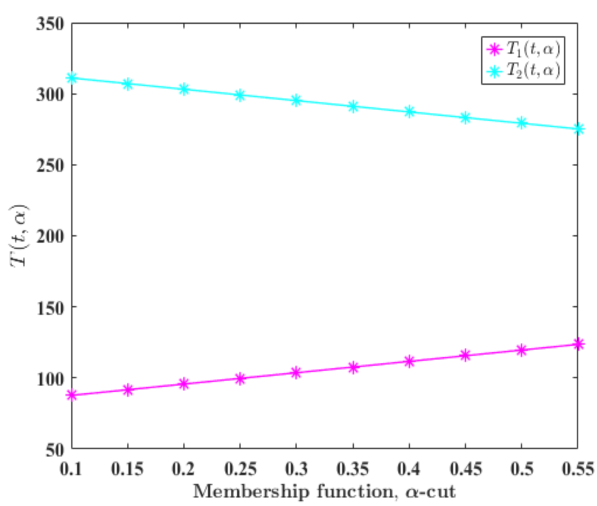
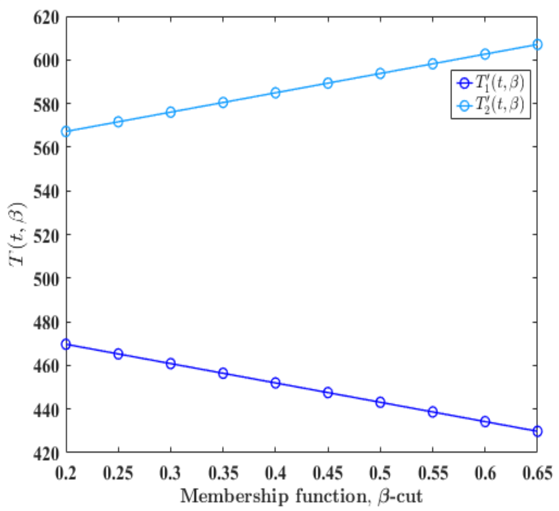


Fig. 10. Impact of t in $T(t, \alpha, \beta, \gamma)$ using $TrapSVN_{number}$

(a)



(b)



(c)

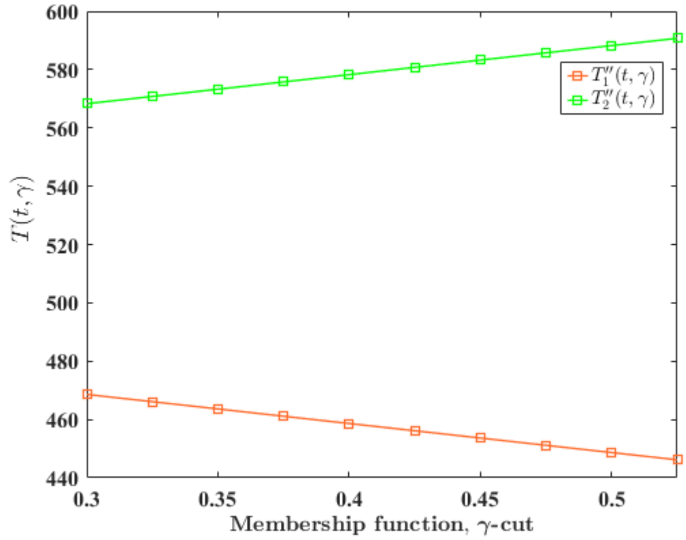
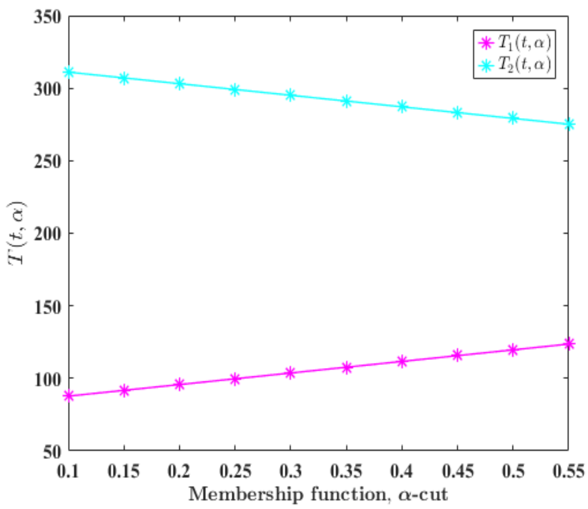


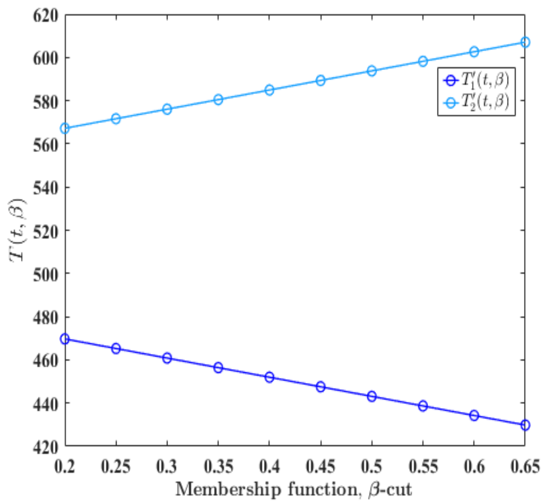
Fig. 11. (a), (b), (c) Impact of t in $T(t,\alpha)$, $T(t,\beta)$, and $T(t,\gamma)$ using $TrapSVN_{number}$

6. Conclusion

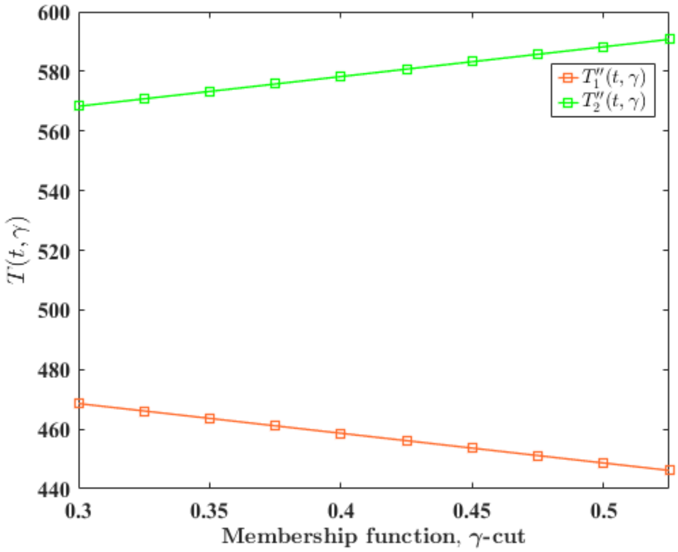
In recent years, mathematical modeling that incorporates uncertainty or vagueness has found significant applications across various industries and engineering fields. In this research paper, we analyze the solution of first-order DE in neutrosophic environment. The $(\alpha,\beta,\gamma)-cut$ neutrosophic set and the strong and weak solutions of neutrosophic DE concepts are also applied for the DE with a $TrapN_{number}$ and a linear generalized $TrapN_{number}$as initial conditions. Furthermore, the application of this first-order DE to the heat convection problem in fluid dynamics provides a more comprehensive interpretation of differential equations in uncertain conditions.



(a)



(b)



(c)

Fig. 12. $(a), (b), (c)$ Impact of t in $T(t,\alpha)$, $T(t,\beta)$, and $T(t,\gamma)$ using $TrapSVN_{number}$

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Conflict of Interest

The authors of this research study declare that they have no competing interests.

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