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Energy of a neutrosophic soft set and its applications to multi-criteria decision-making problems

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Abstract. In our paper, we continue the study of the theory of neutrosophic soft sets and their properties. Based on the singular values of the corresponding matrix, we define the energy of a neutrosophic soft set, as well as the lower energy of a neutrosophic soft set and the upper energy of a neutrosophic soft set, allowing us to introduce an effective method for decision-making. To do this, we first write a neutrosophic soft set as a matrix and multiply it by its transpose with respect to the given product and the given norm t, s. Then, we consider the limits of the defined energies, which are essentially non-negative numerical values. The paper demonstrates through examples how the introduced method can be successfully applied to many problems containing uncertainties.

Keywords: neutrosophic soft set; upper energy; lower energy; decision-making

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1. **Introduction**

In 1978, I. Gutman [1] defined The energy of the graph G is computed as the total of the absolute values of the eigenvalues of the adjacency matrix associated with G. Namely, given the eigenvalues $\theta_1 \geq \theta_2 \geq \ldots \theta_m \geq 0$ of the adjacency matrix of a graph G, the energy of G;

$$E(G) = \sum_{i=1}^{m} |\theta_i|$$

denoted by $E(G)$.

For studies on energy, the following can be mentioned based on the Survey of Graph Energies [2] article by Ivan Gutman and Boris Furtula [3] About 20 years later, energy has become a very favorite field of study. The idea of defining energy in this way and calling it energy comes from quantum chemistry. The other reason is that the right-hand side of the equation is independent of the labeling of the eigenvalues. So there is no need for $\theta_1 \geq \theta_2 \geq \ldots \theta_m \geq 0$. Since graph energy is a symmetric function of the eigenvalues of the graph, it is among the most frequently analyzed functions. The study of the energies of graphs is very active today. More than a hundred energy variants have been proposed on various graphs based on matrices other than the adjacency matrix. There are more than 1000 publications on this topic and these publications have found various fields of application. Beyond graph energy: Norms of graphs and matrices [4], energy of matrices [5], energy of pythagorean fuzzy graphs with applications [6], on incidence energy of graphs [7], energy of some graphs [8–11]. In these studies, lower and upper bounds for energies are found and various applications are given.

Zadeh [12] introduced fuzzy set theory in 1965 to tackle difficulties associated with uncertainty. Since then, fuzzy sets and fuzzy logic have been thoroughly investigated by researchers aiming to address a wide range of real-world problems with ambiguity and uncertainty. Atanassov (1986) [13] extended this framework by introducing intuitionistic fuzzy sets. Subsequently, Smarandache [14] developed the theory of neutrosophic sets, which provides a framework for dealing with issues involving imprecise, indeterminate, and inconsistent data. Neutrosophic sets generalize classical and fuzzy sets and are applied in diverse areas such as decision-making, control theory, medicine, topology, and other practical scenarios. The concept of soft set theory was introduced by Molodtsov [15] in 1999 as a new mathematical approach for managing uncertainties. His soft sets offer a novel method for addressing uncertainty through parameter analysis. Maji et al. [16, 17] later expanded this theory to include fuzzy soft sets and neutrosophic soft sets. Recently, research in neutrosophic theory has gained momentum, with studies focusing on the extension of neutrosophic sets [18–21], neutrosophic algebra [22], decision-making processes [23–27], and neutrosophic environments [28–31].
Studies on energies are not limited to graphs, and energy have also been defined on fuzzy sets. [32] can be given as an example.
Previous literature has not proposed an energy for neutrosophic soft sets by integrating their lower and upper energies. Additionally, there has been no discussion regarding energy in neutrosophic soft sets. The aim of the current research is to propose a new method for a new approach energy using a neutrosophic soft set by in multi criteria decision-making problems. However, when information is endowed with correct, uncertain, and incorrect data, neutrosophic soft sets become a suitable representation for decision-making. The energy defined through this integration is used to solve problems characterized by correct, uncertain, and incorrect data.

The remainder of the paper is organized as follow: In section 2, we gave a brief introduction of key definitions and propositions. In section 3, we introduced a new method for a new approach energy using a neutrosophic soft set and also we discussed their desirable properties.

In section 4, we develop the algorithm for MCDM. In section 5, an illustrative example is developed to describe the effectiveness of the developed method. In section 6, we gave a comparison with some existing methods. Finally, a conclusion is made in Section 7.

2. Preliminaries

Definition 2.1. [33] An eigenvector of a $n \times n$ matrix $\mathcal{N}$ is nonzero vector x such that $\mathcal{N}x = cx$ for some scalar c. A scalar c is called an eigenvalue of $\mathcal{N}$ if there is nontrivial solution x of $\mathcal{N}x = cx$, such an x is called an eigenvector corresponding to c.

Definition 2.2. [14] A neutrosophic sets $\mathcal{K}$ on the universe of discourse $\mathcal{E}$ is defined as: $\mathcal{K} = \{\langle x, (\alpha_{\mathcal{K}}(x), \beta_{\mathcal{K}}(x), \gamma_{\mathcal{K}}(x))\rangle : x \in \mathcal{E}, \alpha_{\mathcal{K}}(x), \beta_{\mathcal{K}}(x), \gamma_{\mathcal{K}}(x) \in [0,1]\}$. There is no restriction on the sum of $\alpha_{\mathcal{K}}(x)$, $\beta_{\mathcal{K}}(x)$ and $\gamma_{\mathcal{K}}(x)$, so $^{-}0 \leq \alpha_{\mathcal{K}}(x) + \beta_{\mathcal{K}}(x) + \gamma_{\mathcal{K}}(x) \leq 3^{+}$.

Definition 2.3. [36] Let $\mathcal{U}$ be a universe set and $\mathcal{E}$ be a set of parameters. Consider $\mathcal{A} \subseteq \mathcal{E}$. Let $\mathcal{N}\int\mathcal{U}$ denotes the set of all neutrosophic sets of $\mathcal{U}$. The collection $(\Phi, \mathcal{A})$ is termed to be the neutrosophic soft set over $\mathcal{U}$, where Φ is a mapping given by $\Phi : \mathcal{A} \to \mathcal{N}\int\mathcal{U}$.

Definition 2.4. [37] Let $\mathcal{N} = \{x, \{< u, \alpha_{\phi_{\mathcal{N}(x)}}(u), \beta_{\phi_{\mathcal{N}(x)}}(u), \gamma_{\phi_{\mathcal{N}}}(u) >: x \in U\} : x \in E\}$ be a neutrosophic soft set. Then the complement of a neutrosophic soft set $\mathcal{N}$ denoted by $\mathcal{N}^{\mathcal{C}}$ and is defined by

$$\mathcal{N}^{\mathcal{C}} = \{(x, \{< u, \gamma_{\phi_{\mathcal{N}(x)}}(u), 1 - \beta_{\phi_{\mathcal{N}(x)}}(u), \alpha_{\phi_{\mathcal{N}(x)}}(u) >: x \in \mathcal{U}\} : x \in \mathcal{E}\}$$

Definition 2.5. [37] Let $\mathcal{N}$ be a neutrosophic soft set over $\mathcal{N}(U)$. Then a subset of $\mathcal{N}(U) \times \mathcal{E}$ is uniquely defined by

$$\mathcal{R}_{\mathcal{N}} = \{(\phi_{\mathcal{N}(x)}, x) : x \in \mathcal{E}, \phi_{\mathcal{N}(x)} \in \mathcal{N}(\mathcal{U})\}$$

which is called a relation form of $(\mathcal{N}, \mathcal{E})$. The characteristic function of $\mathcal{R}_{\mathcal{N}}$ is written by

$$\theta_{\mathcal{R}_{\mathcal{N}}} : \mathcal{N}(\mathcal{U}) \times \mathcal{E} \to [0,1] \times [0,1] \times [0,1], \quad \theta_{\mathcal{R}_{\mathcal{N}}}(u, x) = (\alpha_{\phi_{\mathcal{N}(x)}}(u), \beta_{\phi_{\mathcal{N}(x)}}(u), \gamma_{\phi_{\mathcal{N}(x)}}(u))$$

where $\alpha_{\phi_{\mathcal{N}(x)}}(u)$, $\beta_{\phi_{\mathcal{N}(x)}}(u)$, $\gamma_{\phi_{\mathcal{N}(x)}}(u)$ is the truth-membership, indeterminacy-membership and falsity-membership of $u \in \mathcal{U}$, respectively.

Definition 2.6. [37] The transpose of square neutrosophic soft matrix $[\omega_{ij}]$ of order $m \times n$ is another square neutrosophic soft matrix of order $n \times m$ obtained from $[\omega_{ij}]$ by interchanging its rows and columns. It is denoted by $[\omega_{ij}^{T}]$. Therefore the neutrosophic soft set associated with $[\omega_{ij}^{T}]$ becomes a new neutrosophic soft set over the same universe and over the same set of parameters.

Definition 2.7. [32] The energy of a fuzzy soft set $\Gamma_{\mathcal{A}}$, denoted as $\xi(\Gamma_{\mathcal{A}})$, is defined as $\xi(\Gamma_{\mathcal{A}}) = \sum_{i=1}^{m} \theta_i$ where $\theta_1 \geq \theta_2 \geq \ldots \theta_m \geq 0$ are the singular values of the matrix $\mathcal{A}$ corresponding to the fuzzy soft set $\Gamma_{\mathcal{A}}$.

3. Energy of A Neutrosophic Soft Set

Definition 3.1. [37] Let $\mathcal{U} = \{u_1, \ldots, u_m\}$, $\mathcal{E} = \{x_1, \ldots, x_n\}$ and $\mathcal{N}$ be a neutrosophic soft set over $\mathcal{N}(\mathcal{U})$. Then

$\mathcal{R}_{\mathcal{N}}$	$\phi_{\mathcal{N}}(x_1)$	$\phi_{\mathcal{N}}(x_2)$	$\ldots$	$\phi_{\mathcal{N}}(x_n)$
u_1	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_1, x_1)$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_1, x_2)$	$\ldots$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_1, x_n)$
u_2	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_2, x_1)$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_2, x_2)$	$\ldots$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_m, x_1)$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_m, x_2)$	$\ldots$	$\theta_{\mathcal{R}_{\mathcal{N}}}(u_m, x_m)$

If $\omega_{ij} = \theta_{\mathcal{R}_{\mathcal{N}}}(u_i, x_j)$, we can define a matrix:

$$[\omega_{ij}] = \begin{bmatrix} \omega_{11} & \omega_{12} & \ldots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \ldots & \omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{1m} & \omega_{m2} & \ldots & \omega_{mn} \end{bmatrix}$$

such that $\omega_{ij} = (\alpha_{\phi_{\mathcal{N}}(x_j)}(u_i), \beta_{\phi_{\mathcal{N}}(x_j)}(u_i), \gamma_{\phi_{\mathcal{N}}(x_j)}(u_i)) = (\alpha^a_{ij}, \beta^a_{ij}, \gamma^a_{ij})$ which is called a $m \times n$ *neutrosophic soft matrix* (or namely NS-matrix) of the neutrosophic soft set $\mathcal{N}$ over $\mathcal{N}(\mathcal{U})$.

Example 3.2. Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $\mathcal{E} = \{x_1, x_2, x_3\}$. $\mathcal{N}$ be a neutrosophic soft set over neutrosophic as:

$$\mathcal{N} = \Big\{ (x_1, \{< u_1, (0.3, 0.5, 0.8) >, < u_2, (0.7, 0.4, 0.2) >, < u_3, (0.3, 0.6, 0.1) >\}),$$
$$(x_2, \{< u_1, (0.4, 0.3, 0.9) >, < u_2, (0.8, 0.5, 0.2) >, < u_3, (0.5, 0.2, 0.4) >\}),$$
$$(x_3, \{< u_1, (0.9, 0.4, 0.2) >, < u_2, (0.8, 0.1, 0.5) >, < u_3, (0.4, 0.6, 0.3) >\}) \Big\}$$

Then, the NS-matrix $[\omega_{ij}]$ is written by

$$[\omega_{ij}] = \begin{bmatrix} (0.3, 0.5, 0.8) & (0.7, 0.4, 0.2) & (0.3, 0.6, 0.1) \\ (0.4, 0.3, 0.9) & (0.8, 0.5, 0.2) & (0.5, 0.2, 0.4) \\ (0.9, 0.4, 0.2) & (0.8, 0.1, 0.5) & (0.4, 0.6, 0.3) \end{bmatrix}$$

Definition 3.3. Let $[\omega_{ij}] \in \mathcal{N}_{m\times n}$, $[\vartheta_{jq}] \in \mathcal{N}_{n\times k}$. Then AND-product of $[\omega_{ij}]$ and $[\vartheta_{jq}]$ is defined by

$$\wedge : \mathcal{N}_{m\times n} \times \mathcal{N}_{n\times k} \to \mathcal{N}_{m\times k} \qquad [\omega_{ij}] \wedge [\vartheta_{jq}] = [\eta_{iq}] = (\alpha'_{iq}, \beta'_{iq}, \gamma'_{iq})$$

where

$$\alpha'_{iq} = max(t(\alpha^{\omega}_{i1}, \alpha^{\vartheta}_{1q}), t(\alpha^{\omega}_{i2}, \alpha^{\vartheta}_{2q}), \ldots, t(\alpha^{\omega}_{in}, \alpha^{\vartheta}_{nq})),$$
$$\beta'_{iq} = min(s(\beta^{\omega}_{i1}, \beta^{\vartheta}_{1q}), s(\beta^{\omega}_{i2}, \beta^{\vartheta}_{2q}), \ldots, s(\beta^{\omega}_{in}, \beta^{\vartheta}_{nq})) \text{ and}$$
$$\gamma'_{iq} = min(s(\gamma^{\omega}_{i1}, \gamma^{\vartheta}_{1q}), s(\gamma^{\omega}_{i2}, \gamma^{\vartheta}_{2q}), \ldots, s(\gamma^{\omega}_{in}, \gamma^{\vartheta}_{nq}))$$

such that $1 \leq i \leq m$ and $1 \leq q \leq n$.

Example 3.4. Consider Example 3.2:

$$[\omega_{ij}] = \begin{bmatrix} (0.3, 0.5, 0.8) & (0.7, 0.4, 0.2) & (0.3, 0.6, 0.1) \\ (0.4, 0.3, 0.9) & (0.8, 0.5, 0.2) & (0.5, 0.2, 0.4) \\ (0.9, 0.4, 0.2) & (0.8, 0.1, 0.5) & (0.4, 0.6, 0.3) \end{bmatrix}$$

$$[\omega^T_{ij} = \vartheta_{jq}] = \begin{bmatrix} (0.3, 0.5, 0.8) & (0.4, 0.3, 0.9) & (0.9, 0.4, 0.2) \\ (0.7, 0.4, 0.2) & (0.8, 0.5, 0.2) & (0.8, 0.1, 0.5) \\ (0.3, 0.6, 0.1) & (0.5, 0.2, 0.4) & (0.4, 0.6, 0.3) \end{bmatrix}$$

and for t= einstein product and s= einstein sum;

$$[\omega_{ij}] \wedge [\vartheta_{jq}] = [\Lambda] = \begin{bmatrix} (0.449, 0.689, 0.198) & (0.528, 0.695, 0.384) & (0.528, 0.480, 0.388) \\ (0.528, 0.695, 0.384) & (0.615, 0.384, 0.384) & (0.615, 0.471, 0.201) \\ (0.528, 0.480, 0.388) & (0.615, 0.571, 0.625) & (0.801, 0.198, 0.384) \end{bmatrix}$$

Definition 3.5. Let $[\omega_{ij}] \in \mathcal{N}_{m\times n}$, $[\vartheta_{jq}] \in \mathcal{N}_{n\times k}$. Then OR-product of $[\omega_{ij}]$ and $[\vartheta_{jq}]$ is defined by

$$\vee : \mathcal{N}_{m\times n} \times \mathcal{N}_{n\times k} \to \mathcal{N}_{m\times k} \qquad [\omega_{ij}] \vee [\vartheta_{jq}] = [\eta_{iq}] = (\alpha^c_{iq}, \beta^c_{iq}, \gamma^c_{iq})$$

where

$$\alpha'_{iq} = min(s(\alpha^{\omega}_{i1}, \alpha^{\vartheta}_{1q}), s(\alpha^{\omega}_{i2}, \alpha^{\vartheta}_{2q}), \ldots, s(\alpha^{\omega}_{in}, \alpha^{\vartheta}_{nq})),$$
$$\beta'_{iq} = max(t(\beta^{\omega}_{i1}, \beta^{\vartheta}_{1q}), t(\beta^{\omega}_{i2}, \beta^{\vartheta}_{2q}), \ldots, t(\beta^{\omega}_{in}, \beta^{b}_{nq})) \text{ and}$$
$$\gamma'_{iq} = max(t(\gamma^{\omega}_{i1}, \gamma^{\vartheta}_{1q}), t(\gamma^{\omega}_{i2}, \gamma^{\vartheta}_{2q}), \ldots, t(\gamma^{\omega}_{in}, \gamma^{\vartheta}_{nq}))$$

such that $1 \leq i \leq m$ and $1 \leq q \leq n$.

Example 3.6. Again consider Example 3.2:

$$[\omega_{ij}] = \begin{bmatrix} (0.3, 0.5, 0.8) & (0.7, 0.4, 0.2) & (0.3, 0.6, 0.1) \\ (0.4, 0.3, 0.9) & (0.8, 0.5, 0.2) & (0.5, 0.2, 0.4) \\ (0.9, 0.4, 0.2) & (0.8, 0.1, 0.5) & (0.4, 0.6, 0.3) \end{bmatrix}$$

$$[\omega^T_{ij} = \vartheta_{jq}] = \begin{bmatrix} (0.3, 0.5, 0.8) & (0.4, 0.3, 0.9) & (0.9, 0.4, 0.2) \\ (0.7, 0.4, 0.2) & (0.8, 0.5, 0.2) & (0.8, 0.1, 0.5) \\ (0.3, 0.6, 0.1) & (0.5, 0.2, 0.4) & (0.4, 0.6, 0.3) \end{bmatrix}$$

and for t= einstein product and s= einstein sum;

$$[\omega_{ij}] \vee [\vartheta_{jq}] = [\nu] = \begin{bmatrix} (0.550, 0.310, 0.615) & (0.625, 0.153, 0.705) & (0.625, 0.310, 0.137) \\ (0.625, 0.153, 0.705) & (0.689, 0.201, 0.801) & (0.750, 0.090, 0.166) \\ (0.625, 0.310, 0.137) & (0.750, 0.0909, 0.166) & (0.689, 0.310, 0.201) \end{bmatrix}$$

Proposition 3.7. *Let* $[\omega_{ij}] \in \mathcal{N}_{m\times n}$, $[\vartheta_{jq}] \in \mathcal{N}_{n\times k}$. *Then,*

(1) $([\omega_{ij}] \vee [\vartheta_{jq}])^c = [\omega_{ij}]^c \wedge [\vartheta_{iq}]^c$
(2) $([\omega_{ij}] \wedge [\vartheta_{iq}])^c = [\omega_{ij}]^c \vee [\vartheta_{iq}]^c$

Definition 3.8. Let $\mathcal{N} = \{(x, \{< u, \alpha_{\phi_{\mathcal{N}(x)}}(u), \beta_{\phi_{\mathcal{N}(x)}}(u), \gamma_{\phi N_{(x)}}(u) >: x \in U\} : x \in \mathcal{E}\}$ be a neutrosophic soft set. Then *score function* of $\mathcal{N}$, is denoted by $S^{\mathcal{N}}_{energy}$, is defined as;

$$S^{\mathcal{N}}_{energy} = \frac{\alpha_{\phi_{\mathcal{N}(x)}}(u) - \beta_{\phi_{\mathcal{N}(x)}}(u) - \gamma_{\phi_{\mathcal{N}(x)}}(u)}{\alpha_{\phi_{\mathcal{N}(x)}}(u) + \gamma_{\phi_{\mathcal{N}(x)}}(u)}$$

Example 3.9. Score function of Example 3.4:

$$S^{\Lambda}_{energy} = \begin{bmatrix} -0.676 & -0.604 & -0.371 \\ -0.604 & -0.153 & -0.069 \\ -0.371 & -0.468 & 0.184 \end{bmatrix}$$

and score function of Example 3.6:

$$S^{\nu}_{energy} = \begin{bmatrix} -0.321 & -0.176 & 0.231 \\ -0.176 & -0.209 & 0.537 \\ 0.231 & 0.537 & 0.201 \end{bmatrix}$$

Definition 3.10. The *upper energy* of a neutrosophic soft set $\mathcal{N}(\mathcal{U})$, denoted as $\xi^+(\mathcal{N})$ is defined as $\xi^+(\mathcal{N}) = \sum_{i=1}^{m} |_{\Delta^+}\theta_i^+|$ where $_{\Delta^+}\theta_i$ are eigenvalues of the matrix Δ^+ corresponding to the matrix obtained by $\wedge$-product the neutrosophic matrix and its transpose with the score function.

Example 3.11. To determine upper energy of Example 3.2 from Example 3.9 eigenvalues of S^{Λ}_{energy};

$$_{\Lambda^+}\theta_1^+ = 1 \quad _{\Lambda^+}\theta_2^+ = 0.2 \quad _{\Lambda^+}\theta_3^+ = 0.3$$

and sum of eigenvalues ;

$$\xi^+(\mathcal{N}) = \sum_{i=1}^{3} {_{\Lambda^+}\theta_i^+} = {_{\Lambda^+}\theta_1^+} + {_{\Lambda^+}\theta_2^+} + {_{\Lambda^+}\theta_3^+} = 1 + 0.2 + 0.3 = 1.5$$

therefore upper energy of Example 3.2 is 1.5.

Definition 3.12. The *lower energy* of a neutrosophic soft set $\mathcal{N}(\mathcal{U})$, denoted as $\xi^-(\mathcal{N})$ is defined as $\xi^-(\mathcal{N}) = \sum_{i=1}^{m} |_{\Delta^-}\theta_i^-|$ where θ_i are eigenvalues of the matrix Δ^- corresponding to the matrix obtained by $\vee$-product the neutrosophic matrix and its transpose with the score function.

Example 3.13. To determine lower energy of Example 3.2 from Example 3.9 eigenvalues of S^{ν}_{energy};

$$_{\nu^-}\theta_1^- = 0.8 \quad _{\nu^-}\theta_2^- = 0.3 \quad _{\nu^-}\theta_3^- = 0.1$$

and sum of eigenvalues ;

$$\xi^-(\mathcal{N}) = \sum_{i=1}^{3} {}_{\nu}\text{-}\theta_i^- = {}_{\nu}\text{-}\theta_1^- + {}_{\nu}\text{-}\theta_2^- + {}_{\nu}\text{-}\theta_3^- = 0.8 + 0.3 + 0.1 = 1.2$$

therefore lower energy of Example 3.2 is 1.2.

Definition 3.14. The energy of a neutrosophic soft set $\mathcal{N}(\mathcal{U})$, denoted as $\xi(\mathcal{N})$ is defined as $\xi(\mathcal{N}) = \xi^+(\mathcal{N}) - \xi^-(\mathcal{N})$.

Example 3.15. From Example 3.11 and Example 3.13 energy of given matrix in Example 3.2 is:

$$\xi(\mathcal{N}) = \xi^+(\mathcal{N}) - \xi^-(\mathcal{N}) = 1.5 - 1.2 = 0.3$$

4. Algorithm of Proposed Decision-Making

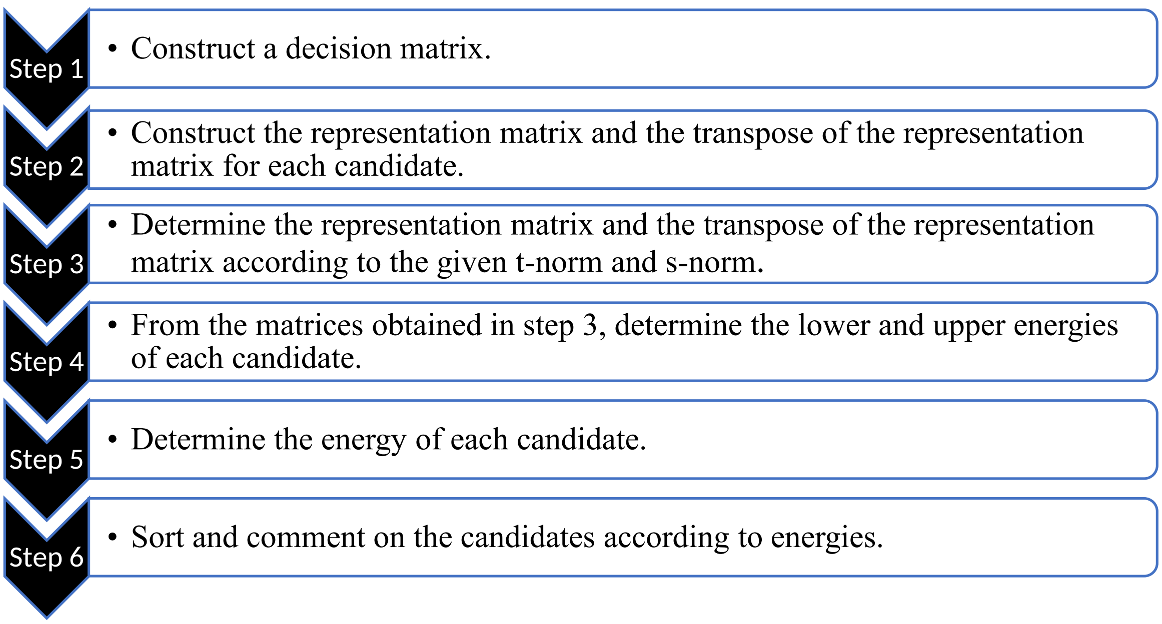

FIGURE 1. Flowchart of Proposed Decision-Making

Algorithm

Step 1: Construct a decision matrix.

Step 2: Construct the representation matrix and the transpose of the representation matrix for each candidate. For construct representation matrix of candidate 1 remove first column and row. For construct representation matrix of candidate 2 remove second column and row, and so on.

Step 3 Determine the representation matrix and the transpose of the representation matrix according to the given t-norm and s-norm.

Step 4 From the matrices obtained in step 3, determine the lower and upper energies of each candidate.

Step 5 Determine the energy of each candidate.

Step 6 Sort and comment on the candidates according to energies.

5. **Application of Proposed Decision-Making**

Example 5.1. A company plans to invest in a new artificial intelligence (AI) to enhance its existing products. Let $\mathcal{U} = \{u_1, u_2, u_3\}$ be the candidate set for the artificial intelligence program. The set of parameters to be considered is $\mathcal{E} = \{x_1, x_2, x_3\}$. For $i = 1, 2, 3$ the parameters x_i represent, in order, security, "budget, and capacity which is represented in the form of single-valued neutrosophic soft set; (In the following example, we use Einstein product.)

$$\mathcal{N} = \Big\{(x_1, \{< u_1, (0.3, 0.4, 0.2) >, < u_2, (0.7, 0.3, 0.2) >, < u_3, (0.2, 0.1, 1.0) >\}),$$
$$(x_2, \{< u_1, (0.1, 0.6, 0.5) >, < u_2, (0.9, 0.2, 0.5) >, < u_3, (0.2, 0.2, 1.0) >\}),$$
$$(x_3, \{< u_1, (0.1, 0.8, 0.4) >, < u_2, (0.5, 0.4, 0.2) >, < u_3, (1.0, 0.2, 0.1) >\})\Big\}$$

and the representation of neutrosophic matrix

$$[\Omega_{ij}] = \begin{bmatrix} (0.3, 0.4, 0.2) & (0.1, 0.6, 0.5) & (0.1, 0.8, 0.4) \\ (0.7, 0.3, 0.2) & (0.9, 0.2, 0.5) & (0.5, 0.4, 0.2) \\ (0.2, 0.1, 1.0) & (0.2, 0.2, 1.0) & (1.0, 0.2, 0.1) \end{bmatrix}$$

for candidate 1

$$\chi_1 = \begin{bmatrix} (0.9, 0.2, 0.5) & (0.5, 0.4, 0.2) \\ (0.2, 0.2, 1.0) & (1.0, 0.2, 0.1) \end{bmatrix}$$

and

$$\chi_1^T = \begin{bmatrix} (0.9, 0.2, 0.5) & (0.2, 0.2, 1.0) \\ (0.5, 0.4, 0.2) & (1.0, 0.2, 0.1) \end{bmatrix}$$

$\wedge$-product of this matrices

$$\chi_1 \wedge \chi_1^T = \chi_1^+ = \begin{bmatrix} (0.801, 0.384, 0.384) & (0.501, 0.384, 0.294) \\ (0.501, 0.384, 0.294) & (1.0, 0.384, 0.198) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_1^+} = \begin{bmatrix} 0.027 & -0.225 \\ -0.225 & 0.348 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$${}_{\chi_1^+}\theta_1^+ = 0.4 \quad {}_{\chi_1^+}\theta_2^+ = 0.1$$

so upper energy for candidate 1

$$\xi^{+}(\chi_1) = {}_{\chi_1^+}\theta_1^+ + {}_{\chi_1^+}\theta_2^+ = 0.4 + 0.1 = 0.5$$

$\vee$-product of this matrices

$$\chi_1 \vee \chi_1^T = \chi_1^- = \begin{bmatrix} (0.8, 0.117, 0.2) & (0.932, 0.054, 0.5) \\ (0.932, 0.054, 0.5) & (0.384, 0.024, 1.0) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_1^-} = \begin{bmatrix} 0.482 & 0.264 \\ 0.264 & -0.462 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$${}_{\chi_1^-}\theta_1^- = 0.7 \quad {}_{\chi_1^-}\theta_2^- = 0.7$$

so lower energy for candidate 1

$$\xi^{-}(\chi_1) = {}_{\chi_1^-}\theta_1^- + {}_{\chi_1^-}\theta_2^- = 0.7 + 0.7 = 1.4$$

hence the energy of candidate 1:

$$\xi(\chi_1) = \xi^{+}(\chi_1) - \xi^{-}(\chi_1) = 0.5 - 1.4 = -0.9$$

and now for candidate 2

$$\chi_2 = \begin{bmatrix} (0.3, 0.4, 0.2) & (0.1, 0.8, 0.4) \\ (0.2, 0.1, 1.0) & (1.0, 0.2, 0.1) \end{bmatrix}$$

and

$$\chi_2^T = \begin{bmatrix} (0.3, 0.4, 0.2) & (0.2, 0.1, 1.0) \\ (0.1, 0.8, 0.4) & (1.0, 0.2, 0.1) \end{bmatrix}$$

$\wedge$-product of this matrices

$$\chi_2 \wedge \chi_2^T = \chi_2^+ = \begin{bmatrix} (0.060, 0.689, 0.384) & (0.1, 0.480, 0.480) \\ (0.1, 0.480, 0.480) & (1.0, 0.198, 0.198) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_2^+} = \begin{bmatrix} -2.278 & -1.483 \\ -1.483 & 0.504 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$${}_{\chi_2^+}\theta_1^+ = 0.7 \quad {}_{\chi_2^+}\theta_2^+ = 0.1$$

so upper energy for candidate 2

$$\xi^{+}(\chi_2) = {}_{\chi_2^+}\theta_1^+ + {}_{\chi_2^+}\theta_2^+ = 0.7 + 0.1 = 0.8$$

$\vee$-product of this matrices

$$\chi_2 \vee \chi_2^T = \chi_2^- = \begin{bmatrix} (0.198, 0.615, 0.117) & (0.471, 0.137, 0.2) \\ (0.471, 0.137, 0.2) & (0.384, 0.024, 1.0) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_2^-} = \begin{bmatrix} -1.694 & 0.199 \\ 0.199 & -0.462 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$${}_{\chi_2^-}\theta_1^- = 1.0 \quad {}_{\chi_2^-}\theta_2^- = 0.3$$

so lower energy for candidate 2

$$\xi^-(\chi_2) = {}_{\chi_2^-}\theta_1^- + {}_{\chi_2^-}\theta_2^- = 1.0 + 0.3 = 1.3$$

hence the energy of candidate 2 :

$$\xi(\chi_2) = \xi^+(\chi_2) - \xi^-(\chi_2) = 0.8 - 1.3 = -0.5$$

and now for candidate 3 :

$$\chi_3 = \begin{bmatrix} (0.3, 0.4, 0.2) & (0.1, 0.6, 0.5) \\ (0.7, 0.3, 0.2) & (0.9, 0.2, 0.5) \end{bmatrix}$$

and

$$\chi_3^T = \begin{bmatrix} (0.3, 0.4, 0.2) & (0.7, 0.3, 0.2) \\ (0.1, 0.6, 0.5) & (0.9, 0.2, 0.5) \end{bmatrix}$$

$\wedge$-product of this matrices

$$\chi_3 \wedge \chi_3^T = \chi_3^+ = \begin{bmatrix} (0.060, 0.689, 0.384) & (0.173, 0.625, 0.384) \\ (0.173, 0.625, 0.384) & (0.890, 0.384, 0.384) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_3^+} = \begin{bmatrix} -2.278 & -1.497 \\ -1.497 & 0.094 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$${}_{\chi_3^+}\theta_1^+ = 1.7 \quad {}_{\chi_3^+}\theta_2^+ = 0.3$$

so upper energy for candidate 3

$$\xi^+(\chi_3) = {}_{\chi_3^+}\theta_1^+ + {}_{\chi_3^+}\theta_2^+ = 1.7 + 0.3 = 2.0$$

$\vee$-product of this matrices

$$\chi_3 \vee \chi_3^T = \chi_3^- = \begin{bmatrix} (0.198, 0.310, 0.2) & (0.864, 0.090, 0.2) \\ (0.826, 0.090, 0.2) & (0.939, 0.060, 0.2) \end{bmatrix}$$

score function of this matrices

$$S_{energy}^{\chi_3^-} = \begin{bmatrix} -0.784 & 0.521 \\ 0.521 & 0.595 \end{bmatrix}$$

therefore eigenvalues of this matrix;

$$_{\chi_3^-}\theta_1^- = 0.7 \quad _{\chi_3^-}\theta_2^- = 0.1$$

so lower energy for candidate 3

$$\xi^-(\chi_3) = {}_{\chi_3^-}\theta_1^- + {}_{\chi_3^-}\theta_2^- = 0.7 + 0.1 = 0.8$$

hence the energy of candidate 3:

$$\xi(\chi_3) = \xi^+(\chi_3) - \xi^-(\chi_3) = 2.0 - 0.8 = 1.2$$

and so

$$u_3 > u_1 > u_2$$

6. Comparison and Analysis Discussion

Nearly all new properties or characteristics of neutrosophic soft sets have spurred further research aimed at establishing comparisons, exploring additional generalizations, proving new properties, or developing applications related to those properties. Alongside the many applications of neutrosophic soft set theory, this paper introduces additional parameters that characterize the nature of neutrosophic soft sets. This contribution aims to advance the application of neutrosophic soft set theory, similar to how graph energy functions in graph theory. The above defined energy of a neutrosophic soft set are the only tool to solve this kind of information and help decision analyst to make a decision. To demonstrate the effectiveness of the proposed method, a comparison of the decision-making results of the MCDM methods based on energy of existing presented by Mudric et al. [32] is carried out, as shown in Table 1.

Table 1. The results from the different operators

Methods	Ranking of alternatives	The optimal alternative
The method in [32]	$U_3 > U_4 > U_2 > U_1$,	U_3
Proposed Method	$U_4 > U_3 > U_2 > U_1$,	U_4

7. Conclusion

Neutrosophic Soft Sets play a very important role in the decision-making process. In addition to numerous applications of the theory of neutrosophic soft sets, this paper defines additional parameters characterizing the nature of neutrosophic soft sets, contributing to further advancements in the application of the theory of neutrosophic soft sets, similar to the role of graph energy in graph theory. A neutrosophic soft energy model provides more precision, flexibility, and compatibility to the system as compared to the classical and fuzzy model. Therefore, developed method have been applied to many real-life decision-making problems. The research significance and value of this paper are highlighted below:

(1) The upper and the lower energies based on neutrosophic soft sets were defined.
(2) In this article presented a new MCDM method for upper and lower energies based on neutrosophic soft sets.
(3) And then, developed method have been applied to a real-life decision-making problems.

Further a numerical example is developed to illustrate the flexibility and applicability of the proposed method. In further work, it is necessary and meaningful to extend the energy of a neutrosophic soft set (1) neutrosophic soft triplet, (2) Interval-valued neutrosophic soft set, and (3) Quadruple neutrosophic theory, and (4) extension of neutrosophic numbers.

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