Generalized Neutrosophic Exponential map

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Abstract: The concept of $\mathfrak{g}\aleph$ compact open topology is introduced. Some characterization of this topology are discussed.

Keywords: $\mathfrak{g}\aleph$ locally Compact Hausdorff space; $\mathfrak{g}\aleph$ product topology; $\mathfrak{g}\aleph$ compact open topology; $\mathfrak{g}\aleph$ homeomorphism; $\mathfrak{g}\aleph$ evaluation map; $\mathfrak{g}\aleph$ Exponential map.

1 Introduction

Ever since the introduction of fuzzy sets by Zadeh [12] and fuzzy topological space by Chang [5], several authors have tried successfully to generalize numerous pivot concepts of general topology to the fuzzy setting. The concept of intuitionistic fuzzy set was introduced are studied by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [[2],[3],[4]]. The concepts of generalized intuitionistic fuzzy closed set was introduced by Dhavaseelan et al[6]. The concepts of Intuitionistic Fuzzy Exponential Map Via Generalized Open Set by Dhavaseelan et al[8]. After the introduction of the neutrosophic set concept [[10], [11]]. The concepts of Neutrosophic Set and Neutrosophic Topological Spaces was introduced by A.A.Salama and S.A.Alblowi[9].

In this paper the concept of $\mathfrak{g}\aleph$ compact open topology are introduced. Some interesting properties are discussed. In this paper the concepts of $\mathfrak{g}\aleph$ local compactness and generalized $\aleph-$ product topology are developed. We have Throughout this paper neutrosophic topological spaces (briefly NTS) (S_1, ξ_1),(S_2, ξ_2) and (S_3, ξ_3) will be replaced by S_1,S_2 and S_3, respectively.

2 Preliminiaries

Definition 2.1. [10, 11] Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are $\aleph-$ components.

Definition 2.2. [10, 11] Let S_1 be a non-empty fixed set. A $\aleph-$ set (briefly N-set) Λ is an object such that $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \gamma_{\Lambda}(x)\rangle : x \in S_1\}$ where $\mu_{\Lambda}(x), \sigma_{\Lambda}(x)$ and $\gamma_{\Lambda}(x)$ which represents the degree of membership function (namely $\mu_{\Lambda}(x)$), the degree of indeterminacy (namely $\sigma_{\Lambda}(x)$) and the degree of non-membership (namely $\gamma_{\Lambda}(x)$) respectively of each element $x \in S_1$ to the set Λ.

Remark 2.1. [10, 11]

(1) An N-set $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$ can be identified to an ordered triple $\langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ in $]0^-, 1^+[$ on S_1.

(2) In this paper, we use the symbol $\Lambda = \langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ for the N-set $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition 2.3. [7]Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be defined as
$\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_{\Gamma}(x), \sigma_{\Gamma}(x), \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$. Then

(a) $\Lambda \subseteq \Gamma$ iff $\mu_{\Lambda}(x) \leq \mu_{\Gamma}(x)$, $\sigma_{\Lambda}(x) \leq \sigma_{\Gamma}(x)$ and $\Gamma_{\Lambda}(x) \geq \Gamma_{\Gamma}(x)$ for all $x \in S_1$;

(b) $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;

(c) $\bar{\Lambda} = \{\langle x, \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \mu_{\Lambda}(x)\rangle : x \in S_1\}$; [Complement of Λ]

(d) $\Lambda \cap \Gamma = \{\langle x, \mu_{\Lambda}(x) \wedge \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \wedge \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \vee \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

(e) $\Lambda \cup \Gamma = \{\langle x, \mu_{\Lambda}(x) \vee \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \vee \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \wedge \gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

(f) $[\,]\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), 1 - \mu_{\Lambda}(x)\rangle : x \in S_1\}$;

(g) $\langle\rangle\Lambda = \{\langle x, 1 - \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition 2.4. [7] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N-sets in S_1. Then

(a) $\bigcap \Lambda_i = \{\langle x, \wedge\mu_{\Lambda_i}(x), \wedge\sigma_{\Lambda_i}(x), \vee\Gamma_{\Lambda_i}(x)\rangle : x \in S_1\}$;

(b) $\bigcup \Lambda_i = \{\langle x, \vee\mu_{\Lambda_i}(x), \vee\sigma_{\Lambda_i}(x), \wedge\Gamma_{\Lambda_i}(x)\rangle : x \in S_1\}$.

Since our main purpose is to construct the tools for developing NTS, we must introduce the $\aleph-$ sets 0_N and 1_N in X as follows:

Definition 2.5. [7] $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.6. [7]A $\aleph-$ topology (briefly N-topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 satisfying the following axioms:

(i) $0_N, 1_N \in \xi_1$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in \xi_1$,

(iii) $\cup G_i \in \xi_1$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is called an NTS and each N-set in ξ_1 is called a $\aleph-$ open set (briefly N-open set) . The complement $\overline{\Lambda}$ of an N-open set Λ in S_1 is called a $\aleph-$ closed set (briefly N-closed set) in S_1.

Definition 2.7. [7] Let Λ be an N-set in an $NTS\ S_1$. Then
$Nint(\Lambda) = \bigcup\{G \mid G$ is an N-open set in S_1 and $G \subseteq \Lambda\}$ is called the $\aleph-$ interior (briefly N-interior) of Λ;
$Ncl(\Lambda) = \bigcap\{G \mid G$ is an N-closed set in S_1 and $G \supseteq \Lambda\}$ is called the $\aleph-$ closure (briefly N-cl) of Λ.

Definition 2.8. [7] Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the $\aleph-$ set $x_{r,t,s}$ is called a $\aleph-$ point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$.where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Definition 2.9. [7] Let (S_1, ξ_1) be a NTS. A $\aleph-$ set Λ in (S_1, ξ_1) is said to be a $\mathfrak{g}\aleph$ closed set if $Ncl\,(\Lambda) \subseteq \Gamma$ whenever $\Lambda \subseteq \Gamma$ and Γ is a $\aleph-$ open set. The complement of a $\mathfrak{g}\aleph$ closed set is called a $\mathfrak{g}\aleph$ open set.

Definition 2.10. [7] Let (X, T) be a $\aleph-$ topological space and Λ be a $\aleph-$ set in X. Then the $\aleph-$ generalized closure and $\aleph-$ generalized interior of Λ are defined by,

(i)$NGcl(\Lambda) = \bigcap\{$G: G is a generalized $\aleph-$ closed set in S_1 and $\Lambda \subseteq G\}$.

(ii)$NGint(\Lambda) = \bigcup\{$G: G is a generalized $\aleph-$ open set in S_1 and $\Lambda \supseteq G\}$.

3 Neutrosophic Compact Open Topology

Definition 3.1. Let S_1 and S_2 be any two NTS. A mapping $f : S_1 \to S_2$ is generalized neutrosophic[briefly $\mathfrak{g}\aleph$] continuous iff for every $\mathfrak{g}\aleph$ open set V in S_2, there exists a $\mathfrak{g}\aleph$ open set U in S_1 such that $f(U) \subseteq V$.

Definition 3.2. A mapping $f : S_1 \to S_2$ is said to be $\mathfrak{g}\aleph$ homeomorphism if f is bijective,$\mathfrak{g}\aleph$ continuous and $\mathfrak{g}\aleph$ open.

Definition 3.3. Let S_1 be a NTS. S_1 is said to be $\mathfrak{g}\aleph$ Hausdorff space or T_2 space if for any two $\aleph-$ sets A and B with $A \cap B = 0_\sim$,there exist $\mathfrak{g}\aleph$ open sets U and V, such that $A \subseteq U$,$B \subseteq V$ and $U \cap V = 0_\sim$.

Definition 3.4. A NTS S_1 is said to be $\mathfrak{g}\aleph$ locally compact iff for any $\aleph$ set A, there exists a $\mathfrak{g}\aleph$ open set G, such that $A \subseteq G$ and G is $\mathfrak{g}\aleph$ compact. That is each $\mathfrak{g}\aleph$ open cover of G has a finite subcover.

Remark 3.1. Let S_1 and S_2 be two NTS with S_2 $\aleph-$ compact. Let $x_{r,t,s}$ be any $\aleph-$ point in S_1. The $\aleph-$ product space $S_1 \times S_2$ containing $\{x_{r,t,s}\} \times S_2$. It is cleat that $\{x_{r,t,s}\} \times S_2$ is $\aleph-$ homeomorphic to S_2

Remark 3.2. Let S_1 and S_2 be two NTS with S_2 $\aleph-$ compact. Let $x_{r,t,s}$ be any $\aleph-$ point in S_1. The $\aleph-$ product space $S_1 \times S_2$ containing $\{x_{r,t,s}\} \times S_2$. $\{x_{r,t,s}\} \times S_2$ is $\aleph-$ compact.

Remark 3.3. A $\aleph-$ compact subspace of a $\aleph-$ Hausdorff space is $\aleph-$ closed.

Proposition 3.1. A $\mathfrak{g}\aleph$ Hausdorff topological space S_1,the following conditions are equivalent.

(a) S_1 is $\mathfrak{g}\aleph$ locally compact

(b) for each $\aleph$ set A, there exists a $\mathfrak{g}\aleph$ open set G in S_1 such that $A \subseteq G$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact

Proof. $(a) \Rightarrow (b)$ By hypothesis for each $\aleph-$ set A in S_1, there exists a $\mathfrak{g}\aleph$ open set G, such that $A \subseteq G$ and G is $\mathfrak{g}\aleph$ compact.Since S_1 is $\mathfrak{g}\aleph$ Hausdorff, by Remark 3.3($\mathfrak{g}\aleph$ compact subspace of $\mathfrak{g}\aleph$ Hausdorff space is $\mathfrak{g}\aleph$ closed),G is $\mathfrak{g}\aleph$ closed,thus $G = NGcl(G)$. Hence $A \subseteq G = NGcl(G)$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact.
$(b) \Rightarrow (a)$ Proof is simple.

Proposition 3.2. Let S_1 be a $\mathfrak{g}\aleph$ Hausdorff topological space. Then S_1 is $\mathfrak{g}\aleph$ locally compact on an $\aleph-$ set A in S_1 iff for every $\mathfrak{g}\aleph$ open set G containing A, there exists a $\mathfrak{g}\aleph$ open set V, such that $A \subseteq V$,$NGcl(V)$ is $\mathfrak{g}\aleph$ compact and $NGcl(V) \subseteq G$.

Proof. Suppose that S_1 is $\mathfrak{g}\aleph$ locally compact on an $\aleph-$ set A. By Definition 3.4,there exists a $\mathfrak{g}\aleph$ open set G, such that $A \subseteq G$ and G is $\mathfrak{g}\aleph$ compact. Since S_1 is $\mathfrak{g}\aleph$ Hausdorff space, by Remark 3.3($\mathfrak{g}\aleph$ compact subspace of $\mathfrak{g}\aleph$ Hausdorff space is $\mathfrak{g}\aleph$ closed),G is $\mathfrak{g}\aleph$ closed,thus $G = NGcl(G)$.Consider an $\aleph-$ set $A \subseteq \overline{G}$.Since S_1 is $\mathfrak{g}\aleph$ Hausdorff space, by Definition 3.3, for any two $\aleph-$ sets A and B with $A \cap B = 0_\sim$, there exist a $\mathfrak{g}\aleph$ open sets C and D,such that $A \subseteq C$, $B \subseteq D$ and $C \cap D = 0_\sim$. Let $V = C \cap G$. Hence $V \subseteq G$ implies $NGcl(V) \subseteq NGcl(G) = G$. Since $NGcl(V)$ is $\mathfrak{g}\aleph$ closed and G is $\mathfrak{g}\aleph$ compact,by Remark 3.3(every $\mathfrak{g}\aleph$ closed subset of a $\mathfrak{g}\aleph$ compact space is $\mathfrak{g}\aleph$ compact) it follows that $NGcl(V)$ is $\aleph-$ compact. Thus $A \subseteq NGcl(V) \subseteq G$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact.

The converse follows from Proposition 3.1(b).

Definition 3.5. Let S_1 and S_2 be two NTS. The function $T : S_1 \times S_2 \to S_2 \times S_1$ defined by $T(x, y) = (y, x)$ for each $(x, y) \in S_1 \times S_2$ is called a $\aleph-$ switching map.

Proposition 3.3. The $\aleph-$ switching map $T : S_1 \times S_2 \to S_2 \times S_1$ defined as above is $\mathfrak{g}\aleph$ continuous.

We now introduce the concept of $\mathfrak{g}\aleph$ compact open topology in the set of all $\mathfrak{g}\aleph$ continuous functions from a NTS S_1 to a NTS S_2.

Definition 3.6. Let S_1 and S_2 be two NTS and let $S_2^{S_1} = \{f : S_1 \to S_2$ such that f is $\mathfrak{g}\aleph$ continuous$\}$. We give this class $S_2^{S_1}$ a topology called the $\mathfrak{g}\aleph$ compact open topology as follows:Let $\mathcal{K} = \{K \in I_1^S : K$ is $\mathfrak{g}\aleph$ compact $S_1\}$ and $\mathcal{V} = \{V \in I_1^S : V$ is $\mathfrak{g}\aleph$ open in $S_2\}$. For any $K \in \mathcal{K}$ and $V \in \mathcal{V}$, let $S_{K,V} = \{f \in S_2^{S_1} : f(K) \subseteq V\}$.

The collection of all such $\{S_{K,V} : K \in \mathcal{K}, V \in \mathcal{V}\}$ generates an $\aleph-$ structure on the class $S_2^{S_1}$.

4 Generalized Neutrosophic Evaluation Map and Generalized Neutrosophic Exponential Map

We now consider the $\mathfrak{g}\aleph$ product topological space $S_2^{S_1} \times S_1$ and define a $\mathfrak{g}\aleph$ continuous map from $S_2^{S_1} \times S_1$ into S_2.

Definition 4.1. The mapping $e : S_2^{S_1} \times S_1 \to S_2$ defined by $e(f, A) = f(A)$ for each $\aleph-$ set A in S_1 and $f \in S_2^{S_1}$ is called the $\mathfrak{g}\aleph$ evaluation map.

Definition 4.2. Let S_1,S_2 and S_3 be three NTS and $f : S_3 \times S_1 \to S_2$ be any function. Then the induced map $\widehat{f} : S_1 \to S_2^{S_3}$ is defined by $(\widehat{f}(A_1))(A_2) = f(A_2, A_1)$ for $\aleph-$ sets A_1 of S_1 and A_2 of S_3.

Conversely,given a function $\widehat{f} : S_1 \to S_2^{S_3}$, a corresponding function f can be also be defined be the same rule.

Proposition 4.1. Let S_1 be a $\mathfrak{g}\aleph$ locally compact Hausdorff space. Then the $\mathfrak{g}\aleph$ evaluation map $e : S_2^{S_1} \times S_1 \to S_2$ is $\mathfrak{g}\aleph$ continuous.

Proof. Consider $(f, A_1) \in S_2^{S_1} \times S_1$,where $f \in S_2^{S_1}$ and $\aleph-$ set A_1 of S_1. Let V be a $\mathfrak{g}\aleph$ open set containing $f(A_1) = e(f, A_1)$ in S_2. Since S_1 is $\mathfrak{g}\aleph$ locally compact and f is $\mathfrak{g}\aleph$ continuous,by Proposition 3.2, there exists an $\mathfrak{g}\aleph$ open set U in S_1, such that $A_1 \subseteq NGcl(U)$ and $NGcl(U)$ is $\mathfrak{g}\aleph$ compact and $f(NGcl(U)) \subseteq V$.

Consider the $\mathfrak{g}\aleph$ open set $S_{NGcl(U),V} \times U$ in $S_2^{S_1} \times S_1$.(f, A_1) is such that $f \in S_{NGcl(U),V}$ and $A_1 \subseteq U$. Let (g, A_2) be such that $g \in S_{NGcl(U),V}$ and $A_2 \subseteq U$ be arbitrary, thus $g(NGcl(U)) \subseteq V$. Since $A_2 \subseteq U$,we have $g(A_2) \subseteq V$ and $e(g, A_2) = g(A_2) \subseteq V$.Thus $e(S_{NGcl(U),V} \times U) \subseteq V$. Hence e is $\mathfrak{g}\aleph$ continuous.

Proposition 4.2. Let S_1 and S_2 be two NTS with S_2 is $\mathfrak{g}\aleph$ compact. Let A_1 be any $\aleph-$ set in S_1 and N be a $\mathfrak{g}\aleph$ open set in the $\mathfrak{g}\aleph$ product space $S_1 \times S_2$ containing $\{A_1\} \times S_2$. Then there exists some $\mathfrak{g}\aleph$ open W with $A_1 \subseteq W$ in S_1, such that $\{A_1\} \times S_2 \subseteq W \times S_2 \subseteq N$.

Proof. It is clear that by Remark 3.1, $\{A_1\} \times S_2$ is $\mathfrak{g}\aleph$ homeomorphism to S_2 and hence by Remark 3.2, $\{A_1\} \times S_2$ is $\mathfrak{g}\aleph$ compact. We cover $\{A_1\} \times S_2$ by the basis elements $\{U \times V\}$(for the $\mathfrak{g}\aleph$ product topology) lying in N.Since $\{A_1\} \times S_2$ is $\mathfrak{g}\aleph$ compact,$\{U \times V\}$ has a finite subcover, say a finite number of basis elements $U_1 \times V_1, ..., U_n \times V_n$. Without loss of generality we assume that $\{A_1\} \subseteq U_i$ for each $i = 1, 2, ..., n$.Since otherwise the basis elements would be superfluous.

Let $W = \bigcap_{i=1}^{n} U_i$. Clearly W is $\mathfrak{g}\aleph$ open and $A_1 \subseteq W$. We show that $W \times S_2 \subseteq \bigcup_{i=1}^{n}(U_i \times V_i)$. Let (A_1, B) be an $\aleph-$ set in $W \times S_2$. Now $(A_1, B) \subseteq U_i \times V_i$ for some i, thus $B \subseteq V_i$. But $A_1 \subseteq U_i$ for every $i = 1, 2, ..., n$(because $A_1 \subseteq W$). Therefore, $(A_1, B) \subseteq U_i \times V_i$ as desired. But $U_i \times V_i \subseteq N$ for all $i = 1, 2, ..., n$ and $W \times S_2 \subseteq \bigcup_{i=1}^{n}(U_i \times V_i)$, therefore $W \times S_2 \subseteq N$.

Proposition 4.3. Let S_3 be a $\mathfrak{g}\aleph$ locally compact Hausdorff space and S_1, S_2 be arbitrary NTS. Then a map $f : S_3 \times S_1 \to S_2$ is $\mathfrak{g}\aleph$ continuous iff $\widehat{f} : S_1 \to S_2^{S_3}$ is $\mathfrak{g}\aleph$ continuous,where $\widehat{f}$ is defined by the rule $(\widehat{f}(A_1))(A_2) = f(A_2, A_1)$.

Proof. Suppose that $\widehat{f}$ is $\mathfrak{g}\aleph$ continuous. Consider the functions $S_3 \times S_1 \xrightarrow{i_Z} \times \widehat{f} S_3 \times S_2^{S_3} \xrightarrow{t} S_2^{S_3} \times S_3 \xrightarrow{e} S_2$, where i_Z denote the $\aleph-$ identity function on Z,t denote the $\aleph-$ switching map and e denote the $\mathfrak{g}\aleph$ evaluation map. Since $et(i_Z \times \widehat{f})(A_2, A_1) = et(A_2, \widehat{f}(A_1)) = e(\widehat{f}(A_1), A_2) = (\widehat{f}(A_1))(A_2) = f(A_2, A_1)$ it follows that $f = et(i_Z \times \widehat{f})$ and f being the composition of $\mathfrak{g}\aleph$ continuous functions is itself $\mathfrak{g}\aleph$.

Conversely,suppose that f is $\mathfrak{g}\aleph$ continuous,let A_1 be any arbitrary $\aleph-$ set in S_1. We have $\widehat{f}(A_1) \in S_2^{S_3}$.Consider $S_{K,U} = \{g \in S_2^{S_3} : g(K) \subseteq U, K \in I^{S_3}$ is $\mathfrak{g}\aleph$ compact and $U \in I^{S_2}$ is $\mathfrak{g}\aleph$ open$\}$,containing $\widehat{f}(A_1)$. We need to find a $\mathfrak{g}\aleph$ open W with $A_1 \subseteq W$,such that $\widehat{f}(A_1) \subseteq S_{K,U}$; this will suffice to prove $\widehat{f}$ to be a $\mathfrak{g}\aleph$ continuous map.

For any $\aleph-$ set A_2 in K, we have $(\widehat{f}(A_1))(A_2) = f(A_2, A_1) \in U$ thus $f(K \times \{A_1\}) \subseteq U$, that is $K \times \{A_1\} \subseteq f^{-1}(U)$. Since f is $\mathfrak{g}\aleph$ continuous,$f^{-1}(U)$ is a $\mathfrak{g}\aleph$ open set in $S_3 \times S_1$.Thus $f^{-1}(U)$ is a $\mathfrak{g}\aleph$ open set $S_3 \times S_1$ containing $K \times \{A_1\}$. Hence by Proposition 4.2, there exists a $\mathfrak{g}\aleph$ open W with $A_1 \subseteq W$ in S_1, such that $K \times \{A_1\} \subseteq K \times W \subseteq f^{-1}(U)$. Therefore $f(K \times W) \subseteq U$. Now for any $A_1 \subseteq W$ and $A_2 \subseteq K$,$f(A_2, A_1) = (\widehat{f}(A_1))(A_2) \subseteq U$.Therefore $\widehat{f}(A_1)(K) \subseteq U$ for all $A_1 \subseteq W$. That is $\widehat{f}(A_1) \in S_{K,U}$ for all $A_1 \subseteq W$.Hence $\widehat{f}(W) \subseteq S_{K,U}$ as desired.

Proposition 4.4. Let S_1 and S_3 be two $\mathfrak{g}\aleph$ locally compact Hausdorff spaces. Then for any NTS S_2, the function $E : S_2^{S_3 \times S_1} \to (S_2^{S_3})^{S_1}$ defined by $E(f) = \widehat{f}$(that is $E(f)(A_1)(A_2) = f(A_2, A_1) = (\widehat{f}(A_1))(A_2))$ for all $f : S_3 \times X \to S_2$ is a $\mathfrak{g}\aleph$ homeomorphism.

Proof.

(a) Clearly E is onto.

(b) For E to be injective. Let $E(f) = E(g)$ for $f, g : S_3 \times S_1 \rightarrow S_2$.Thus $\widehat{f} = \widehat{g}$,where $\widehat{f}$ and $\widehat{g}$ are the induced maps of f and g respectively. Now for any $\aleph-$ set A_1 in S_1 and any $\aleph-$ set A_2 in S_3, $f(A_2, A_1) = (\widehat{f}(A_1))(A_2) = (\widehat{g}(A_1))(A_2) = g(A_2, A_1)$;thus $f = g$.

(c) For proving the $\mathfrak{g}\aleph$ continuity of E,consider any $\mathfrak{g}\aleph$ subbasis neighbourhood V of $\widehat{f}$ in $(S_2^{S_3})^{S_1}$, that is V is of the form $S_{K,W}$ where K is a $\mathfrak{g}\aleph$ compact subset of S_1 and W is $\mathfrak{g}\aleph$ open in $S_2^{S_3}$. Without loss of generality we may assume that $W = S_{L,U}$, where L is a $\mathfrak{g}\aleph$ compact subset of S_3 and U is a $\mathfrak{g}\aleph$ open set in S_2.Then $\widehat{f}(K) \subseteq S_{L,U} = W$ and this implies that $\widehat{f}(K)(L) \subseteq U$. Thus for any $\aleph-$ set $A_1 \subseteq K$ and for all $\aleph-$ sets $A_2 \subseteq L$. We have $(\widehat{f}(A_1))(A_2) \subseteq U$, that is $f(A_2, A_1) \subseteq U$ and therefore $f(L \times K) \subseteq U$. Now since L is $\mathfrak{g}\aleph$ compact in S_3 and K is $\mathfrak{g}\aleph$ compact in S_1,$L \times K$ is also $\mathfrak{g}\aleph$ compact in $S_3 \times S_1$[6] and since U is a $\mathfrak{g}\aleph$ open set in S_2, we conclude that $f \in S_{L\times K,U} \subseteq S_2^{S_3 \times S_1}$. We assert that $E(S_{L\times K,U}) \subseteq S_{K,W}$.Let $g \in S_{L\times K,U}$ be arbitrary. Thus $g(L \times K) \subseteq U$, that is $g(A_2, A_1) = (\widehat{g}(A_1))(A_2) \subseteq U$ for all $\aleph-$ sets $A_2 \subseteq L$ in S_3 and for all $\aleph-$ sets $A_1 \subseteq K$ in S_1.So $(\widehat{g}(A_1))(L) \subseteq U$ for all $\aleph-$ sets $A_1 \subseteq K$ in S_1 , that is $\widehat{g}(A_1) \subseteq S_{L,U} = W$ for all $\aleph-$ sets $A_1 \subseteq K$ in U.Hence we have $\widehat{g}(K) \subseteq W$,that is $\widehat{g} = E(g) \in S_{K,W}$ for any $g \in S_{L\times K,U}$. Thus $E(S_{L\times K,U}) \subseteq S_{K,W}$. This proves that E is $\mathfrak{g}\aleph$ continuous.

(d) For proving the $\mathfrak{g}\aleph$ continuity of E^{-1}, we consider the following $\mathfrak{g}\aleph$ evaluation maps: $e_1 : (S_2^{S_3})^{S_1} \times S_1 \rightarrow S_2^{S_3}$ defined by $e_1(\widehat{f}, A_1) = \widehat{f}(A_1)$ where $\widehat{f} \in (S_2^{S_3})^{S_1}$ and A_1 is an $\aleph-$ set in S_1 and $e_2 : S_2^{S_3} \times S_3 \rightarrow S_2$ defined by $e_2(g, A_2) = g(A_2)$ where $g \in S_2^{S_3}$ and A_2 is a $\aleph-$ set in S_3. Let ψ denote the composition of the following $\mathfrak{g}\aleph$ continuous functions $\psi : (S_3 \times S_1) \times (S_2^{S_3})^{S_1} \xrightarrow{T} (S_2^{S_3})^{S_1} \times (S_3 \times S_1) \xrightarrow{i\times t} (S_2^{S_3})^{S_1} \times (S_1 \times S_3) \xrightarrow{=} ((S_2^{S_3})^{S_1} \times S_1) \times S_3 \xrightarrow{e_1\times i_Z} (S_2^{S_3}) \times S_3 \xrightarrow{e_2} S_2$, where i, i_Z denote the $\aleph-$ identity maps on $(S_2^{S_3})^{S_1}$ and S_3 respectively and T, t denote the $\aleph-$ switching maps.Thus $\psi : (S_3 \times S_1) \times (S_2^{S_3})^{S_1} \rightarrow S_2$ that is $\psi \in S_2^{(S_3\times S_1)\times(S_2^{S_3})^{S_1}}$. We consider the map $\widetilde{E} : S_2^{(S_3\times S_1)\times(S_2^{S_3})^{S_1}} \rightarrow (S_2^{(S_3\times S_1)})^{(S_2^{S_3})^{S_1}}$ (as defined in the statement of the proposition in fact it is E). So $\widetilde{E}(\psi) : (S_2^{S_3})^{S_1} \rightarrow S_2^{(S_3\times S_1)}$.Now for any $\aleph-$ sets A_2 in S_3,A_1 in S_1 and $f \in S_2^{(S_3\times S_1)}$, again to check that $(\widetilde{E}(\psi) \circ E)(f)(A_2, A_1) = f(A_2, A_1)$; hence $\widetilde{E}(\psi) \circ E$=identity. Similarly for any $\widehat{g} \in (S_2^{S_3})^{S_1}$ and $\aleph-$ sets A_1 in S_1,A_2 in S_3, again to check that $(E \circ \widetilde{E}(\psi))(\widehat{g})(A_1, A_2) = (\widehat{g}(A_1))(A_2)$;hence $E \circ \widetilde{E}(\psi)$=identity. Thus E is a $\mathfrak{g}\aleph$ homeomorphism.

Definition 4.3. The map E in Proposition 4.4 is called the $\mathfrak{g}\aleph$ exponential map.

As easy consequence of Proposition 4.4 is as follows.

Proposition 4.5. Let S_1, S_2 and S_3 be three $\mathfrak{g}\aleph$ locally compact Hausdorff spaces. Then the map $N : S_2^{S_1} \times {S_3}^{S_2} \rightarrow {S_3}^{S_1}$ defined by $N(f, g) = g \circ f$ is $\mathfrak{g}\aleph$ continuous.

Proof. Consider the following compositions: $S_1 \times S_2^{S_1} \times {S_3}^{S_2} \xrightarrow{T} S_2^{S_1} \times {S_3}_2^S \times S_1 \xrightarrow{t\times i_X} {S_3}_2^S \times S_2^{S_1} \times S_1 \xrightarrow{=} {S_3}_2^S \times (S_2^{S_1} \times S_1) \xrightarrow{i\times e_2} {S_3}^{S_2} \times S_2 \xrightarrow{e_2} S_3$ where T, t denote the $\aleph-$ switching maps, i_X, i denote the $\aleph-$ identity functions on S_1 and ${S_3}_2^S$ respectively and e_2 denote the $\mathfrak{g}\aleph$ evaluation maps. Let $\varphi = e_2 \circ (i \times e_2) \circ (t \times i_X) \circ T$. By proposition 4.4, we have an exponential map. $E : {S_3}^{S_1\times S_2^{S_1}\times {S_3}_2^S} \rightarrow ({S_3}^{S_1})^{S_2^{S_1}\times {S_3}_2^S}$.Since $\varphi \in {S_3}^{S_1\times S_2^{S_1}\times {S_3}_2^S}$,$E(\varphi) \in ({S_3}^{S_1})^{S_2^{S_1}\times {S_3}_2^S}$. Let $N = E(\varphi)$,that is $N : S_2^{S_1} \times {S_3}_2^S \rightarrow {S_3}^{S_1}$ is an $\mathfrak{g}\aleph$ continuous. For $f \in S_2^{S_1}, g \in {S_3}_2^S$ and for any $\aleph-$ set A_1 in S_1, it is easy to see that $N(f, g)(A_1) = g(f(A_1))$.

5 Conclusions

In this paper, we introduced the concept of $\mathfrak{g}\aleph$ compact open topology and Some characterization of this topology are discussed.

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