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Neutrosophic Statistical Interval Methods for Reliability Analysis: A Failure Censored LifeTest with Repetitive Group Sampling Plan

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Abstract. The Reliability Sampling Plans based on Failure-Censored data assume that all failure data is reliable, clear, and definitive, as noted in the literature.If the failure data measurements are unclear, indistinct, and unsure in a practical sense, we opt to adopt a new form of reliability theory known as Neutrosophic Reliability Theory. In this paper deals with Repetitive Group Sampling Plan based on failure-censored life tests using the Neutrosophic Statistical Interval Method. This plan assumes that failure times follow a Neutrosophic Generalized Exponential Distribution and also consider the Neutrosophic plan parameters for a given producer's risk and a given consumer's risk. We present the tables and results for various levels of Neutrosophic plan parameterization. The numerical illustrations of real data study provide further clarity, making it easier to interpret the concept.

Keywords: Neutrosophic Generalized Exponential Distribution; Failure Censored Reliability Lifetest; Acceptable Quality Level; Limiting Quality Level

1. Introduction

Quality monitoring and control of products are carried out through Statistical Quality Control (SQC). It is used to ensure that product quality is met or exceeded by measuring and analysing data gathered during manufacturing or service delivery. Statistical Quality Control can be divided into two major areas: one is process control and another one is product control.

- Process control, Process requirements and performance changes may be detected using them over time for the monitoring of manufacturing processes.
- Product control techniques focus on testing the end product for defects. It is also called acceptance sampling or sampling inspection, are used to draw inferences about how lots of manufactured goods are being handled, including acceptance or rejection of an individual lot or group of lots, based on specified rules.

Industrial sectors place a high priority on product reliability and consistency of performance. Using a Reliability Sampling Plan, We can ensure the product meets standards based on the Lifetime of products. When there is a certain number of failures that are observed during a life test, Time-censoring is used to suppress this number of failures followed by a predetermined period of time. Time-censoring involves terminating the test after a predetermined period of time.In failure-censoring, which is also referred as Type II censoring, the test is terminated once a certain number of failures have occurred or after a predetermined number of test hours have been completed. Both types of censoring help reduce the cost of testing and minimize the testing time.Schneider(1989) [1] developed prediction models using failure-censored variables. Seo and Yum (1993) [2]developed a method of failure censoring life test failures in exponential distributions. A failure-censored reliability sampling plan can be developed when items should be tested in sets of a fixed size, according to Balasooriya (1995) [3].It assumes that all failure data are determinant. In real life situation, failure data measurement is unclear, uncertainty and indeterminate. Neutrosophic Statistics are useful in such situations and uncertainty or indeterminacy associated with data may be handled more effectively in a flexible manner. Smarandache(2014) [4] introduced the concept of Neutrosophic statistics. Using Neutrosophic statistics, Aslam and Arif(2018) [5] have been able to test grouped products for the Weibull distribution. Neutrosophic statistics developed by Aslam and Raza(2019) [6] are intended to sample the loss function of a process.Utilizing Neutrosophic statistics, Aslam (2019) [7] introduced a novel method based on Neutrosophic statistics to assess roughness coefficients of rock joints. Chen (2017) [8] presented a technique for representing a rock joint's roughness coefficient via the neutrosophic interval. Aslam (2018) [9] investigated the Neutrosophic statistical interval methodology. Gupta and Kundu (2007) [10] formulated the Generalized Exponential Distribution.

According to Aslam(2019) [11], Neutrosophic statistical intervals are used to develop acceptance sampling plans. Isik and Kaya(2022) [12] has designed Single and Double Sampling Plan based on Neutrosophic Statistics.A method for Sudden Death Testing utilizing Repetitive Sampling within the Neutrosophic Statistical Interval Method was created by Aslam et al. (2020). [13].

This paper deals with Repetitive Group sampling plan based on Failure Censored Reliabilty life test using Neutrosophic Statistical Interval method.Alhasan(2021) [14] introduced a concept of Neutrosophic Reliability Theory. Rao et.al(2023) [15] have developed a Neutrosophic Generalized Exponential Distribution. Sherman(1965) [16]has introduced the Sampling Plan of Repetitive Sampling Group Plan. It improves resource management by enabling more effective resource allocation. Additionally, it aids in decreasing bias during the sampling process by inspecting the items multiple times.

Based on literature review using single sampling plans and failure censored lifetest in Neutrosophic Statistics, we did not came across any studies on the Repetitive Group sampling plan using Neutrosophic Statistical Interval method for failure censored Reliability lifetest. In this article, we will explain how the plan was created utilizing Repetitive Group sampling plan within the neutrosophic statistical interval method. The purpose of this study is used to optimize the neutrosophic plan parameters of the proposed sampling plan for the two specified points: one is Acceptable quality level (AQL) and other one is limiting quality level (LQL) levels. The tables and results are used to explain the data taken from the industrial sectors. In this method, various applications are used like in engineering, economics, medical and other fields. Aslam (2019) [17] has developed a new failure-censored reliability test for Neutrosophic Weibull distributions. In a single sampling plan with failure-censored reliability, inspection costs are high, and we need to remove a large number of test units.

The proposed study aims to minimize the degree of censoring so that we can reduce the number of tests removed as much as possible while minimizing the amount of time and cost involved in inspections. According to Jun et al. (2010) [18], a repetitive group sampling plan under a failure-censored reliability life test can result in a substantial reduction in failures when compared with single sampling plans. Therefore, we develop the Repetitive Group Sampling Plan with a Failure-Censored Reliability Life test using the Neutrosophic Statistical Interval Method. For the proposed study, the life test follows a Neutrosophic Generalized Exponential Distribution.

Using these results, the proposed study can optimize the sampling size, plan parameters, and minimize inspection time and costs.Finally, we compare the results from the proposed sampling plan to those under classical statistics to calculate its effectiveness.

2. Nomenclature/Notation

$x_i \rightarrow$ random sample

$\theta \rightarrow$ Scale parameter of Generalized Exponential Distribution

$\delta \rightarrow$ Shape parameter of Generalized Exponential Distribution

$\theta_N \rightarrow$ Neutrosophic Scale parameter of Generalized Exponential Distribution

$\delta_N \rightarrow$ Neutrosophic Shape parameter of Generalized Exponential Distribution

$x_{N_i} \to$ Neutrosophic random sample
$T_{N_i} \to$ Neutrosophic random sample of time
$L \to$ Lowest Specification Limit
$n_N \to$ Neutrosophic random sample size
$r_N \to$ performs test until occurs failures
$k_{a_N} \to$ Neutrosophic acceptance region
$k_{r_N} \to$ Neutrosophic rejection region
$(ASN)_N \to$ Neutrosophic Average Sample Number
$AQL \to$ Acceptable Quality Level
$LQL \to$ Limiting Quality Level
$\alpha \to$ Producer's risk
$\beta \to$ Consumer's risk
$v_N \to$ Neutrosophic Quantity Statistic

3. Basic Terminologies

Definition 3.1. Neutrosophic Statistics

Neutrosophic Statistics pertains to a collection of data where the data or a portion of it possesses some level of indeterminacy, along with techniques employed to examine the data. In Classical Statistics, all data are fixed; this differentiates neutrosophic statistics from classical statistics.

Definition 3.2. Classical Reliability

Reliability function: Let T denote the lifetime of a system, the reliability of that system at the point in time t , that $R(T) = (T > t)$, it is called the reliability at the time t , and we can define it as the probability that the time at which the system could fail is greater than t. The reliability by cumulative distribution function for a random variable as:

$$R(t) = \int_t^\infty f(t)dt = 1 - P(\mathcal{T} > t) = 1 - F(t). \tag{1}$$

Definition 3.3. Neutrosophic Reliability

Let T denote a Neutrosophic random variable that signifies the time of system failure, while t represents the duration of this system's operation. We established reliability based on Neutrosophic probability in the following manner:

$$NR(t) = \int_t^\infty f_N(t)dt = NP(\mathcal{T} > t), \tag{2}$$

where there can exist an interval, set, or Neutrosophic number that may have some indeterminacy, and t represents the Neutrosophic probability distribution. Then, $f_N(t)$ represents the

Neutrosophic reliability concerning a Neutrosophic probability distribution. The Neutrosophic reliability by cumulative distribution function for a random variable as:

$$R_N(t) = \int_t^{\infty} f_N(t)dt = 1 - P_N(\mathcal{T}_{\mathcal{N}} > t) = 1 - F_N(t). \tag{3}$$

Definition 3.4. Failure Censored Reliability Life Test
The Failure Censored Reliability test involves placing the items on the test simultaneously and continuing the test until a specified number of failures are observed.

4. **Life Time Distribution**

4.1. *Generalized Exponential Distribution*

Probabilistic statistics concerns the distribution of time intervals between events in a continuous, independent Poisson process occurring at a steady average rate, referred to as the exponential distribution. We select a random sample of n items from a generalized exponential distribution , then arrange the samples in ascending order, beginning with the smallest observation, before testing the experiment. A life test experiment is potentially concluded when all r failures have been observed recorded after the initial failures. The primary advantages of possibly halting before completing all n observations arise from the observations occurring in a particular organizing and reaching a conclusion more quickly or with fewer observations than a method that requires examining all the items under evaluation. Consequently, this study concentrates on failure-censored data, where in n items undergo testing, and the experiment concludes when a specified number of items ,like $r(< n)$, have failed.

Let x_j, where j=1,2...n, be a continuous random variable that adheres to the Generalized Exponential distribution, characterized by the shape parameter δ and the scale parameter θ. The Cumulative Distribution Function of the Generalized Exponential distribution is

$$F(x) = \left(1 - \exp\left\{-\frac{x}{\theta}\right\}\right)^{\delta} \text{x} > 0, \delta > 0, v > 0 \tag{4}$$

The Probability Density Function of Generalized Exponential distribution is,

$$f(x) = \frac{\delta}{\theta}\left(1 - e^{-x/\theta}\right)^{\delta-1} \cdot e^{-x/\theta}, \text{x} > 0, \delta > 0, v > 0 \tag{5}$$

The information gained from this kind of testing is crucial when working with expensive, advanced products. To provide a more precise depiction of time until failure, the Generalized Exponential Distribution is utilized in place of the frequently used exponential distribution. While including the Generalized Exponential Distribution in life testing modeling makes modeling and estimation more complicated, it provides a more precise fit to life data compared to the exponential distribution because of its flexibility.

4.2. *Neutrosophic Generalized Exponential Distribution*

The Neutrosophic Generalized Exponential Distribution is defined as the following: If $x_{N_i} \in [x_L, x_U] = i = 1; 2; 3; ...; n_N$ is a random sample whose distribution follows the Neutrosophic Generalized Exponential Distribution with Neutrosophic scale parameters $\theta_N \in [\theta_L, \theta_U]$ and a Neutrosophic shape parameter $\delta_N \in [\delta_L, \delta_U]$. The Neutrosophic Fuzzy Generalized Exponential Distribution with the Probability Density function is defined as follows:

$$f(x_N) = \frac{\delta_N}{\theta_N}\left(1 - e^{-x_N/\theta_N}\right)^{\delta_N - 1} e^{-x_N/\theta_N}, \quad \theta_N > 0, \theta_N \in \{\theta_L, \theta_U\}, \delta_N \in \{\delta_L, \delta_U\} \tag{6}$$

The Neutrosophic Cumulative Distribution Function is

$$F(x_N) = \left(1 - e^{-x_N/\theta_N}\right)^{\delta_N} \tag{7}$$

Neutrosophic Survival function:

$$S(x_N) = 1 - \left(1 - e^{-x_N/\theta_N}\right)^{\delta_N} \tag{8}$$

Neutrosophic Hazard function:

$$h(x_N) = \frac{\frac{\delta_N}{\theta_N}\left(1 - e^{-x_N/\theta_N}\right)^{\delta_N - 1} e^{-x_N/\theta_N}}{1 - \left(1 - e^{-x_N/\theta_N}\right)^{\delta_N}} \tag{9}$$

5. **Design of the Proposed Plan**

Let us assume that $T_{Ni} \in \{T_L, T_U\} = i = 1; 2; 3; ...; n_N$ be a random sample follows the Neutrosophic Generalized exponential distribution having a Neutrosophic scale parameter $\theta_N \in \{\theta_L, \theta_U\}$ and Neutrosophic shape parameter $\delta_N \in \{\delta_L, \delta_U\}$, then following the Neutrosophic Probability Density Function is

$$f(T_N) = \frac{\delta_N}{\theta_N}\left(1 - e^{-t_N/\theta_N}\right)^{\delta_N - 1} e^{-t_N/\theta_N}, \quad \theta_N > 0, \theta_N \in \{\theta_L, \theta_U\}, \delta_N \in \{\delta_L, \delta_U\} \tag{10}$$

The Neutrosophic Cumulative Distribution function is

$$F(T_N) = \left(1 - e^{-t_N/\theta_N}\right)^{\delta_N} \tag{11}$$

When L is the lowest specification limit, then a product outside of this range is considered defective. As a result, item at time L is unreliable, as provided by

$$P_N = p_{r_N}(T_N < L) = F_N(L) \tag{12}$$

Then the following equation 11, this formula can be used to calculate the fraction of non-conforming units before time L:

$$P_N = 1 - \left(1 - e^{-L/\theta_N}\right)^{\delta_N} \tag{13}$$

From equation 13, we have

$$w_N = -\ln(1 - P_N) = (L/\theta_N)^{\delta_N}, \quad \theta_N \in \{\theta_L, \theta_U\} \tag{14}$$

6. **Operating Procedure of Proposed Plan**

We suggest the following sampling plan under the Neutrosophic Statistical Interval Method based upon the information above.

- Step 1: Randomly take a size of sample $n_N \in [n_L, n_U]$ from a lot.
- Step 2: If we have $n_N \in [n_L, n_U]$ items for inspection and carry out tests up to r_N, where $n_N \in [n_L, n_U]$ and $r_N \in [r_L, r_U]$, the outcome is r_N, with $r_N \in \{r_L, r_U\}$.
- Step 3: Let X_{Nj} be the j^{th} failure time $(j = 1, 2, 3, \ldots, r_N)$. Calculate the statistic value
$$v_N = \sum_{i=1}^{r_N} \left(X_{N(j)}\right)^{\delta_N} + (n_N - r_N)\left(X_{N(j)}\right)^{\delta_N}; \tag{15}$$
$$v_N \in \{v_L, v_U\}.$$
- Step 4: The lot will be accepted if $v_N < k_{a_N} L^{(\delta_N)}$; $v_N < k_{r_N} L^{(\delta_N)}$ where $k_{a_N} \in [k_{a_L}, k_{a_U}]$ is a Neutrosophic Fuzzy acceptance number and the lot will be rejected if $v_N \geq k_{a_N} L^{(\delta_N)}$; $v_N \geq k_{r_N} L^{(\delta_N)}$ where $k_{aN} \in [k_{aL}, k_{aU}]$, $k_{a_N} \in [k_{rL}, k_{rU}]$, is a Neutrosophic Fuzzy rejection number. Otherwise, the procedure will move to Step 1.

To implement the proposed plan, the values of Neutrosophic Fuzzy parameters $r_N \in [r_L, r_U]$, $k_{a_N} \in [k_{a_L}, k_{a_U}]$ and $k_{r_N} \in [k_{r_L}, k_{r_U}]$ should be known. Based on this assumption,

$$v_N = \sum_{i=1}^{r_N} \left(X_{N(j)}\right)^{\delta_N} + (n_N - r_N)\left(X_{N(j)}\right)^{\delta_N};$$
$$v_N \in \{v_L, v_U\}$$

has a Neutrosophic Chi-square distribution associated with failure censored reliability v_N with a Neutrosophic degree of freedom $2r_N$.Using Chi-square distribution, we can calculate Neutrosophic Fuzzy Operating Characteristics for a Single Sampling Plan as follows:

$$P_a(p_N) = 1 - G_{2r_N}(2k_{a_N} w_N); k_{a_N} \in [k_{a_L}, k_{a_U}] \tag{16}$$

The probability of rejecting a lot under the Neutrosophic Statistical Interval Method is calculated as follows:

$$P_r(p_N) = G_{2r_N}(2k_{rN} w_N); k_{r_N} \in [k_{r_L}, k_{r_U}] \tag{17}$$

The Neutrosophic Operating Characteristic of Repetitive Group Sampling Plan is derived for the proposed sampling plan and given as follows:

$$L(P_N) = \frac{P_a(p_N)}{P_a(p_N) + P_r(p_N)} \tag{18}$$

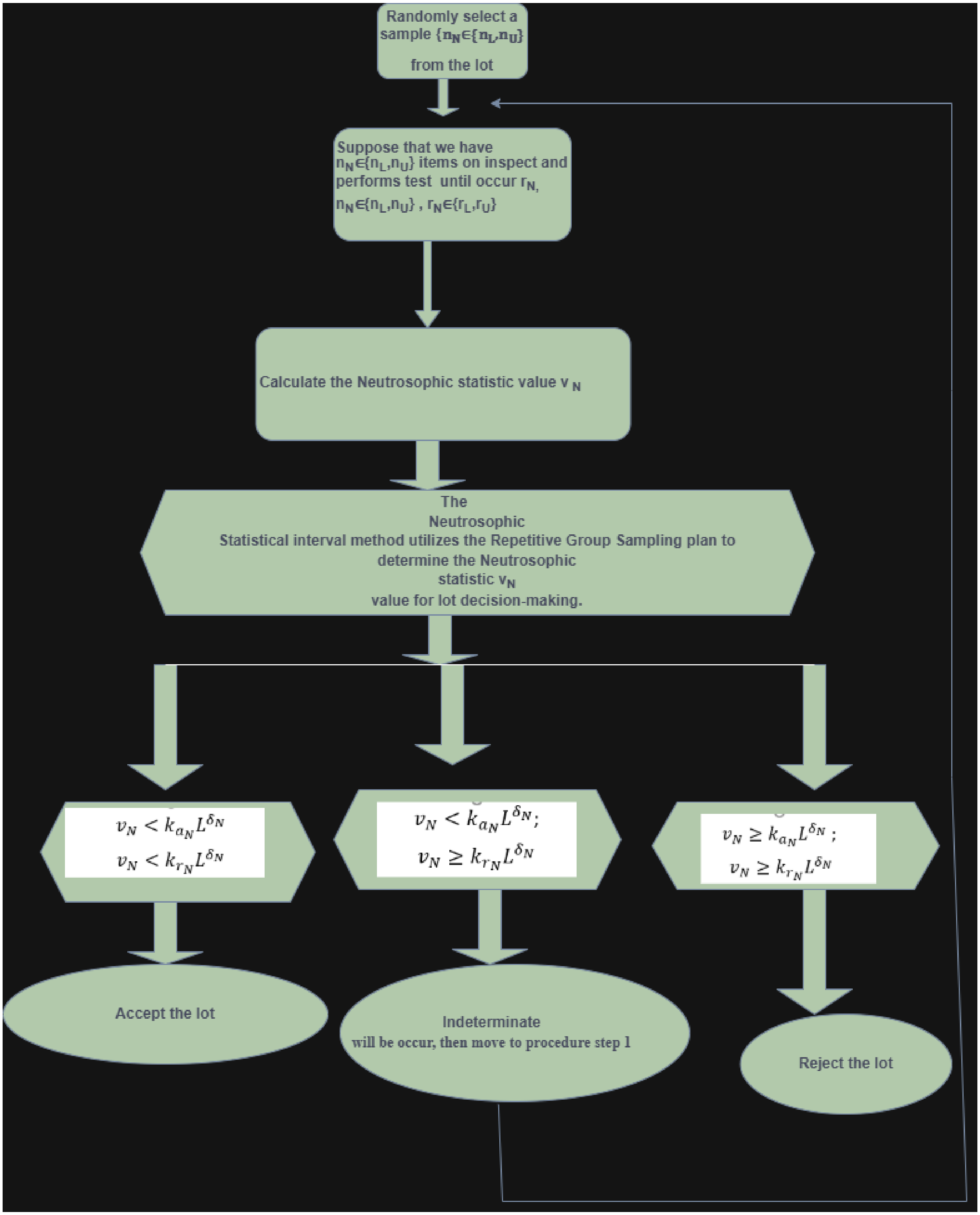

FIGURE 1. Flow chart for proposed plan

$$L(P_N) = \frac{(1 - G_{2r_N}(2k_{a_N} w_N))}{(1 - G_{2r_N}(2k_{a_N} w_N)) + G_{2r_N}(2k_{r_N} w_N)} \tag{19}$$

According to the suggested plan, the Neutrosophic Average Sample Number (NASN) will be calculated as follows:

$$ASN_N(P_N) = \frac{n_N}{(1 - G_{2r_N}(2k_{a_N} w_N)) + G_{2r_N}(2k_{r_N} w_N)} \tag{20}$$

The Neutrosophic plan parameters are α, β, AQL, and LQL respectively. It comprises the producer's risk, the consumer's risk, and the acceptable and limiting quality levels. A Neutrosophic fuzzy non-linear optimization grid search technique will be employed using the Neutrosophic Statistical Interval Method to establish the parameters for the suggested Failure Censored Life test plan:

$$\begin{gathered} Minimize ASN_N \in [ASN_L, ASN_U] \\ L(C_{AQL}) = \\ \frac{(1 - G_{2r_N}(2k_{a_N} w_N))}{(1 - G_{2r_N}(2k_{a_N} w_N)) + G_{2r_N}(2k_{r_N} w_N)} \leq 1 - \alpha \\ L(C_{LQL}) = \\ \frac{(1 - G_{2r_N}(2k_{a_N} w_N))}{(1 - G_{2r_N}(2k_{a_N} w_N)) + G_{2r_N}(2k_{r_N} w_N)} \geq \beta \end{gathered} \tag{21}$$

7. Algorithm

Failure Censored Lifetest based on Neutrosophic Statistical Interval model designing :

Step 1. Choose the random sample from the given lot

Step 2. Inspect the items based on the proposed sampling plan and continue the test until occurs failures of inspection time of product.

Step 3. Compute the Failure Censored Reliability lifetest based on Neutrosophic quantity statistic v_N and Make a decision of accept and reject the product.

A non-linear optimization procedure under Neutrosophic Statistical Interval Method determines the plan parameters $k_{a_N} \in [k_{a_L}, k_{a_U}]$, $k_{r_N} \in [k_{r_L}, k_{r_U}]$ and $r_N \in [r_L, r_U]$.

It gives the multiple combinations that satisfy the constraints provided during the simulation process. The Neutrosophic plan parameter combinations with the minimum of Neutrosophic Average Sample Number are chosen among all of these combinations. The following tables1,2,3 shows the various Neutrosophic parametric values obtained based on $(\alpha = 0.10, \beta = 0.10)$, $(\alpha = 0.05, \beta = 0.05)$ and $(\alpha = 0.05, \beta = 0.10)$ From the given table results are,

(1) For the fixed values of Neutrosophic parameters, $r_N \in r_L, r_U$ and $k_{a_N} \in \{k_{a_L}, k_{a_U}\}$,$k_{r_N} \in \{k_{r_L}, k_{r_U}\}$ decreases.
(2) For the fixed values of Neutrosophic parameters, $r_N \in \{r_L, r_U\}$ and $k_{a_N} \in \{k_{a_L}, k_{a_U}\}$,$k_{r_N} \in \{k_{r_L}, k_{r_U}\}$ decreases as LQL increases.

TABLE 1. Proposed Sampling plan parameter when α=0.10,β=0.10

AQL	LQL	r_N	ASN_N	k_{a_N}	k_{r_N}	$L(C_{AQL})$	$L(C_{(LQL)})$
0.01	0.035	[3,4]	[4.00,4.42]	[200.9,221.5]	[149.39,167.39]	[0.89,0.89]	[0.03,0.05]
0.01	0.05	[2,3]	[2.55,3.13]	[108.15,127.53]	[45.68,107.76]	[0.90,0.89]	[0.03,0.04]
0.01	0.1	[1,2]	[1.24,2.06]	[37.59,61.87]	[12.45,61.87]	[0.90,0.90]	[0.02.0.01]
0.01	0.2	[1,2]	[1.01,2.05]	[12.05,45.6]	[9.94,34.98]	[0.90,0.94]	[0.07,0.0004]
0.01	0.3	[1,2]	[1.00,2.00]	[6.86,11.14]	[8.98,9.08]	[0.91,0.99]	[0.08,0.100]
0.02	0.06	[4,6]	[4.87,6.05]	[129.33,147.56]	[80.19,142.63]	[0.90,0.92]	[0.05,0.10]
0.02	0.09	[3,4]	[3.23,5.03]	[69.56,119.56]	[52.85,117.14]	[0.89,0.90]	[0.04,0.01]
0.02	0.1	[2,3]	[2.43,4.10]	[49.45,92.02]	[23.34,92.02]	[0.90,0.90]	[0.04,0.01]
0.02	0.2	[1,2]	[1.27,3.00]	[17.30,30.44]	[4.04,28.9]	[0.89,0.90]	[0.03,0.03]
0.02	0.5	[1,2]	[1.00,2.00]	[4.27,5.97]	[2.04,3.76]	[0.90,0.97]	[0.05,0.10]
0.05	0.15	[5,7]	[5.33,7.83]	[56.42,91.69]	[46.28,73.76]	[0.89,0.89]	[0.05,0.008]
0.05	0.2	[4,6]	[4.03,6.09]	[35.22,53.98]	[33.23,49.88]	[0.90,0.95]	[0.04,0.02]
0.05	0.25	[3,5]	[3.30,4.99]	[29.40,30.01]	[20.54,28.21]	[0.89,0.98]	[0.01,0.07]
0.05	0.35	[2,4]	[2.03,3.96]	[11.66,23.34]	[10.47,18.76]	[0.89,0.98]	[0.04,0.01]
0.05	0.5	[1,3]	[1.05,3.00]	[3.60,7.75]	[1.92,7.03]	[0.90,0.99]	[0.10,0.10]

TABLE 2. Proposed Sampling plan parameters when α=0.05,β=0.05

AQL	LQL	r_N	ASN_N	k_{a_N}	k_{r_N}	$L(C_{AQL})$	$L(C_{(LQL)})$
0.01	0.035	[4,6]	[4.79,6.27]	[232.14,296.80]	[167.77,243.95]	[0.95,0.95]	[0.05,0.05]
0.01	0.05	[3,5]	[4.79,6.27]	[127.69,178.87]	[79.44,197.79]	[0.95,0.96]	[0.05,0.04]
0.01	0.1	[2,3]	[2.07,4.00]	[46.10,94.64]	[32.84,91.6]	[0.95,0.98]	[0.05,0.01]
0.01	0.2	[1,3]	[1.11,3.00]	[15.39,30.64]	[4.23,13.98]	[0.95,0.99]	[0.05,0.05]
0.01	0.3	[1,2]	[1.04,2.00]	[8.97,13.12]	[4.23,9.89]	[0.95,0.99]	[0.04,0.05]
0.02	0.06	[6,8]	[6.61,8.30]	[174.04,215.07]	[130.69,180.73]	[0.95,0.96]	[0.05,0.04]
0.02	0.09	[5,6]	[5.04,5.92]	[95.43,109.20]	[91.29,122.41]	[0.95,0.96]	[0.05,0.04]
0.02	0.1	[4,5]	[4.05,5.02]	[72.93,86.72]	[68.45,72.57]	[0.95,0.97]	[0.04,0.04]
0.02	0.2	[2,4]	[2.03,4.01]	[21.43,34.44]	[17.83,27.9]	[0.95,0.99]	[0.05,0.04]
0.02	0.5	[1,2]	[1.03,2.01]	[4.49,6.79]	[2.55,3.56]	[0.95,0.96]	[0.05,0.05]
0.05	0.15	[7,9]	[7.24,9.05]	[72.93,87.69]	[64.94,86.08]	[0.94,0.99]	[0.05,0.04]
0.05	0.2	[6,7]	[6.03,7.03]	[45.91,51.78]	[44.88,49.78]	[0.94,0.96]	[0.05,0.05]
0.05	0.25	[5,6]	[4.91,6.05]	[29.5,35.29]	[23.06,29.21]	[0.95,0.99]	[0.05,0.05]
0.05	0.35	[3,5]	[2.99,5.00]	[14.55,21.06]	[16.28,18.76]	[0.95,0.99]	[0.05,0.04]
0.05	0.5	[2,3]	[2.01,3.02]	[6.84,8.98]	[6.08,7.03]	[0.95,0.99]	[0.05,0.05]

TABLE 3. Proposed Sampling plan parameters when α=0.05,β=0.10

AQL	LQL	r_N	ASN_N	k_{a_N}	k_{r_N}	$L(C_{AQL})$	$L(C_{(LQL)})$
0.01	0.035	[4,6]	[4.51,6.04]	[205.8,269.3]	[137.7,260.32]	[0.95,0.95]	[0.10,0.08]
0.01	0.05	[3,5]	[3.19,5.04]	[113.16,193.67]	[79.44,182.22]	[0.95,0.96]	[0.08,0.03]
0.01	0.1	[2,3]	[2.03,4.13]	[38.9,146.8]	[32.84,114.83]	[0.95,0.96]	[0.08,0.0001]
0.01	0.2	[1,3]	[1.07,3.00]	[12.05,74.1]	[4.23,73.99]	[0.95,0.96]	[0.09,0]
0.01	0.3	[1,2]	[1.07,2.01]	[12.05,38.9]	[4.23,36.21]	[0.95,0.95]	[0.02,0.009]
0.02	0.06	[6,8]	[6.30,8.27]	[156.15,212.94]	[130.69,185.73]	[0.95,0.96]	[0.09,0.01]
0.02	0.09	[5,6]	[5.02,6.05]	[94.03,129.18]	[91.29,122.41]	[0.96,0.96]	[0.06,0.02]
0.02	0.1	[4,5]	[4.04,5.03]	[73.2,96.94]	[68.45,92.57]	[0.95,0.95]	[0.05,0]
0.02	0.2	[2,4]	[2.02,4.04]	[20.39,68.11]	[17.83,63.37]	[0.95,0.95]	[0.06,0]
0.02	0.5	[1,2]	[1.02,2.01]	[3.606,19.6]	[2.55,18.00]	[0.95,0.95]	[0.09,0]
0.05	0.15	[7,9]	[7.07,9.07]	[67.71,91.5]	[64.94,88.08]	[0.95,0.95]	[0.08,0.04]
0.05	0.2	[6,7]	[6.18,7.09]	[58.26,65.26]	[51.69,60.88]	[0.95,0.95]	[0.01,0.01]
0.05	0.25	[5,6]	[5.02,6.04]	[40.21,50.21]	[39.02,48.21]	[0.95,0.95]	[0.01,0.004]
0.05	0.35	[3,5]	[3.01,5.06]	[17.82,36.31]	[16.28,36.31]	[0.95,0.95]	[0.01,0]
0.05	0.5	[2,3]	[1.90,3.02]	[1.32,14.53]	[7.09,13.088]	[0.95,0.95]	[0.03,0.002]

TABLE 4. Dataset of failure times for the air conditioning system

Failure times data on the air cooling system in airplane
23, 261, 87, 7, [121, 125] 14, 62, 47, 225, [71, 74] ,
246, 21,42, 20, 5, 12, 120, [11, 13] 3, 14,
71, 11, 14, 11, 16,90, 1, 16, [52, 55], 95

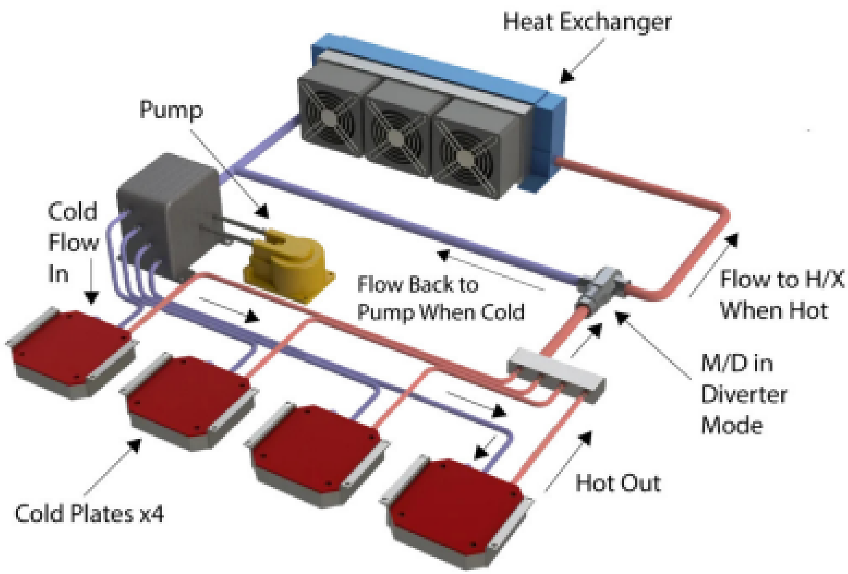

FIGURE 2. Example for Cooler system in an aeroplane

8. Real Data Study

In this study involving real data, we focus on assessing applications according to the proposed sampling plan utilizing the Neutrosophic Generalized Exponential distribution. It was used to analyze the real dataset, which was taken from khan et.al(2021) [19]. This dataset has been used by several authors in their studies, including the following:Linhart, Zucchini(1986) [20],Magalhães et.al(2020) [21].The implementation of the proposed plan indicates the failure occurrences of the air cooling system utilized in an aircraft. Figure 2 represents an example of air coolers. The experiment was conducted and only the failure time of the products was recorded. Table 4 represents the failure time of given data sets. To check the normality of data using Empirical Cumulative Distribution Function plot, Histogram and Q-Q Plots. From figure 3 , 4 and 5 represents the figure of the empirical cumulative distribution function, the histogram and the Q-Q plots give the results of the Neutrosophic Generalized Exponential distribution and its goodness of fit in the data sets.

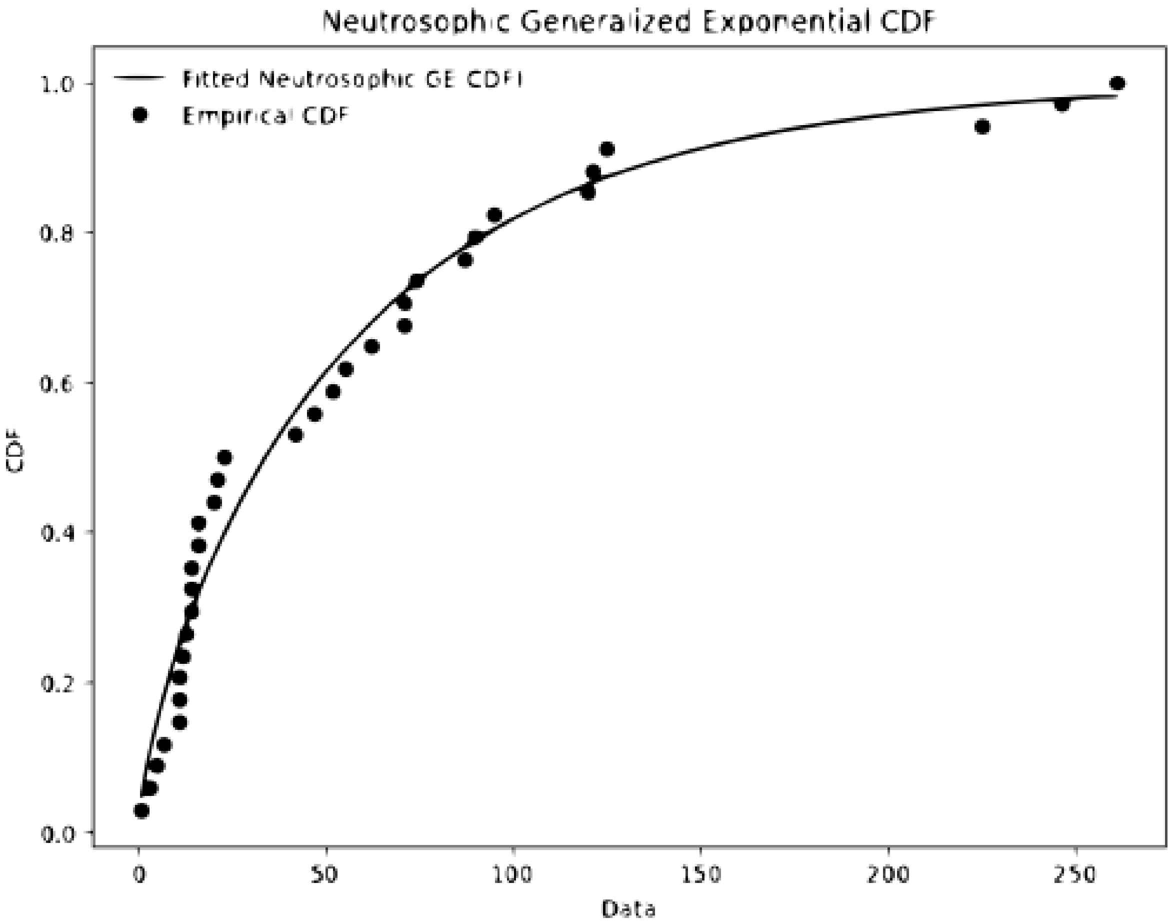

FIGURE 3. Curve fitting for given data based on Neutrosophic Generalized Exponential Distribution

Let us examine that the minimum specification for the product is L=25. Some observations during the experiment testing time were unclear, imprecise, or inconsistent.The producer risk associated with the aircraft cooling system cannot establish an appropriate sample size for product testing due to some unclear values or metrics. The air cooling system of the aircraft is characterized by the Neutrosophic Generalized Exponential Distribution involving a shape parameter $\delta_N \in \{2, 2.2\}$. Let AQL=0.01, LQL=0.035, $\alpha = 0.10$ and $\beta = 0.10$. From Table1 1, we get $r_N \in \{3, 4\}$,$k_{a_N} \in \{200.9, 221.5\}$,$k_{r_N} \in \{149.39, 167.39\}$.

- Step 1: Randomly select a sample $n_N \in [30, 30]$ from a lot.
- Step 2: We have $n_N = \{30, 30\}$ items on inspect and performs test till the failure of run time occurs. Note the failure time [121,125], [71, 74], [11, 13] and [52, 55].
- Step 3: Compute the Failure Censored Reliability Lifetest based on Neutrosophic quantity statistic v_N using equation15

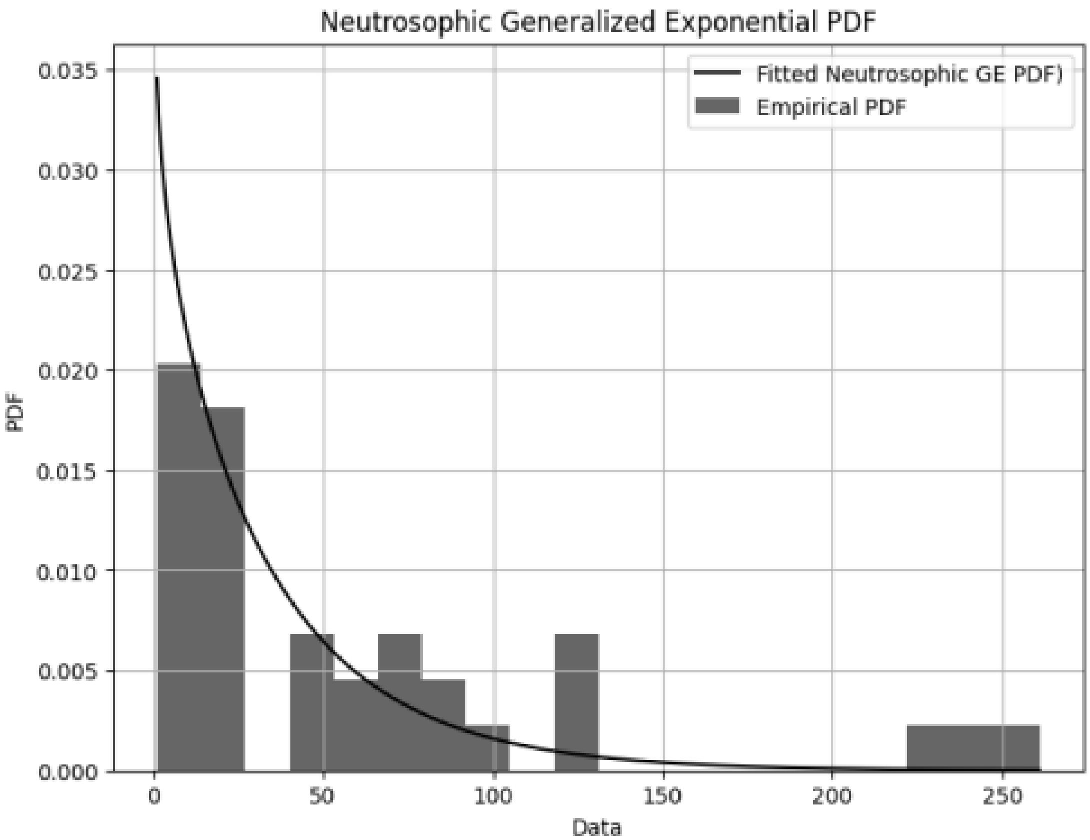

FIGURE 4. Normality using Histogram for given data follows Neutrosophic Generalized Exponential Distribution

- The Neutrosophic quantity v_N will be computed as:

$$\begin{aligned}
&= (121,125)^2 + (71,74)^2 + (11,13)^2 + (52,55)^2 + \\
&\quad \left[(30-4)(52,55)^2\right] \\
&= (22507, 24295) + (70304, 78650) \\
&\quad v_N \in (92811, 102945)
\end{aligned}$$

- Step 4: We consider the values of $v_N \in (92811, 102945) < k_{a_N} L^{\delta_N} = [125562.5, 138437.5]$,$v_N \in (92811, 102945) < k_{a_N} L^{\delta_N} = [93368.75, 104618.75]$. As a result, the cooling system of the airplane product is accepted. The Neutrosophic statistical approach utilizes statistical values of cooling system in order to select the lot that should be submitted with the product.

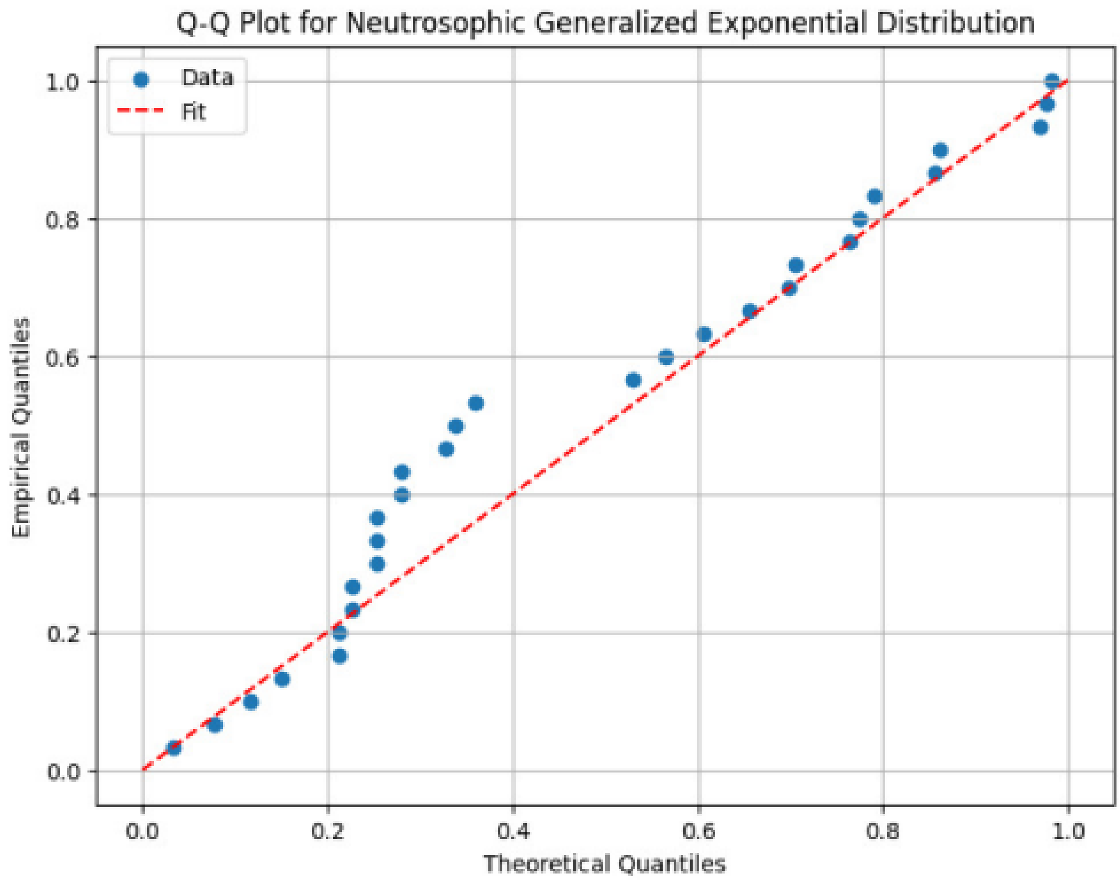

FIGURE 5. Q-Q plot for given data follows Neutrosophic Generalized Exponential Distribution

9. Comparative Study:

This proposed study compares the Neutrosophic interval method with classical statistics in order to determine its efficiency. Jun et al. (2010) use a Weibull distribution to estimate the values of plan parameters under classical statistics. Using classical statistics, we calculated the plan parameters of the proposed study. Table 5 presents plan parameters for selected AQL and LQL values when $\alpha = 0.05, \beta = 0.10$. It is important to use the same parameters in both plans when comparing them.

Table 5 indicates that the suggested plan features lesser interval values compared to the current plan according to traditional statistics. It is essential to highlight that the suggested plan is a range plan, while the current sampling plan is a fixed value. Consequently, the suggested sampling plan enables a more efficient examination of uncertain information. It is better suited for practical applications, and it is also more economical.

TABLE 5. Comparison of Proposed Sampling plan parameters when α=0.05,β=0.10

AQL	LQL	r_N	r
0.01	0.035	[4,6]	46
0.01	0.05	[3,5]	44
0.01	0.1	[2,3]	39
0.01	0.2	[1,3]	28
0.01	0.3	[1,2]	27
0.02	0.06	[6,8]	63
0.02	0.09	[5,6]	56
0.02	0.1	[4,5]	46
0.02	0.2	[2,4]	42
0.02	0.5	[1,2]	34
0.05	0.15	[7,9]	66
0.05	0.2	[6,7]	62
0.05	0.25	[5,6]	53
0.05	0.35	[3,5]	48
0.05	0.5	[2,3]	44

10. Conclusion

Our research aims to develop a failure-censored reliability testing plan using Neutrosophic Statistical Interval Method with Repetitive Group Sampling. The suggested sampling plan uses non-linear optimization grid search techniques to compute the tables. We provide a real data example to clarify our study's concept and present the findings clearly, comprehensibly and also this method provides increased flexibility in parameter selection and delivers more precise results.

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