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Pentapartitioned Neutrosophic Soft Set with Interval Membership

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Abstract. In this article, the concept of Interval valued pentapartitioned neutrosophic soft set (IVPNSS) has been introduced, which is a hybrid method by combining Pentapartitioned Neutrosophic set and Interval valued soft set. IVPNSS is an ordered pair of a function and an interval valued pentapartitioned neutrosophic set, which is the domain of the function and co-domain is the assembly of all the interval valued pentapartitioned neutrosophic sets (IVPNS). Additionally, some basic operations on IVPNSSs have been defined which plays an imperative role in decision making processes. Some useful properties of IVPNSS are also presented in the article. In the end, the article addresses the strength and drawbacks of the proposed method and the concluding remarks are presented to signify the future scope in this area.

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1. **Introduction**

Numerous frequent issues in the fields of economics, science and engineering, arise in real life that cannot be conquered with classical mathematical methods. The introduction of Fuzzy Sets (FSs) (Zadeh, 1965) and Intuitionistic FSs (IFSs) (Atanassov, 1986) became crucial for addressing this kind of real-world problem. Since IFS (Atanassov, 1986) theory takes into account both membership and non-membership values, it has numerous of applications in the subject of uncertainty, whereas FS (Zadeh, 1965) primarily addresses membership values and is therefore less significant in practical matters than IFS (Atanassov, 1986). However, IFS theory does not fully address indeterminacy as an independent factor or inconsistent observations in belief systems. To tackle these issues, Smarandache (1998) introduced the Neutrosophic Set (NS) in 1998, extending the IFS theory to handle indeterminacy. The foundational aspects of NS (Smarandache, 1998) and Single-Valued NS (SVNS) (Wang et al., 2010) are explored in the works of Broumi et al. (2018) and Pramanik (2022a) respec tively. Chatterjee et al. (2016) developed the concept of the Quadripartition Single-Valued NS (QSVNS), which was later extended to interval quadripartitioned neutrosophic sets by Pramanik (2022b). In 2013 Smarandache (2013) characterized indeterminacy into functions which are unknown membership function, contradiction membership function and ignorance membership function to propose Five Symbol Valued Neutrosophic Logic (FSVNL). Using the notion of FSVNL Pentapartitioned Neutrosophic Set (PNS) has been introduced by Mallick and Pramanik (2020), where truth, contradiction, ignorance, unknown and falsity membership functions have been considered. Molodtsov (1999) proposed soft set (SS) theory in 1999, a whole new way to model uncertainty and ambiguities. Shil et al. (2024) combined PNS (Mallick Pramanik, 2020) and soft sot (Molodtsov (1999) and developed Single Valued Pentapartitioned Neutrosophic Soft Set (SVPNSS). Giving the membership value in an interval becomes convenient since decision-makers are often finding themselves unable to provide a fixed membership value in real-life uncertain scenarios, and as a necessity, Interval Pentapartitioned Neutrosophic Set (IVPNS) was first developed by Pramanik (2023) and later Datta (2023) found some interesting results. The motivation for this work stems from the following fact:

- The existing fuzzy sets and soft sets are unable to address uncertainty in three different independent ways.
- In decision-making processes, experts often struggle to assign a precise value when rating an object using membership values. Therefore, to simplify the decision-making process, it is sometimes necessary to rate an object in terms of membership values that fall within a specific interval.
- Existing SS approaches are unable to address five membership values.
- It is crucial to examine the fundamental operational laws and properties of the proposed method to implement it effectively in real-life situations. The remaining portion of the article organized as follows: Section 2 comprises of some basic literature survey, section 3 includes the new notion of IVPNSS where IVPNS has been combined with SS theory and in which five significant membership values (truthiness, contradiction, ignorance, unknown/ lack of knowledge, falsity) are considered using soft set theory. Section 3 also includes some basic characteristic of IVPNSS, where soft set theory was used to examine various significant features

of IVPNSS. Section 4 presented the strength and drawbacks of the induced method and section 5 presented the concluding remarks and future scope of this study.

2. Preliminaries

In this section, we first lay out the fundamental ideas that serve as the basis for this analysis before getting into the primary sections of this research article. We hope to provide a clear grasp of the framework driving our inquiry by providing this foundation.

2.1. *Definition [15]*

A NS $\mathfrak{L}$ on the universe $\mathfrak{W}$ is $\mathfrak{L} = \{(\gimel, T_{\mathfrak{L}}(\gimel), I_{\mathfrak{L}}(\gimel), F_{\mathfrak{L}}(\gimel)) : \gimel \in \mathfrak{W}\}$, where $T_{\mathfrak{L}}, I_{\mathfrak{L}}, F_{\mathfrak{L}} : \mathfrak{W} \to]^{-}0, 1^{+}[$ are functions such that the condition: $\forall \gimel \in \mathfrak{W}$, $^{-}0 \leq T_{\mathfrak{L}}(\gimel) + I_{\mathfrak{L}}(\gimel) + F_{\mathfrak{L}}(\gimel) \leq 3^{+}$ is satisfied. Here $T_{\mathfrak{L}}, I_{\mathfrak{L}}, F_{\mathfrak{L}}$ represent the truth, indeterminacy and falsity membership function respectively.

2.2. *Definition [4]*

A PNS $\mathfrak{L}$ on the universe $\mathfrak{W}$ is $\mathfrak{L} = \{(\gimel, T_{\mathfrak{L}}(\gimel), C_{\mathfrak{L}}(\gimel), G_{\mathfrak{L}}(\gimel), U_{\mathfrak{L}}(\gimel), F_{\mathfrak{L}}(\gimel)) : \gimel \in \mathfrak{W}\}$, where $T_{\mathfrak{L}}(\gimel), C_{\mathfrak{L}}(\gimel), G_{\mathfrak{L}}(\gimel), U_{\mathfrak{L}}(\gimel), F_{\mathfrak{L}}(\gimel) : \gimel \in \mathfrak{W} \to [0, 1]$ are functions such that the condition: $\forall \gimel \in \mathfrak{W}$, $0 \leq T_{\mathfrak{L}}(\gimel)+C_{\mathfrak{L}}(\gimel)+G_{\mathfrak{L}}(\gimel)+U_{\mathfrak{L}}(\gimel)+F_{\mathfrak{L}}(\gimel) \leq 5$ is satisfied. Here $T_{\mathfrak{L}}, C_{\mathfrak{L}}, G_{\mathfrak{L}}, U_{\mathfrak{L}}, F_{\mathfrak{L}}$ represent the truth, contradiction, ignorance, unknown and falsity membership functions respectively.

2.3. *Definition [3]*

An interval valued pentapartitioned NS $\mathfrak{L}$ on the universe $\mathfrak{W}$ is defined as $\mathfrak{L} = \{(\gimel, T_{\mathfrak{L}}(\gimel), C_{\mathfrak{L}}(\gimel), G_{\mathfrak{L}}(\gimel), U_{\mathfrak{L}}(\gimel), F_{\mathfrak{L}}(\gimel)) : \gimel \in \mathfrak{W}\}$, where $T_{\mathfrak{L}}, C_{\mathfrak{L}}, G_{\mathfrak{L}}, U_{\mathfrak{L}}, F_{\mathfrak{L}} : \mathfrak{W} \to \mathrm{Int}([0, 1])$ are functions satisfying the condition: $\forall \gimel \in \mathfrak{W}$, $0 \leq supT_{\mathfrak{L}}(\gimel) + supC_{\mathfrak{L}}(\gimel) + supG_{\mathfrak{L}}(\gimel) + supU_{\mathfrak{L}}\gimel) + supF_{\mathfrak{L}}(\gimel) \leq 5$. Here $T_{\mathfrak{L}}(\gimel) = [infT_{\mathfrak{L}}(\gimel), supT_{\mathfrak{L}}(\gimel)]$ and similarly, $C_{\mathfrak{L}}(\gimel), G_{\mathfrak{L}}(\gimel), U_{\mathfrak{L}}(\gimel)$ and $F_{\mathfrak{L}}(\gimel)$ all are sub intervals of $[0, 1]$. Here $T_{\mathfrak{L}}, C_{\mathfrak{L}}, G_{\mathfrak{L}}, U_{\mathfrak{L}}, F_{\mathfrak{L}}$ represent the truth, contradiction, ignorance, unknown and falsity membership functions respectively.

2.4. *Definition [5]*

Let $\mathfrak{W}$ be an universe set and $\mathfrak{E}$ be a set of attributes. Now let us Consider $\mathfrak{L}$ as a non empty subset of $\mathfrak{E}$ and $P(\mathfrak{W})$ be power set of $\mathfrak{W}$. By soft set we mean the collection $(\mathfrak{f}, \mathfrak{L})$ over $\mathfrak{W}$, where $\mathfrak{f} : \mathfrak{L} \to P(\mathfrak{W})$.

3. **Interval Valued Pentapartitioned Neutrosophic Soft Set**

3.1. *Definition:*

Let $\mathfrak{W}$ be the universe whereas $\mathfrak{E}$ be a set of attributes. Let $IVPNS^{\mathfrak{W}}$ be the collection of all interval valued pentapartitioned neutrosophic set on $\mathfrak{W}$. The pair $(\mathfrak{f}, \mathfrak{L})$, $\mathfrak{L} \subseteq \mathfrak{E}$ is called a interval valued pentapartitioned neutrosophic soft set over $\mathfrak{W}$, $\mathfrak{f} : \mathfrak{L} \rightarrow IVPNS^{\mathfrak{W}}$ is a mapping. The collection of all $IVPNSS$ over $\mathfrak{W}$ is denoted by $IVPNSS^{\mathfrak{W}}$.

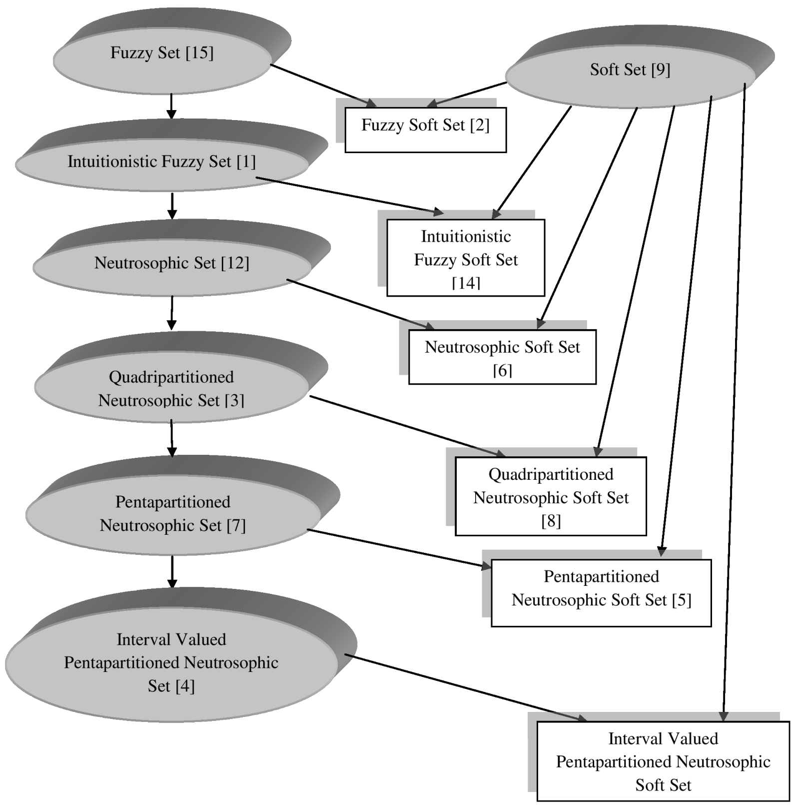

FIGURE 1. Different types of hybrid models

3.2. *Definition:*

Let $\mathfrak{W}$ be the universe $\mathfrak{E}$ be a set of attributes. Let $\mathfrak{g} : \mathfrak{L} \rightarrow IVPNS^{\mathfrak{W}}$ and $\mathfrak{h} : \mathfrak{M} \rightarrow IVPNS^{\mathfrak{W}}$ defined by $\mathfrak{g}(a) = \{(\gimel, T_{\mathfrak{g}(a)}(\gimel), C_{\mathfrak{g}(a)}(\gimel), G_{\mathfrak{g}(a)}(\gimel), U_{\mathfrak{g}(a)}(\gimel), F_{G(a)}(\gimel)) : \gimel \in \mathfrak{W}\}$, $\mathfrak{h}(b) = \{(\gimel, T_{\mathfrak{h}(b)}(\gimel), C_{\mathfrak{h}(b)}(\gimel), G_{\mathfrak{h}(b)}(\gimel), U_{\mathfrak{h}(b)}(\gimel), F_{\mathfrak{h}(b)}(\gimel)) : \gimel \in \mathfrak{W}\}$, $a \in \mathfrak{L} \subseteq \mathfrak{E}$, $b \in \mathfrak{M} \subseteq \mathfrak{E}$ are two $IVPNSS$ over $\mathfrak{W}$ and its denoted by $(\mathfrak{g}, \mathfrak{L}), (\mathfrak{h}, \mathfrak{M})$ respectively. Here $T_{\mathfrak{g}(a)}(\gimel), C_{\mathfrak{g}(a)}(\gimel), G_{\mathfrak{g}(a)}(\gimel), U_{\mathfrak{g}(a)}(\gimel), F_{\mathfrak{g}(a)}(\gimel), T_{\mathfrak{h}(b)}(\gimel), C_{\mathfrak{h}(b)}(\gimel), G_{\mathfrak{h}(b)}(\gimel), U_{\mathfrak{h}(b)}(\gimel), F_{\mathfrak{h}(b)}(\gimel) \in$

$Int([0,1])$. Also $0 \leq supT_{\mathfrak{g}(a)}(\daleth) + supC_{\mathfrak{g}(a)}(\daleth) + supG_{\mathfrak{g}(a)}(\daleth) + supU_{\mathfrak{g}(a)}(\daleth) + supF_{\mathfrak{g}(a)}(\daleth) \leq 5$
$0 \leq supT_{\mathfrak{h}(b)}(\daleth) + supC_{\mathfrak{h}(b)}(\daleth) + supG_{\mathfrak{h}(b)}(\daleth) + supU_{\mathfrak{h}(b)}(\daleth) + supF_{\mathfrak{h}(b)}(\daleth) \leq 5$.

(i) The union is $(\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{M}) = (\mathfrak{K}, \mathfrak{N})$, is an $IVPNSS$ over $\mathfrak{W}$, where $\mathfrak{N} = \mathfrak{L} \cup \mathfrak{M}$ and for $\vartheta \in \mathfrak{N}$, $\mathfrak{K} : \mathfrak{N} \to IVPNS^{\mathfrak{W}}$ is defined by $\mathfrak{K}(\vartheta) = \{\daleth, T_{\mathfrak{K}(\vartheta)}\daleth), C_{\mathfrak{K}(\vartheta)}(\daleth), G_{\mathfrak{K}(\vartheta)}(\daleth), U_{\mathfrak{K}(\vartheta)}(\daleth), F_{\mathfrak{K}(\vartheta)}(\daleth) : \daleth \in \mathfrak{W}\}$, where for $\daleth \in \mathfrak{W}$

$$T_{\mathfrak{K}(\vartheta)}(\daleth) = \begin{cases} T_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} - \mathfrak{M} \\ T_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M} - \mathfrak{L} \\ T_{\mathfrak{g}(\vartheta)}(\daleth) \vee T_{\mathfrak{h}(\vartheta)}(\daleth) = max\{T_{\mathfrak{g}(\vartheta)}(\daleth), T_{\mathfrak{h}(\vartheta)}(\daleth)\} & if\, \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$C_{\mathfrak{K}(\vartheta)}(\daleth) = \begin{cases} C_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} - \mathfrak{M} \\ C_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M} - \mathfrak{L} \\ C_{\mathfrak{g}(\vartheta)}(\daleth) \vee C_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$G_{\mathfrak{K}(\vartheta)}(\daleth) = \begin{cases} G_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} - \mathfrak{M} \\ G_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M} - \mathfrak{L} \\ G_{\mathfrak{g}(\vartheta)}(\daleth) \Box G_{\mathfrak{h}(\vartheta)}(\daleth) = \frac{G_{\mathfrak{g}(\vartheta)}(\daleth) + G_{\mathfrak{h}(\vartheta)}\daleth)}{2} & if\, \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$U_{\mathfrak{K}(\vartheta)}(\daleth) = \begin{cases} U_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} - \mathfrak{M} \\ U_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M} - \mathfrak{L} \\ U_{\mathfrak{g}(\vartheta)}(\daleth) \wedge U_{\mathfrak{h}(\vartheta)}(\daleth) = min\{U_{\mathfrak{g}(\vartheta)}(\daleth), U_{\mathfrak{h}(\vartheta)}(\daleth)\} & if\, \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$F_{\mathfrak{K}(\vartheta)}(\daleth) = \begin{cases} F_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} - \mathfrak{M} \\ F_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M} - \mathfrak{L} \\ F_{\mathfrak{g}(\vartheta)}(\daleth) \wedge F_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}.$$

(ii) The intersection is $(\mathfrak{g}, \mathfrak{L}) \cap (\mathfrak{h}, \mathfrak{M}) = (\mathfrak{K}, \mathfrak{N})$ (say), is an $IVPNSS$ over $\mathfrak{W}$, where $\mathfrak{N} = \mathfrak{L} \cap \mathfrak{M}$ and for $\vartheta \in \mathfrak{N}$, $\mathfrak{K} : \mathfrak{N} \to IVPNS^{\mathfrak{W}}$ is defined by $\mathfrak{K}(\vartheta)=\{\daleth, T_{\mathfrak{K}(\vartheta)}(\daleth), C_{\mathfrak{K}(\vartheta)}(\daleth), G_{\mathfrak{K}(\vartheta)}(\daleth), U_{\mathfrak{K}(\vartheta)}(\daleth), F_{\mathfrak{K}(\vartheta)}(\daleth) : \daleth \in \mathfrak{W}\}$, where for $\daleth \in \mathfrak{W}$
$T_{\mathfrak{K}(\vartheta)}(\daleth)=T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{h}(\vartheta)}(\daleth)$, $C_{\mathfrak{K}(\vartheta)}(\daleth)=C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{h}(\vartheta)}(\daleth)$, $G_{\mathfrak{K}(\vartheta)}(\daleth)=G_{\mathfrak{g}(\vartheta)}(\daleth)\Box G_{\mathfrak{h}(\vartheta)}(\daleth)$, $U_{\mathfrak{K}(\vartheta)}(\daleth)=U_{\mathfrak{g}(\vartheta)}(\daleth) \vee U_{\mathfrak{h}(\vartheta)}(\daleth)$, $F_{\mathfrak{K}(\vartheta)}(\daleth)=F_{\mathfrak{g}(\vartheta)}(\daleth) \vee F_{\mathfrak{h}(\vartheta)}(\daleth)$.

(iii) The complement of $(\mathfrak{g}, \mathfrak{L})$, $(\mathfrak{g}, \mathfrak{L})^c$ is an $IVPNSS$ over $\mathfrak{W}$ and defined as $(\mathfrak{g}, \mathfrak{L})^c=(\mathfrak{g}^c, \rceil\mathfrak{L})$, where $\mathfrak{g}^c : \rceil\mathfrak{L} \to IVPNS^{\mathfrak{W}}$ is defined by
$\mathfrak{g}^c(\neg a) = \{(\daleth, T_{\mathfrak{g}^c(\neg a)}(\daleth), C_{\mathfrak{g}^c(\neg a)}(\daleth), G_{\mathfrak{g}^c(\neg a)}(\daleth), U_{\mathfrak{g}^c(\neg a)}(\daleth), F_{\mathfrak{g}^c(\neg a)}(\daleth)) : \daleth \in \mathfrak{W}\}$ for $a \in \mathfrak{L}$, where $T_{\mathfrak{g}^c(\neg a)}(\daleth) = F_{\mathfrak{g}(a)}(\daleth)$, $C_{\mathfrak{g}^c(\neg a)}(\daleth) = U_{\mathfrak{g}(a)}(\daleth)$, $G_{\mathfrak{g}^c(\neg a)}(\daleth) = 1 - G_{\mathfrak{g}(a)}(\daleth)$, $U_{\mathfrak{g}^c(\neg a)}(\daleth) = C_{\mathfrak{g}(a)}(\daleth)$ and $F_{\mathfrak{g}^c(\neg a)}(\daleth) = T_{\mathfrak{g}(a)}(\daleth)$.
We can write, $\mathfrak{g}^c(\neg a)=\{(\daleth, F_{\mathfrak{g}(a)}(\daleth), U_{\mathfrak{g}(a)}(\daleth), 1 - G_{\mathfrak{g}(a)}(\daleth), C_{\mathfrak{g}(a)}(\daleth), T_{\mathfrak{g}(a)}(\daleth)) : \daleth \in \mathfrak{W}\}$. Here $\neg a$ means 'not a', for example if a is 'beautiful' then $\neg a$ will be 'not beautiful'. Similarly, $\rceil\mathfrak{L}$ stands for the 'NOT set of $\mathfrak{L}$'.

3.3. *Example*

Let $\mathfrak{W}$ be the set of houses that are being studied and $\mathfrak{E}$ is the collection of attributes. Consider $\mathfrak{E}$ ={beautiful, wooden, costly, moderate}. In this instance, to define a $IVPNSS$ means to point out beautiful houses, Costly houses. let, there are three houses in the universe $\mathfrak{W}$ given by, $\mathfrak{W} = \{h_1, h_2, h_3\}$ and the set of attributes $\mathfrak{L} = \{e_1, e_2\}$, where e_1 and e_2 denote the attributes 'beautiful' and 'costly' respectively.

The tabular depiction of $(\mathfrak{g}, \mathfrak{L})$

$\mathfrak{W}$	e_1	e_2
h_1	([.1,.2], [0,.2], [.3,.4], [.5,.6], [.7,.9])	([.2,.3], [.1,.2], [.4,.5], [.6,.7], [.8,.9])
h_2	([.7,.8], [.1,.2], [.3,.4], [.5,.6], [.8,.9])	([.1,.2], [.3,.5], [.4,.6], [.7,.9], [.5,.8])
h_3	([.2,.3], [.4,.5], [.6,.7], [.1,.2], [.8,.9])	([.2,.4], [.1,.3], [.6,.7], [.8,.9], [.5,.6])

The tabular depiction of $(\mathfrak{h}, \mathfrak{M})$

$\mathfrak{W}$	e_2
h_1	([.6,.7], [.1,.2], [.8,.9], [.3,.4], [.4,.5])
h_2	([.4,.5], [.2,.3],[.5,.6], [.8,.9],[.3,.7])
h_3	([.2,.4], [.1,.2], [.4,.6], [.2,.9], [.1,.7])

Union of $(\mathfrak{g}, \mathfrak{L})$ and $(\mathfrak{h}, \mathfrak{M})$ is as follows:

$\mathfrak{W}$	e_1	e_2
h_1	([.1,.2], [0,.2], [.3,.4], [.5,.6], [.7,.9])	([.6,.7], [.1,.2], [.6,.7], [.3,.4], [.4,.5])
h_2	([.7,.8], [.1,.2], [.3,.4], [.5,.6], [.8,.9])	([.4,.5], [.3,.5], [.45,.6], [.7,.9], [.3,.7])
h_3	([.2,.3], [.4,.5], [.6,.7], [.1,.2], [.8,.9])	([.2,.4], [.1,.3], [.5,.65], [.2,.9], [.1,.6])

Intersection of $(\mathfrak{g}, \mathfrak{L})$ and $(\mathfrak{h}, \mathfrak{M})$ is as follows:

$\mathfrak{W}$	e_2
h_1	([.2,.3], [.1,.2], [.6,.7], [.6,.7], [.8,.9])
h_2	([.1,.2], [.2,.3],[.45,.6], [.8,.9],[.5,.8])
h_3	([.2,.4], [.1,.2], [.5,.65], [.8,.9], [.5,.7])

The tabular depiction of $(\mathfrak{g}, \mathfrak{L})^c$

$\mathfrak{W}$	e_1	e_2
h_1	([.7,.9], [.5,.6], [.6,.7], [0,.2], [.1,.2])	([.8,.9], [.6,.7], [.5,.6], [.1,.2], [.2,.3])
h_2	([.8,.9], [.5,.6], [.6,.7], [.1,.2], [.7,.8])	([.5,.8], [.7,.9], [.4,.6], [.3,.5], [.1,.2])
h_3	([.8,.9], [.1,.2], [.3,.4], [.4,.5], [.2,.3])	([.5,.6], [.8,.9], [.3,.4], [.1,.3], [.2,.4])

3.4. *Theorem*

For any three $IVPNSS$s $(\mathfrak{g}, \mathfrak{L}), (\mathfrak{h}, \mathfrak{M})$ and $(\mathfrak{K}, \mathfrak{N})$ the followings are true:

(i) Associative Law:

$$((\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{M})) \cup (\mathfrak{K}, \mathfrak{N}) = (\mathfrak{g}, \mathfrak{L}) \cup ((\mathfrak{h}, \mathfrak{M}) \cup (\mathfrak{K}, \mathfrak{N}))$$

$((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M}))\cap(\mathfrak{K},\mathfrak{N})=(\mathfrak{g},\mathfrak{L})\cap((\mathfrak{h},\mathfrak{M})\cap(\mathfrak{K},\mathfrak{N}))$

(ii) Distributive law:

$(\mathfrak{g},\mathfrak{L})\cap((\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N}))=((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M}))\cup((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{K},\mathfrak{N}))$

$(\mathfrak{g},\mathfrak{L})\cup((\mathfrak{h},\mathfrak{M})\cap(\mathfrak{K},\mathfrak{N}))=((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M}))\cap((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{K},\mathfrak{N}))$

(iii) Commutative Law:

$(\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M})=(\mathfrak{h},\mathfrak{M})\cup(\mathfrak{g},\mathfrak{L})$

$(\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M})=(\mathfrak{h},\mathfrak{M})\cap(\mathfrak{g},\mathfrak{L})$

(iv) De Morgan's Law:

De Morgan's law holds only when $\mathfrak{M}=\mathfrak{L}$ as in such cases only we will get $\mathfrak{L}\cup\mathfrak{M}=\mathfrak{L}\cap\mathfrak{M}$

$((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{L}))^c=(\mathfrak{g},\mathfrak{L})^c\cap(\mathfrak{h},\mathfrak{L})^c$.

$((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{L}))^c=(\mathfrak{g},\mathfrak{L})^c\cup(\mathfrak{h},\mathfrak{L})^c$.

(v) Idempotent Law:

$(\mathfrak{g},\mathfrak{L})\cup(\mathfrak{g},\mathfrak{L})=(\mathfrak{g},\mathfrak{L})$

$(\mathfrak{g},\mathfrak{L})\cap(\mathfrak{g},\mathfrak{L})=(\mathfrak{g},\mathfrak{L})$

(vi) Absorption Law:

$(\mathfrak{g},\mathfrak{L})\cup((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M}))=(\mathfrak{g},\mathfrak{L})$

$(\mathfrak{g},\mathfrak{L})\cap((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M}))=(\mathfrak{g},\mathfrak{L})$

(vii) Involution Law:

$((\mathfrak{g},\mathfrak{L})^c)^c=(\mathfrak{g},\mathfrak{L})$

Proof: Let $\mathfrak{W}$ be the universe and $\mathfrak{E}$ be a set of attributes. Let us also consider $(\mathfrak{g},\mathfrak{L})$ and $(\mathfrak{h},\mathfrak{M})$ be defined as in definition 3.2 and $(\mathfrak{K},\mathfrak{N})$ be another $IVPNSS$ where $\mathfrak{K}:\mathfrak{N}\to IVPNS^{\mathfrak{W}}$, $\mathfrak{K}(c)=\{(\beth,T_{\mathfrak{K}(c)}(\beth),C_{\mathfrak{K}(c)}(\beth),G_{\mathfrak{K}(c)}(\beth),U_{k(c)}(\beth),F_{k(c)}(\beth)):\beth\in\mathfrak{W}\}$, $c\in\mathfrak{N}\subseteq\mathfrak{E}$, over $\mathfrak{W}$. Here, $T_{\mathfrak{K}(c)}(\beth),C_{\mathfrak{K}(c)}(\beth),G_{\mathfrak{K}(c)}(\beth),U_{\mathfrak{K}(c)}(\beth),F_{\mathfrak{K}(c)}(\beth)\in Int([0,1])$ satisfying the condition $0\le supT_{\mathfrak{K}(a)}(\beth)+supC_{\mathfrak{K}(a)}(\beth)+supG_{\mathfrak{K}(a)}(\beth)+supU_{\mathfrak{K}(a)}(\beth)+supF_{\mathfrak{K}(a)}(\beth)\le 5$.

(i) Associative Law:

Let $(\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M})=(\mathfrak{s},\mathfrak{D})$, where $\mathfrak{D}=\mathfrak{L}\cup\mathfrak{M}$ and $\mathfrak{s}:\mathfrak{D}\to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta\in\mathfrak{D}$, $\mathfrak{s}(\vartheta)=\{(\beth,T_{\mathfrak{s}(\vartheta)}(\beth),C_{\mathfrak{s}(\vartheta)}(\beth),G_{\mathfrak{s}(\vartheta)}(\beth),U_{\mathfrak{s}(\vartheta)}(\beth),F_{\mathfrak{s}(\vartheta)}(\beth)):\beth\in\mathfrak{W}\}$

$$T_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases} T_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}-\mathfrak{M}\\ T_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{L}\\ T_{\mathfrak{g}(\vartheta)}(\beth)\vee T_{h(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\end{cases}$$

$$C_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases} C_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}-\mathfrak{M}\\ C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{L}\\ C_{\mathfrak{g}(\vartheta)}(\beth)\vee C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\end{cases}$$

$$G_{\mathfrak{s}(\vartheta)}(\gimel)=\begin{cases} G_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-\mathfrak{M} \\ G_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-\mathfrak{L} \\ G_{\mathfrak{g}(\vartheta)}(\gimel)\square G_{h(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$U_{\mathfrak{s}(\vartheta)}(\gimel)=\begin{cases} U_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-\mathfrak{M} \\ U_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-\mathfrak{L} \\ U_{\mathfrak{g}(\vartheta)}(\gimel)\wedge U_{h(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$F_{\mathfrak{s}(\vartheta)}(\gimel)=\begin{cases} F_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-\mathfrak{M} \\ F_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-\mathfrak{L} \\ F_{\mathfrak{g}(\vartheta)}(\gimel)\wedge F_{\gimel(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

Now let $((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M}))\cup(\mathfrak{K},\mathfrak{N})=(\mathfrak{s},\mathfrak{D})\cup(\mathfrak{K},\mathfrak{N})=(\mathfrak{t},\mathfrak{E})$, where $\mathfrak{E}=\mathfrak{D}\cup\mathfrak{N}=\mathfrak{L}\cup\mathfrak{M}\cup\mathfrak{N}$ and $\mathfrak{t}:\vartheta\to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta\in\mathfrak{E}$,
$\mathfrak{t}(\vartheta)=\left\{\left(\gimel, T_{\mathfrak{t}(\vartheta)}(\gimel), C_{\mathfrak{t}(\vartheta)}(\gimel), G_{\mathfrak{t}(\vartheta)}(\gimel), U_{\mathfrak{t}(\vartheta)}(\gimel), F_{\mathfrak{t}(\vartheta)}(\gimel)\right):\gimel\in\mathfrak{W}\right\}$.

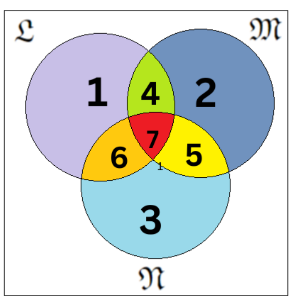

FIGURE 2. Venn diagram of seven disjoint regions

Here $\mathfrak{E}$ can be divided into seven disjoint regions and the membership values will be different in each of these seven regions. The seven disjoint regions are

1. $\mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N})$
2. $\mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L})$
3. $\mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M})$
4. $(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N}$
5. $(\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L}$
6. $(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M}$
7. $\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N}$

$$T_{\mathfrak{t}(\vartheta)}(\gimel)=\begin{cases} T_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ T_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ T_{\mathfrak{g}(\vartheta)}(\gimel)\vee T_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ T_{\mathfrak{h}(\vartheta)}(\gimel)\vee T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ T_{\mathfrak{K}(\vartheta)}(\gimel)\vee T_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ T_{\mathfrak{g}(\vartheta)}(\gimel)\vee T_{\mathfrak{h}(\vartheta)}(\gimel)\vee T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap N \end{cases}$$

$$C_{\mathfrak{t}(\vartheta)}(\gimel)=\begin{cases} C_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ C_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ C_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ C_{\mathfrak{g}(\vartheta)}(\gimel)\vee C_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ C_{\mathfrak{h}(\vartheta)}(\gimel)\vee C_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ C_{\mathfrak{K}(\vartheta)}(\gimel)\vee C_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ C_{\mathfrak{g}(\vartheta)}(\gimel)\vee C_{\mathfrak{h}(\vartheta)}(\gimel)\vee C_{k(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{C} \end{cases}$$

$$G_{\mathfrak{t}(\vartheta)}(\gimel)=\begin{cases} G_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ G_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ G_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ G_{\mathfrak{g}(\vartheta)}(\gimel)\square G_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ G_{\mathfrak{h}(\vartheta)}(\gimel)\square G_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ G_{\mathfrak{K}(\vartheta)}(\gimel)\square G_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ G_{\mathfrak{g}(\vartheta)}(\gimel)\square G_{\mathfrak{h}(\vartheta)}(\gimel)\square G_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$U_{\mathfrak{t}(\vartheta)}(\gimel)=\begin{cases} U_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ U_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ U_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ U_{\mathfrak{g}(\vartheta)}(\gimel)\wedge U_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ U_{\mathfrak{h}(\vartheta)}(\gimel)\wedge U_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ U_{\mathfrak{K}(\vartheta)}(\gimel)\wedge U_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ U_{\mathfrak{g}(\vartheta)}(\gimel)\wedge U_{\mathfrak{h}(\vartheta)}(\gimel)\wedge U_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$F_{\mathfrak{t}(\vartheta)}(\gimel)=\begin{cases} F_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ F_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ F_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ F_{\mathfrak{g}(\vartheta)}(\gimel)\wedge F_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ F_{\mathfrak{h}(\vartheta)}(\gimel)\wedge F_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ F_{\mathfrak{K}(\vartheta)}(\gimel)\wedge F_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ F_{\mathfrak{g}(\vartheta)}(\gimel)\wedge F_{\mathfrak{h}(\vartheta)}(\gimel)\wedge F_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

Again assume that $(\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N})=(\mathfrak{u},\mathfrak{M})$, where $\mathfrak{M}=\mathfrak{M}\cup\mathfrak{N}$ and $\mathfrak{u}:\mathfrak{M}\to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta\in\mathfrak{M}$, $\mathfrak{u}(\vartheta)$=$\mathfrak{s}(\vartheta)$=$\left\{\left(\gimel, T_{\mathfrak{u}(\vartheta)}(\gimel), C_{\mathfrak{u}(\vartheta)}(\gimel), G_{\mathfrak{u}(\vartheta)}(\gimel), U_{\mathfrak{u}(\vartheta)}(\gimel), F_{\mathfrak{u}(\vartheta)}(\gimel)\right):\gimel\in\mathfrak{W}\right\}$

$$T_{\mathfrak{u}(\vartheta)}(\gimel)= \begin{cases} T_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - \mathfrak{M} \\ T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - \mathfrak{L} \\ T_{\mathfrak{h}(\vartheta)}(\gimel) \vee T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$C_{\mathfrak{u}(\vartheta)}(\gimel)= \begin{cases} C_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - \mathfrak{M} \\ C_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - \mathfrak{L} \\ C_{\mathfrak{h}(\vartheta)}(\gimel) \vee C_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$G_{\mathfrak{u}(\vartheta)}(\gimel)= \begin{cases} G_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - \mathfrak{M} \\ G_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - \mathfrak{L} \\ G_{\mathfrak{h}(\vartheta)}(\gimel) \square G_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$U_{\mathfrak{u}(\vartheta)}(\gimel)= \begin{cases} U_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - \mathfrak{M} \\ U_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - \mathfrak{L} \\ U_{\mathfrak{h}(\vartheta)}(\gimel) \wedge U_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

$$F_{\mathfrak{u}(\vartheta)}(\gimel)= \begin{cases} F_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - \mathfrak{M} \\ F_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - \mathfrak{L} \\ F_{\mathfrak{h}(\vartheta)}(\gimel) \wedge F_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \end{cases}$$

Now if $(\mathfrak{g}, \mathfrak{L}) \cup ((\mathfrak{h}, \mathfrak{M}) \cup (\mathfrak{K}, \mathfrak{N})) = (\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{u}, \mathfrak{M}) = (\mathfrak{v}, \mathfrak{E})$, where $\mathfrak{E} = \mathfrak{L} \cup \mathfrak{M} = \mathfrak{L} \cup \mathfrak{M} \cup \mathfrak{N}$ and $\mathfrak{v} : \vartheta \to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta \in \mathfrak{E}$,
$\mathfrak{v}(\vartheta)=\left\{ \left(\gimel, \gimel_{\mathfrak{v}(\vartheta)}(\gimel), C_{\mathfrak{v}(\vartheta)}(\gimel), G_{\mathfrak{v}(\vartheta)}(\gimel), U_{\mathfrak{v}(\vartheta)}(\gimel), F_{\mathfrak{v}(\vartheta)}(\gimel)\right) : \gimel \in \mathfrak{W}\right\}$

$$T_{\mathfrak{v}(\vartheta)}(\gimel)= \begin{cases} T_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} - (\mathfrak{M} \cup \mathfrak{N}) \\ T_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{M} - (\mathfrak{N} \cup \mathfrak{L}) \\ T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{N} - (\mathfrak{L} \cup \mathfrak{M}) \\ T_{\mathfrak{g}(\vartheta)}(\gimel) \vee T_{\mathfrak{h}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{L} \cap \mathfrak{M}) - \mathfrak{N} \\ T_{\mathfrak{h}(\vartheta)}(\gimel) \vee T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{M} \cap \mathfrak{N}) - \mathfrak{L} \\ T_{\mathfrak{K}(\vartheta)}(\gimel) \vee T_{\mathfrak{g}(\vartheta)}(\gimel) & if\ \vartheta \in (\mathfrak{N} \cap \mathfrak{L}) - \mathfrak{M} \\ T_{\mathfrak{g}(\vartheta)}(\gimel) \vee T_{\mathfrak{h}(\vartheta)}(\gimel) \vee T_{\mathfrak{K}(\vartheta)}(\gimel) & if\ \vartheta \in \mathfrak{L} \cap \mathfrak{M} \cap \mathfrak{N} \end{cases}$$

$$C_{\mathfrak{v}(\vartheta)}(\daleth)= \begin{cases} C_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ C_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ C_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ C_{\mathfrak{g}(\vartheta)}(\daleth)\vee C_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ C_{\mathfrak{h}(\vartheta)}(\daleth)\vee C_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ C_{\mathfrak{K}(\vartheta)}(\daleth)\vee C_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ C_{\mathfrak{g}(\vartheta)}(\daleth)\vee C_{\mathfrak{h}(\vartheta)}(\daleth)\vee C_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$G_{\mathfrak{v}(\vartheta)}(\daleth)= \begin{cases} G_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ G_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ G_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ G_{\mathfrak{g}(\vartheta)}(\daleth)\Box G_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ G_{\mathfrak{h}(\vartheta)}(\daleth)\Box G_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ G_{\mathfrak{K}(\vartheta)}(\daleth)\Box G_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ G_{\mathfrak{g}(\vartheta)}(\daleth)\Box G_{\mathfrak{h}(\vartheta)}(\daleth)\Box G_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$U_{\mathfrak{v}(\vartheta)}(\daleth)= \begin{cases} U_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ U_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ U_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ U_{\mathfrak{g}(\vartheta)}(\daleth)\wedge U_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ U_{\mathfrak{h}(\vartheta)}(\daleth)\wedge U_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ U_{\mathfrak{K}(\vartheta)}(\daleth)\wedge U_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ U_{\mathfrak{g}(\vartheta)}(\daleth)\wedge U_{\mathfrak{h}(\vartheta)}(\daleth)\wedge U_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$F_{\mathfrak{v}(\vartheta)}(\daleth)= \begin{cases} F_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}-(\mathfrak{M}\cup\mathfrak{N}) \\ F_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{M}-(\mathfrak{N}\cup\mathfrak{L}) \\ F_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{N}-(\mathfrak{L}\cup\mathfrak{M}) \\ F_{\mathfrak{g}(\vartheta)}(\daleth)\wedge F_{\mathfrak{h}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ F_{\mathfrak{h}(\vartheta)}(\daleth)\wedge F_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{M}\cap\mathfrak{N})-\mathfrak{L} \\ F_{\mathfrak{K}(\vartheta)}(\daleth)\wedge F_{\mathfrak{g}(\vartheta)}(\daleth) & if\, \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ F_{\mathfrak{g}(\vartheta)}(\daleth)\wedge F_{\mathfrak{h}(\vartheta)}(\daleth)\wedge F_{\mathfrak{K}(\vartheta)}(\daleth) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$\therefore (\mathfrak{t},\mathfrak{E}) = (\mathfrak{v},\mathfrak{E})$

i.e, $((\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M}))\cup(\mathfrak{K},\mathfrak{N}) = (\mathfrak{g},\mathfrak{L})\cup((\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N}))$

The second one can be proved similarly.

(ii) Distributive Law:

Let $(\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N})=(\mathfrak{s},\mathfrak{D})$ (say), is an $IVPNSS$ over $\mathfrak{W}$, where $\mathfrak{D}=\mathfrak{M}\cup\mathfrak{N}$ and for $\vartheta\in\mathfrak{D}$, $\mathfrak{s}:\mathfrak{D}\to IVPNS^{\mathfrak{W}}$ is given by

$\mathfrak{s}(\vartheta)=\{(\beth,T_{\mathfrak{s}(\vartheta)}(\beth),C_{\mathfrak{s}(\vartheta)}(\beth),G_{\mathfrak{s}(\vartheta)}(\beth),U_{\mathfrak{s}(\vartheta)}(\beth),F_{\mathfrak{s}(\vartheta)}(\beth)):\beth\in\mathfrak{W}\}$

$$T_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases}T_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{N}\\ T_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{N}-\mathfrak{M}\\ T_{\mathfrak{h}(\vartheta)}(\beth)\vee T_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$C_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases}C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{N}\\ C_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{N}-\mathfrak{M}\\ C_{\mathfrak{h}(\vartheta)}(\beth)\vee C_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$G_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases}G_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{N}\\ G_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{N}-\mathfrak{M}\\ G_{\mathfrak{h}(\vartheta)}(\beth)\square G_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$U_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases}U_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{N}\\ U_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{N}-\mathfrak{M}\\ U_{\mathfrak{h}(\vartheta)}(\beth)\wedge U_{\mathfrak{K}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$F_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases}F_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{N}\\ F_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{N}-\mathfrak{M}\\ F_{\mathfrak{g}(\vartheta)}(\beth)\wedge F_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

Now, let $(\mathfrak{g},\mathfrak{L})\cap((\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N}))=(\mathfrak{g},\mathfrak{L})\cap(\mathfrak{s},\mathfrak{D})=(\mathfrak{t},\mathfrak{E})$ where $\mathfrak{E}=\mathfrak{L}\cap\mathfrak{D}=\mathfrak{L}\cap(\mathfrak{M}\cup\mathfrak{N})=(\mathfrak{L}\cap\mathfrak{M})\cup(\mathfrak{L}\cap\mathfrak{N})$ and $\mathfrak{t}:\vartheta\to IVPNS^{\mathfrak{W}}$ is given by for $\vartheta\in E$,

$\mathfrak{t}(\vartheta)=\{(x,T_{\mathfrak{t}(\vartheta)}(x),C_{\mathfrak{t}(\vartheta)}(x),G_{\mathfrak{t}(\vartheta)}(x),U_{\mathfrak{t}(\vartheta)}(x),F_{\mathfrak{t}(\vartheta)}(x):)\,\beth\in\mathfrak{W}\}$

$$T_{\mathfrak{t}(\vartheta)}(\beth)=\begin{cases}T_{\mathfrak{g}(\vartheta)}(\beth)\wedge T_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N}\\ T_{\mathfrak{K}(\vartheta)}(\beth)\wedge T_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M}\\ T_{\mathfrak{g}(\vartheta)}(\beth)\wedge\left(T_{\mathfrak{h}(\vartheta)}(\beth)\vee T_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$C_{\mathfrak{t}(\vartheta)}(\beth)=\begin{cases}C_{\mathfrak{g}(\vartheta)}(\beth)\wedge C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N}\\ C_{\mathfrak{K}(\vartheta)}(\beth)\wedge C_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M}\\ C_{\mathfrak{g}(\vartheta)}(\beth)\wedge\left(C_{\mathfrak{h}(\vartheta)}(\beth)\vee C_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$G_{\mathfrak{t}(\vartheta)}(\beth)=\begin{cases}G_{\mathfrak{g}(\vartheta)}(\beth)\square G_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N}\\ G_{\mathfrak{K}(\vartheta)}(\beth)\square G_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M}\\ G_{\mathfrak{g}(\vartheta)}(\beth)\square\left(G_{\mathfrak{h}(\vartheta)}(\beth)\square G_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N}\end{cases}$$

$$U_{\mathfrak{t}(\vartheta)}(\daleth)= \begin{cases} U_{\mathfrak{g}(\vartheta)}(\daleth) \vee U_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ U_{\mathfrak{K}(\vartheta)}(\daleth) \vee U_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ U_{\mathfrak{g}(\vartheta)}(\daleth) \vee \left(U_{\mathfrak{h}(\vartheta)}(\daleth) \wedge U_{\mathfrak{K}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$F_{\mathfrak{t}(\vartheta)}(\daleth)= \begin{cases} F_{\mathfrak{g}(\vartheta)}(\daleth) \vee F_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ F_{\mathfrak{K}(\vartheta)}(\daleth) \vee F_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ F_{\mathfrak{g}(\vartheta)}(\daleth) \vee \left(F_{\mathfrak{h}(\vartheta)}(\daleth) \wedge F_{\mathfrak{K}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

Again assume that $(\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M}) = (\mathfrak{u},\mathfrak{M})$ (say), is an $IVPNSS$ over $\mathfrak{W}$,where $\mathfrak{M} = \mathfrak{L}\cap\mathfrak{M}$ and for $\vartheta \in \mathfrak{M}$, $\mathfrak{u} : \mathfrak{M} \to IVPNS^{\mathfrak{W}}$ is given by

$\mathfrak{u}(\vartheta) = \left\{ \left(\daleth, T_{\mathfrak{u}(\vartheta)}(\daleth), C_{\mathfrak{u}(\vartheta)}(\daleth), G_{\mathfrak{u}(\vartheta)}(\daleth), U_{\mathfrak{u}(\vartheta)}(\daleth), F_{\mathfrak{u}(\vartheta)}(\daleth)\right) : \daleth \in \mathfrak{W}\right\}$

where, $T_{\mathfrak{u}(\vartheta)}(\daleth)$=$T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{h}(\vartheta)}(\daleth)$, $C_{\mathfrak{u}(\vartheta)}(\daleth)$=$C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{h}(\vartheta)}(\daleth)$, $G_{\mathfrak{u}(\vartheta)}(\daleth)$=$G_{\mathfrak{g}(\vartheta)}(\daleth)\Box G_{\mathfrak{h}(\vartheta)}(\daleth)$, $U_{\mathfrak{u}(\vartheta)}(\daleth)$=$U_{\mathfrak{g}(\vartheta)}(\daleth) \vee U_{\mathfrak{h}(\vartheta)}(\daleth)$, $F_{\mathfrak{u}(\vartheta)}(\daleth)$=$F_{\mathfrak{g}(\vartheta)}(\daleth) \vee F_{\mathfrak{h}(\vartheta)}(\daleth)$

Again $(\mathfrak{g},\mathfrak{L})\cap(\mathfrak{K},\mathfrak{N}) = (\mathfrak{v},\mathfrak{N})$ (say), is an $IVPNSS$ over $\mathfrak{W}$, where $\mathfrak{N} = \mathfrak{L}\cap\mathfrak{N}$ and for $\vartheta \in \mathfrak{N}$, $\mathfrak{v} : \mathfrak{N} \to IVPNS^{\mathfrak{W}}$ is given by

$\mathfrak{v}(\vartheta)$=$\left\{ \left(\daleth, T_{\mathfrak{v}(\vartheta)}(\daleth), C_{\mathfrak{v}(\vartheta)}(\daleth), G_{\mathfrak{v}(\vartheta)}(\daleth), U_{\mathfrak{v}(\vartheta)}(\daleth), F_{\mathfrak{v}(\vartheta)}(\daleth)\right) : \daleth \in \mathfrak{W}\right\}$

where, $T_{\mathfrak{v}(\vartheta)}(\daleth)$=$T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{K}(\vartheta)}(\daleth)$, $C_{\mathfrak{v}(\vartheta)}(\daleth)$=$C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{K}(\vartheta)}(\daleth)$, $G_{\mathfrak{v}(\vartheta)}(\daleth)$=$G_{\mathfrak{g}(\vartheta)}(\daleth)\Box G_{\mathfrak{K}(\vartheta)}(\daleth)$, $U_{\mathfrak{v}(\vartheta)}(\daleth)$=$U_{\mathfrak{g}(\vartheta)}(\daleth) \vee U_{\mathfrak{K}(\vartheta)}(\daleth)$, $F_{(\vartheta)}(\daleth)$=$F_{\mathfrak{g}(\vartheta)}(\daleth) \vee F_{\mathfrak{K}}(\vartheta)(\daleth)$

Now, $((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M})) \cup ((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{K},\mathfrak{N}))$=$(\mathfrak{u},\mathfrak{M}) \cup (\mathfrak{v},\mathfrak{N}) = (\mathfrak{j},\mathfrak{E})$ where $\mathfrak{E} = \mathfrak{M}\cup\mathfrak{N} = (\mathfrak{L}\cap\mathfrak{M})\cup(\mathfrak{L}\cap\mathfrak{N})$ and $\mathfrak{j} : \vartheta \to IVPNS^{\mathfrak{W}}$ is given by for $\vartheta \in \mathfrak{E}$,
$\mathfrak{j}(\vartheta)$=$\left\{ \left(\daleth, T_{\mathfrak{j}(\vartheta)}(\daleth), C_{\mathfrak{j}(\vartheta)}(\daleth), G_{\mathfrak{j}(\vartheta)}(\daleth), U_{\mathfrak{j}(\vartheta)}(\daleth), F_{\mathfrak{j}(\vartheta)}(\daleth) :\right) \daleth \in \mathfrak{W}\right\}$
where,

$$T_{\mathfrak{j}(\vartheta)}(\daleth)= \begin{cases} T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ T_{\mathfrak{K}(\vartheta)}(\daleth) \wedge T_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ \left(T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{g}(\vartheta)}(\daleth)\right) \vee \left(T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{h}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$= \begin{cases} T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge T_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ T_{\mathfrak{K}(\vartheta)}(\daleth) \wedge T_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ T_{\mathfrak{g}(\vartheta)}(\daleth) \wedge \left(T_{\mathfrak{h}(\vartheta)}(\daleth) \vee T_{\mathfrak{K}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$C_{\mathfrak{j}(\vartheta)}(\daleth)= \begin{cases} C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ C_{\mathfrak{K}(\vartheta)}(\daleth) \wedge C_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ \left(C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{h}(\vartheta)}(\daleth)\right) \vee \left(C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{K}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$= \begin{cases} C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge C_{\mathfrak{h}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ C_{\mathfrak{K}(\vartheta)}(\daleth) \wedge C_{\mathfrak{g}(\vartheta)}(\daleth) & if\ \vartheta \in (\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ C_{\mathfrak{g}(\vartheta)}(\daleth) \wedge \left(C_{\mathfrak{h}(\vartheta)}(\daleth) \vee C_{\mathfrak{K}(\vartheta)}(\daleth)\right) & if\ \vartheta \in \mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$G_{\mathfrak{j}(\vartheta)}(\beth)=\begin{cases} G_{\mathfrak{g}(\vartheta)}(\beth)\Box G_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ G_{\mathfrak{K}(\vartheta)}(\beth)\Box G_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ \left(G_{\mathfrak{g}(\vartheta)}(\beth)\Box G_{\mathfrak{h}(\vartheta)}(\beth)\right)\Box\left(G_{\mathfrak{g}(\vartheta)}(\beth)\Box G_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$=\begin{cases} G_{\mathfrak{g}(\vartheta)}(\beth)\Box G_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ G_{\mathfrak{K}(\vartheta)}(\beth)\Box G_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ G_{\mathfrak{g}(\vartheta)}(\beth)\Box\left(G_{\mathfrak{h}(\vartheta)}(\beth)\Box G_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$U_{\mathfrak{j}(\vartheta)}(\beth)=\begin{cases} U_{\mathfrak{g}(\vartheta)}(\beth)\vee U_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ U_{\mathfrak{K}(\vartheta)}(\beth)\vee U_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ \left(U_{\mathfrak{g}(\vartheta)}(\beth)\vee U_{\mathfrak{h}(\vartheta)}(\beth)\right)\wedge\left(U_{\mathfrak{g}(\vartheta)}(\beth)\vee U_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$=\begin{cases} U_{\mathfrak{g}(\vartheta)}(\beth)\vee U_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ U_{\mathfrak{K}(\vartheta)}(\beth)\vee U_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ U_{\mathfrak{g}(\vartheta)}(\beth)\vee\left(U_{\mathfrak{h}(\vartheta)}(\beth)\wedge U_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$F_{\mathfrak{j}(\vartheta)}(\beth)=\begin{cases} F_{\mathfrak{g}(\vartheta)}(\beth)\vee F_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ F_{\mathfrak{K}(\vartheta)}(\beth)\vee F_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ \left(F_{\mathfrak{g}(\vartheta)}(\beth)\vee F_{\mathfrak{h}(\vartheta)}(\beth)\right)\wedge\left(F_{\mathfrak{g}(\vartheta)}(\beth)\vee F_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$$=\begin{cases} F_{\mathfrak{g}(\vartheta)}(\beth)\vee F_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{L}\cap\mathfrak{M})-\mathfrak{N} \\ F_{\mathfrak{K}(\vartheta)}(\beth)\vee F_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in(\mathfrak{N}\cap\mathfrak{L})-\mathfrak{M} \\ F_{\mathfrak{g}(\vartheta)}(\beth)\vee\left(F_{\mathfrak{h}(\vartheta)}(\beth)\wedge F_{\mathfrak{K}(\vartheta)}(\beth)\right) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M}\cap\mathfrak{N} \end{cases}$$

$\therefore\ \mathfrak{t}(\vartheta)=\mathfrak{j}(\vartheta)$

i.e, $(\mathfrak{g},\mathfrak{L})\cap((\mathfrak{h},\mathfrak{M})\cup(\mathfrak{K},\mathfrak{N}))=((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{h},\mathfrak{M}))\cup((\mathfrak{g},\mathfrak{L})\cap(\mathfrak{K},\mathfrak{N}))$

The other one can be proved in similar way.

(iii) Commutative Law:

Let $(\mathfrak{g},\mathfrak{L})\cup(\mathfrak{h},\mathfrak{M})=(\mathfrak{s},\mathfrak{D})$, where $\mathfrak{D}=\mathfrak{L}\cup\mathfrak{M}$ and $\mathfrak{s}:\mathfrak{D}\to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta\in\mathfrak{D}$, $\mathfrak{s}(\vartheta)=\left\{\left(\beth,T_{\mathfrak{s}(\vartheta)}(\beth),C_{\mathfrak{s}(\vartheta)}(\beth),G_{\mathfrak{s}(\vartheta)}(\beth),U_{\mathfrak{s}(\vartheta)}(\beth),F_{\mathfrak{s}(\vartheta)}(\beth)\right):\beth\in\mathfrak{W}\right\}$

$$T_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases} T_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}-\mathfrak{M} \\ T_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{L} \\ T_{\mathfrak{g}(\vartheta)}(\beth)\vee T_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$C_{\mathfrak{s}(\vartheta)}(\beth)=\begin{cases} C_{\mathfrak{g}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}-\mathfrak{M} \\ C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{M}-\mathfrak{L} \\ C_{\mathfrak{g}(\vartheta)}(\beth)\vee C_{\mathfrak{h}(\vartheta)}(\beth) & if\ \vartheta\in\mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$G_{\mathfrak{s}(\vartheta)}(\gimel)= \begin{cases} G_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ G_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ G_{\mathfrak{g}(\vartheta)}(\gimel)\square G_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$U_{\mathfrak{s}(\vartheta)}(\gimel)= \begin{cases} U_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ U_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ U_{\mathfrak{g}(\vartheta)}(\gimel)\wedge U_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$F_{\mathfrak{s}(\vartheta)}(\gimel)= \begin{cases} F_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ F_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ F_{\mathfrak{g}(\vartheta)}(\gimel)\wedge F_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

Again assume that, Let $(\mathfrak{h},\mathfrak{M})\cup(\mathfrak{g},\mathfrak{L})=(\mathfrak{t},\mathfrak{D})$, where $\mathfrak{D}=\mathfrak{M}\cup\mathfrak{L}$ and $\mathfrak{t}:\mathfrak{D}\to IVPNS^{\mathfrak{W}}$ is given by, for $\vartheta\in\mathfrak{W}$, $\mathfrak{t}(\vartheta)=\left\{\left(\gimel, T_{\mathfrak{t}(\vartheta)}(\gimel), C_{\mathfrak{t}(\vartheta)}(\gimel), G_{\mathfrak{t}(\vartheta)}(\gimel), U_{\mathfrak{t}(\vartheta)}(\gimel), F_{\mathfrak{t}(\vartheta)}(\gimel\gimel)\right):\gimel\in\mathfrak{W}\right\}$

$$T_{\mathfrak{t}(\vartheta)}(\gimel)= \begin{cases} T_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ T_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ T_{\mathfrak{h}(\vartheta)}(\gimel)\vee T_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}\cap\mathfrak{L} \end{cases} = \begin{cases} T_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ T_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ T_{\mathfrak{g}(\vartheta)}(\gimel)\vee T_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$C_{\mathfrak{t}(\vartheta)}(\gimel)= \begin{cases} C_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ C_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ C_{\mathfrak{h}(\vartheta)}(\gimel)\vee C_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}\cap\mathfrak{L} \end{cases} = \begin{cases} C_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ C_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ C_{\mathfrak{g}(\vartheta)}(\gimel)\vee C_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$G_{\mathfrak{t}(\vartheta)}(\gimel)= \begin{cases} G_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ G_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ G_{\mathfrak{h}(\vartheta)}(\gimel)\square G_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}\cap\mathfrak{L} \end{cases} = \begin{cases} G_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ G_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ G_{\mathfrak{g}(\vartheta)}(\gimel)\square G_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$U_{\mathfrak{t}(\vartheta)}(\gimel)= \begin{cases} U_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ U_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ U_{\mathfrak{h}(\vartheta)}(\gimel)\wedge U_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}\cap\mathfrak{L} \end{cases} = \begin{cases} U_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ U_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ U_{\mathfrak{g}(\vartheta)}(\gimel)\wedge U_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$$F_{\mathfrak{t}(\vartheta)}(\gimel)= \begin{cases} F_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ F_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ F_{\mathfrak{h}(\vartheta)}(\gimel)\wedge F_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}\cap\mathfrak{L} \end{cases} = \begin{cases} F_{\mathfrak{g}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}-\mathfrak{M} \\ F_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{M}-\mathfrak{L} \\ F_{\mathfrak{g}(\vartheta)}(\gimel)\wedge F_{\mathfrak{h}(\vartheta)}(\gimel) & if\, \vartheta \in \mathfrak{L}\cap\mathfrak{M} \end{cases}$$

$\therefore \mathfrak{s}(\vartheta) = \mathfrak{t}(\vartheta)$

$i.e, (\mathfrak{s}, \mathfrak{D}) = (\mathfrak{t}, \mathfrak{D})$

$i.e, (\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{M}) = (\mathfrak{h}, \mathfrak{M}) \cup (\mathfrak{g}, \mathfrak{L})$

Similarly the remaining one can be proved.

(iv) De Morgans Law

Let, $\mathfrak{g} : \mathfrak{L} \rightarrow IVPNS^{\mathfrak{W}}$ and $\mathfrak{h} : \mathfrak{L} \rightarrow IVPNS^{\mathfrak{W}}$ and $(\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{L}) = (\mathfrak{s}, \mathfrak{L})$, where $\mathfrak{s} : \mathfrak{L} \rightarrow IVPNS^{\mathfrak{W}}$ is given by, for $a \in \mathfrak{L}$, $\mathfrak{s}(a)=\left\{\left(\beth, T_{\mathfrak{s}(a)}(\beth), C_{\mathfrak{s}(a)}(\beth), G_{\mathfrak{s}(a)}(\beth), U_{\mathfrak{s}(a)}(\beth), F_{\mathfrak{s}(a)}(\beth)\right) : \beth \in \mathfrak{W}\right\}$

where, $T_{\mathfrak{s}(a)}(\beth) = T_{\mathfrak{g}(a)}(\beth) \vee T_{\mathfrak{h}(a)}(\beth)$,

$C_{\mathfrak{s}(a)}(\beth) = C_{\mathfrak{g}(a)}(\beth) \vee C_{\mathfrak{h}(a)}(\beth)$,

$G_{\mathfrak{s}(a)}(\beth)=G_{\mathfrak{g}(a)}(\beth)\Box G_{\mathfrak{h}(a)}(\beth), U_{\mathfrak{s}(a)}(\beth)=U_{\mathfrak{g}(a)}(\beth) \wedge U_{\mathfrak{h}(a)}(\beth), F_{\mathfrak{s}(a)}(\beth)=F_{\mathfrak{g}(a)}(\beth) \wedge F_{\mathfrak{h}(a)}(\beth)$

Now,$((\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{M}))^c= (\mathfrak{s}, \mathfrak{L})^c= (\mathfrak{s}^c, \rceil\mathfrak{L})$, where for $a \in \mathfrak{L}$,

$\mathfrak{s}^c(\neg a)= \left\{\left(\beth, F_{\mathfrak{s}(a)}(\beth), U_{\mathfrak{s}(a)}(\beth), G_{\mathfrak{s}(a)}(\beth), C_{\mathfrak{s}(a)}(\beth), T_{\mathfrak{s}(a)}(\beth)\right) : \beth \in W\right\}$

Now, $(\mathfrak{g}, \mathfrak{L})^c = (\mathfrak{g}^c, \rceil\mathfrak{L})$ where

$\mathfrak{g}^c(\neg a)=\left\{\left(\beth, F_{\mathfrak{g}(a)}(\beth), U_{\mathfrak{g}(a)}(\beth), G_{\mathfrak{g}(a)}(\beth), C_{\mathfrak{g}(a)}(\beth), T_{\mathfrak{g}(a)}(\beth) : \beth \in \mathfrak{W}\right)\right\}$

$(\mathfrak{h}, \mathfrak{L})^c = (\mathfrak{h}^c, \rceil\mathfrak{L})$ wherϑ

$\text{h}^c(\neg a)=\left\{\left(\beth, F_{\mathfrak{h}(a)}(\beth), U_{\mathfrak{h}(a)}(\beth), G_{\mathfrak{h}(a)}(\beth), C_{\mathfrak{h}(a)}(\beth), T_{\mathfrak{h}(a)}(\beth) : \beth \in \mathfrak{W}\right)\right\}$

$\therefore (\mathfrak{g}, \mathfrak{L})^c \cap (\mathfrak{h}, \mathfrak{L})^c = (\mathfrak{g}^c, \rceil\mathfrak{L}) \cap (\mathfrak{h}^c, \rceil\mathfrak{L}) = (t^c, \rceil\mathfrak{L})$ (say) where,

$\mathfrak{t}(\neg a) = \left\{\left(\beth, F_{\mathfrak{g}(a)}(\beth) \wedge F_{\mathfrak{h}(a)}(\beth), U_{\mathfrak{g}(a)}(\beth) \wedge U_{\mathfrak{h}(a)}(\beth), G_{\mathfrak{g}(a)}(\beth)\Box G_{\mathfrak{h}(a)}(\beth), C_{\mathfrak{g}(a)}(\beth) \vee C_{\mathfrak{h}(a)}(\beth), T_{\mathfrak{g}(a)}(\beth) \vee T_{\mathfrak{h}(a)}(\beth) : \beth \in \mathfrak{W}\right)\right\}$

$=\mathfrak{s}^c(\neg a)$

Therefore, $(\mathfrak{s}, \mathfrak{L})^c = (\mathfrak{t}^c, \rceil\mathfrak{L})$

Consequently, $((\mathfrak{g}, \mathfrak{L}) \cup (\mathfrak{h}, \mathfrak{L}))^c =(\mathfrak{g}, \mathfrak{L})^c \cap (\mathfrak{h}, \mathfrak{L})^c$.

The other part can be proved similarly.

(v) Idempotent Law:

The proof follows directly.

(vi) Absorption Law:

The proof follows directly.

(vii) Involution Law:

Follows directly from definition.

4. Strength and drawbacks of the induced method:

Identifying the strength helps an individual to leverage the method effectively in a specific suitable situation whereas drawbacks of a method provide opportunities for refinement and development of a better method. So to ensure the capability of the method strength and drawbacks of the induced method is highlighted below:

Strength:

• Strength of the method lies in the fact that it can addresses five significant membership values which are very much essential during rating of an object or to describe a object efficiently.

• By using intervals, IVPNSS provides the decision-maker flexibility by allowing membership values to fall within a range.

• IVPNSS can capture contradictory and incomplete information effectively and making it suitable for complex systems where uncertainty is inherent.

Drawbacks:

• Compared to other current approaches, the calculating process employing basic operating laws is time-consuming because IVPNSS includes five membership functions.

• During decision making process, some expert may get confused when they have to provide a rating using IVPNSS.

However, the aforementioned disadvantage can be eliminated if the equation for transforming linguistic terms into IVPNS is created. In this case, decision-makers can express their opinions in linguistic terms, which can then be converted into IVPNS.

5. **Conclusions**

In conclusion, while Pentapartitioned Fuzzy Soft Sets ($PNSS$) offer a valuable tool for decision making problems, their effectiveness is limited when dealing with membership values that fall outside a crisp number. This paper addressed this limitation by proposing a new upgraded soft set theory specifically designed to handle such scenarios. By incorporating this new theory, decision makers can leverage $IVPNSS$ to achieve a higher degree of precision and accuracy in situations involving uncertain or interval-based membership values. Relations on $IVPNSS$s can be defined as [7] along with a new hybrid model by combining this concept with rough set as established in [6]. Topological structures can also be defined as in [8], [9], [14]. The decision-making difficulty can be resolved with the help of the recently developed approach. It can be utilized directly in multi-criteria decision-making problems, and the generalized form can also be used by employing an aggregation operator. This conclusion reiterates the importance of $IVPNSS$ and highlights the need for this upgraded soft set theory. It emphasizes the benefit of using the new theory for achieving more precise decisions.

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