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Fuzzy Logic vs. Neutrosophic Logic: Operations Logic

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Abstract. The goal of this research is first to show how different, thorough, widespread and effective are the operations logic of the neutrosophic logic compared to the fuzzy logic's operations logical. The second aim is to observe how a fully new logic, the neutrosophic logic, is established starting by changing the previous logical perspective fuzzy logic, and by changing that, we mean changing the truth values from the truth and falsity degrees membership in fuzzy logic, to the truth, falsity and indeterminacy degrees membership in neutrosophic logic; and thirdly, to observe that there is no limit to the logical discoveries - we only change the principle, then the system changes completely.

Keywords: Fuzzy Logic, Neutrosophic Logic, Logical Connectives, Operations Logic, New Logic.

1 Introduction:

There is no doubt in the fact that the mathematical logic as an intellectual practice has not been far from contemplation and the philosophical discourse, and disconnecting it from philosophy seems to be more of a systematic disconnection than a real one, because throughout the history of philosophy, the philosophers and what they have built as intellectual landmark, closed or opened, is standing on a logical foundation even if it did not come out as a symbolic mathematical logic.

Since the day Aristotle established the first logic theory which combines the first rules of the innate conclusion mechanism of the human being, it was a far-reaching step-forward to all those who came after him up till today, and that led to the epiphany that : the universe with all its physical and metaphysical notions is in fact a logical structure that needs an incredible accuracy in abstraction to show it for the beauty of the different notions in it, and the emotional impressions it makes in the common sense keeps the brain from the real perception of its logical structure. Many scientists and philosophers paid attention to the matter which is reflected in the variety and the difference of the systems, the logical references and mathematics in the different scientific fields. Among these scientists and philosophers who have strived to find this logical structure are: Professor Lotfi A. Zadeh, founder of the fuzzy logic (FL) idea, which he established in 1965 [7], and Professor Florentin Smarandache, founder of the neutrosophic logic (NL) idea, which he established in 1995 [1]. In this research and using the logical operations only of the two theories that we have sampled from the two systems, we will manage to observe which one is wider and more comprehensive to express more precisely the hidden logical structure of the universe.

2 Definition of Fuzzy and Neutrosophic Logical Connectives (Operations Logic):

The connectives (rules of inference, or operators), in any non-bivalent logic, can be defined in various ways, giving rise to lots of distinct logics. A single change in one of any connective's truth table is enough to form a (completely) different logic [2]. For example, Fuzzy Logic and Neutrosophic Logic.

2.1 One notes the fuzzy logical values of the propositions (A) and $(B)$by:

$$FL(A) = (T_A, F_A)\text{ , and } FL(B) = (T_B, F_B)$$

A fuzzy propositions (A) and (B) are real standard subsets in universal set(U), which is characterized by a truth-membership function T_A , T_B , and a falsity-membership function F_A , F_B, of $[0,1]$. That is

$$T_A : U \rightarrow [0,1]$$
$$F_A : U \rightarrow [0,1]$$
And
$$T_B : U \rightarrow [0,1]$$
$$F_B : U \rightarrow [0,1]$$

There is no restriction on the sum of T_A, F_A or T_B, F_B, so $0 \leq supT_A + supF_A \leq 1$, and $0 \leq supT_B + supF_B \leq 1$.

2.2 Two notes the neutrosophic logical values of the propositions (A) and (B) by[2]:

$$NL(A) = (T_A, I_A, F_A)\text{ , and } NL(B) = (T_B, I_B, F_B)$$

A neutrosophic propositions (A) and (B) are real standard or non-standard subsets in universal set(U), which is characterized by a truth-membership function T_A , T_B , a indeterminacy-membership function I_A , I_B and a falsity-membership function F_A , F_B, of $]^-0,1^+[$. That is

$$T_A : U \to]^-0,1^+[$$
$$I_A : U \to]^-0,1^+[$$
$$F_A : U \to]^-0,1^+[$$
And
$$T_B : U \to]^-0,1^+[$$
$$I_B : U \to]^-0,1^+[$$
$$F_B : U \to]^-0,1^+[$$

There is no restriction on the sum of T_A, I_A, F_A or T_B, I_B, F_B, so $^-0 \leq supT_A + supI_A + supF_A \leq 3^+$, and $^-0 \leq supT_B + supI_B + supF_B \leq 3^+$.[3]

2.3 Negation:

2.3.1 In Fuzzy Logic:

Negation the fuzzy propositions (A) and (B) is the following :

$$FL(\neg A) = (\{1\} - T_A, \{1\} - F_A)$$
And
$$FL(\neg B) = (\{1\} - T_B, \{1\} - F_B)$$

The negation link of the two fuzzy propositions (A) and (B) in the following truth table [6]:

(1,0)	(1,0)	(0,1)	(0,1)
(1,0)	(0,1)	(0,1)	(1,0)
(0,1)	(1,0)	(1,0)	(0,1)
(0,1)	(0,1)	(1,0)	(1,0)

2.3.2 In Neutrosophic Logic:

Negation the neutrosophic propositions (A) and (B) is the following [4]:

$$NL(\neg A) = (\{1\} \ominus T_A, \{1\} \ominus I_A, \{1\} \ominus F_A)$$
And
$$NL(\neg B) = (\{1\} \ominus T_B, \{1\} \ominus I_B, \{1\} \ominus F_B)$$

The negation link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$\neg A$	$\neg B$
(1,0,0)	(1,0,0)	(0,1,1)	(0,1,1)
(1,0,0)	(0,0,1)	(0,1,1)	(1,1,0)
(0,0,1)	(0,1,0)	(1,1,0)	(1,0,1)
(0,0,1)	(1,0,0)	(1,1,0)	(0,1,1)
(0,1,0)	(0,0,1)	(1,0,1)	(1,1,0)
(0,1,0)	(0,1,0)	(1,0,1)	(1,0,1)

2.4 Conjunction :

2.4.1 In Fuzzy Logic:

Conjunction the fuzzy propositions (A) and (B) is the following :

$$FL(A \wedge B) = (T_A \cdot T_B, F_A \cdot F_B)$$

(And, in similar way, generalized for n propositions)

The conjunction link of the two fuzzy propositions (A) and (B) in the following truth table [6] :

A	B	$A \wedge B$
(1,0)	(1,0)	(1,0)
(1,0)	(0,1)	(0,0)
(0,1)	(1,0)	(0,0)
(0,1)	(0,1)	(0,1)

2.4.2 In Neutrosophic Logic:

Conjunction the neutrosophic propositions (A) and (B) is the following [5]:

$$NL(A \wedge B) = (T_A \odot T_B, I_A \odot I_B, F_A \odot F_B)$$

(And, in similar way, generalized for n propositions)

The conjunction link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A \wedge B$
(1,0,0)	(1,0,0)	(1,0,0)
(1,0,0)	(0,0,1)	(0,0,0)
(0,0,1)	(0,1,0)	(0,0,0)
(0,0,1)	(1,0,0)	(0,0,0)
(0,1,0)	(0,0,1)	(0,0,0)
(0,1,0)	(0,1,0)	(0,1,0)

2.5 Weak or inclusive disjunction:

2.5.1 In Fuzzy Logic:

Inclusive disjunction the fuzzy propositions (A) and (B) is the following :

$$FL(A \vee B) = \big((T_A + T_B) - (T_A \cdot T_B), (F_A + F_B) - (F_A \cdot F_B)\big)$$

(And, in similar way, generalized for n propositions)

The inclusive disjunction link of the two fuzzy propositions (A) and (B) in the following truth table [6]:

A	B	$A \vee B$
(1,0)	(1,0)	(1,0)
(1,0)	(0,1)	(1,1)
(0,1)	(1,0)	(1,1)
(0,1)	(0,1)	(0,1)

2.5.2 In Neutrosophic Logic:

Inclusive disjunction the neutrosophic propositions (A) and (B) is the following [4]:

$$NL(A \vee B) = (T_A \oplus T_B \ominus T_A \odot T_B, I_A \oplus I_B \ominus I_A \odot I_B, F_A \oplus F_B \ominus F_A \odot F_B)$$

(And, in similar way, generalized for n propositions)

The inclusive disjunction link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A \vee B$
$(1,0,0)$	$(1,0,0)$	$(1,0,0)$
$(1,0,0)$	$(0,0,1)$	$(1,0,1)$
$(0,0,1)$	$(0,1,0)$	$(0,1,1)$
$(0,0,1)$	$(1,0,0)$	$(1,0,1)$
$(0,1,0)$	$(0,0,1)$	$(0,1,1)$
$(0,1,0)$	$(0,1,0)$	$(0,1,0)$

2.6Strong or exclusive disjunction:

2.6.1 In Fuzzy Logic:

Exclusive disjunction the fuzzy propositions (A) and (B) is the following :

$$\boldsymbol{FL(A \vee\vee B)} = \begin{pmatrix} (\boldsymbol{T_A} \cdot (\{\boldsymbol{1}\} - \boldsymbol{T_B}) + \boldsymbol{T_B} \cdot (\{\boldsymbol{1}\} - \boldsymbol{T_A}) - \boldsymbol{T_A} \cdot \boldsymbol{T_B} \cdot (\{\boldsymbol{1}\} - \boldsymbol{T_A}) \cdot (\{\boldsymbol{1}\} - \boldsymbol{T_B}), \\ (\boldsymbol{F_A} \cdot (\{\boldsymbol{1}\} - \boldsymbol{F_B}) + \boldsymbol{F_B} \cdot (\{\boldsymbol{1}\} - \boldsymbol{F_A}) - \boldsymbol{F_A} \cdot \boldsymbol{F_B} \cdot (\{\boldsymbol{1}\} - \boldsymbol{F_A}) \cdot (\{\boldsymbol{1}\} - \boldsymbol{F_B}) \end{pmatrix}$$

(And, in similar way, generalized for n propositions)

The exclusive disjunction link of the two fuzzy propositions (A) and (B) in the following truth table [6]:

A	B	$A \vee\vee B$
$(1,0)$	$(1,0)$	$(0,0)$
$(1,0)$	$(0,1)$	$(1,1)$
$(0,1)$	$(1,0)$	$(1,1)$
$(0,1)$	$(0,1)$	$(0,0)$

2.6.2 In Neutrosophic Logic:

Exclusive disjunction the neutrosophic propositions (A) and (B) is the following [5]:

$$\boldsymbol{NL(A \vee\vee B)} = \begin{pmatrix} (\boldsymbol{T_A} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{T_B}) \oplus \boldsymbol{T_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{T_A}) \ominus \boldsymbol{T_A} \odot \boldsymbol{T_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{T_A}) \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{T_B}), \\ (\boldsymbol{I_A} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{I_B}) \oplus \boldsymbol{I_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{I_A}) \ominus \boldsymbol{I_A} \odot \boldsymbol{I_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{I_A}) \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{I_B}), \\ (\boldsymbol{F_A} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{F_B}) \oplus \boldsymbol{F_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{F_A}) \ominus \boldsymbol{F_A} \odot \boldsymbol{F_B} \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{F_A}) \odot (\{\boldsymbol{1}\} \ominus \boldsymbol{F_B}) \end{pmatrix}$$

(And, in similar way, generalized for n propositions)

The exclusive disjunction link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A \vee\vee B$
$(1,0,0)$	$(1,0,0)$	$(0,0,0)$
$(1,0,0)$	$(0,0,1)$	$(1,0,1)$
$(0,0,1)$	$(0,1,0)$	$(0,1,1)$
$(0,0,1)$	$(1,0,0)$	$(1,0,1)$
$(0,1,0)$	$(0,0,1)$	$(0,1,1)$
$(0,1,0)$	$(0,1,0)$	$(0,0,0)$

2.7 Material conditional (implication) :

2.7.1 In Fuzzy Logic:

Implication the fuzzy propositions (A) and (B) is the following :

$$FL(A \to B) = (\{1\} - T_A + T_A \cdot T_B, \{1\} - F_A + F_A \cdot F_B)$$

The implication link of the two fuzzy propositions (A) and (B) in the following truth table [6]:

A	B	$A \to B$
$(1,0)$	$(1,0)$	$(1,0)$
$(1,0)$	$(0,1)$	$(0,1)$
$(0,1)$	$(1,0)$	$(1,0)$
$(0,1)$	$(0,1)$	$(0,1)$

2.7.2 In Neutrosophic Logic:

Implication the neutrosophic propositions (A) and (B) is the following [4]:

$$\boldsymbol{NL(A \to B)} = (\{\boldsymbol{1}\} \ominus \boldsymbol{T_A} \oplus \boldsymbol{T_A} \odot \boldsymbol{T_B}, \{\boldsymbol{1}\} \ominus \boldsymbol{I_A} \oplus \boldsymbol{I_A} \odot \boldsymbol{I_B}, \{\boldsymbol{1}\} \ominus \boldsymbol{F_A} \oplus \boldsymbol{F_A} \odot \boldsymbol{F_B})$$

The implication link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A \to B$
$(1,0,0)$	$(1,0,0)$	$(1,1,1)$
$(1,0,0)$	$(0,0,1)$	$(0,1,1)$
$(0,0,1)$	$(0,1,0)$	$(1,1,0)$
$(0,0,1)$	$(1,0,0)$	$(1,1,0)$
$(0,1,0)$	$(0,0,1)$	$(1,0,1)$
$(0,1,0)$	$(0,1,0)$	$(1,1,1)$

2.8 Material biconditional (equivalence) :

2.8.1 In Fuzzy Logic:

Equivalencethe fuzzy propositions (A) and (B) is the following :

$$FL(A \leftrightarrow B) = \begin{pmatrix} ((\{1\} - T_A + T_A \cdot T_B) \cdot (\{1\} - T_B + T_A \cdot T_B)), \\ ((\{1\} - F_A + F_A \cdot F_B) \cdot (\{1\} - F_B + F_A \cdot F_B)) \end{pmatrix}$$

The equivalence link of the two fuzzy propositions (A) and (B) in the following truth table :

A	B	$A \leftrightarrow B$
$(1,0)$	$(1,0)$	$(1,1)$
$(1,0)$	$(0,1)$	$(0,0)$
$(0,1)$	$(1,0)$	$(0,0)$
$(0,1)$	$(0,1)$	$(1,1)$

2.8.2 In Neutrosophic Logic:

Equivalencethe neutrosophic propositions (A) and (B) is the following [5]:

$$NL(A \leftrightarrow B) = \begin{pmatrix} ((\{1\} \ominus T_A \oplus T_A \odot T_B) \odot (\{1\} \ominus T_B \oplus T_A \odot T_B)), \\ ((\{1\} \ominus I_A \oplus I_A \odot I_B) \odot (\{1\} \ominus I_B \oplus I_A \odot I_B)), \\ ((\{1\} \ominus F_A \oplus F_A \odot F_B) \odot (\{1\} \ominus F_B \oplus F_A \odot F_B)) \end{pmatrix}$$

The equivalence link of the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A \leftrightarrow B$
$(1,0,0)$	$(1,0,0)$	$(1,1,1)$
$(1,0,0)$	$(0,0,1)$	$(0,1,0)$
$(0,0,1)$	$(0,1,0)$	$(1,0,0)$
$(0,0,1)$	$(1,0,0)$	$(0,1,0)$
$(0,1,0)$	$(0,0,1)$	$(1,0,0)$
$(0,1,0)$	$(0,1,0)$	$(1,1,1)$

2.9 Sheffer's connector:

2.9.1 In Fuzzy Logic:

The result of the sheffer's connector between the two fuzzy propositions (A) and (B) :

$$FL(A|\ B) = FL(\neg A \vee \neg B) = (\{1\} - T_A \cdot T_B, \{1\} - F_A \cdot F_B)$$

The result of the sheffer's connector between the two fuzzy propositions (A) and (B) in the following truth table :

A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$A\|B$
(1,0)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)
(1,0)	(0,1)	(0,1)	(1,0)	(1,1)	(1,1)
(0,1)	(1,0)	(1,0)	(0,1)	(1,1)	(1,1)
(0,1)	(0,1)	(1,0)	(1,0)	(1,0)	(1,0)

2.9.2 In Neutrosophic Logic:

The result of the sheffer's connector between the two neutrosophic propositions (A) and (B)[4]:

$$\boldsymbol{NL(A|\ B) = NL(\neg A \vee \neg B) = (\{1\} \ominus T_A \odot T_B, \{1\} \ominus I_A \odot I_B, \{1\} \ominus F_A \odot F_B)}$$

The result of the sheffer's connector between the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$A\|B$
(1,0,0)	(1,0,0)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)
(1,0,0)	(0,0,1)	(0,1,1)	(1,1,0)	(1,1,1)	(1,1,1)
(0,0,1)	(0,1,0)	(1,1,0)	(1,0,1)	(1,1,1)	(1,1,1)
(0,0,1)	(1,0,0)	(1,1,0)	(0,1,1)	(1,1,1)	(1,1,1)
(0,1,0)	(0,0,1)	(1,0,1)	(1,1,0)	(1,1,1)	(1,1,1)
(0,1,0)	(0,1,0)	(1,0,1)	(1,0,1)	(1,0,1)	(1,0,1)

2.10 Peirce's connector:

2.10.1 In Fuzzy Logic:

The result of the Peirce's connectorbetween the two fuzzy propositions $(A)$and (B) :

$$\boldsymbol{FL(A \downarrow B) = FL(\neg A \wedge \neg B) = \big((\{1\} - T_A) \cdot (\{1\} - T_B), (\{1\} - F_A) \cdot (\{1\} - F_B)\big)}$$

The result of the peirce's connectorbetween the two fuzzy propositions (A) and (B) in the following truth table :

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$	$A \downarrow B$
(1,0)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)
(1,0)	(0,1)	(0,1)	(1,0)	(0,0)	(0,0)
(0,1)	(1,0)	(1,0)	(0,1)	(0,0)	(0,0)
(0,1)	(0,1)	(1,0)	(1,0)	(1,0)	(1,0)

2.10.2 In Neutrosophic Logic:

The result of the Peirce's connectorbetween the two neutrosophic propositions (A) and (B)[5]:

$$\boldsymbol{NL(A \downarrow B) = NL(\neg A \wedge \neg B) = \big((\{1\} \ominus T_A) \odot (\{1\} \ominus T_B), (\{1\} \ominus I_A) \odot (\{1\} \ominus I_B), (\{1\} \ominus F_A) \odot (\{1\} \ominus F_B)\big)}$$

The result of the peirce's connectorbetween the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$	$A \downarrow B$
(1,0,0)	(1,0,0)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)
(1,0,0)	(0,0,1)	(0,1,1)	(1,1,0)	(0,1,0)	(0,1,0)
(0,0,1)	(0,1,0)	(1,1,0)	(1,0,1)	(1,0,1)	(1,0,1)
(0,0,1)	(1,0,0)	(1,1,0)	(0,1,1)	(0,1,0)	(0,1,0)
(0,1,0)	(0,0,1)	(1,0,1)	(1,1,0)	(1,0,0)	(1,0,0)
(0,1,0)	(0,1,0)	(1,0,1)	(1,0,1)	(1,0,1)	(1,0,1)

3 Conclusion :

From what has been discussed previously, we can ultimately reach three points :

3.1 We see that the logical operations of the neutrosophic logic (NL) are different from the logical operations of the fuzzy logic (FL) in terms of width, comprehensiveness and effectiveness. The reason behind that is the addition of professor Florentin Smarandache of anew field to the real values, the truth and falsity interval in (FL) and that is what he called « the indeterminacy interval » which is expressed in the function I_A or I_B in the logical operations of (NL) as we have seen, and that is what makes (NL) the closest and most precise image of the hidden logical structure of the universe.

3.2 We see that (NL) is a fully new logic, that has been established starting by changing a principle (FL), we mean by this principle changing the real values of the truth and falsity membership degrees only in (FL) to the truth and indeterminacy then falsity membership degrees in (NL).

3.3 We see that there is no limit to the logical discoveries, we only have to change the principle and that leads to completely change the system. So what if we also change the truth values from the truth and indeterminacy and falsity membership degrees in (NL), and that is by **doubling** it, as follows :

The neutrosophic propositions (A) is real standard or non-standard subsets in universal set(U), which is characterized by a truth-membership function T_A , a indeterminacy-membership function I_A , and a falsity-membership function F_A , of $]^-0, 1^+[$. That is

$$T_A : U \to]^-0, 1^+[$$
$$I_A : U \to]^-0, 1^+[$$
$$F_A : U \to]^-0, 1^+[$$

Let T_A , is real standard or non-standard subset in universal set(U), which is characterized by a truth-truth membership function T_{T_A} , a indeterminacy-truth membership function I_{T_A} , and a falsity-truth membership function F_{T_A} , of $]^-0, 1^+[$. That is

$$T_{T_A} : U \to]^-0, 1^+[$$
$$I_{T_A} : U \to]^-0, 1^+[$$
$$F_{T_A} : U \to]^-0, 1^+[$$

There is no restriction on the sum of $T_{T_A}, I_{T_A}, F_{T_A}$, so $^{-}0 \leq supT_{T_A} + supI_{T_A} + supF_{T_A} \leq 3^{+}$.
Let I_A, is real standard or non-standard subset in universal set(U), which is characterized by a truth-indeterminacy membership function T_{I_A}, a indeterminacy-indeterminacy membership function I_{I_A}, and a falsity-indeterminacy membership function F_{I_A}, of $]^{-}0, 1^{+}[$. That is

$$T_{I_A} : U \to]^{-}0, 1^{+}[$$
$$I_{I_A} : U \to]^{-}0, 1^{+}[$$
$$F_{I_A} : U \to]^{-}0, 1^{+}[$$

There is no restriction on the sum of $T_{I_A}, I_{I_A}, F_{I_A}$, so $^{-}0 \leq supT_{I_A} + supI_{I_A} + supF_{I_A} \leq 3^{+}$.
Let F_A, is real standard or non-standard subset in universal set(U), which is characterized by a truth-falsity membership function T_{F_A}, a indeterminacy-falsity membership function I_{F_A}, and a falsity-falsity membership function F_{F_A}, of $]^{-}0, 1^{+}[$. That is

$$T_{F_A} : U \to]^{-}0, 1^{+}[$$
$$I_{F_A} : U \to]^{-}0, 1^{+}[$$
$$F_{F_A} : U \to]^{-}0, 1^{+}[$$

There is no restriction on the sum of $T_{F_A}, I_{F_A}, F_{F_A}$,so $^{-}0 \leq supT_{F_A} + supI_{F_A} + supF_{F_A} \leq 3^{+}$.
Therefore :

$$T_{TA} + I_{T_A} + F_{T_A} : U \to]^{-}0, 3^{+}[$$

$$T_{I_A} + I_{I_A} + F_{I_A} : U \to]^{-}0, 3^{+}[$$

$$T_{F_A} + I_{F_A} + F_{F_A} : U \to]^{-}0, 3^{+}[$$

There is no restriction on the sum of $T_{T_A}, I_{T_A}, F_{T_A}$, and of$T_{I_A}, I_{I_A}, F_{I_A}$, and of$T_{F_A}, I_{F_A}, F_{F_A}$, so $^{-}0 \leq supT_{T_A} + supI_{T_A} + supF_{T_A} + supT_{I_A} + supI_{I_A} + supF_{I_A} + supT_{F_A} + supI_{F_A} + supF_{F_A} \leq 9^{+}$.
Therefore :

$$(T_{T_A}, I_{T_A}, F_{T_A}), (T_{I_A}, I_{I_A}, F_{I_A}), (T_{F_A}, I_{F_A}, F_{F_A})) : U \to]^{-}0, 1^{+}[\text{^}9$$

This example: we suggest to be named: **Double Neutrosophic Logic** (**DNL**).

This is a particular case of Neutrosophic Logic and Set of Type-2 (and Type-n), introduced by Smarandache [8] in 2017, as follows:

> *"Definition of Type-2 (and Type-n) Neutrosophic Set (and Logic).*
> Type-2 Neutrosophic Set is actually a neutrosophic set of a neutrosophic set.
> See an example for a type-2 single-valued neutrosophic set below:
> Let x(0.4 <0.3, 0.2, 0.4>, 0.1 <0.0, 0.3, 0.8>, 0.7 <0.5, 0.2, 0.2>) be an element in the neutrosophic set A, which means the following: x(0.4, 0.1, 0.7) belongs to the neutrosophic set A in the following way, the truth value of x is 0.4, the indeterminacy value of x is 0.1, and the falsity value of x is 0.7 [this is type-1 neutrosophic set]; but the neutrosophic probability that the truth value of x is 0.4 with respect to the neutrosophic set A is <0.3, 0.2, 0.4>, the neutrosophic probability that the indeterminacy value of x is 0.1 with respect to the neutrosophic set A is <0.0, 0.3, 0.8>, and the neutrosophic probability that the falsity value of x is 0.7 with respect to the neutrosophic set A is <0.5, 0.2, 0.2> [now this is type-2 neutrosophic set].
>
> So, in a type-2 neutrosophic set, when an element x(t, i, f) belongs to a neutrosophic set A, we are not sure about the values of t, i, f, we only get each of them with a given neutrosophic probability.
>
> Neutrosophic Probability (NP) of an event E is defined as: NP(E) = (chance that E occurs, indeterminate chance about E occurrence, chance that E does not occur).
> Similarly, a type-2 fuzzy set is a fuzzy set of a fuzzy set.
> And a type-2 intuitionistic fuzzy set is an intuitionistic fuzzy set of an intuitionistic fuzzy set.
> Surely, one can define a type-3 neutrosophic set (which is a neutrosophic set of a neutrosophic set of a neutrosophic set), and so on (*type-n neutrosophic set*, for n ≥ 2), but they become useless and confusing.
> Neither in fuzzy set nor in intuitionistic fuzzy set the researchers went further that type-2."

Hence : $(FL) \to (NL) \to (\boldsymbol{DNL}) \to NLn.$

Especially in quantum theory, there is an uncertainty about the energy and the momentum of particles. And, because the particles in the subatomic world don't have exact positions, we better calculate their **double neutrosophic probabilities** (i.e. computation a truth-truth percent, indeterminacy-truth percent, falsity-truth percent, and truth-indeterminacy percent, indeterminacy-indeterminacy percent, falsity-indeterminacy percent, and truth-falsity percent, indeterminacy-falsity percent, falsity-falsity percent) of being at some particular points than their neutrosophic probabilities.

3.4 Definition of Double Neutrosophic Logical Connectives (Operations Logic) :

One notes the double neutrosophic logical values of the propositions (A) and (B) by:

$$DNL(A) = \left((T_{T_A}, I_{T_A}, F_{T_A}), (T_{I_A}, I_{I_A}, F_{I_A}), (T_{F_A}, I_{F_A}, F_{F_A})\right)$$

And

$$DNL(B) = \left((T_{T_B}, I_{T_B}, F_{T_B}), (T_{I_B}, I_{I_B}, F_{I_B}), (T_{F_B}, I_{F_B}, F_{F_B})\right)$$

3.4.1 Negation:

$$\boldsymbol{DNL(\neg A)} =$$
$$\left((\{1\} \ominus \boldsymbol{T_{T_A}}, \{1\} \ominus \boldsymbol{I_{T_A}}\{1\} \ominus \boldsymbol{F_{T_A}}), (\{1\} \ominus \boldsymbol{T_{I_A}}, \{1\} \ominus \boldsymbol{I_{I_A}}\{1\} \ominus \boldsymbol{F_{I_A}}), (\{1\} \ominus \boldsymbol{T_{F_A}}, \{1\} \ominus \boldsymbol{I_{F_A}}\{1\} \ominus \boldsymbol{F_{F_A}})\right)$$

And

$$\boldsymbol{DNL(\neg B)} =$$
$$\left((\{1\} \ominus \boldsymbol{T_{T_B}}, \{1\} \ominus \boldsymbol{I_{T_B}}\{1\} \ominus \boldsymbol{F_{T_B}}), (\{1\} \ominus \boldsymbol{T_{I_B}}, \{1\} \ominus \boldsymbol{I_{I_B}}\{1\} \ominus \boldsymbol{F_{I_B}}), (\{1\} \ominus \boldsymbol{T_{F_B}}, \{1\} \ominus \boldsymbol{I_{F_B}}\{1\} \ominus \boldsymbol{F_{F_B}})\right)$$

3.4.2 Conjunction :

$$DNL(A \wedge B) =$$

$$(T_{T_A} \odot T_{T_B}, I_{T_A} \odot I_{T_B}, F_{T_A} \odot F_{T_B}), (T_{I_A} \odot T_{I_B}, I_{I_A} \odot I_{I_B}, F_{I_A} \odot F_{I_B}), (T_{F_A} \odot T_{F_B}, I_{F_A} \odot I_{F_B}, F_{F_A} \odot F_{F_B})$$

(And, in similar way, generalized for n propositions)

3.4.3 Weak or inclusive disjunction :

$$DNL(A \vee B) =$$

$$\left. \begin{matrix} (T_{T_A} \oplus T_{T_B} \ominus T_{T_A} \odot T_{T_B}, I_{T_A} \oplus I_{T_B} \ominus I_{T_A} \odot I_{T_B}, F_{T_A} \oplus F_{T_B} \ominus F_{T_A} \odot F_{T_B}), \\ (T_{I_A} \oplus T_{I_B} \ominus T_{I_A} \odot T_{I_B}, I_{I_A} \oplus I_{I_B} \ominus I_{I_A} \odot I_{I_B}, F_{I_A} \oplus F_{I_B} \ominus F_{I_A} \odot F_{I_B}), \\ (T_{F_A} \oplus T_{F_B} \ominus T_{F_A} \odot T_{F_B}, I_{F_A} \oplus I_{F_B} \ominus I_{F_A} \odot I_{F_B}, F_{F_A} \oplus F_{F_B} \ominus F_{F_A} \odot F_{F_B}) \end{matrix} \right)$$

(And, in similar way, generalized for n propositions)

3.4.4 Strong or exclusive disjunction :

$$DNL(A \vee\vee B) =$$

$$\begin{pmatrix} \begin{pmatrix} (T_{T_A} \odot (\{1\} \ominus T_{T_B}) \oplus T_{T_B} \odot (\{1\} \ominus T_{T_A}) \ominus T_{T_A} \odot T_{T_B} \odot (\{1\} \ominus T_{T_A}) \odot (\{1\} \ominus T_{T_B}), \\ (I_{T_A} \odot (\{1\} \ominus I_{T_B}) \oplus I_{T_B} \odot (\{1\} \ominus I_{T_A}) \ominus I_{T_A} \odot I_{T_B} \odot (\{1\} \ominus I_{T_A}) \odot (\{1\} \ominus I_{T_B}), \\ F_{T_A} \odot (\{1\} \ominus F_{T_B}) \oplus F_{T_B} \odot (\{1\} \ominus F_{T_A}) \ominus F_{T_A} \odot F_{T_B} \odot (\{1\} \ominus F_{T_A}) \odot (\{1\} \ominus F_{T_B} \end{pmatrix}, \\ \begin{pmatrix} T_{I_A} \odot (\{1\} \ominus T_{I_B}) \oplus T_{I_B} \odot (\{1\} \ominus T_{I_A}) \ominus T_{I_A} \odot T_{I_B} \odot (\{1\} \ominus T_{I_A}) \odot (\{1\} \ominus T_{I_B}), \\ I_{I_A} \odot (\{1\} \ominus I_{I_B}) \oplus I_{I_B} \odot (\{1\} \ominus I_{I_A}) \ominus I_{I_A} \odot I_{I_B} \odot (\{1\} \ominus I_{I_A}) \odot (\{1\} \ominus I_{I_B}), \\ F_{I_A} \odot (\{1\} \ominus F_{I_B}) \oplus F_{I_B} \odot (\{1\} \ominus F_{I_A}) \ominus F_{I_A} \odot F_{I_B} \odot (\{1\} \ominus F_{I_A}) \odot (\{1\} \ominus F_{I_B} \end{pmatrix}, \\ \begin{pmatrix} (T_{F_A} \odot (\{1\} \ominus T_{F_B}) \oplus T_{F_B} \odot (\{1\} \ominus T_{F_A}) \ominus T_{F_A} \odot T_{F_B} \odot (\{1\} \ominus T_{F_A}) \odot (\{1\} \ominus T_{F_B}), \\ (I_{F_A} \odot (\{1\} \ominus I_{F_B}) \oplus I_{F_B} \odot (\{1\} \ominus I_{F_A}) \ominus I_{F_A} \odot I_{F_B} \odot (\{1\} \ominus I_{F_A}) \odot (\{1\} \ominus I_{F_B}), \\ (F_{F_A} \odot (\{1\} \ominus F_{F_B}) \oplus F_{F_B} \odot (\{1\} \ominus F_{F_A}) \ominus F_{F_A} \odot F_{F_B} \odot (\{1\} \ominus F_{F_A}) \odot (\{1\} \ominus F_{F_B} \end{pmatrix} \end{pmatrix}$$

(And, in similar way, generalized for n propositions)

3.4.5 Material conditional (implication) :

$$DNL(A \rightarrow B) =$$

$$\begin{pmatrix} \{1\} \ominus T_{T_A} \oplus T_{T_A} \odot T_{T_B}, \{1\} \ominus I_{T_A} \oplus I_{T_A} \odot I_{T_B}, \{1\} \ominus F_{T_A} \oplus F_{T_A} \odot F_{T_B}), \\ \{1\} \ominus T_{I_A} \oplus T_{I_A} \odot T_{I_B}, \{1\} \ominus I_{I_A} \oplus I_{I_A} \odot I_{I_B}, \{1\} \ominus F_{I_A} \oplus F_{I_A} \odot F_{I_B}), \\ \{1\} \ominus T_{F_A} \oplus T_{F_A} \odot T_{F_B}, \{1\} \ominus I_{F_A} \oplus I_{F_A} \odot I_{F_B}, \{1\} \ominus F_{F_A} \oplus F_{F_A} \odot F_{F_B} \end{pmatrix}$$

3.4.6 Material biconditional (equivalence) :

$$DNL(A \leftrightarrow B) =$$

$$\begin{pmatrix} \begin{pmatrix} ((\{1\} \ominus T_{T_A} \oplus T_{T_A} \odot T_{T_B}) \odot (\{1\} \ominus T_{T_B} \oplus T_{T_A} \odot T_{T_B})), \\ ((\{1\} \ominus I_{T_A} \oplus I_{T_A} \odot I_{T_B}) \odot (\{1\} \ominus I_{T_B} \oplus I_{T_A} \odot I_{T_B})), \\ ((\{1\} \ominus F_{T_A} \oplus F_{T_A} \odot F_{T_B}) \odot (\{1\} \ominus F_{T_B} \oplus F_{T_A} \odot F_{T_B}) \end{pmatrix}, \\ \begin{pmatrix} ((\{1\} \ominus T_{I_A} \oplus T_{I_A} \odot T_{I_B}) \odot (\{1\} \ominus T_{I_B} \oplus T_{I_A} \odot T_{I_B})), \\ ((\{1\} \ominus I_{I_A} \oplus I_{I_A} \odot I_{I_B}) \odot (\{1\} \ominus I_{I_B} \oplus I_{I_A} \odot I_{I_B})), \\ ((\{1\} \ominus F_{I_A} \oplus F_{I_A} \odot F_{I_B}) \odot (\{1\} \ominus F_{I_B} \oplus F_{I_A} \odot F_{I_B}) \end{pmatrix}, \\ \begin{pmatrix} ((\{1\} \ominus T_{F_A} \oplus T_{F_A} \odot T_{F_B}) \odot (\{1\} \ominus T_{F_B} \oplus T_{F_A} \odot T_{F_B})), \\ ((\{1\} \ominus I_{F_A} \oplus I_{F_A} \odot I_{F_B}) \odot (\{1\} \ominus I_{F_B} \oplus I_{F_A} \odot I_{F_B})), \\ ((\{1\} \ominus F_{F_A} \oplus F_{F_A} \odot F_{F_B}) \odot (\{1\} \ominus F_{F_B} \oplus F_{F_A} \odot F_{F_B}) \end{pmatrix} \end{pmatrix}$$

3.4.7 Sheffer's connector :

$$DNL(A | B) = DNL(\neg A \vee \neg B) =$$

$$\begin{pmatrix} \{1\} \ominus T_{T_A} \odot T_{T_B}, \{1\} \ominus I_{T_A} \odot I_{T_B}, \{1\} \ominus F_{T_A} \odot F_{T_B}), \\ \{1\} \ominus T_{I_A} \odot T_{I_B}, \{1\} \ominus I_{I_A} \odot I_{I_B}, \{1\} \ominus F_{I_A} \odot F_{I_B}), \\ \{1\} \ominus T_{F_A} \odot T_{F_B}, \{1\} \ominus I_{F_A} \odot I_{F_B}, \{1\} \ominus F_{F_A} \odot F_{F_B} \end{pmatrix}$$

3.4.8 Peirce's connector :

$$DNL(A \downarrow B) = DNL(\neg A \wedge \neg B) =$$

$$\begin{pmatrix} \{1\} \ominus T_{T_A} \odot \{1\} \ominus T_{T_B}, \{1\} \ominus I_{T_A} \odot \{1\} \ominus I_{T_B}, \{1\} \ominus F_{T_A} \odot \{1\} \ominus F_{T_B}), \\ \{1\} \ominus T_{I_A} \odot \{1\} \ominus T_{I_B}, \{1\} \ominus I_{I_A} \odot \{1\} \ominus I_{I_B}, \{1\} \ominus F_{I_A} \odot \{1\} \ominus F_{I_B}), \\ \{1\} \ominus T_{F_A} \odot \{1\} \ominus T_{F_B}, \{1\} \ominus I_{F_A} \odot \{1\} \ominus I_{F_B}, \{1\} \ominus F_{F_A} \odot \{1\} \ominus F_{F_B} \end{pmatrix}$$

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