NSS

Neutrosophic Sets and Systems, Vol. 80, 2025

University of New Mexico

Hypersoft Semi-Connected Spaces

Florentin Smarandache[1], V.Inthumathi[2] and M.Amsaveni[3,*]

[1] "Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA;
smarand@unm.edu

[2] Department of Mathematics, Nallamuthu Gounder Mahalingam College,
Tamil Nadu, India;
inthumathi65@gmail.com

[3] Department of Mathematics, Nallamuthu Gounder Mahalingam College,
Tamil Nadu, India;
amsavenim2000@gmail.com

[*]Correspondence: amsavenim2000@gmail.com"

Abstract. Neutrosophic sets are an important field of topology that deals with inconsistent, indeterminate, and uncertain circumstances in real life. Hypersoft sets have grown in popularity due to their ability to represent data as a collection of trait-valued disjoint sets that combine various attributes. Motivated by this concept, we introduce a new hyper-approach known as hypersoft semi-separated, which employs hypersoft semi-open sets [HSO]. This article presents several local properties of HSO sets, such as hypersoft semi-neighborhoods of hypersoft points and hypersoft semi-first countable spaces. Additionally, definitions of hypersoft semi-connected [HSCO] spaces, hypersoft locally semi-connected [HLSCO] spaces, and associated properties are provided. The comparison between hypersoft-connected [HCO] spaces, HSCO, and HLSCO spaces is presented along with their examples.

Keywords: HSO sets, hypersoft semi-first countable spaces, hypersoft semi-separation, HSCO, HLSCO.

---

1. Introduction

Molodtsov [18] developed the mathematical method of soft set theory to deal with uncertainty. The study of soft set theory and its applications has been explored by numerous researchers [6,7,17,33]. Shabir and Naz [36] established the concept of soft topological spaces by building soft sets over an initial universe set with a certain set of parameters.

Sasikala and Sivaraj [35] discovered the soft semi-open and soft semi-closed sets in soft topological spaces [STS]. Bin Chen [4] described the local properties of soft semi-open sets. Krishnaveni and Sekar [16] defined the characteristics of soft semi-connected and soft locally semi-connected spaces.

Smarandache [37] devised a clever strategy for dealing with ambiguity. By switching to a multiple decision function, the soft sets were extended into hypersoft sets. Based on the hypersoft sets and their expansion, several researchers have developed a wide range of operators, features, and applications [1, 2, 10, 12]. Hypersoft topology, hypersoft closure, hypersoft interior, hypersoft connected spaces, and hypersoft separation axioms were investigated by Musa and Asaad [21–23]. Inthumathi and Amsaveni [14] introduced hypersoft subspace topology, hypersoft basis, hypersoft limit point, and hypersoft Hausdorff spaces. Inthumathi et al. [13] presented HSO sets, hypersoft semi-closed [HSC] sets, hypersoft semi-interior, and hypersoft semi-closure.

As an extension of hypersoft sets, the N-Hypersoft set was introduced by Musa et al. In [32], the N-Hypersoft set was intended to manage analyses using binary and non-binary data while maintaining an intuitive sense of structural symmetry. The superhypersoft set [41] is an extension of the hypersoft set that was first introduced by Smarandache. In addition, he introduced superhyperstructure, superhyperfunction [39], and superhypertopology [40]. An extension of the hypersoft set, from determinate to indeterminate data, is the indetermhypersoft set [8, 38]. The concept of hypersoft sets is expanded to Pythagorean neutrosophic hypersoft sets, and double-frame soft sets are extended to double-frame hypersoft sets by Muhammad Saeed et al. [19,31]. With the help of this extension, we may manage intricate decision-making situations, including several qualities and their interactions, in a strong framework. Novel approaches have been demonstrated by recent studies that have advanced a variety of hypersoft sectors [3, 11, 20, 24–29, 34].

In recent days, Neutrosophic sets play a dominant role in addressing uncertainty in decision-making difficulties. In [9, 30, 42, 43], Smarandache expanded his ideas through neutrosophic sets in various domains. In future, we plan to concentrate on strengthening our concept for the aforementioned extended neutrosophic domains and their applications.

The current work is structured into the subsequent sections: In Section 2, some fundamental concepts regarding hypersoft sets and hypersoft topological spaces (HTS) are summarized. Section 3 defines the hypersoft semi-neighborhood system, hypersoft semi-first countable spaces, and associated properties. New ideas like hypersoft semi-separation and hypersoft semi-connectedness via hypersoft semi-open sets in an HTS are presented in Section 4. Section 5 explains the concept of HLSCO spaces and the connection between HCO, HSCO, and HLSCO spaces. Section 6 narrates the conclusion.

2. Preliminaries

This section aims to provide a clear overview of the planned study by going over some fundamental hypersoft set attributes. For further information and fundamental terminology, see [5, 15]

The following table describes the various abbreviations and definitions used throughout the paper.

Abbreviation	Definition
STS	Soft topological spaces
HTS	Hypersoft topological spaces
HSO	Hypersoft semi-open
HSC	Hypersoft semi-closed
HCO	Hypersoft connected
HSCO	Hypersoft semi-connected
HLSCO	Hypersoft locally semi-connected
HS.S.C	Hypersoft semi component

Definition 2.1. [18] “Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U. The pair (F, E) or simply F_E, is called a soft set over U, where F is a mapping given by $F : E \rightarrow P(U)$.

Definition 2.2. [37] Let U be a universe of discourse, P(U) the power set of U and $E_1, E_2, \cdots, E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each $i = 1, 2, ..., n$. A hypersoft set can be identified by the pair $(\mathcal{F}, A_1 * A_2 * \cdots * A_n)$, where $\mathcal{F} : A_1 * A_2 * \cdots * A_n \rightarrow P(U)$. For sake of simplicity, we write the symbols $\mathbf{E}$ for $E_1 * E_2 * \cdots * E_n$, $\mathbf{A}$ for $A_1 * A_2 * \cdots * A_n$.

Definition 2.3. [21] Let τ be a collection of hypersoft sets over U, then τ is called a hypersoft topology over U if

(1) $(\emptyset, \mathbf{A})$ and$(\mathcal{F}, \mathbf{A})$ belongs to τ,
(2) The intersection of any two hypersoft sets in τ belongs to τ,
(3) The union of any number of a hypersoft sets in τ belongs to τ.
Then $((\mathcal{F}, \mathbf{A}), \tau)$ is called a HTS over U.

Definition 2.4. [21] Let $((\mathcal{F}, \mathbf{A}), \tau)$ be a HTS and let $(\mathcal{F}, \mathbf{B})$ be a hypersoft set then

(1) The hypersoft interior of $(\mathcal{F}, \mathbf{B})$ is the hypersoft set
$h\text{-}int(\mathcal{F}, \mathbf{B}) = \bigcup\{(\mathcal{F}, \mathbf{C}) : (\mathcal{F}, \mathbf{C})$ is hypersoft open and $(\mathcal{F}, \mathbf{C}) \subseteq (\mathcal{F}, \mathbf{B})\}$.
(2) The hypersoft closure of $(\mathcal{F}, \mathbf{B})$ is the hypersoft set
$h\text{-}cl(\mathcal{F}, \mathbf{B}) = \bigcap\{(\mathcal{F}, \mathbf{C}) : (\mathcal{F}, \mathbf{C})$ is hypersoft closed and $(\mathcal{F}, \mathbf{B}) \subseteq (\mathcal{F}, \mathbf{C})\}$.

Definition 2.5. [13] Let $(\mathcal{F}, \mathbf{B})$ be a hypersoft set of a HTS $((\mathcal{F}, \mathbf{A}), \tau)$. $(\mathcal{F}, \mathbf{B})$ is called a HSO set if $(\mathcal{F}, \mathbf{B}) \subseteq h\text{-}cl(h\text{-}int(\mathcal{F}, \mathbf{B}))$.

Definition 2.6. [13] A hypersoft set $(\mathcal{F}, \mathbf{B})$ in a HTS $((\mathcal{F}, \mathbf{A}), \tau)$ is called a HSC set if its relative complement is a HSO set.

Definition 2.7. [13] Let $((\mathcal{F}, \mathbf{A}), \tau)$ be a HTS and let $(\mathcal{F}, \mathbf{B})$ be a hypersoft set in $(\mathcal{F}, \mathbf{A})$.

(1) The hypersoft semi-interior of $(\mathcal{F}, \mathbf{B})$ is the hypersoft set
$\bigcup\{(\mathcal{F}, \mathbf{C}) : (\mathcal{F}, \mathbf{C})$ is HSO and $(\mathcal{F}, \mathbf{C}) \subseteq (\mathcal{F}, \mathbf{B})\}$ and it is denoted by $h\text{-}sint(\mathcal{F}, \mathbf{B})$.

(2) The hypersoft semi-closure of $(\mathcal{F}, \mathbf{B})$ is the hypersoft set
$\bigcap\{(\mathcal{F}, \mathbf{C}) : (\mathcal{F}, \mathbf{C})$ is HSC and $(\mathcal{F}, \mathbf{B}) \subseteq (\mathcal{F}, \mathbf{C})\}$ and it is denoted by $h\text{-}scl(\mathcal{F}, \mathbf{B})$.
Clearly, $h\text{-}scl(\mathcal{F}, \mathbf{B})$ is the smallest HSC set containing $(\mathcal{F}, \mathbf{B})$ and
$h\text{-}sint(\mathcal{F}, \mathbf{B})$ is the largest HSO set contained in $(\mathcal{F}, \mathbf{B})$.

Definition 2.8. [22] Let $((\mathcal{F}, \mathbf{A}), \tau)$ be a HTS. Two non-null hypersoft sets $(\mathcal{F}, \mathbf{A})$ and $(\mathcal{F}, \mathbf{B})$ are said to be separated hypersoft sets iff $(\mathcal{F}, \mathbf{A}) \bigcap h{-}cl(\mathcal{F}, \mathbf{B}) = \Phi$ and $h{-}cl(\mathcal{F}, \mathbf{A}) \bigcap (\mathcal{F}, \mathbf{B}) = \Phi$.

Note that any two separated hypersoft sets are disjoint hypersoft sets.

Definition 2.9. [22] A HTS $((\mathcal{F}, \mathbf{A}), \tau)$ is called hypersoft disconnected iff $(\mathcal{F}, \mathbf{A})$ can be expressed as the union of two non-null separated hypersoft sets. Otherwise, $((\mathcal{F}, \mathbf{A}), \tau)$ is called hypersoft connected."

3. **Hypersoft Semi-Neighborhoods**

Definition 3.1. A hypersoft set $(\Omega, \mathfrak{A})$ in a HTS $((\Omega, \mathcal{P}), \tau)$ is called a hypersoft semi-neighborhood of the hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$ if there is a HSO set $(\Omega, \mathfrak{B}) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B}) \subseteq (\Omega, \mathfrak{A})$.

The semi-neighborhood system of a hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$ which is denoted by $\mathcal{H}^{\mathcal{N}}{}_{(x, \Omega_{\mathfrak{A}}(x))}$ is the set of all its semi-neighborhoods.

Example 3.2. Let $Y = \{y_1, y_2, y_3\}$, $P_1 = \{p_1, p_2\}$, $P_2 = \{p_3\}$, $P_3 = \{p_4\}$ and let Ω is a mapping from $\mathcal{P} \longrightarrow P(Y)$.

Then the hypersoft set is defined by

$$(\Omega, \mathcal{P}) = \left\{ \begin{array}{c} (p_1, p_3, p_4), \{y_1, y_2\} \\ ((p_2, p_3, p_4), \{y_2, y_3\}) \end{array} \right\}$$

Consider all the hypersoft subsets of $(\Omega, \mathcal{P})$ as follows:

$(\Omega, \mathcal{P})_1 = \{(p_1, p_3, p_4), \{y_1\}\}$

$(\Omega, \mathcal{P})_2 = \{(p_1, p_3, p_4), \{y_2\}\}$

$(\Omega, \mathcal{P})_3 = \{(p_1, p_3, p_4), \{y_1, y_2\}\}$

$(\Omega, \mathcal{P})_4 = \{(p_2, p_3, p_4), \{y_2\}\}$

$(\Omega,\mathcal{P})_5 = \{(p_2,p_3,p_4),\{y_3\}\}$

$(\Omega,\mathcal{P})_6 = \{(p_2,p_3,p_4),\{y_2,y_3\}\}$

$(\Omega,\mathcal{P})_7 = \{((p_1,p_3,p_4),\{y_1\}),((p_2,p_3,p_4),\{y_2\})\}$

$(\Omega,\mathcal{P})_8 = \{((p_1,p_3,p_4),\{y_1\}),((p_2,p_3,p_4),\{y_3\})\}$

$(\Omega,\mathcal{P})_9 = \{((p_1,p_3,p_4),\{y_1\}),((p_2,p_3,p_4),\{y_2,y_3\})\}$

$(\Omega,\mathcal{P})_{10} = \{((p_1,p_3,p_4),\{y_2\}),((p_2,p_3,p_4),\{y_2\})\}$

$(\Omega,\mathcal{P})_{11} = \{((p_1,p_3,p_4),\{y_2\}),((p_2,p_3,p_4),\{y_3\})\}$

$(\Omega,\mathcal{P})_{12} = \{((p_1,p_3,p_4),\{y_2\}),((p_2,p_3,p_4),\{y_2,y_3\})\}$

$(\Omega,\mathcal{P})_{13} = \{((p_1,p_3,p_4),\{y_1,y_2\}),((p_2,p_3,p_4),\{y_2\})\}$

$(\Omega,\mathcal{P})_{14} = \{((p_1,p_3,p_4),\{y_1,y_2\}),((p_2,p_3,p_4),\{y_3\})\}$

$(\Omega,\mathcal{P})_{15} = (\Omega,\mathcal{P})$

$(\Omega,\mathcal{P})_{16} = (\Omega,\mathcal{P})_\Phi$.

$\tau = \{(\Omega,\mathcal{P}),(\Omega,\mathcal{P})_\Phi,(\Omega,\mathcal{P})_2,(\Omega,\mathcal{P})_{11},(\Omega,\mathcal{P})_{13}\}$

Then $((\Omega,\mathcal{P}),\tau)$ is a HTS. The family of all hypersoft open sets is

$\{(\Omega,\mathcal{P}),(\Omega,\mathcal{P})_\Phi,(\Omega,\mathcal{P})_2,(\Omega,\mathcal{P})_{11},(\Omega,\mathcal{P})_{13}\}$.

The family of all hypersoft closed sets is

$\{(\Omega,\mathcal{P})_\Phi,(\Omega,\mathcal{P}),(\Omega,\mathcal{P})_9,(\Omega,\mathcal{P})_7,(\Omega,\mathcal{P})_5\}$.

The family of all HSO sets is

$\{(\Omega,\mathcal{P})_\Phi,(\Omega,\mathcal{P}),(\Omega,\mathcal{P})_2,(\Omega,\mathcal{P})_3,(\Omega,\mathcal{P})_{10},(\Omega,\mathcal{P})_{11},(\Omega,\mathcal{P})_{12},(\Omega,\mathcal{P})_{13},(\Omega,\mathcal{P})_{14}\}$.

The family of all HSC sets is

$\{(\Omega,\mathcal{P})_\Phi,(\Omega,\mathcal{P}),(\Omega,\mathcal{P})_9,(\Omega,\mathcal{P})_6,(\Omega,\mathcal{P})_8,(\Omega,\mathcal{P})_7,(\Omega,\mathcal{P})_1,(\Omega,\mathcal{P})_5,(\Omega,\mathcal{P})_4\}$.

Consider the hypersoft point $\{(p_1,p_3,p_4),\{y_2\}\}$.

Then the family of hypersoft semi-neighborhood of $\{(p_1,p_3,p_4),\{y_2\}\}$ is

$\mathcal{H}^{\mathcal{N}}{}_{\{(p_1,p_3,p_4),\{y_2\}\}} = \{(\Omega,\mathcal{P})_{10},(\Omega,\mathcal{P})_{11},(\Omega,\mathcal{P})_{12},(\Omega,\mathcal{P})_{13},(\Omega,\mathcal{P})_{14},(\Omega,\mathcal{P})_{15}\}$

Proposition 3.3. *The semi-neighborhood system $\mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ at a hypersoft point $(x,\Omega_{\mathfrak{A}}(x))$ in the HTS $((\Omega,\mathcal{P}),\tau)$ has the following results.*

(1) *If $(\Omega,\mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then $(x,\Omega_{\mathfrak{A}}(x)) \in (\Omega,\mathfrak{A})$*

(2) *If $(\Omega,\mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ and $(\Omega,\mathfrak{A}) \subseteq (\Omega,\mathfrak{B})$, then $(\Omega,\mathfrak{B}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$.*

(3) *If $(\Omega,\mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then there exists a hypersoft set $(\Omega,\mathfrak{B}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))} \ni (\Omega,\mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x_1,\Omega_{\mathfrak{A}}(x_1))}$ for each $(x_1,\Omega_{\mathfrak{A}}(x_1)) \in (\Omega,\mathfrak{B})$.*

proof:

(1) If $(\Omega,\mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then we have a HSO set $(\Omega,\mathfrak{B}) \ni (x,\Omega_{\mathfrak{A}}(x)) \in (\Omega,\mathfrak{B}) \subseteq (\Omega,\mathfrak{A})$. So we have $(x,\Omega_{\mathfrak{A}}(x)) \in (\Omega,\mathfrak{A})$.

(2) If $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ and $(\Omega, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$. Because $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, there exists a HSO set $(\Omega, \mathfrak{F}) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$. So we have $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$ and we have $(\Omega, \mathfrak{B}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$.

(3) If $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then there is a HSO set $(\Omega, \mathfrak{F}) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$. Let $(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{F})$ and for each $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{B})$,
$(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{B}) \subseteq (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$.
This shows that $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{N}}{}_{(x_1,\Omega_{\mathfrak{A}}(x_1))}$.

Definition 3.4. Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $\mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ be a hypersoft semi-neighborhood of a hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$. If for every hypersoft semi-neighborhood $(\Omega, \mathfrak{A})$ of $(x, \Omega_{\mathfrak{A}}(x))$, there is a $(\Omega, \mathfrak{F}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))} \subseteq \mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))} \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$, then $\mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ is called a hypersoft semi-neighborhoods base of $\mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ at $(x, \Omega_{\mathfrak{A}}(x))$.

Example 3.5. From example 3.2, a hypersoft semi-neighborhood base of $\mathcal{H}^{\mathcal{N}}{}_{\{(p_1,p_3,p_4),\{y_2\}\}}$ at $\{(p_1, p_3, p_4), \{y_2\}\}$ is $\mathcal{H}^{\mathcal{B}}{}_{\{(p_1,p_3,p_4),\{y_2\}\}} = (\Omega, \mathcal{P})_2$

Proposition 3.6. *Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $\mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ be a hypersoft semi-neighborhood of a hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$. $\mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ is the hypersoft semi-neighborhoods base of $\mathcal{H}^{\mathcal{N}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ at $(x, \Omega_{\mathfrak{A}}(x))$. Then one has the following:*

(1) *If $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{A})$.*

(2) *If $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$ and $(\Omega, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$, then $(\Omega, \mathfrak{B}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$.*

(3) *If $(\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))}$, then there exists a hypersoft set $(\Omega, \mathfrak{B}) \in \mathcal{H}^{\mathcal{B}}{}_{(x,\Omega_{\mathfrak{A}}(x))} \ni (\Omega, \mathfrak{A}) \in \mathcal{H}^{\mathcal{B}}{}_{(x_1,\Omega_{\mathfrak{A}}(x_1))}$ for each $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{B})$.*

proof: By looking into the relevant properties of hypersoft semi-neighborhoods in proposition 3.2, one can readily verify these properties.

Definition 3.7. Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $(x, \Omega_{\mathfrak{A}}(x))$ be a hypersoft point in $((\Omega, \mathcal{P}), \tau)$. If $(x, \Omega_{\mathfrak{A}}(x))$ has a countable hypersoft semi-neighborhoods base, then we say that $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-first-countable at $(x, \Omega_{\mathfrak{A}}(x))$. If each hypersoft point in $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-first-countable, then we say that $((\Omega, \mathcal{P}), \tau)$ is a hypersoft semi-first-countable space.

Example 3.8. From example 3.5, the hypersoft point $\{(p_1, p_3, p_4), \{y_2\}\}$ has a countable hypersoft semi-neighborhood base $(\Omega, \mathcal{P})_2$. Therefore a HTS $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-first-countable at $\{(p_1, p_3, p_4), \{y_2\}\}$.

Proposition 3.9. *Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $(x, \Omega_{\mathfrak{A}}(x))$ be a hypersoft point in $((\Omega, \mathcal{P}), \tau)$. Then $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-first countable at $(x, \Omega_{\mathfrak{A}}(x))$ iff there is a countable hypersoft semi-neighborhoods base $\{(\Omega_n, \mathfrak{A}), n \in N\}$ at $(x, \Omega_{\mathfrak{A}}(x)) \ni (\Omega_{n+1}, \mathfrak{A}) \subseteq (\Omega_n, \mathfrak{A})$ for each $n \in N$.*

proof: $\Leftarrow$Obvious.
$\Rightarrow$ Let $\{(\sigma_n, \mathfrak{A}), n \in N\}$ be a countable hypersoft semi-neighborhoods base at $(x, \Omega_{\mathfrak{A}}(x))$. For each $n \in N$ put $(\Omega_n, \mathfrak{A}) = \bigcap_{i=1}^{n} (\sigma_i, \mathfrak{A})$. Then it is easy to see that $\{(\Omega_n, \mathfrak{A}), n \in N\}$ is a hypersoft semi-neighborhoods base at $(x, \Omega_{\mathfrak{A}}(x))$ and $(\Omega_{n+1}, \mathfrak{A}) \subseteq (\Omega_n, \mathfrak{A})$ for each $n \in N$.

Definition 3.10. A hypersoft set $(\Omega, \mathfrak{A})$ in a HTS $((\Omega, \mathcal{P}), \tau)$ is called a hypersoft semi-neighborhood of the hypersoft set $(\Omega, \mathfrak{F})$ if there is a HSO set $(\Omega, \mathfrak{B}) \ni$
$(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{B}) \subseteq (\Omega, \mathfrak{A})$.

Example 3.11. From example 3.2, consider the hypersoft set $(\Omega, \mathcal{P})_1$. The family of all hypersoft semi-neighborhoods of the hypersoft set $(\Omega, \mathcal{P})_1 = \{(\Omega, \mathcal{P})_{13}, (\Omega, \mathcal{P})_{14}, (\Omega, \mathcal{P})_{15}\}$.

Proposition 3.12. *Let $((\Omega, \mathcal{P}), \tau)$ be a HTS, $(\Omega, \mathfrak{B})$ be a hypersoft set. Then $(\Omega, \mathfrak{B})$ is HSO iff for each hypersoft set $(\Omega, \mathfrak{A})$ contained in $(\Omega, \mathfrak{B})$, $(\Omega, \mathfrak{B})$ is a hypersoft semi-neighborhood of $(\Omega, \mathfrak{A})$.*

proof: $\Rightarrow$Obvious
$\Leftarrow$ Because $(\Omega, \mathfrak{B}) \subseteq (\Omega, \mathfrak{B})$, then we have a HSO set $(\Omega, \mathfrak{F}) \ni (\Omega, \mathfrak{B}) \subseteq (\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{B})$. So we have $(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{F})$ and $(\Omega, \mathfrak{B})$ is HSO.

Definition 3.13. Let the sequence $\{(\Omega_n, \mathfrak{A}), n \in N\}$ be a hypersoft sequence in a HTS $((\Omega, \mathcal{P}), \tau)$. Then $\{(\Omega_n, \mathfrak{A}), n \in N\}$ is eventually contained in a hypersoft set $(\Omega, \mathfrak{B})$ iff there is an integer $m \ni$ if $n \geq m$, then $(\Omega_n, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$.
The sequence is frequently contained in $(\Omega, \mathfrak{B})$ iff for each integer m, there is an integer $n \ni$ $n \geq m$ and $(\Omega_n, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$.

Definition 3.14. Let the sequence $\{(\Omega_n, \mathfrak{A}), n \in N\}$ be a hypersoft sequence in a HTS $((\Omega, \mathcal{P}), \tau)$, then one says $\{(\Omega_n, \mathfrak{A}), n \in N\}$ semi converges to a hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$ if it is eventually contained in each semi-neighborhood of $(x, \Omega_{\mathfrak{A}}(x))$.
The hypersoft point $(x, \Omega_{\mathfrak{A}}(x))$ is called a semi-cluster hypersoft point of $\{(\Omega_n, \mathfrak{A}), n \in N\}$ if the sequence is frequently contained in every semi-neighborhood of $(x, \Omega_{\mathfrak{A}}(x))$.

Proposition 3.15. *Let $((\Omega, \mathcal{P}), \tau)$ be a hypersoft semi-first countable, then one has the following:*

(1) *$(\Omega, \mathfrak{B})$ is HSO iff for every hypersoft sequence $\{(\Omega_n, \mathfrak{A}), n \in N\}$ which semi-converges to $(x, \Omega_{\mathfrak{A}}(x))$ in $(\Omega, \mathfrak{B})$ is eventually contained in $(\Omega, \mathfrak{B})$.*
(2) *If $(x, \Omega_{\mathfrak{A}}(x))$ is a semi-cluster hypersoft point of the hypersoft sequence $\{(\Omega_n, \mathfrak{A}), n \in N\}$, then one has a subsequence of $\{(\Omega_n, \mathfrak{A}), n \in N\}$ which semi converges to $(x, \Omega_{\mathfrak{A}}(x))$.*

Proof :

(1) Because $(\Omega, \mathfrak{B})$ is HSO, $(\Omega, \mathfrak{B})$ is a semi-neighborhood of $(x, \Omega_{\mathfrak{A}}(x))$, and $\{(\Omega_n, \mathfrak{A}), n \in N\}$ semi-converges to $(x, \Omega_{\mathfrak{A}}(x))$. Then we have $\{(\Omega_n, \mathfrak{A}), n \in N\}$ is eventually contained in $(\Omega, \mathfrak{B})$.

For each $(x, \Omega_{\mathfrak{A}}(x))$ contained in $(\Omega, \mathfrak{B})$, let $\{(\Omega_n, \mathfrak{A}), n \in N\}$ be the semi-neighborhood systems $\ni (\Omega_{n+1}, \mathfrak{A}) \subseteq (\Omega_n, \mathfrak{A})$ for each $n \in N$ by proposition 3.6.

Then $\{(\Omega_n, \mathfrak{A}), n \in N\}$ is eventually contained in each semi-neighborhood of $(x, \Omega_{\mathfrak{A}}(x))$. ie) $\{(\Omega_n, \mathfrak{A}), n \in N\}$ semi-converges to $(x, \Omega_{\mathfrak{A}}(x))$. So we have an integer $m \ni$ if $n \geq m$, $(\Omega_n, \mathfrak{A}) \subseteq (\Omega, \mathfrak{B})$. Then $(\Omega, \mathfrak{B})$ is semi-neighborhood of $(x, \Omega_{\mathfrak{A}}(x))$ and by proposition 3.8, $(\Omega, \mathfrak{B})$ is HSO.

(2) Let $\{(\Omega_n, \mathfrak{A}), n \in N\}$ be the semi-neighborhoods $\ni (\Omega_{n+1}, \mathfrak{A}) \subseteq (\Omega_n, \mathfrak{A})$ for each $n \in N$ by proposition 3.6. For every non-negative integer i, find n_i satisfies $n_i \geq i$ and $(\Omega_{n_i}, \mathfrak{A}) \subseteq (\Omega_i, \mathfrak{A})$. Then $\{(\Omega_{n_i}, \mathfrak{A}), i \in N\}$ is a subsequence of the sequence $\{(\Omega_n, \mathfrak{A}), n \in N\}$. Obviously this subsequence semi-converges to $(x, \Omega_{\mathfrak{A}}(x))$.

4. **HSCO Spaces**

Definition 4.1. Let $((\Omega, \mathcal{P}), \tau)$ be a HTS. A hypersoft semi separation on $(\Omega, \mathcal{P})$ is a pair $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ of non- null HSO sets $\ni (\Omega, \mathcal{P}) = (\Omega, \mathfrak{A}) \cup (\Omega, \mathfrak{B})$,$(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$. In otherwords, The two non-null hypersoft subsets $(\Omega, \mathfrak{A}), (\Omega, \mathfrak{B})$ of a HTS $((\Omega, \mathcal{P}), \tau)$ are said to be hypersoft semi-separated iff $h - scl(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$ and $(\Omega, \mathfrak{A}) \cap h - scl(\Omega, \mathfrak{B}) = \Phi$.

Definition 4.2. A HTS $((\Omega, \mathcal{P}), \tau)$ is called HSCO space if there is no hypersoft semi-separations on $(\Omega, \mathcal{P})$. If $((\Omega, \mathcal{P}), \tau)$ has hypersoft semi-separations then $((\Omega, \mathcal{P}), \tau)$ is called hypersoft semi disconnected space.

Example 4.3. From example 3.2, we see that $(\Omega, \mathcal{P})$ cannot be expressed as the union of two hypersoft semi-separated sets and hence $((\Omega, \mathcal{P}), \tau)$ is HSCO.

Theorem 4.4. *Let $((\Omega, \mathcal{P}), \tau)$ be a HTS. $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ are hypersoft semi-separations on $(\Omega, \mathcal{P})$. If $(\Omega, \mathfrak{F})$ is a HSCO subspace of $((\Omega, \mathcal{P}), \tau)$, then one has $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$ or $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{B})$.*

proof: Since $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ are HSO sets, then we have $(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{F})$ and $(\Omega, \mathfrak{B}) \cap (\Omega, \mathfrak{F})$ are also HSO sets. Hence $(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{F})$ and $(\Omega, \mathfrak{B}) \cap (\Omega, \mathfrak{F})$ are hypersoft semi-separations of $(\Omega, \mathfrak{F})$ and this is contradiction. So one of $(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{F})$ or $(\Omega, \mathfrak{B}) \cap (\Omega, \mathfrak{F})$ is Φ and thus $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$ or $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{B})$.

Theorem 4.5. *Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $(\Omega, \mathfrak{F})$ is a HSCO subspace of $((\Omega, \mathcal{P}), \tau)$. If $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{C}) \subseteq \overline{(\Omega, \mathfrak{F})}$, then $(\Omega, \mathfrak{C})$ is HSCO.*

proof: Suppose $(\Omega, \mathfrak{C})$ is not HSCO, then there exist non-null HSO sets $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ which form a hypersoft semi-separation of $(\Omega, \mathfrak{C})$. Then by theorem 4.4, we have $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$ or $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{B})$. Suppose $(\Omega, \mathfrak{F}) \subseteq (\Omega, \mathfrak{A})$, then we have $(\Omega, \mathfrak{C}) \subseteq \overline{(\Omega, \mathfrak{F})} \subseteq \overline{(\Omega, \mathfrak{A})}$. So $(\Omega, \mathfrak{C}) \cap (\Omega, \mathfrak{B}) \subseteq \overline{(\Omega, \mathfrak{A})} \cap (\Omega, \mathfrak{B}) = (\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$. Hence $(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{B}) \cap (\Omega, \mathfrak{C}) = \Phi$ a contradiction. Thus, we have $(\Omega, \mathfrak{C})$ is HSCO.

Theorem 4.6. *A HTS $((\Omega, \mathcal{P}), \tau)$ is HSCO iff the both HSO and HSC sets are only Φ and $(\Omega, \mathcal{P})$.*

proof: $\Rightarrow$ Let the HTS $((\Omega, \mathcal{P}), \tau)$ be HSCO if $(\Omega, \mathfrak{A})$ is both HSO and HSC in $((\Omega, \mathcal{P}), \tau)$ which is different from Φ and $(\Omega, \mathcal{P})$. So $(\Omega, \mathfrak{A})^c$ is HSO set in $((\Omega, \mathcal{P}), \tau)$ which is different from Φ and $(\Omega, \mathcal{P})$. Then $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{A})^c$ is a hypersoft semi-separation of $(\Omega, \mathcal{P})$. This is a contradiction. So the both HSO and HSC hypersoft sets are only Φ and $(\Omega, \mathcal{P})$.

$\Leftarrow$ Let $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ be a hypersoft semi-separation of $((\Omega, \mathcal{P}), \tau)$. Let $(\Omega, \mathfrak{A}) \neq (\Omega, \mathcal{P})$ and by definition $(\Omega, \mathfrak{A}) = (\Omega, \mathfrak{B})^c$. This proves that $(\Omega, \mathfrak{A})$ is both HSO and HSC in $((\Omega, \mathcal{P}), \tau)$ which is different from Φ and $(\Omega, \mathcal{P})$. This is a contradiction. So $((\Omega, \mathcal{P}), \tau)$ is HSCO.

Proposition 4.7. *A HTS $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-disconnected iff there exists non-null, non-whole hypersoft set which is both HSO and HSC.*

Proof: Let $(\Omega, \mathfrak{A})$ be a non-null, non-whole hypersoft set which is both HSO and HSC. We have to show that $((\Omega, \mathcal{P}), \tau)$ is hypersoft semi-disconnected. Let $(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{A})^c$. Then, $(\Omega, \mathfrak{B})$ is non-null since $(\Omega, \mathfrak{A})$ is non-whole hypersoft set. Moreover, $(\Omega, \mathfrak{A}) \cup (\Omega, \mathfrak{B}) = (\Omega, \mathcal{P})$ and $(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$. Since $(\Omega, \mathfrak{A})$ is both HSO and HSC, then $(\Omega, \mathfrak{B})$ is also HSO and HSC. Hence $h-scl(\Omega, \mathfrak{A}) = (\Omega, \mathfrak{A})$, $h-scl(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{B})$. It follows that $(\Omega, \mathfrak{A}) \cap h-scl(\Omega, \mathfrak{B}) = \Phi$ and $h-scl(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$. Thus $(\Omega, \mathcal{P})$ has been expressed as the union of two hypersoft semi-separated sets and so $((\Omega, \mathcal{P}), \tau)$ is a hypersoft disconnected.

Conversely, let $((\Omega, \mathcal{P}), \tau)$ be a hypersoft semi-disconnected. Then there exists non-null hypersoft sets $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ suth that $(\Omega, \mathfrak{A}) \cap h - scl(\Omega, \mathfrak{B}) = \Phi$ and $h - scl(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$ and $(\Omega, \mathcal{P}) = (\Omega, \mathfrak{A}) \cup (\Omega, \mathfrak{B})$. Since $(\Omega, \mathfrak{A}) \subseteq h - scl(\Omega, \mathfrak{A})$, $h - scl(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$ then $(\Omega, \mathfrak{A}) \cap (\Omega, \mathfrak{B}) = \Phi$. Hence $(\Omega, \mathfrak{A}) = (\Omega, \mathfrak{B})^c$. Since $(\Omega, \mathfrak{B})$ is non-null and $(\Omega, \mathfrak{B}) \cup (\Omega, \mathfrak{B})^c = (\Omega, \mathcal{P})$, it follows that $(\Omega, \mathfrak{B}) = (\Omega, \mathfrak{A})^c$ is a non-whole hypersoft set. Now, $(\Omega, \mathfrak{A}) \cup h - scl(\Omega, \mathfrak{B}) = (\Omega, \mathcal{P})$. Also $(\Omega, \mathfrak{A}) \cap h - scl(\Omega, \mathfrak{B}) = \Phi$ then $(\Omega, \mathfrak{A}) = (h - scl(\Omega, \mathfrak{B}))^c$ and similarly $(\Omega, \mathfrak{B}) = (h - scl(\Omega, \mathfrak{A}))^c$. Since $h - scl(\Omega, \mathfrak{A})$ and $h - scl(\Omega, \mathfrak{B})$ are HSC sets, it follows that $(\Omega, \mathfrak{A})$ and $(\Omega, \mathfrak{B})$ are HSO sets. Since $(\Omega, \mathfrak{A}) = (\Omega, \mathfrak{B})^c, (\Omega, \mathfrak{A})$ is also HSC set. Thus, $(\Omega, \mathfrak{A})$ is non-null, non-whole hypersoft set which is both HSO and HSC. We have shown incidentally that $(\Omega, \mathfrak{B})$ is also a non-null, non-whole hypersoft set which is both HSO and HSC.

Theorem 4.8. *The union of a collection of HSCO subspaces of $((\Omega,\mathcal{P}),\tau)$ that have non-null intersection is HSCO.*

proof: Let $((\Omega,\mathfrak{A}_\alpha),\tau_{\mathfrak{A}_\alpha})_{\alpha\in J}$ be an arbitrary collection of hypersoft semi connected subspaces of $((\Omega,\mathcal{P}),\tau)$. Suppose that $(\Omega,\mathfrak{A}) = \bigcup_{\alpha\in J}(\Omega,\mathfrak{A}_\alpha) = (\Omega,\mathfrak{B})\cup(\Omega,\mathfrak{C})$. Where $(\Omega,\mathfrak{B})$ and $(\Omega,\mathfrak{C})$ form a hypersoft semi-separation of $(\Omega,\mathfrak{A})$. By hypothesis, we may choose a hypersoft point $(x,\Omega_{\mathfrak{A}_\alpha}(x)) \ni (x,\Omega_{\mathfrak{A}_\alpha}(x)) \in \bigcap_{\alpha\in J}(\Omega,\mathfrak{A}_\alpha)$ and it must belong to either a hypersoft subset$(\Omega,\mathfrak{B})$ or a hypersoft subset $(\Omega,\mathfrak{C})$. Since $(\Omega,\mathfrak{B}),(\Omega,\mathfrak{C})$ are disjoint, we must have $(\Omega,\mathfrak{A}_\alpha)\subset(\Omega,\mathfrak{B})$ for all $\alpha\in J$, and so $(\Omega,\mathfrak{A})\subset(\Omega,\mathfrak{B})$. From this we obtain that $(\Omega,\mathfrak{C})=\Phi$, which is a contradiction. This proves the theorem.

Proposition 4.9. *If $(\Omega,\mathfrak{A})$ and $(\Omega,\mathfrak{B})$ are hypersoft semi-separated sets and $(\Pi,\mathfrak{A})\subseteq(\Omega,\mathfrak{A})$ and $(\Pi,\mathfrak{B})\subseteq(\Omega,\mathfrak{B})$, then $(\Pi,\mathfrak{A})$ and $(\Pi,\mathfrak{B})$ are also hypersoft semi-separated sets.*

Proof: We are given that $(\Omega,\mathfrak{A})\cap h-scl(\Omega,\mathfrak{B})=\Phi$ and $(\Omega,\mathfrak{B})\cap h-scl(\Omega,\mathfrak{A})=\Phi$. Also $(\Pi,\mathfrak{A})\subseteq(\Omega,\mathfrak{A})$ implies $h-scl(\Pi,\mathfrak{A})\subseteq h-scl(\Omega,\mathfrak{A})$ and $(\Pi,\mathfrak{B})\subseteq(\Omega,\mathfrak{B})$ implies $h-scl(\Pi,\mathfrak{B})\subseteq h-scl(\Omega,\mathfrak{B})$. It follows that $(\Pi,\mathfrak{A})\cap h-scl(\Pi,\mathfrak{B})=\Phi$ and $(\Pi,\mathfrak{A})\cap h-scl(\Pi,\mathfrak{B})=\Phi$. Hence $(\Pi,\mathfrak{A})$ and $(\Pi,\mathfrak{B})$ are hypersoft semi-separated sets.

Proposition 4.10. *Let $(\Omega,\mathfrak{A})$ and $(\Omega,\mathfrak{B})$ are hypersoft semi-separated sets of a HTS $((\Omega,\mathcal{P}),\tau)$. If $(\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B})$ is a HSC, then $(\Omega,\mathfrak{A})$ and $(\Omega,\mathfrak{B})$ are HSC.*

Proof: Suppose that $(\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B})$ is a HSC so that $h-scl((\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B}))=(\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B})$. To prove that $(\Omega,\mathfrak{A})$ and $(\Omega,\mathfrak{B})$ are HSC, we have to prove that $h-scl(\Omega,\mathfrak{A})=(\Omega,\mathfrak{A})$ and $h-scl(\Omega,\mathfrak{B})=(\Omega,\mathfrak{B})$. Since we have $h-scl((\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B}))=h-scl(\Omega,\mathfrak{A})\cup h-scl(\Omega,\mathfrak{B})$, then $h-scl(\Omega,\mathfrak{A})\cup h-scl(\Omega,\mathfrak{B})=(\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B})$.

Evidently, $h-scl(\Omega,\mathfrak{A})=h-scl(\Omega,\mathfrak{A})\cap(h-scl(\Omega,\mathfrak{A})\cup h-scl(\Omega,\mathfrak{B}))=h-scl(\Omega,\mathfrak{A})\cap((\Omega,\mathfrak{A})\cup(\Omega,\mathfrak{B}))=(h-scl((\Omega,\mathfrak{A})\cap(\Omega,\mathfrak{A}))\cup(h-scl((\Omega,\mathfrak{A})\cap(\Omega,\mathfrak{B}))=(\Omega,\mathfrak{A})\cup\Phi=(\Omega,\mathfrak{A})$. Thus $h-scl(\Omega,\mathfrak{A})=(\Omega,\mathfrak{A})$. Similarly, we can prove that $h-scl(\Omega,\mathfrak{B})=(\Omega,\mathfrak{B})$.

Proposition 4.11. *Let $((\Omega,\mathcal{P}),\tau_1)$ and $((\Omega,\mathcal{P}),\tau_2)$ be two HSCO spaces, then $((\Omega,\mathcal{P}),\tau_1\cap\tau_2)$ is a HSCO space.*

Proof: By contradiction, we assume that $((\Omega,\mathcal{P}),\tau_1\cap\tau_2)$ is not a HSCO space. Then there exist two non-empty disjoint HSO sets $(\Omega,\mathfrak{A}),(\Omega,\mathfrak{B})\in(\tau_1\cap\tau_2)\ni$ their union is $(\Omega,\mathcal{P})$ in $((\Omega,\mathcal{P}),\tau_1\cap\tau_2)$. This implies that $(\Omega,\mathfrak{A})$, $(\Omega,\mathfrak{B})$ are two non-null disjoint hypersoft sets $\ni$ their union is $(\Omega,\mathcal{P})$ in $((\Omega,\mathcal{P}),\tau_1)$ and $((\Omega,\mathcal{P}),\tau_2)$. Which is contradiction to given hypothesis. Thus $((\Omega,\mathcal{P}),\tau_1\cap\tau_2)$ is a HSCO space.

Proposition 4.12. *Let $((\Omega,\mathcal{P}),\tau_1)$ and $((\Omega,\mathcal{P}),\tau_2)$ be two hypersoft semi-disconnected space, then $((\Omega,\mathcal{P}),\tau_1\cup\tau_2)$ is a hypersoft semi-disconnected space.*

proof: This is straight forward.

Proposition 4.13. *Let $((\Omega, \mathcal{P}), \tau_1)$ and $((\Omega, \mathcal{P}), \tau_2)$ be two HTS. If $((\Omega, \mathcal{P}), \tau_1)$ is hypersoft semi-disconnected and $\tau_1 \subseteq \tau_2$, then $((\Omega, \mathcal{P}), \tau_2)$ is hypersoft semi-disconnected.*

Proof: Since $((\Omega, \mathcal{P}), \tau_1)$ is hypersoft semi-disconnected, then there exists non-null, non-whole hypersoft set $(\Omega, \mathfrak{A})$ which is both HSO and HSC in $((\Omega, \mathcal{P}), \tau_1)$. Since τ_2 is finer than τ_1, then $(\Omega, \mathfrak{A})$ is a HSO set belonging to $((\Omega, \mathcal{P}), \tau_2)$. Again, since $(\Omega, \mathfrak{A})$ is a HSC in $((\Omega, \mathcal{P}), \tau_1)$, then $(\Omega, \mathfrak{A})^c$ is a HSO set. Since τ_2 is finer than τ_1, then $(\Omega, \mathfrak{A})^c$ is a HSO set belonging to $((\Omega, \mathcal{P}), \tau_2)$ and consequently $(\Omega, \mathfrak{A})$ is a HSC set in $((\Omega, \mathcal{P}), \tau_2)$. Thus $(\Omega, \mathfrak{A})$ is a non-null, non-whole hypersoft set which is both HSO and HSC in $((\Omega, \mathcal{P}), \tau_2)$. If follows that $((\Omega, \mathcal{P}), \tau_2)$ is hypersoft semi-disconnected.

Corollary 4.14. *Let $((\Omega, \mathcal{P}), \tau_1)$ and $((\Omega, \mathcal{P}), \tau_2)$ be two HTS. If $((\Omega, \mathcal{P}), \tau_1)$ is HSCO and $\tau_2 \subseteq \tau_1$, then $((\Omega, \mathcal{P}), \tau_2)$ is HSCO.*

Definition 4.15. Let $((\Omega, \mathcal{P}), \tau)$ be a HTS and $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{A})$. The hypersoft semi component of $(x, \Omega_{\mathfrak{A}}(x))$ is the union of all hypersoft semi connected subsets of $(\Omega, \mathfrak{A})$ containing $(x, \Omega_{\mathfrak{A}}(x))$ and it is denoted by $HS.S.C(x, \Omega_{\mathfrak{A}}(x))$
From the above theorem, we see that the hypersoft semi component of $(x, \Omega_{\mathfrak{A}}(x))$ is HSCO.

Example 4.16. Consider a HTS $((\Omega, \mathcal{P}), \tau)$ in example 3.2, let $\{(p_1, p_3, p_4), \{y_2\}\} \in (\Omega, \mathcal{P})_{12}$. The hypersoft semi-connected subsets of $(\Omega, \mathcal{P})_{12}$ containing $\{(p_1, p_3, p_4), \{y_2\}\}$ are $(\Omega, \mathcal{P})_2, (\Omega, \mathcal{P})_{10}, (\Omega, \mathcal{P})_{11}$.
The hypersoft semi-component of $\{(p_1, p_3, p_4), \{y_2\}\} = (\Omega, \mathcal{P})_2 \cup (\Omega, \mathcal{P})_{10} \cup (\Omega, \mathcal{P})_{11} = (\Omega, \mathcal{P})_{12}$.

Theorem 4.17. *In a HTS $((\Omega, \mathcal{P}), \tau)$,*

(1) *each hypersoft semi component of $(x, \Omega_{\mathfrak{A}}(x))$ is a maximal HSCO set in $(\Omega, \mathcal{P})$.*
(2) *the set of all distinct hypersoft semi components of hypersoft points of $(\Omega, \mathcal{P})$ form a partition of $(\Omega, \mathcal{P})$.*
(3) *each hypersoft semi-component of $(x, \Omega_{\mathfrak{A}}(x))$ is HSC in $(\Omega, \mathcal{P})$.*

proof:

(1) From the definition, the proof is obvious.
(2) Let $HS.S.C(x, \Omega_{\mathfrak{A}}(x))$ and $HS.S.C(x_1, \Omega_{\mathfrak{A}}(x_1))$ be two hypersoft semi components of distinct hypersoft points $(x, \Omega_{\mathfrak{A}}(x))$ and $(x_1, \Omega_{\mathfrak{A}}(x_1))$ in $(\Omega, \mathcal{P})$. If $HS.S.C(x, \Omega_{\mathfrak{A}}(x)) \cap HS.S.C(x_1, \Omega_{\mathfrak{A}}(x_1)) \neq \Phi$, then by theorem 4.8, $HS.S.C(x, \Omega_{\mathfrak{A}}(x)) \cup HS.S.C(x_1, \Omega_{\mathfrak{A}}(x_1))$ is HSCO. But $HS.S.C(x, \Omega_{\mathfrak{A}}(x)) \subset HS.S.C(x, \Omega_{\mathfrak{A}}(x)) \cup HS.S.C(x_1, \Omega_{\mathfrak{A}}(x_1))$, which contradicts the maximality of $HS.S.C(x, \Omega_{\mathfrak{A}}(x))$. Now for any hypersoft point $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P}), (x, \Omega_{\mathfrak{A}}(x)) \in$

$HS.S.C(x, \Omega_{\mathfrak{A}}(x))$

and $\bigcup_{(x,\Omega_{\mathfrak{A}}(x))\in(\Omega,\mathcal{P})} (x, \Omega_{\mathfrak{A}}(x)) \subset \bigcup_{(x,\Omega_{\mathfrak{A}}(x))\in(\Omega,\mathcal{P})} HS.S.C(x, \Omega_{\mathfrak{A}}(x))$.

This implies that $(x, \Omega_{\mathfrak{A}}(x)) \in \bigcup_{(x,\Omega_{\mathfrak{A}}(x))\in(\Omega,\mathcal{P})} HS.S.C(x, \Omega_{\mathfrak{A}}(x)) \subseteq (\Omega, \mathcal{P})$.

(3) Let $(x, \Omega_{\mathfrak{A}}(x))$ be any hypersoft point in $(\Omega, \mathcal{P})$. Then $h - scl(HS.S.C(x, \Omega_{\mathfrak{A}}(x)))$is a HSCO set containing $(x, \Omega_{\mathfrak{A}}(x))$. But $HS.S.C(x, \Omega_{\mathfrak{A}}(x))$ is the maximal HSCO set containing $(x, \Omega_{\mathfrak{A}}(x))$. so $h - scl(HS.S.C(x, \Omega_{\mathfrak{A}}(x))) \subset HS.S.C(x, \Omega_{\mathfrak{A}}(x))$. Hence $HS.S.C(x, \Omega_{\mathfrak{A}}(x))$ is HSC.

5. HLSCO Spaces

Definition 5.1. A HTS $((\Omega, \mathcal{P}), \tau)$ is called HLSCO at $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P})$ iff for every HSO set (Ω, U) containing $(x, \Omega_{\mathfrak{A}}(x))$ there exists a HSCO open set $(\Omega, \mathfrak{C}) \ni$
$(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{C}) \subset (\Omega, U)$.
$((\Omega, \mathcal{P}), \tau)$ is called HLSCO iff it is a HLSCO at every hypersoft point of $(\Omega, \mathcal{P})$.

Example 5.2. Let $((\Omega, \mathcal{P}), \tau)$ be the HTS, where $(\Omega, \mathcal{P})$ and its hypersoft subsets are consider as in example 3.2. Consider $\tau = \{(\Omega, \mathcal{P}), (\Omega, \mathcal{P})_\Phi, (\Omega, \mathcal{P})_2, (\Omega, \mathcal{P})_3, (\Omega, \mathcal{P})_{11}, (\Omega, \mathcal{P})_{12}, (\Omega, \mathcal{P})_{14}\}$
Then $((\Omega, \mathcal{P}), \tau)$ is a HTS.
The family of all HSO sets is
$\{(\Omega, \mathcal{P})_\Phi, (\Omega, \mathcal{P}), (\Omega, \mathcal{P})_2, (\Omega, \mathcal{P})_3, (\Omega, \mathcal{P})_{10}, (\Omega, \mathcal{P})_{11}, (\Omega, \mathcal{P})_{12}, (\Omega, \mathcal{P})_{13}, (\Omega, \mathcal{P})_{14}\}$.
The family of all HSC sets is
$\{(\Omega, \mathcal{P})_\Phi, (\Omega, \mathcal{P}), (\Omega, \mathcal{P})_9, (\Omega, \mathcal{P})_6, (\Omega, \mathcal{P})_8, (\Omega, \mathcal{P})_7, (\Omega, \mathcal{P})_1, (\Omega, \mathcal{P})_5, (\Omega, \mathcal{P})_4\}$.
The HSO sets containing $((p_1, p_3, p_4), \{y_1, y_2\})$ are $(\Omega, \mathcal{P})_{13}, (\Omega, \mathcal{P})_{14}$ and $(\Omega, \mathcal{P})$. Clearly the hypersoft set $(\Omega, \mathcal{P})_3$ is HSCO and open. Therefore $(\Omega, \mathcal{P})$ is HLSCO at $((p_1, p_3, p_4), \{y_1, y_2\})$.
The HSO sets containing $((p_2, p_3, p_4), \{y_2, y_3\})$ are $(\Omega, \mathcal{P})_{12}$ and $(\Omega, \mathcal{P})$. We can easily show that $(\Omega, \mathcal{P})_{12}$ is HSCO. Since $(\Omega, \mathcal{P})_{12}$ is HSCO and hypersoft open therefore $(\Omega, \mathcal{P})$ is HLSCO at $((p_2, p_3, p_4), \{y_2, y_3\})$. Therefore $(\Omega, \mathcal{P})$ is HLSCO.

Theorem 5.3. *A HTS $((\Omega, \mathcal{P}), \tau)$ is HLSCO iff the hypersoft semi-components of HSO set are hypersoft open sets.*

proof: Suppose that $((\Omega, \mathcal{P}), \tau)$ is HLSCO. Let $(\Omega, \mathfrak{A}) \subset (\Omega, \mathcal{P})$ be HSO and $(\Omega, \mathfrak{B})$ be a hypersoft semi-component of $(\Omega, \mathfrak{A})$. If $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B})$, then because $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{A})$, there is a HSCO open set $(\Omega, U) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, U) \subset (\Omega, \mathfrak{A})$. Since $(\Omega, \mathfrak{B})$ is the hypersoft semi component of $(x, \Omega_{\mathfrak{A}}(x))$ and (Ω, U) is HSCO, we have $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, U) \subset (\Omega, \mathfrak{B})$. This shows that $(\Omega, \mathfrak{B})$ is hypersoft open.
Conversely, let $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P})$ be arbitrary and let $(\Omega, \mathfrak{A})$ be a HSO set containing

$(x, \Omega_{\mathfrak{A}}(x))$. Let $(\Omega, \mathfrak{B})$ be the hypersoft semi component of $(\Omega, \mathfrak{A}) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B})$. Now $(\Omega, \mathfrak{B})$ is HSCO open set $\ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B}) \subset (\Omega, \mathfrak{A})$. This proves the theorem.

Theorem 5.4. *A HTS $((\Omega, \mathcal{P}), \tau)$ is HLSCO iff given any hypersoft point $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{A})$ and a HSO set (Ω, U) containing $(x, \Omega_{\mathfrak{A}}(x))$, there is a hypersoft open set $(\Omega, \mathfrak{B})$ containing $(x, \Omega_{\mathfrak{A}}(x)) \ni (\Omega, \mathfrak{B})$ is contained in a single hypersoft semi component of (Ω, U).*

proof: Let $(\Omega, \mathcal{P})$ be HLSCO, $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P})$ and (Ω, U) be a HSO set containing $(x, \Omega_{\mathfrak{A}}(x))$. Let (Ω, V) be a hypersoft semi component of (Ω, U) that contains $(x, \Omega_{\mathfrak{A}}(x))$. Since $(\Omega, \mathcal{P})$ is HLSCO and (Ω, U) is HSO, there is a HSCO open set $(\Omega, \mathfrak{B}) \ni (x, \Omega_{\mathfrak{A}}(x))$ and so $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B}) \subset (\Omega, V) \subset (\Omega, U)$. Since hypersoft semi components are disjoint sets, it follows that $(\Omega, \mathfrak{B})$ is not contained in any other hypersoft semi component of (Ω, U).

Conversely, we suppose that given any hypersoft point $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P})$ and any HSO set (Ω, U) containing $(x, \Omega_{\mathfrak{A}}(x))$, there is a hypersoft open set $(\Omega, \mathfrak{B})$ containing $(x, \Omega_{\mathfrak{A}}(x))$ which is contained in a single hypersoft semi component $(\Omega, \mathfrak{F})$ of (Ω, U). Then $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{B}) \subset (\Omega, \mathfrak{F}) \subset (\Omega, U)$. Let $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{F})$, then $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, U)$. Thus there is a hypersoft open set $(\Omega, \mathfrak{Q}) \ni (x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{Q})$ and $(\Omega, \mathfrak{Q})$ is contained in a single hypersoft semi component of (Ω, U). As the hypersoft semi components are disjoint hypersoft sets and $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{F})$, $(x_1, \Omega_{\mathfrak{A}}(x_1)) \in (\Omega, \mathfrak{Q}) \subset (\Omega, \mathfrak{F})$. Thus $(\Omega, \mathfrak{F})$ is hypersoft open. Thus for every $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F})$ and for every HSO set (Ω, U) containing $(x, \Omega_{\mathfrak{A}}(x))$, there is a HSCO open set $(\Omega, \mathfrak{F}) \ni (x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathfrak{F}) \subset (\Omega, U)$. Thus $((\Omega, \mathcal{P}), \tau)$ is HLSCO at $(x, \Omega_{\mathfrak{A}}(x))$. Since $(x, \Omega_{\mathfrak{A}}(x)) \in (\Omega, \mathcal{P})$ is arbitrary $((\Omega, \mathcal{P}), \tau)$ is HLSCO. This proves the theorem.

Remark 5.5. HLSCO does not imply HSCO as shown by the following example.

Example 5.6. From example 5.2, $(\Omega, \mathcal{P})$ is HLSCO. But $(\Omega, \mathcal{P})$ can be expressed as the union of two hypersoft semi separated sets $(\Omega, \mathcal{P})_4$ and $(\Omega, \mathcal{P})_9$. Therefore $(\Omega, \mathcal{P})$ is not HSCO.

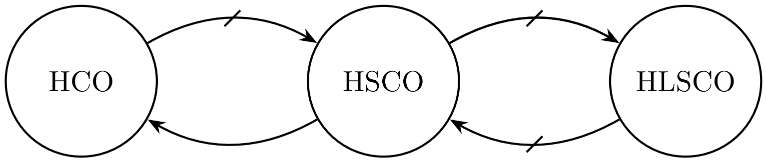
Remark 5.7. HSCO does not imply HLSCO as shown by the following example.

Example 5.8. From example 4.3, $((\Omega, \mathcal{P}), \tau)$ is HSCO. We show that it is HLSCO. Here $(\Omega, \mathcal{P})_3$ is a hypersoft semi-open set containing $((p_1, p_3, p_4), \{y_1, y_2\})$. But there is no hypersoft open subset of $(\Omega, \mathcal{P})_3$ containing $((p_1, p_3, p_4), \{y_1, y_2\})$ and so $((\Omega, \mathcal{P}), \tau)$ is not HLSCO at $((p_1, p_3, p_4), \{y_1, y_2\})$. Therefore $((\Omega, \mathcal{P}), \tau)$ is not HLSCO.

Remark 5.9. Since every HSCO is HCO. But the converse is not true as shown by the following example.

Example 5.10. It is evident from example 5.2 that $((\Omega, \mathcal{P}), \tau)$ is HCO but it is not HSCO.

Remark 5.11. Below is an outline of our research results from the earlier examples:



6. Conclusion

In this study, the concepts of hypersoft semi-neighbourhood of hypersoft point, hypersoft semi-first countable spaces, semi-converges, and semi-cluster hypersoft point were introduced. After that, using HSO sets, we have defined hypersoft semi-separation, HSCO, and HLSCO for strengthening the field of hypersoft set theory. In this work, we also demonstrated the relationship between the HCO, HSCO, and HLSCO spaces through examples. Future research may investigate the concepts of hypersoft pre-separation, hypersoft pre-neighborhood, hypersoft pre-connectedness, and hypersoft locally pre-connectedness. The resulting data will be immensely beneficial for future researchers to improve this work on the Neutrosophic hypersoft set, Plithogenic hypersoft set, Indetermhypersoft set, and Superhypersoft set.

Acknowledgments:

The authors are highly thankful to the editor and referees for the valuable comments and suggestions for improving the quality of the paper.

Conflict of interest :

The authors declare that there is no conflict of interest in the research.

Ethical approval :

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Abbas, B., Murtaza, G., Smarandache, F., "*Basic Operations on Hypersoft Sets and Hypersoft Point,*" Neutrosophic Sets and Systems, 35, 407-421, 2020.
2. Ajay, D., Joseline Charisma, J., Boonsatit, N., Hammachukiattikul, P., Rajchakit, G., "*Neutrosophic Semi-open Hypersoft Sets with an Application to MAGDM under the COVID-19 Scenario,*" Journal of Mathematics, Special Issue, 1-16, 2021.
3. Angel, N., Pandiammal, P., Nivetha Martin: "*Plithogenic Hypersoft Based Plithogenic Cognitive Maps in Sustainable Industrial Development*", Neutrosophic Sets and Systems, 73, 52-65, 2024.
4. Bin Chen, "*Some Local Properties of Soft Semi open Sets,*" Discrete Dynamics in Nature and Society, 57, 1547-1553, 2013.
5. Cagman, N., Karatas, S., Enginoglu, S., "*SoftTopology,*" Computers and Mathematics With Applications, 62, 351-358, 2011.
6. Cagman, N., Enginoglu S., "*Soft Set Theory and uni-int Decision Making,*" European Journal of Operational Research, 207, 848-855, 2010.

7. Chan, B,.“*Soft Semi-open Sets and Related Properties in Soft Topological spaces*,” Applied Mathematics and Information sciences, 7(1), 287-294, 2013.
8. Florentin Smarandache., “*New Types of Soft Sets HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set: An Improved Version*,” Neutrosophic Systems With Applications, 8, 35-41, 2023.
9. Florentin Smarandache., “ *Foundation of SuperHyperStructure & Neutrosophic SuperHyperStructure (review paper)*,” Neutrosophic Sets and Systems, 63, 367-381, 2024.
10. Florentin Smarandache., Inthumathi, V., Amsaveni, M., “*Hypersoft Sets in a Game Theory-Based Decision Making Model*,” International Journal of Neutrosophic Science (IJNS), 24(1), 74-86, 2024
11. Faisal Al-Sharqi,. Ashraf Al-Quran,. and Zahari Md. Rodzi,. “*Multi-Attribute Group Decision-Making Based on Aggregation Operator and Score Function of Bipolar Neutrosophic Hypersoft Environment*,” Neutrosophic Sets and Systems, 61, 465-492, 2023.
12. Inthumathi, V., Amsaveni, M., Arun vignesh, M., “*Application of Hypersoft Sets in Covid-19 Decision Making Model,*” International Journal of Research and Analytical Reviews, 9, 2022.
13. Inthumathi, V., Amsaveni, M., Nathibrami,M., “*On Hypersoft Semi-open Sets,*” Neutrosophic Sets and Systems, 57, 294-305, 2023.
14. Inthumathi, V., Amsaveni, M., “*On Hypersoft Topology,* ” Indian Journal of Natural sciences, 15(86), 82107-82116, 2024.
15. Keun min, W., “*A Note on Soft Topological Spaces,*” Computers and Mathematics with Applications, 62, 3524-3528, 2011.
16. Krishnaveni, J., Sekar, C., “*Soft Semi Connected and Soft Locally Semi Connected Properties in Soft Topological Spaces,*” International Journal of Mathematics and Soft Computing, 3(3), 85-91, 2013.
17. Maji, P. K., Biswas, R., Roy, A. R., “*Soft Set Theory,*” Computers and Mathematics with Applications, 45(4-5), 555-562, 2013.
18. Molodstov, D., “*Soft Set Theory-first Results,*” Computers and Mathematics with Applications, 37(4-5), 19-31, 1999.
19. Muhammad Saeed., Hafiz Inam ul Haq., Mubashir Ali., “*Extension of Double Frame Soft Set to Double Frame Hypersoft Set,*” HyperSoft Set Methods in Engineering, 2, 18-27, 2024.
20. Muhammad Saqlain., Poom Kumam., Wiyada Kumam., “*Neutrosophic Linguistic Valued Hypersoft Set with Application: Medical Diagnosis and Treatment,*” Neutrosophic Sets and System, 63, 130-152, 2023.
21. Musa, S. Y., Asaad, B. A., “*Hypersoft Topological Spaces,*” Neutrosophic Sets and Systems, 49, 397-415, 2022.
22. Musa, S. Y., Asaad, B. A., “*Connectedness on Hypersoft Topological Spaces*,” Neutrosophic Sets and System, 51,667-679, 2022.
23. Musa, S. Y., Asaad, B. A., “*Hypersoft Separation Axioms*,” Filomat, 36(19), 6679–6686, 2023.
24. Muhammad Saqlain., Poom Kumam., Wiyada Kumam., “*Multi-Polar Interval-Valued Neutrosophic Hypersoft Set with Multi-Criteria Decision Making of Cost-Effective Hydrogen Generation Technology Evaluation*,” Neutrosophic Sets and Systems, 61, 100-117, 2023.
25. Mona Mohamed., Alaa Elmor., Florentin Smarandache., Ahmed, A., Metwaly., “*An Efficient SuperHyperSoft Framework for Evaluating LLMs-based Secure Blockchain Platforms*,” Neutrosophic Sets and Systems, 72, 1-21, 2024.
26. Mahima Thakur, Smarandache, F., Thakur, S,. “*Neutrosophic Semi* $\delta-$*Preopen Sets and Neutrosophic Semi* δ*-Pre Continuity*,” Neutrosophic Sets and Systems, 73, 400-414, 2024.
27. Muhammad Arshad,. Muhammad Saeed,. Atiqe Ur Rahman,. “*Interval Complex Single-valued Neutrosophic Hypersoft Set with Application in Decision Making*,” Neutrosophic Sets and Systems, 60, 396-419, 2023.
28. Nouran Ajabnoor,. “*Evaluation of Distributed Leadership in Education Using Neutrosophic HyperSoft Set*,” Neutrosophic Sets and Systems, 72, 326-340, 2024.

29. Rashmi. S.Chaudhry., Anil Chandhok., "*Evaluation of E-Commerce Sites using Novel Similarity Measure of Neutrosophic Hypersoft Sets*," Neutrosophic Sets and Systems, 61, 165-176, 2023.
30. Ranulfo Paiva Barbosa (Sobrinho)., Smarandache, F., "*Pura Vida Neutrosophic Algebra*," Neutrosophic Systems With Applications, 9, 101-106, 2023.
31. Saeed, M., Ashraf, M., Ijaz, A., Jaffar, M. N., "*Development of Pythagorean Neutrosophic Hypersoft Set with Application in Work Life Balance Problem*," HyperSoft Set Methods in Engineering, 1, 59-70, 2024.
32. Sagvan Y. Musa., Ramadhan A. Mohammed., Baravan A. Asaad., "*N-Hypersoft Sets: An Innovative Extension of Hypersoft Sets and Their Applications*," Symmetry, 15, 1-18, 2023.
33. Sai, B.V.S.T., Srinivasa kumar, V., "*On Soft Semi-open Sets and Soft Semi-Topology,*" International Journal of Mathematical Archieve, 4(4), 114-117, 2013.
34. Sathya, P,. Nivetha Martin,. Florentin Smarandache,. "*Plithogenic Forest Hypersoft Sets in Plithogenic Contradiction Based Multi-Criteria Decision Making*," Neutrosophic Sets and Systems, 73, 668-693, 2024.
35. Sasikala, V. E., Sivaraj, D., "*On Soft Semi-open Sets,*" International Journal of Management and Applied Science, 3(9), 2017.
36. Shabir, M., Naz, M., "*On soft topological spaces,*" Computers and Mathematics with Applications, 61(7), 1786-1799, 2011.
37. Smarandache, F., "*Extension of Soft Set to Hypersoft Set and then to Plithogenic Hypersoft Set,*" Neutrosophic Sets and Systems, 22, 168-170, 2018.
38. Smarandache, F., "*New Types of Soft Sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set*," International Journal of Neutrosophic Science (IJNS), 20(4), 58-64, 2023.
39. Smarandache, F., " *Foundation of the SuperHyperSoft Set and the Fuzzy Extension SuperHyperSoft Set: A New Vision* ," Neutrosophic Systems With Applications, 11, 48-51, 2023.
40. Smarandache, F.,"*New Types of Topologies and Neutrosophic Topologies*," Neutrosophic Systems With Applications, 1, 1-3, 2023.
41. Smarandache, F., "*SuperHyperFunction, SuperHyperStructure, Neutrosophic SuperHyperFunction and Neutrosophic SuperHyperStructure: Current understanding and future directions*," Neutrosophic Systems With Applications, 12, 68-76, 2023.
42. Smarandache, F.,"*Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics*," Neutrosophic Systems With Applications, 15, 16-32, 2024.
43. Smarandache, F.,"*Foundation of Revolutionary Topologies*," Neutrosophic Systems With Applications, 13, 45-66, 2024.

Received: Oct 2, 2024. Accepted: Jan 14, 2025