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# Neutrosophic Mean Estimators Using Extreme Indeterminate Observations in Sample Surveys

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**Abstract**. In classical statistics, research typically relies on precise data to estimate the population mean, especially when auxiliary information is available. However, in the presence of outliers, conventional statistical approaches that depend on accurate data and auxiliary information encounter challenges. The primary objective is to attain the most accurate population mean estimates while minimizing the mean square error. Neutrosophic statistics, a more attractive framework than classical statistics, deals with data characterized by imprecision and uncertainty. In this current article, we adapt Särndal's strategy and introduce neutrosophic mean estimators, applying them to meteorological data, specifically stratified dew point data. In these proposed estimators, the incorporation of auxiliary information and the application of robust techniques address issues that arise due to outliers and imprecise observations. These factors can otherwise undermine the effectiveness of neutrosophic estimation methods. The article also suggests combining auxiliary information with extremely indeterminate neutrosophic observations, utilizing robust regression methods (Huber-M, Hampel-M, and Tukey-M), as well as the quantile regression technique. These approaches enhance the neutrosophic mean estimation process. The outcomes, which include the utilization of dew point data, showcase the superior performance of the proposed estimators compared to adapted estimators in a neutrosophic context. Ultimately, this study provides valuable insights by taking an initial step in defining and utilizing the concept of neutrosophic indeterminate extreme observations.

**Keywords:** Neutrosophic Mean estimation, Särndal Approach, Extreme Values, Neutrosophic Robust Regression, Neutrosophic Robust Quantile Regression.

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## 1. Introduction

The term ‘dew point’ holds significant meaning in the field of meteorology, as it represents the crucial temperature at which moisture begins to condense, leading to the formation of visible dew, mist, or clouds. While traditionally viewed as a single value, the concept of dew point gains a fascinating complexity when examined through the lens of neutrosophic logic—a theory that accounts for uncertainty and indeterminacy. Traditionally, the dew point is considered the temperature at which the air can no longer absorb moisture, leading to visible condensation. However, neutrosophic logic challenges this binary perspective. In neutrosophy, the dew point is seen as a triadic truth, consisting of the region of true saturation (where condensation is inevitable), the region of indeterminate saturation (where the distinction between saturation and unsaturation is unclear), and the region of false saturation (indicating a lack of saturation). This trinity captures the intricate interplay between temperature, moisture, and atmospheric dynamics.

In the neutrosophic context, the relationship between the dew point and moisture undergoes a transformative shift. Humidity, as a measure of atmospheric moisture, transcends a simple binary concept. Neutrosophic humidity encompasses the presence of moisture, a hazy realm of uncertainty where the state of moisture’s existence remains unclear, as well as the absence of moisture. This aligns with the complex nature of humidity levels, where distinguishing between moist and dry conditions isn’t always straightforward. Neutrosophic dew point introduces an element of uncertainty into predictions. High dew point often indicate favorable conditions for precipitation, while low dew point typically signal drier circumstances. Neutrosophic logic incorporates this uncertainty. High dew point not only suggest the possibility of precipitation but also acknowledge the inherent uncertainty in meteorological forecasts. Forecasts become increasingly unpredictable when the value indeterminate comes into play. Even low dew point, commonly viewed as clear indicators of dryness, find representation within the neutrosophic spectrum. This underscores the idea that outcomes may not always align with expectations based on dryness.

Viewing human comfort through the neutrosophic lens adds a layer of depth, especially in relation to humidity. High humidity typically leads to discomfort because evaporative cooling becomes less effective. Neutrosophy accommodates varying degrees of ambiguity in this context. The indeterminate element acknowledges that while high humidity often coincides with discomfort as a true condition, comfort isn’t solely determined by humidity levels. Similar to the false aspect, which represents lower humidity, it doesn’t guarantee everyone’s comfort. Amidst fluctuating humidity, neutrosophic reflection allows us to explore the intricate range of human sensations. The dew point serves as the conductor of the neutrosophic atmosphere, illustrating atmospheric events. Within this realm of uncertainty, weather patterns are shaped by warm currents and colder masses. In this symphony, the neutrosophic triad is expressed. The true value signifies air saturation at specific times, while the indeterminate section records transitions from one definitive condition to another. The false aspect pertains to situations where saturation is still absent. This triadic dance mirrors the complex choreography of atmospheric components.

In statistical inference, estimators that incorporate auxiliary data have proven to be essential. Researchers have developed various estimation approaches, including ratio estimation methods, to compute population metrics like the mean. These techniques have received extensive research attention and have shown promise in reducing sampling error, particularly when a strong connection exists between the study variable and auxiliary variable. By considering both the auxiliary variable and the primary research variable, these estimators can enhance precision and potentially reduce the required sample size. Cochran [9] emphasized the advantages of ratio estimation techniques in obtaining more accurate estimates.

Fuzzy logic has emerged as an effective tool for data analysis, particularly when dealing with data that is fuzzy, ambiguous, uncertain, or imprecise— in contrast to classical statistics, which assumes precise data. Such data can be effectively handled and processed using fuzzy statistics. However, fuzzy statistics do not explicitly consider the level of data uncertainty. Neutrosophic logic, an extension of fuzzy logic, offers a more comprehensive approach by incorporating both the determinate and indeterminate aspects of observations. Smarandache [30] highlights that neutrosophic logic enables the measurement and interpretation of data with hazy or unclear observations. Fuzzy logic techniques, involving complex fuzzy sets and their broadening to complex neutrosophic sets, have discovered practical applications in decision-making contexts. Researchers have also investigated the use of different kinds of auxiliary information to improve the performance of ratio-type estimators, including incorporating the coefficient of variation, as noted by Robson [17] applications.

The growing demand for statistical approaches capable of handling and analyzing data in a more robust and flexible manner is evident in the application of neutrosophic numbers in decision-making scenarios. Neutrosophic statistics open up new avenues for research, enabling a deeper understanding and analysis of data with varying degrees of indeterminacy. Motivated by these insights, we propose Särndal-type neutrosophic robust mean estimators that utilize extreme indeterminate observations. We assess the performance of these estimators through an application to a dew point data set.

The article is organized into several sections to provide a structured exploration of the proposed concepts and methodologies. In Section 2, we delve into the notion of Extreme Observations and robust mean estimation. Section 3 delves into the realm of Neutrosophic

Statistics and Adapted Estimators. Section 4 introduces our proposal for Neutrosophic Estimators, encompassing Neutrosophic Ratio Estimators, Neutrosophic Product Estimators (both in combined and separate versions), and the Särndal method. Further, Section 4.1 presents our proposed Neutrosophic OLS Regression and its Proposed Estimators, while Section 4.2 covers the intricacies of the proposed Neutrosophic Robust Regression and its Proposed Estimators. Subsection 4.3 provides insights into Neutrosophic Robust Quantile Regression along with its Proposed Estimators. Sections 5 and 6 are dedicated to showcasing practical applications and discussing the outcomes of our proposed methodologies. Finally, in Section 7, we draw the article to a close by outlining the potential utility of neutrosophic estimators in addressing the complexities and ambiguities inherent in real-world data analysis.

## 2. Extreme Observations and Neutrosophic Robust Estimation of Mean

The characteristics of neutrosophic observations high, low include ambiguity, indeterminacy, and paraconsistency. As a result, it is impossible to determine with certainty whether a neutrosophic observation is high, low, or neither. When there is a lot of uncertainty or when the data is insufficient, this kind of observation can be helpful. A neutrosophic observation of high and low, for instance, may be used to characterise the stock market, which is renowned for being erratic. Additionally, it might be used to the possibility of a natural disaster like a tsunami or hurricane. A neutrosophic number, which is a number with three elements including truth, indeterminacy, and falsehood, may be used to describe neutrosophic observations such as high and low. The degree to which an observation is true is represented by its truth component, while its degree of uncertainty is represented by its indeterminacy component and its degree of falsehood by its falsity component. The observation "the stock market will go up tomorrow" might be represented by a neutrosophic number such as (0.6, 0.2, 0.2). In other words, there is a 60% chance the stock market will rise tomorrow, a 20% chance it won't rise or fall, and a 20% chance it will fall.

The inclusion of unusual observations can significantly impact mean estimators' outcomes, which is not uncommon in sample survey data. These estimators, designed to calculate a variable's average value, are particularly sensitive to extremely large or small values when present in the sample. The presence of extreme values can introduce bias into estimations, leading researchers to often consider excluding them from the dataset entirely. However, an alternative approach is to retain known extreme values in the data and use them as supplementary information to enhance estimation accuracy. By incorporating these high values as additional data points, estimators can leverage a broader range of information, potentially leading to more accurate and reliable results. This approach underscores the importance of considering auxiliary variables to improve estimation precision alongside the utilization of extreme values.

When available, auxiliary variables provide additional data that can enhance the accuracy of the estimation process. It becomes clear that these estimators are often based on single-point estimate when looking for estimators that make use of the maximum and lowest values of X. They concentrate to include the extreme values of the variable to offer insights into the data distribution or to draw conclusions about the characteristics of the population. Such as Ali et al. [1], Zaman et al. [41] [43], Bulut and Zaman [7] have done numerious contributions in this field.

Reserchers such as Tahir et al. [31], Kumar et al. [14], Singh et al. [29], Singh et al. [27], Yadav and Smarandache [33], Alomair and Shahzad [4], Yadav and Prasad ( [34], [35]), Kumari et al. [15], Singh and Kumari [28], Alqudah et al. [3] and Ullah et al. [32] present a variety of different neutrosophic mean estimators and discuss their applications.

It is important to note that there aren't many research especially looking at how to use extreme observations in neutrosophic mean estimators. The current article focuses on this particular point.

## 3. Neutrosophic Statistics and Adapted Estimators

Neutrosophic statistical approaches have grown in popularity for analysing datasets with variable degrees of uncertainty, often known as neutrosophic data. This statistical technique recognises the difficulty of precisely establishing sample sizes, as noted in Smarandache's study [30]. Smarandache's [30] work has proved the use of neutrosophic statistics in investigating systems with inherent uncertainty and ambiguity. In addition to more general uses, neutrosophic statistics are useful in the discipline of rock engineering. Researchers have utilised neuosophic numbers to investigate issues like the scale effect and the anisotropy of joint roughness coefficients. According to Chen et al. [8], these efforts have resulted in the development of effective methods for addressing information loss and creating well-fitting functions.

A fresh method for analysing neutrosophic data has also been made available through neutrosophic analysis of variance. This method broadens the scope of data analysis in the context of ambiguous or missing data by allowing researchers to draw important conclusions from datasets with varied degrees of ambiguity. Neutosophic interval value statistics is a particular method within Neutosophic Statistics that expresses data using Neutosophic Intervals in the context of sample surveys. A generalisation of conventional intervals, neutrosophic intervals allow for indeterminacy and partial truth but lack sharp boundary definitions. To handle such ambiguous interval data, neutrosophic interval value statistics expands on conventional statistical techniques.

To demonstrate neutrosophic interval value statistics in a representative survey, let's look at the following example:

Let's say you were to survey a population to find out how tall each person was. The conventional method would entail gathering exact height measurements, such as 165 cm, 172 cm, and so on. The data that is currently accessible, nevertheless, could occasionally be ambiguous or inaccurate. For instance, during the survey, individuals may give estimations of their height that are general or ambiguous, such as "around 170 cm" or "between 160 cm and 180 cm." These replies muddy the data and leave it up to interpretation. These ambiguous height measurements would be represented by neutrosophic intervals in neutrosophic interval value statistics. For instance, the response "around 170 cm" may be represented as a neutrosophic interval [165 cm, 175 cm, 0.6], where the bounds of the interval denote the range of feasible heights and the truth-membership value 0.6 denotes the degree of trust that the height falls within that range. The response "between 160 cm and 180 cm" might be expressed similarly as a neutrosophic interval [155 cm, 185 cm, 0.8], suggesting a broader range with a greater level of conviction. After gathering a collection of neutrosophic interval data, you may analyse the data using statistical methods designed specifically for neutrosophic statistics. This might entail doing hypothesis testing using neutrosophic intervals or computing other statistical measures, such as the mean, median, mode, and variance. By using neutrosophic interval value statistics, it is possible to include ambiguous and imprecise data in the analysis and gain a deeper knowledge of the survey data.

Consider a population of the size $N$ is divided into $L$ mutually exclusive strata of sizes $N_h (h = 1, 2, 3, ..., L)$ such that $\sum_{h=1}^{L} N_h = N$. We consider a neutrosophic variable, denoted by $Y_{(\delta N)} \epsilon [Y_{(\delta N)_L}, Y_{(\delta N)_U}]$, on a finite population $U_{(\delta N)}$ consisting of N identifiable units. To estimate the unknown mean of neutosophic study variable $Y_{(\delta N)}$, denoted by $\bar{Y}_{(\delta N)} \epsilon [\bar{Y}_{(\delta N)_L}, \bar{Y}_{(\delta N)_U}]$, a neutrosophic auxiliary variable $X_{(\delta N)} \epsilon [X_{(\delta N)_L}, X_{(\delta N)_U}]$ is used. Note that additional information may be available for the entire population through knowledge of $X_{(\delta N)}$ or $\bar{X}_{(\delta N)} \epsilon [\bar{X}_{(\delta N)_L}, \bar{X}_{(\delta N)_U}]$, or on a sample from $U_{(\delta N)}$ in the absence of population information related to study variable.

In stratified random sampling $Y_{hi_{(\delta N)}}$ and $X_{hi_{(\delta N)}}$ be the values of the neutrosophic study variable and the neutrosophic auxiliary variable at $i^{th}$ unit $(i = 1, 2, 3, ..., N_h)$ in the $h^{th}$ $(h = 1, 2, 3, ..., L)$ stratum respectively. Let a sample of size $n_h$ $(h = 1, 2, 3, ..., L)$ is drawn from each stratum independently by simple random sampling without replacement (SRSWOR) such that $\sum_{h=1}^{L} n_h = n$ where $n$ is total number of units in a sample. Let $y_{hi_{(\delta N)}}$ and $x_{hi_{(\delta N)}}$ be the values of the neutrosophic study variable and the neutrosophic auxiliary variable of the $i^{th}$ unit $(i = 1, 2, 3, ..., N_h)$ in a sample.

Define:

$\bar{Y}_{(\delta N)} = \frac{\sum_{i=1}^{N} Y_{i_{(\delta N)}}}{N}$, $\bar{X}_{(\delta N)} = \frac{\sum_{i=1}^{N} X_{i_{(\delta N)}}}{N}$ = Population means of the neutrosophic study and the neutrosophic auxiliary variables respectively,

$\bar{Y}_{h_{(\delta N)}} = \frac{\sum_{i=1}^{N} Y_{hi_{(\delta N)}}}{N}$, $\bar{X}_{h_{(\delta N)}} = \frac{\sum_{i=1}^{N} X_{hi_{(\delta N)}}}{N}$ = Population means of the neutrosophic study and the neutrosophic auxiliary variables in the $h^{th}$ statum,
$\bar{y}_{h_{(\delta N)}} = \frac{\sum_{i=1}^{N} y_{hi_{(\delta N)}}}{N}$, $\bar{x}_{h_{(\delta N)}} = \frac{\sum_{i=1}^{N} x_{hi_{(\delta N)}}}{N}$ = Sample means of the neutrosophic study and the neutrosophic auxiliary variables in the $h^{th}$ statum,
$S^2_{hx_{(\delta N)}}$ = Neutrosophic auxiliary variable Population variance in the $h^{th}$ stratum,
$S^2_{hy_{(\delta N)}}$ = Neutrosophic Population study variable variance in the $h^{th}$ stratum,
$S_{hyx_{(\delta N)}}$ = Neutrosophic Population covariance of $Y_{(\delta N)}$ and $X_{(\delta N)}$ in the $h^{th}$ stratum,
$\beta_{h_{(\delta N)}}$ = Neutrosophic Population regression coefficient for the $h^{th}$ stratum,
$\beta_{c_{(\delta N)}}$ = Neutrosophic Population regression coefficient across the strata,
$W_h$ = Neutrosophic Stratum weight in the $h^{th}$ stratum,
$f_h$ = Neutrosophic Sampling fraction in the $h^{th}$ stratum,
$R_{(\delta N)}$ = Neutrosophic Population ratio,
$R_{h_{(\delta N)}}$ = Neutrosophic Population ratio in the $h^{th}$ stratum.
when the study variable's and the auxiliary variables' regression line does not pass through the origin, it indicates that the regression model has an intercept term. The intercept term causes a change in the regression line, expressing the intrinsic link between the study variable and the auxiliary variables that goes beyond a simple proportionate relationship. In these circumstances, the regression estimate outperforms the ratio and product estimators.
Hansen and Hurwitz [12] suggested two distinct strategies for building ratio estimators in stratified random sampling. Stratified random sampling entails segmenting the population into various groups or strata and then sampling from each stratum separately. Ratio estimators estimate population parameters by calculating the ratio of auxiliary variables to the study variable within each stratum. The methods suggested in have introduced innovative methodologies or adjustments to current ratio estimating systems, therefore improving the estimates' accuracy.
In this paper, we consider the use of maximum and minimum values in neutrosophic statistics using both combined and separate estimators. To deal with uncertainty and inconsistencies, neutrosophic statistics mixes fuzzy logic, intuitionistic logic, and paraconsistent logic. We can investigate the upper and lower boundaries of neutrosophic statistics by employing maximum and minimum values.
The Neutrosopic mean per unit estimator in stratified random sampling are given by:

$$\bar{y}_{st_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{y}_{h_{(\delta N)}} \tag{1}$$

$$Var(\bar{y}_{st_{(\delta N)}}) = \sum_{h=1}^{L} W_h^2 \lambda_h S^2_{hy_{(\delta N)}} \tag{2}$$

where $\lambda_h = \frac{1-f_h}{n_h}$

**Adapted Neutrosophic Combined Estimators**

Neutrosophic Combined ratio estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\bar{Y}}_{RC0_{(\delta N)}} = \frac{\bar{y}_{st_{(\delta N)}}}{\bar{x}_{st_{(\delta N)}}}\bar{X}_{(\delta N)} \tag{3}$$

$$MSE(\hat{\bar{Y}}_{RC0_{(\delta N)}}) \cong \sum_{h=1}^{L} W_h^2\lambda_h\left(S^2_{hy_{(\delta N)}} + R^2_{(\delta N)}S^2_{hx_{(\delta N)}} - 2R_{(\delta N)}S_{hyx_{(\delta N)}}\right) \tag{4}$$

Neutrosophic Combined product estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\bar{Y}}_{PC0_{(\delta N)}} = \frac{\bar{y}_{st_{(\delta N)}}}{\bar{X}_{(\delta N)}}\bar{x}_{st_{(\delta N)}} \tag{5}$$

$$MSE(\hat{\bar{Y}}_{PC0_{(\delta N)}}) \cong \sum_{h=1}^{L} W_h^2\lambda_h\left(S^2_{hy_{(\delta N)}} + R^2_{(\delta N)}S^2_{hx_{(\delta N)}} + 2R_{(\delta N)}S_{hyx_{(\delta N)}}\right) \tag{6}$$

Neutrosophic Combined regression estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\bar{Y}}_{lrC0_{(\delta N)}} = \bar{y}_{st_{(\delta N)}} + b_{c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st_{(\delta N)}}\right) \tag{7}$$

where $b_{c_{(\delta N)}}$ is a neutrosophic combined sample regression coefficient.

$$MSE(\hat{\bar{Y}}_{lrC0_{(\delta N)}}) = \sum_{h=1}^{L} W_h^2\lambda_h S^2_{hy_{(\delta N)}}\left(1 - p_c^2\right) \tag{8}$$

where $p_c = \frac{\sum_{h=1}^{L} W_h^2\lambda_h S_{hyx_{(\delta N)}}}{\sqrt{\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hy_{(\delta N)}}\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hx_{(\delta N)}}}}$

We can also write $MSE(\hat{\bar{Y}}_{lrC0_{(\delta N)}})$ expression as

$$MSE(\hat{\bar{Y}}_{lrC0_{(\delta N)}}) = \left\{\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hy_{(\delta N)}} + \beta^2_{c_{(\delta N)}}\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hx_{(\delta N)}} - 2\beta_{c_{(\delta N)}}\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hyx_{(\delta N)}}\right\} \tag{9}$$

Where $\beta_{c_{(\delta N)}} = \frac{\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hyx_{(\delta N)}}}{\sum_{h=1}^{L} W_h^2\lambda_h S^2_{hx_{(\delta N)}}}$

**Adapted Neutrosophic Separate Estimators**

Neutrosophic Separate ratio estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\bar{Y}}_{RS0_{(\delta N)}} = \sum_{h=1}^{L} W_h\bar{y}_{hR_{(\delta N)}},\ where\ \bar{y}_{hR_{(\delta N)}} = \frac{\bar{y}_{h_{(\delta N)}}}{\bar{x}_{h_{(\delta N)}}}\bar{X}_{h_{(\delta N)}} \tag{10}$$

$$MSE(\hat{\bar{Y}}_{RS0_{(\delta N)}}) \cong \sum_{h=1}^{L} W_h^2\lambda_h\left(S^2_{hy_{(\delta N)}} + R^2_{h_{(\delta N)}}S^2_{hx_{(\delta N)}} - 2R_{h_{(\delta N)}}S_{hyx_{(\delta N)}}\right) \tag{11}$$

Neutrosophic Separate product estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{PS_{0_{(\delta N)}}} = \sum_{h=1}^{L} W_h \bar{y}_{hP_{(\delta N)}},\ where\ \bar{y}_{hP_{(\delta N)}} = \frac{\bar{y}_{h_{(\delta N)}}}{\bar{X}_{h_{(\delta N)}}} \bar{x}_{h_{(\delta N)}} \tag{12}$$

$$MSE(\hat{\bar{Y}}_{PS_{0_{(\delta N)}}}) \cong \sum_{h=1}^{L} W_h^2 \lambda_h \Big( S^2_{hy_{(\delta N)}} + R^2_{h_{(\delta N)}} S^2_{hx_{(\delta N)}} + 2R_{h_{(\delta N)}} S_{hyx_{(\delta N)}} \Big) \tag{13}$$

Neutrosophic Separate regression estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{lrS_{0_{(\delta N)}}} = \sum_{h=1}^{L} W_h \bar{y}_{hlr_{(\delta N)}} \tag{14}$$

where $\bar{y}_{hlr_{(\delta N)}} = \bar{y}_{h_{(\delta N)}} + b_{h_{(\delta N)}} (\bar{X}_{h_{(\delta N)}} - \bar{x}_{h_{(\delta N)}})$ and $b_{h_{(\delta N)}} = \frac{S_{hyx_{(\delta N)}}}{S^2_{hx_{(\delta N)}}}$

$$MSE(\hat{\bar{Y}}_{lrS_{0_{(\delta N)}}}) = \sum_{h=1}^{L} W_h^2 \lambda_h S^2_{hy_{(\delta N)}} \Big(1 - p_h^2\Big) \tag{15}$$

where $p_h = \frac{S_{hyx_{(\delta N)}}}{S_{hy_{(\delta N)}} S_{hx_{(\delta N)}}}$

We can also write $MSE(\hat{\bar{Y}}_{lrS_{0_{(\delta N)}}})$ expression as

$$MSE(\hat{\bar{Y}}_{lrS_{0_{(\delta N)}}}) = \sum_{h=1}^{L} W_h^2 \lambda_h \Big[ S^2_{hy_{(\delta N)}} + \beta^2_{h_{(\delta N)}} S^2_{hx_{(\delta N)}} - 2\beta_{h_{(\delta N)}} S^2_{hyx_{(\delta N)}} \Big] \tag{16}$$

Where $\beta_{h_{(\delta N)}} = \frac{S^2_{hyx_{(\delta N)}}}{S^2_{hx_{(\delta N)}}}$

## 4. Särndal approach and proposed neutrosophic estimators

In many cases, real databases contain instances of exceptionally big or tiny numbers. This is also true in the context of neutrosophic statistics, which considers the existence of indeterminacy, ambiguity, and inconsistency in data. Hybrid seed production firms, for example, produce new seed kinds and provide specified productivity ranges per acre that might help farmers. These ranges enable the detection of maximum and lowest values within the collection. Similarly, when calculating the average income of families, it is typically well-known what the income of the richest individual (maximum) in a community is, but determining the income of the lowest person (minimum) is simple. Furthermore, several surveys undertaken at regular intervals give significant information on both the maximum and minimum values.

However, a problem in neutrosophic statistics is that the mean per unit estimate for the mean of a finite population is very sensitive to extreme values. Accordingly, the estimator can yield false findings if any of these unexpected values are chosen for the sample. Särndal [19] provided an unbiased estimator to handle this issue precisely within the context of extreme observations in the population. The purpose of this estimate is to lessen the influence of the extreme values seen in the dataset. Särndal's method can be used to enhance the precision and dependability of statistical analysis in neutrosophic statistics.

Let $y_{h_{max_{(\delta N)}}}$ ($y_{h_{min_{(\delta N)}}}$) and $x_{h_{max_{(\delta N)}}}$ ($x_{h_{min_{(\delta N)}}}$) be the neutrosophic study's maximum (minimum) and auxiliary variables' respective maximum (minimum) values in stratum h. The suggested estimate for finite population mean is ($\bar{Y}_{(\delta N)}$) under neutrosophic stratified random sampling are as follows:

$$\bar{y}_{st_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{y}_{h_{c_{(\delta N)}}} \tag{17}$$

$$\bar{y}_{h_{c_{(\delta N)}}} = \begin{cases} \bar{y}_{h_{(\delta N)}} + c_{h_{(\delta N)}} & \text{if a sample from the } h^{th} \text{ stratum contains } y_{h_{min_{(\delta N)}}} \text{ but not } y_{h_{max_{(\delta N)}}}, \\ \bar{y}_{h_{(\delta N)}} - c_{h_{(\delta N)}} & \text{if a sample from the } h^{th} \text{ stratum contains } y_{h_{max_{(\delta N)}}} \text{ but not } y_{h_{min_{(\delta N)}}}, \\ \bar{y}_{h_{(\delta N)}} & \text{in other cases,} \end{cases} \tag{18}$$

where $\bar{y}_{h_{c_{(\delta N)}}}$ $(h = 1, 2, 3, ..., L)$ are arbitrary constants.
The selection of values in a sample is based on the relationship between the neutrosophic study variable $Y_{h_{(\delta N)}}$ and the neutrosophic auxiliary variable $X_{h_{(\delta N)}}$. When the relationship is positive, it is predicted that a big value of $Y_{h_{(\delta N)}}$ will be picked for large values of $X_{h_{(\delta N)}}$ and a small value of $Y_{h_{(\delta N)}}$ for small values of $X_{h_{(\delta N)}}$. The definition of ratio estimators follows from this. In contrast, when $Y_{h_{(\delta N)}}$ and $X_{h_{(\delta N)}}$ have a negative relationship, a big value of $Y_{h_{(\delta N)}}$ is anticipated for small values of $X_{h_{(\delta N)}}$ and a small value of $Y_{h_{(\delta N)}}$ is anticipated for large values of $X_{h_{(\delta N)}}$. Consequently, product estimators are defined. These estimators are based on the assumption that the sample's selected values will be consistent.

**(1) : Neutrosophic Ratio Estimators**

Neutrosophic Combined ratio estimators in stratified random sampling using extreme values of data.

$$\hat{\bar{Y}}_{RC_{1_{(\delta N)}}} = \frac{\bar{y}_{st.c11_{(\delta N)}}}{\bar{x}_{st.c21_{(\delta N)}}} \bar{X}_{(\delta N)} \tag{19}$$

Neutrosophic Separate ratio estimators in stratified random sampling using extreme values of data.

$$\hat{\bar{Y}}_{RS_{1_{(\delta N)}}} = \sum_{h=1}^{L} W_h \frac{\bar{y}_{h.c11_{(\delta N)}}}{\bar{x}_{h.c21_{(\delta N)}}} \bar{X}_{h_{(\delta N)}} \tag{20}$$

where; $\bar{y}_{st.c11_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{y}_{h.c11_{(\delta N)}}$ and $\bar{x}_{st.c21_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{x}_{h.c21_{(\delta N)}}$
and

$$(\bar{y}_{h_{c.11_{(\delta N)}}}, \bar{x}_{h_{c.21_{(\delta N)}}}) = \begin{cases} \Delta_{R1} & \text{if a sample contains } y_{h_{min_{(\delta N)}}} \text{ but not } y_{h_{max_{(\delta N)}}}, \\ \Delta_{R2} & \text{if a sample contains } y_{h_{max_{(\delta N)}}} \text{ but not } y_{h_{min_{(\delta N)}}}, \\ \Delta_{R3} & \text{in other cases,} \end{cases} \tag{21}$$

where $\Delta_{R1} = \left(\bar{y}_{h_{(\delta N)}} + c_{1h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}} + c_{2h_{(\delta N)}}\right)$, $\Delta_{R2} = \left(\bar{y}_{h_{(\delta N)}} - c_{1h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}} - c_{2h_{(\delta N)}}\right)$, $\Delta_{R3} = \left(\bar{y}_{h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}}\right)$, $c_{1h_{(\delta N)}}$ and $c_{2h_{(\delta N)}}$ $(h = 1, 2, 3, ..., L)$ are arbitrary constant.

**(2) : Neutrosophic Product Estimators**

Neutrosophic combined product estimators in stratified random sampling using extreme values of data.

$$\hat{\bar{Y}}_{PC_{1_{(\delta N)}}} = \frac{\bar{y}_{st.c12_{(\delta N)}}}{\bar{X}_{(\delta N)}} \bar{x}_{st.c22_{(\delta N)}} \tag{22}$$

Neutrosophic Separate product estimators in stratified random sampling using extreme values of data.

$$\hat{\bar{Y}}_{PS_{1_{(\delta N)}}} = \sum_{h=1}^{L} W_h \frac{\bar{y}_{h.c12_{(\delta N)}}}{\bar{X}_{h_{(\delta N)}}} \bar{x}_{h.c22_{(\delta N)}} \tag{23}$$

where; $\bar{y}_{st.c11_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{y}_{h.c11_{(\delta N)}}$ and $\bar{x}_{st.c21_{(\delta N)}} = \sum_{h=1}^{L} W_h \bar{x}_{h.c21_{(\delta N)}}$

and

$$(\bar{y}_{h_{c.12_{(\delta N)}}}, \bar{x}_{h_{c.22_{(\delta N)}}}) = \begin{cases} \Delta_{P1} & \text{if a sample contains } y_{h_{min_{(\delta N)}}} \text{ but not } y_{h_{max_{(\delta N)}}}, \\ \Delta_{P2} & \text{if a sample contains } y_{h_{max_{(\delta N)}}} \text{ but not } y_{h_{min_{(\delta N)}}}, \\ \Delta_{P3} & \text{in other cases,} \end{cases} \tag{24}$$

where $\Delta_{P1} = \left(\bar{y}_{h_{(\delta N)}} + c_{1h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}} - c_{2h_{(\delta N)}}\right)$, $\Delta_{P2} = \left(\bar{y}_{h_{(\delta N)}} - c_{1h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}} + c_{2h_{(\delta N)}}\right)$, $\Delta_{P3} = \left(\bar{y}_{h_{(\delta N)}}, \bar{x}_{h_{(\delta N)}}\right)$, $c_{1h_{(\delta N)}}$ and $c_{2h_{(\delta N)}}$ $(h = 1, 2, 3, ..., L)$ are arbitrary constant.

**Properties of the Neutrosophic estimators**

To study the large sample properties of neutrosophic estimators let's consider the following transformations:

$e_{st.c_{1j_{(\delta N)}}} = \frac{\left(\bar{y}_{st.c1j_{(\delta N)}} - \bar{Y}_{(\delta N)}\right)}{\bar{Y}_{(\delta N)}}$, $e_{st.c_{2j_{(\delta N)}}} = \frac{\left(\bar{x}_{st.c2j_{(\delta N)}} - \bar{X}_{(\delta N)}\right)}{\bar{X}_{(\delta N)}}$, $j = 1, 2$

$e_{h.c_{1j_{(\delta N)}}} = \frac{\left(\bar{y}_{h.c1j_{(\delta N)}} - \bar{Y}_{h_{(\delta N)}}\right)}{\bar{Y}_{h_{(\delta N)}}}$, $e_{h.c_{2j_{(\delta N)}}} = \frac{\left(\bar{x}_{h.c2j_{(\delta N)}} - \bar{X}_{h_{(\delta N)}}\right)}{\bar{X}_{h_{(\delta N)}}}$, $j = 1, 2$

Up to the first order of the approximation, we will get

$E(e_{st.c_{1j_{(\delta N)}}}) = E(e_{st.c_{2j_{(\delta N)}}}) = E(e_{h.c_{1j_{(\delta N)}}}) = E(e_{h.c_{2j_{(\delta N)}}}) = 0,$

$E(e^2_{st.c_{1j_{(\delta N)}}}) = \frac{1}{\bar{Y}^2_{(\delta N)}} \sum_{h=1}^{L} W_h^2 \lambda_h \left\{ S^2_{hy_{(\delta N)}} - \frac{2n_h c_{1h_{(\delta N)}}}{N_h - 1} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}} - n_h c_{1h_{(\delta N)}}) \right\},$

$E(e^2_{st.c_{2j_{(\delta N)}}}) = \frac{1}{\bar{X}^2_{(\delta N)}} \sum_{h=1}^{L} W_h^2 \lambda_h \left\{ S^2_{hx_{(\delta N)}} - \frac{2n_h c_{2h_{(\delta N)}}}{N_h - 1} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}} - n_h c_{2h_{(\delta N)}}) \right\},$

$E(e^2_{h.c_{1j_{(\delta N)}}}) = \frac{1}{\bar{Y}^2_{h_{(\delta N)}}} \lambda_h \left\{ S^2_{hy_{(\delta N)}} - \frac{2n_h c_{1h_{(\delta N)}}}{N_h - 1} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}} - n_h c_{1h_{(\delta N)}}) \right\},$

$$E(e^2_{h.c_{2j_{(\delta N)}}}) = \frac{1}{\bar{X}^2_{h_{(\delta N)}}}\lambda_h \left\{S^2_{hx_{(\delta N)}} - \frac{2n_h c_{2h_{(\delta N)}}}{N_h-1}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}} - n_h c_{2h_{(\delta N)}})\right\},$$

$$\begin{aligned} E(e_{st.c_{1j_{(\delta N)}}}, e_{st.c_{2j_{(\delta N)}}}) &= \frac{1}{\bar{Y}_{(\delta N)}\bar{X}_{(\delta N)}}\sum_{h=1}^{L} W_h^2 \lambda_h \Big[S_{hyx_{(\delta N)}} - \frac{n_h}{N_h-1}\Big\{c_{1h_{(\delta N)}} \\ &\quad (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}}(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) \\ &\quad -2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}}\Big\}\Big] \end{aligned}$$

$$\begin{aligned} E(e_{h.c_{1j_{(\delta N)}}}, e_{h.c_{2j_{(\delta N)}}}) &= \frac{1}{\bar{Y}_{h_{(\delta N)}}\bar{X}_{h_{(\delta N)}}}\lambda_h \Big[S_{hyx_{(\delta N)}} - \frac{n_h}{N_h-1}\Big\{c_{1h_{(\delta N)}} \\ &\quad (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}}(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) \\ &\quad -2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}}\Big\}\Big] \end{aligned}$$

for $j = 1, 2$.

**Properties of Neutrosophic Combined Estimators :**

Neutrosophic combined ratio estimators $\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}$ in terms of relatives error is given by

$$\hat{\bar{Y}}_{RC_{1_{(\delta N)}}} = \bar{Y}_{(\delta N)}\Big(1 + e_{st.c_{11_{(\delta N)}}}\Big)\Big(1 + e_{st.c_{21_{(\delta N)}}}\Big)^{-1}, \tag{25}$$

Approximation upto first order, we have

$$\begin{aligned} (\hat{\bar{Y}}_{RC_{1_{(\delta N)}}} - \bar{Y}_{(\delta N)}) &\cong \bar{Y}_{(\delta N)}\Big(e_{st.c_{11_{(\delta N)}}} + e_{st.c_{21_{(\delta N)}}} - e_{st.c_{11_{(\delta N)}}} e_{st.c_{21_{(\delta N)}}} \\ &\quad + e^2_{st.c_{21_{(\delta N)}}}\Big), \end{aligned} \tag{26}$$

The Bias of $\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}$ is given by

$$\begin{aligned} Bias(\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}) &\cong \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{\bar{X}_{(\delta N)}}\Big[R_{(\delta N)}\Big\{S^2_{hx_{(\delta N)}} - \frac{2n_h c_{2h_{(\delta N)}}}{N_h-1}(x_{h_{max_{(\delta N)}}} \\ &\quad -x_{h_{min_{(\delta N)}}} - n_h c_{2h_{(\delta N)}})\Big\} - \Big\{S_{hyx_{(\delta N)}} - \frac{n_h}{N_h-1}(c_{1h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} \\ &\quad -x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}}(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) \\ &\quad -2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}})\Big\}\Big] \end{aligned} \tag{27}$$

The MSE of $\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}$ is given by

$$\begin{aligned} MSE(\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}) &\cong MSE(\hat{\bar{Y}}_{RC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h-1}(c_{1h_{(\delta N)}} - R_{(\delta N)} c_{2h_{(\delta N)}}) \\ &\quad \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - R_{(\delta N)}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) \\ &\quad -n_h(c_{1h_{(\delta N)}} - Rc_{2h_{(\delta N)}})\Big\} \end{aligned} \tag{28}$$

Differentiating $MSE(\hat{\bar{Y}}_{RC_{1_{(\delta N)}}})$ with respect to $c_{1h_{(\delta N)}}$ and $c_{2h_{(\delta N)}}$ $(h = 1, 2, 3, ..., L)$ respectively and equate to zero, we have

$$\frac{\partial MSE(\hat{\bar{Y}}_{RC_{1_{(\delta N)}}})}{\partial c_{1h_{(\delta N)}}} = 0 \tag{29}$$

$$(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - R_{(\delta N)}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) - 2n_h(c_{1h_{(\delta N)}} - Rc_{2h_{(\delta N)}}) = 0 \tag{30}$$

Since there are $2L$ unknowns in these equations, a unique solution for the constants is not feasible. We believe that $c_{1h_{(\delta N)}} = \frac{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right)}{2n_h}$ and it implies that $c_{2h_{(\delta N)}} = \frac{\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)}{2n_h}$. For the optimum values of $c_{1h_{(\delta N)}}$ and $c_{2h_{(\delta N)}}$ , the MSE of ( $\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}$), is given by

$$\begin{aligned} MSE(\hat{\bar{Y}}_{RC_{1_{(\delta N)}}})_{min} &= MSE(\hat{\bar{Y}}_{RC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h-1)}\Big\{(y_{h_{max_{(\delta N)}}} \\ &\quad -y_{h_{min_{(\delta N)}}}) - R_{(\delta N)}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}})\Big\}^2 \end{aligned} \tag{31}$$

Similarly, the combined product estimator's bias and MSE $\hat{\bar{Y}}_{PC_{1_{(\delta N)}}}$ , are given by

$$\begin{aligned} Bias(\hat{\bar{Y}}_{PC_{1_{(\delta N)}}}) &\cong \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{\bar{X}_{(\delta N)}}\Big[S_{hyx_{(\delta N)}} - \frac{n_h}{N_h-1}\Big\{c_{1h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} \\ &\quad -x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}}(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) \\ &\quad -2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}}\Big\}\Big] \end{aligned} \tag{32}$$

$$\begin{aligned} MSE(\hat{\bar{Y}}_{PC_{1_{(\delta N)}}}) &\cong MSE(\hat{\bar{Y}}_{PC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h-1}(c_{1h_{(\delta N)}} - R_{(\delta N)}c_{2h_{(\delta N)}}) \\ &\quad \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) + R_{(\delta N)}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) \\ &\quad -n_h(c_{1h_{(\delta N)}} + R_{(\delta N)}c_{2h_{(\delta N)}})\Big\} \end{aligned} \tag{33}$$

Utlizing values of $c_{1h_{(\delta N)}}$ and $c_{2h_{(\delta N)}}$, MSE of $\hat{\bar{Y}}_{PC_{1_{(\delta N)}}}$ , is given as

$$\begin{aligned} MSE(\hat{\bar{Y}}_{PC_{1_{(\delta N)}}})_{min} &= MSE(\hat{\bar{Y}}_{PC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h-1)}\Big\{\Big(y_{h_{max_{(\delta N)}}} \\ &\quad -y_{h_{min_{(\delta N)}}}\Big) - R_{(\delta N)}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{34}$$

**Properties of Neutrosophic Separate Estimators :**

Neutrosophic separate ratio estimators $\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}$ in terms of relatives error is given by

$$\hat{\bar{Y}}_{RS_{1_{(\delta N)}}} = \sum_{h=1}^{L} W_h \bar{Y}_{h_{(\delta N)}}\Big(1 + e_{h.c_{11_{(\delta N)}}}\Big)\Big(1 + e_{h.c_{21_{(\delta N)}}}\Big)^{-1}, \tag{35}$$

Under the above considered transformations on expanding the expression we have

$$(\hat{\bar{Y}}_{RS_{1_{(\delta N)}}} - \bar{Y}_{(\delta N)}) \cong \sum_{h=1}^{L} W_h \bar{Y}_{h_{(\delta N)}} \Big( e_{h.c_{11_{(\delta N)}}} + e_{h.c_{21_{(\delta N)}}} - e_{h.c_{11_{(\delta N)}}} e_{h.c_{21_{(\delta N)}}} + e^2_{h.c_{21_{(\delta N)}}} \Big), \tag{36}$$

The Bias of $\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}$ is given by

$$Bias(\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}) \cong \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{\bar{X}_{h_{(\delta N)}}} \Big[ R_{h_{(\delta N)}} \Big\{ S^2_{hx_{(\delta N)}} - \frac{2n_h c_{h_{(\delta N)}}}{N_h - 1} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}} - n_h c_{2h_{(\delta N)}}) \Big\} - \Big\{ S_{hyx_{(\delta N)}} - \frac{n_h}{N_h - 1} (c_{1h_{(\delta N)}} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - 2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}}) \Big\} \Big] \tag{37}$$

The MSE of $\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}$ is given by

$$MSE(\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}) \cong \sum_{h=1}^{L} W_h^2 \lambda_h \Big[ S^2_{hy_{(\delta N)}} - \frac{2n_h c_{1h_{(\delta N)}}}{N_h - 1} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}} - n_h c_{1h_{(\delta N)}}) + R^2_{h_{(\delta N)}} \{ S^2_{hx_{(\delta N)}} - \frac{2n_h c_{2h_{(\delta N)}}}{N_h - 1} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}} - n_h c_{2h_{(\delta N)}}) \} - 2R_{h_{(\delta N)}} \{ S_{hyx_{(\delta N)}} - \frac{n_h}{N_h - 1} ( c_{1h_{(\delta N)}} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - 2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}}) \} \Big] \tag{38}$$

Min MSE of $\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}$ , is as follows

$$MSE(\hat{\bar{Y}}_{RS_{1_{(\delta N)}}})_{min} = MSE(\hat{\bar{Y}}_{RS_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \Big\{ \Big( y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}} \Big) - R_{h_{(\delta N)}} \Big( x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}} \Big) \Big\}^2 \tag{39}$$

Similarly bias and MSE of estimator $\hat{\bar{Y}}_{PS_{1_{(\delta N)}}}$ , are as follows

$$Bias(\hat{\bar{Y}}_{PS_{1_{(\delta N)}}}) \cong \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{\bar{X}_{(\delta N)}} \Big[ S_{hyx_{(\delta N)}} - \frac{n_h}{N_h - 1} \Big\{ c_{1h_{(\delta N)}} (x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) + c_{2h_{(\delta N)}} (y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - 2n_h c_{1h_{(\delta N)}} c_{2h_{(\delta N)}} \Big\} \Big] \tag{40}$$

$$MSE(\hat{\hat{Y}}_{PS_{1_{(\delta N)}}}) \cong MSE(\hat{\hat{Y}}_{PS_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} + R_{h_{(\delta N)}} c_{2h_{(\delta N)}}) \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) + R_{h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) - n_h(c_{1h_{(\delta N)}} + R_{h_{(\delta N)}} c_{2h_{(\delta N)}})\Big\} \quad (41)$$

Min MSE of $\hat{\hat{Y}}_{PS_{1_{(\delta N)}}}$ , is as follows

$$MSE(\hat{\hat{Y}}_{PS_{1_{(\delta N)}}})_{min} = MSE(\hat{\hat{Y}}_{PS_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)}\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - R_{h_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \quad (42)$$

4.1. *Neutrosophic OLS Regression and Proposed Estimators*

In the discipline of statistics, parametric regression techniques are widely used, providing useful tools for modeling and analysing correlations between variables. Among these strategies, Ordinary Least Squares (OLS) stands out as one of the most often used. The primary goal of OLS is to estimate model parameters by minimising the sum of squared residuals, which indicate the differences between observed and projected values. We are utilising the Särndal method for addressing extreme values as we pursue thorough statistical analysis.

Neutrosophic statistics addresses uncertainty that is characterised by ambiguity, vagueness, and indeterminacy, posing particular difficulties for data interpretation. Neutosophic statistics incorporates the Särndal technique, and we want to create novel approaches that efficiently manage high values in the presence of uncertain data. This combination strategy has a great deal of promise to enhance the reliability and validity of statistical analysis, facilitating better informed decision-making across a variety of disciplines.

4.1.1. *When Positive Correlated*

**(1) : Neutrosophic Combined Regression Estimators**

Neutrosophic combined regression estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}} = \bar{y}_{st.c11_{(\delta N)}} + b_{c_{(\delta N)}}\Big(\bar{X}_{(\delta N)} - \bar{x}_{st.c21_{(\delta N)}}\Big) \quad (43)$$

where $b_{c_{(\delta N)}}$ is a neutrosophic combined sample regression coefficient.

**(2) : Neutrosophic Separate Regression Estimators**

Neutrosophic separate regression estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}} = \sum_{h=1}^{L} W_h \left[\bar{y}_{h.c11_{(\delta N)}} + b_{h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c21_{(\delta N)}}\right)\right] \quad (44)$$

4.1.2. *When Negative Correlated*

**(1) : Neutrosophic Combined Regression Estimators**

Neutrosophic combined regression estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\hat{Y}}_{lrC_{12_{(\delta N)}}} = \bar{y}_{st.c12_{(\delta N)}} + b_{c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st.c22_{(\delta N)}}\right) \quad (45)$$

**(2) : Neutrosophic Separate Regression Estimators**

Neutrosophic separate regression estimators in stratified random sampling of a finite population mean $(\bar{Y}_{(\delta N)})$ are given as:

$$\hat{\hat{Y}}_{lrS_{12_{(\delta N)}}} = \sum_{h=1}^{L} W_h \left[\bar{y}_{h.c12_{(\delta N)}} + b_{h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c22_{(\delta N)}}\right)\right] \quad (46)$$

**Properties of Neutrosophic Combined Regression Estimators**

Bias and MSE of combined neutrosophic regression estimator $\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias(\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}}) = -Cov\left(\bar{x}_{st.c21_{(\delta N)}}, b_{c_{(\delta N)}}\right) \quad (47)$$

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}}) \;&\cong\; MSE(\hat{\hat{Y}}_{lrC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} - \beta_{c_{(\delta N)}} c_{2h_{(\delta N)}}) \\ &\Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - \beta_{c_{(\delta N)}}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) \\ &-n_h(c_{1h_{(\delta N)}} - \beta_{c_{(\delta N)}} c_{2h_{(\delta N)}})\Big\} \end{aligned} \quad (48)$$

where $\beta_{c_{(\delta N)}} = \frac{Cov\left(\bar{y}_{st_{(\delta N)}}, \bar{x}_{st_{(\delta N)}}\right)}{Var\left(\bar{x}_{st_{(\delta N)}}\right)}$ is population regression coefficient.

Min MSE of $\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrC_{11_{(\delta N)}}})_{min} \;&=\; MSE(\hat{\hat{Y}}_{lrC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - \beta_{c_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \end{aligned} \quad (49)$$

When there is negative correlation , the MSE of neutrosophic combined regression estimator $\hat{\hat{Y}}_{lrC_{12_{(\delta N)}}}$, is given by

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrC_{12_{(\delta N)}}}) &\cong MSE(\hat{\hat{Y}}_{lrC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\Big(c_{1h_{(\delta N)}} + \beta_{c_{(\delta N)}} c_{2h_{(\delta N)}}\Big) \\ &\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) + \beta_{c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big) \\ &-n_h\Big(c_{1h_{(\delta N)}} + \beta_{c_{(\delta N)}} c_{2h_{(\delta N)}}\Big)\Big\} \qquad (50) \end{aligned}$$

Min MSE of $\hat{\hat{Y}}_{lrC_{12_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrC_{12_{(\delta N)}}})_{min} &\cong MSE(\hat{\hat{Y}}_{lrC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) + \beta_{c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \qquad (51) \end{aligned}$$

Equations (49) and (51)'s findings for the variances of combined regression estimators allow us to calculate the minimum mean square error of the combined regression estimator as follows.

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrC_{1_{(\delta N)}}})_{min} &\cong MSE(\hat{\hat{Y}}_{lrC_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - |\beta_{c_{(\delta N)}}|\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \qquad (52) \end{aligned}$$

**Properties of Neutrosophic Separate Regression Estimators**

Bias and MSE of separate neutrosophic regression estimator $\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias(\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}}) = -\sum_{h=1}^{L} W_{h_{(\delta N)}} Cov\Big(\bar{x}_{h.c21_{(\delta N)}}, b_{h_{(\delta N)}}\Big) \qquad (53)$$

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}}) &\cong MSE(\hat{\hat{Y}}_{lrS_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\Big(c_{1h_{(\delta N)}} - \beta_{h_{(\delta N)}} c_{2h_{(\delta N)}}\Big) \\ &\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - \beta_{h_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big) \\ &-n_h\Big(c_{1h_{(\delta N)}} - \beta_{h_{(\delta N)}} c_{2h_{(\delta N)}}\Big)\Big\} \qquad (54) \end{aligned}$$

where $\beta_{h_{(\delta N)}} = \frac{S_{hyx(\delta N)}}{S^2_{hx_{(\delta N)}}}$ is population regression coefficient in $h^{th}$ stratum.

Min MSE of $\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrS_{11_{(\delta N)}}})_{min} &= MSE(\hat{\hat{Y}}_{lrS_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - \beta_{h_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \qquad (55) \end{aligned}$$

When there is negative correlation , the MSE of neutrosophic Separate regression estimator $\hat{\hat{Y}}_{lrS_{12(\delta N)}}$, is given by

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrS_{12(\delta N)}}) &\cong MSE(\hat{\hat{Y}}_{lrS_{0(\delta N)}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\Big(c_{1h_{(\delta N)}} + \beta_{h_{(\delta N)}} c_{2h_{(\delta N)}}\Big) \\ &\Big\{\Big(y_{h_{max(\delta N)}} - y_{h_{min(\delta N)}}\Big) + \beta_{h_{(\delta N)}}\Big(x_{h_{max(\delta N)}} - x_{h_{min(\delta N)}}\Big) \\ &-n_h\Big(c_{1h_{(\delta N)}} + \beta_{h_{(\delta N)}} c_{2h_{(\delta N)}}\Big)\Big\} \qquad (56) \end{aligned}$$

Min MSE of $\hat{\hat{Y}}_{lrS_{12(\delta N)}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrS_{12(\delta N)}})_{min} &= MSE(\hat{\hat{Y}}_{lrS_{0(\delta N)}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\Big\{\Big(y_{h_{max(\delta N)}} - y_{h_{min(\delta N)}}\Big) + \beta_{h_{(\delta N)}}\Big(x_{h_{max(\delta N)}} - x_{h_{min(\delta N)}}\Big)\Big\}^2 \qquad (57) \end{aligned}$$

Generally the minimum mean square error of Separate regression estimator can be written as

$$\begin{aligned} MSE(\hat{\hat{Y}}_{lrS_{1(\delta N)}})_{min} &= MSE(\hat{\hat{Y}}_{lrS_{0(\delta N)}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ &\Big\{\Big(y_{h_{max(\delta N)}} - y_{h_{min(\delta N)}}\Big) - |\beta_{h_{(\delta N)}}|\Big(x_{h_{max(\delta N)}} - x_{h_{min(\delta N)}}\Big)\Big\}^2 \qquad (58) \end{aligned}$$

### 4.2. *Neutrosophic Robust Regression and Proposed Estimators*

When working with datasets, the presence of outliers or exceptional data points may affect the accuracy of estimates obtained through the ordinary least squares (OLS) approach. OLS, a commonly employed approach for establishing relationships between variables, becomes susceptible in the face of outliers, as even a single outlier can exert undue influence on the results of investigation. The vulnerability has been demonstrated by researchers Hampel et al. [11] and Rousseeuw and Leroy [18],

Seber and Lee [21] explain the causes of OLS's susceptibility to outliers. First off, greater residuals have a disproportionately strong impact on the total error assessment, dominating the contributions of smaller residuals, when mistakes are quantified using the squared deviations among observed and predicted results (residuals). Furthermore, common measures of central tendency, such as the arithmetic mean, are not designed to deal with outliers. Due to the impact of greater squared values, the existence of outliers causes significant distortions in error estimates, magnifying their effect on regression findings.

Alternative techniques, including robust regression, are used to mitigate the negative impact of outliers on regression results. These methods are made to be less impacted by outliers and to give more accurate predictions even when there are odd data points present. Interested parties can read Yu and Yao's study [37] for an in-depth explanation of these robust regression techniques. Regression using ordinary least squares (OLS) is a popular method for estimating a linear regression model's parameters. Its primary process entails minimising the sum of squared differences between the dependent variable's expected and actual values. Finding the best line that best explains the relationship between the independent and dependent variables while reducing model errors is the major goal. However, OLS relies on several presumptions that could not hold true in all real-world situations, such as the normal distribution of the residuals (the differences between estimated and observed values) and a constant variance.

OLS's increased sensitivity to outliers—data values that significantly deviate from the norm—is an important attribute. Natural variances or mistakes in the data collection process might lead to outliers. Even though it could be irregular, their influence might be significant, skewing predictions of regression coefficients and producing inaccurate forecasts. Enter robust regression, a strategy that has gained interest to address OLS's sensitivity to outliers. Robust regression offers a more robust analysis since it is made to be less impacted by these unusual instances. This is especially helpful when working with real-world data when OLS's presumptions might not hold true. Robust regression allows for the diversity and complexity that are frequently seen in real-world data, allowing analysts to gain more trustworthy conclusions. The differences between the two approaches highlight the constantly developing approaches in statistics and data analysis, highlighting the demand for flexible tools that can negotiate the complexities of real datasets and produce accurate findings even in the face of anomalies. Within robust statistical techniques, robust regression is a subset that focuses on minimising the sum of absolute residues instead of the sum of squared residuals. This novel strategy avoids the weaknesses to outliers that afflict conventional approaches. Extreme data points have less disproportionate effect on results when the absolute values of residuals are highlighted. In robust regression, outliers are given less weight rather than being given the same weight as other data points. This tactic greatly reduces their influence on the regression's estimated coefficients. Ordinary Least Squares (OLS) analysis frequently falls short of robust regression because it is less adept at navigating the turbulent seas of extreme values, especially in circumstances rife with outliers.

The homoscedasticity assumption, which underlies OLS regression, states that residual variance should be consistent over the range of the independent variable. This principle, however, can be undermined by real-world data, particularly if the distribution of the dependent variable is atypical. The difference may result in inaccurate coefficient estimations and uncertain projections. Another obstacle for OLS is sensitivity to outliers, which can reduce precision. With its weighted least squares technique, robust regression intervenes in this situation as a solution to the problem. With this strategy, observations with smaller variations receive heavier weights while those with bigger variances receive lesser weights. The end result is a more accurate estimation of the coefficients, which holds up well even when there are outliers or important data points. Regression analyses are therefore kept reliable and perceptive even while dealing with the complexity and uncertainties present in real-world data thanks to this strong technique.

When dealing with data that vary from a normal distribution and hence violate the assumption of normality, robust regression outperforms ordinary least squares (OLS) regression. In order for OLS regression to work, the dependent variable must have a normal distribution, which is a requirement that is frequently not satisfied by real-world data. As a result, predictions may turn out to be inaccurate and the coefficients obtained using OLS may be biased. Contrastingly, robust regression does not rely on the normality assumption and has a better ability to deal with non-normal data. Huber-M, Hampel-M, and Tukey-M are three robust regression approaches that are specially used in this work. For a complete understanding of these strategies, please refer to the in-depth study conducted by Zaman [38], Zaman and Bulut ( [39], [40], [42]), Shahzad et al.( [24], [26], [25]), Sajjad et al. [20], Ali et al. [2], Mukhtar and Shahzad [16], Bulut and Zaman [6].

4.2.1. *When Positive Correlated*

**(1) : Neutrosophic Combined Robust Regression Estimators**

Neutrosophic combined regression estimators ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}} = \bar{y}_{st.c11_{(\delta N)}} + \beta_{(rob_i).c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st.c21_{(\delta N)}}\right) \tag{59}$$

**(2) : Neutrosophic Separate Robust Regression Estimators**

Neutrosophic separate regression estimators ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}} = \sum_{h=1}^{L} W_h\left[\bar{y}_{h.c11_{(\delta N)}} + \beta_{(rob_i).h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c21_{(\delta N)}}\right)\right] \tag{60}$$

4.2.2. *When Negative Correlated*

**(1) : Neutrosophic Combined Robust Regression Estimators**

Neutrosophic combined regression estimators ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(P_i).C_{12_{(\delta N)}}} = \bar{y}_{st.c12_{(\delta N)}} + \beta_{(rob_i).c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st.c22_{(\delta N)}}\right) \tag{61}$$

**(2) : Neutrosophic Separate Robust Regression Estimators**

Neutrosophic separate regression estimators ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(P_i).S_{12_{(\delta N)}}} = \sum_{h=1}^{L} W_h\left[\bar{y}_{h.c12_{(\delta N)}} + \beta_{(rob_i).h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c22_{(\delta N)}}\right)\right] \tag{62}$$

where $\beta_{(rob_i).c_{(\delta N)}}$ and $\beta_{(rob_i).h_{(\delta N)}}$ are computed from robust regression tools like Huber-M, Hample-M, and Tukey-M for $i = 1, 2, 3$.

**Properties of Neutrosophic combined Robust Regression Estimators**

The bias and MSE of combined neutrosophic regression estimator $\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias\left(\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}}\right) = -Cov\left(\bar{x}_{st.c21_{(\delta N)}}, b_{c_{(\delta N)}}\right) \tag{63}$$

$$\begin{aligned} MSE\left(\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}}\right) \cong\ & MSE(\hat{\bar{Y}}_{(P_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\left(c_{1h_{(\delta N)}} - \hat{\beta}_{(rob_i).c_{(\delta N)}} c_{2h_{(\delta N)}}\right) \\ & \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - \hat{\beta}_{(rob_i).c_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right. \\ & \left. -n_h\left(c_{1h_{(\delta N)}} - \hat{\beta}_{(rob_i).c_{(\delta N)}} c_{2h_{(\delta N)}}\right)\right\} \end{aligned} \tag{64}$$

where $\hat{\beta}_{(rob_i).c_{(\delta N)}}$ is population regression coefficient.

Min MSE of $\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).C_{11_{(\delta N)}}})_{min} =\ & MSE(\hat{\bar{Y}}_{(P_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - \hat{\beta}_{(rob_i).c_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \end{aligned} \tag{65}$$

When there is negative correlation , the MSE of neutrosophic combined robust regression estimator $\hat{\bar{Y}}_{(P_i).C_{12_{(\delta N)}}}$, is as follows

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).C_{12_{(\delta N)}}}) \cong\ & MSE(\hat{\bar{Y}}_{(P_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\left(c_{1h_{(\delta N)}} + \hat{\beta}_{(rob_i).c_{(\delta N)}} c_{2h_{(\delta N)}}\right) \\ & \left\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) + \hat{\beta}_{(rob_i).c_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right. \\ & \left. -n_h\left(c_{1h_{(\delta N)}} + \hat{\beta}_{(rob_i).c_{(\delta N)}} c_{2h_{(\delta N)}}\right)\right\} \end{aligned} \tag{66}$$

Min MSE of $\hat{\bar{Y}}_{(P_i).C_{12_{(\delta N)}}}$, is given by:

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).C_{12_{(\delta N)}}})_{min} \cong\ & MSE(\hat{\bar{Y}}_{(P_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) + \hat{\beta}_{(rob_i).c_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \end{aligned} \tag{67}$$

Equations (65) and (67)'s findings for the variances of combined regression estimators allow us to calculate the minimum mean square error of the combined regression estimator as follows.

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).C_{1_{(\delta N)}}})_{min} \cong\ & MSE(\hat{\bar{Y}}_{(P_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - |\hat{\beta}_{(rob_i).c_{(\delta N)}}|\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \end{aligned} \tag{68}$$

**Properties of Neutrosophic Separate Robust Regression Estimators**

Bias and MSE of separate neutrosophic regression estimator $\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias(\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}}) = -\sum_{h=1}^{L} W_{h_{(\delta N)}} Cov\Big(\bar{x}_{h.c21_{(\delta N)}}, b_{h_{(\delta N)}}\Big) \tag{69}$$

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}}) \cong\ & MSE(\hat{\bar{Y}}_{(P_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} - \hat{\beta}_{(rob_i).h_{(\delta N)}} \\ & \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - \hat{\beta}_{(rob_i).h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) \\ & -n_h(c_{1h_{(\delta N)}} - \hat{\beta}_{(rob_i).h_{(\delta N)}})\Big\} \end{aligned} \tag{70}$$

where $\hat{\beta}_{(rob_i).h_{(\delta N)}}$ is population regression coefficient in $h^{th}$ stratum.

Min MSE of $\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).S_{11_{(\delta N)}}})_{min} =\ & MSE(\hat{\bar{Y}}_{(P_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) - \hat{\beta}_{(rob_i).h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}})\Big\}^2 \end{aligned} \tag{71}$$

When there is negative correlation , the MSE of neutrosophic separate robust regression estimator $\hat{\bar{Y}}_{(P_i).S_{12_{(\delta N)}}}$, is given by

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).S_{12_{(\delta N)}}}) \cong\ & MSE(\hat{\bar{Y}}_{(P_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} + \hat{\beta}_{(rob_i).h_{(\delta N)}} \\ & \Big\{(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}) + \hat{\beta}_{(rob_i).h_{(\delta N)}}(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}) \\ & -n_h(c_{1h_{(\delta N)}} + \hat{\beta}_{(rob_i).h_{(\delta N)}})\Big\} \end{aligned} \tag{72}$$

Min MSE of $\hat{\bar{Y}}_{(P_i).S_{12_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE\Big(\hat{\bar{Y}}_{(P_i).S_{12_{(\delta N)}}}\Big)_{min} =\ & MSE\Big(\hat{\bar{Y}}_{(P_i).S_{0_{(\delta N)}}}\Big) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) + \hat{\beta}_{(rob_i).h_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{73}$$

Generally the Min MSE of Separate Robust neutrosophic regression estimator can be written as

$$\begin{aligned} MSE(\hat{\bar{Y}}_{(P_i).S_{1_{(\delta N)}}})_{min} =\ & MSE(\hat{\bar{Y}}_{(P_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - |\hat{\beta}_{(rob_i).h_{(\delta N)}}|\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{74}$$

4.3. *Neutrosophic Robust Quantile Regression and Proposed Estimators*

In contrast to the conventional linear regression strategy, quantile regression presents an alternative that does not rely on the assumptions of constant variance and a normal distribution in error terms. The adaptability of this alternative method avoids the necessity for certain presumptions to be true. Quantile regression predicts the conditional values within the dependent variable's distribution in a linear model rather than depending on averages [10]. The quartiles used to calculate the coefficients are frequently 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, and 0.85. Robust quantile coefficients for regression take center stage in place of the typical coefficients generated from Ordinary Least Squares (OLS) regression, in contrast to the traditional emphasis on averages.

In our suggestion, neutrosophic estimators are used, providing improved resilience to the disruptive influence of data outliers. When datasets are impacted by outlier data points, these estimators should be used in estimation approaches due to their exceptional resilience. The robust quantile regression method that is suggested delivers estimates that are more accurate even when outliers are present, therefore it is not necessary to exclude these data points from the study.

Together, these viewpoints provide a more comprehensive understanding of the trans formative properties of quantile regression, its stable coefficients, and the novel incorporation of neutrosophic estimators, all of which support a more flexible and accurate analysis of data that may be affected by outliers researchers such as Anas et al. [5], Yadav and Prasad [36], Shahzad et al. ( [22], [23]) and Koç and Koç [13] have made significant contributions in the filed of robust quantile regression.

4.3.1. *When Positive Correlated*

**(1) : Neutrosophic Combined Robust Quantile Regression Estimators**

Neutrosophic combined robust quantile regression estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(Q_i).C_{11_{(\delta N)}}} = \bar{y}_{st.c11_{(\delta N)}} + \beta_{(q_j).c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st.c21_{(\delta N)}}\right) \tag{75}$$

**(2) : Neutrosophic Separate Robust Quantile Regression Estimators**

Neutrosophic Separate robust quantile regression estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(Q_i).S_{11_{(\delta N)}}} = \sum_{h=1}^{L} W_h\left[\bar{y}_{h.c11_{(\delta N)}} + \beta_{(q_j).h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c21_{(\delta N)}}\right)\right] \tag{76}$$

4.3.2. *When Negative Correlated*

**(1) : Neutrosophic Combined Robust Quantile Regression Estimators**

Neutrosophic combined robust quantile regression estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(Q_i).C_{12_{(\delta N)}}} = \bar{y}_{st.c12_{(\delta N)}} + \beta_{(q_j).c_{(\delta N)}}\left(\bar{X}_{(\delta N)} - \bar{x}_{st.c22_{(\delta N)}}\right) \tag{77}$$

**(2) : Neutrosophic Separate Robust Quantile Regression Estimators**

Neutrosophic separate robust quantile regression estimators in stratified random sampling of a finite population mean ($\bar{Y}_{(\delta N)}$) are given as:

$$\hat{\bar{Y}}_{(Q_i).S_{12_{(\delta N)}}} = \sum_{h=1}^{L} W_h\left[\bar{y}_{h.c12_{(\delta N)}} + \beta_{(q_j).h_{(\delta N)}}\left(\bar{X}_{h_{(\delta N)}} - \bar{x}_{h.c22_{(\delta N)}}\right)\right] \tag{78}$$

where $\beta_{(q_j).c_{(\delta N)}}$ and $\beta_{(q_i).h_{(\delta N)}}$ are computed from robust quantile regression tools for $j = 0.15, 0.25, 0.35, ..., 0.85$.

**Properties of Neutrosophic Combined Robust Quantile Regression Estimators**

Bias and MSE of combined neutrosophic regression estimator $\hat{\hat{Y}}_{(Q_i).C_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias(\hat{\hat{Y}}_{(Q_i).C_{11_{(\delta N)}}}) = -Cov\Big(\bar{x}_{st.c21_{(\delta N)}}, b_{c_{(\delta N)}}\Big) \tag{79}$$

$$\begin{aligned} MSE(\hat{\hat{Y}}_{(Q_i).C_{11_{(\delta N)}}}) \cong\ & MSE\Big(\hat{\hat{Y}}_{(Q_i).C_{0_{(\delta N)}}}\Big) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\Big(c_{1h_{(\delta N)}} - \hat{\beta}_{(q_j).c_{(\delta N)}} c_{2h_{(\delta N)}}\Big) \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - \hat{\beta}_{(q_j).c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big) \\ & -n_h\Big(c_{1h_{(\delta N)}} - \hat{\beta}_{(q_j).c_{(\delta N)}} c_{2h_{(\delta N)}}\Big)\Big\} \end{aligned} \tag{80}$$

where $\hat{\beta}_{(q_j).c_{(\delta N)}}$ is population regression coefficient.
Min MSE of $\hat{\hat{Y}}_{(Q_i).C_{11_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{(Q_i).C_{11_{(\delta N)}}})_{min} =\ & MSE(\hat{\hat{Y}}_{(Q_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - \hat{\beta}_{(q_j).c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{81}$$

When there is negative correlation , the MSE of neutrosophic combined robust quantile regression estimator $\hat{\hat{Y}}_{(Q_i).C_{12_{(\delta N)}}}$, is as follows

$$\begin{aligned} MSE(\hat{\hat{Y}}_{(Q_i).C_{12_{(\delta N)}}}) \cong\ & MSE(\hat{\hat{Y}}_{(Q_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}\Big(c_{1h_{(\delta N)}} + \hat{\beta}_{(q_j).c_{(\delta N)}} c_{2h_{(\delta N)}}\Big) \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) + \hat{\beta}_{(q_j).c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big) \\ & -n_h\Big(c_{1h_{(\delta N)}} + \hat{\beta}_{(q_j).c_{(\delta N)}} c_{2h_{(\delta N)}}\Big)\Big\} \end{aligned} \tag{82}$$

Min MSE of $\hat{\hat{Y}}_{(Q_i).C_{12_{(\delta N)}}}$, is as follows:

$$\begin{aligned} MSE(\hat{\hat{Y}}_{(Q_i).C_{12_{(\delta N)}}})_{min} \cong\ & MSE(\hat{\hat{Y}}_{(Q_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) + \hat{\beta}_{(q_j).c_{(\delta N)}}\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{83}$$

Equations (81) and (83)'s findings for the variances of combined regression estimators allow us to calculate the minimum mean square error of the combined regression estimator as follows.

$$\begin{aligned} MSE(\hat{\hat{Y}}_{(Q_i).C_{1_{(\delta N)}}})_{min} \cong\ & MSE(\hat{\hat{Y}}_{(Q_i).C_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \\ & \Big\{\Big(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\Big) - |\hat{\beta}_{(q_j).c_{(\delta N)}}|\Big(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\Big)\Big\}^2 \end{aligned} \tag{84}$$

**Properties of Neutrosophic Separate Robust Quantile Regression Estimators**

Bias and MSE of separate neutrosophic robust quantile regression estimator $\hat{\hat{Y}}_{(Q_i).S_{11_{(\delta N)}}}$ in case positive correlation given by

$$Bias(\hat{\hat{Y}}_{(Q_i).S_{11_{(\delta N)}}}) = -\sum_{h=1}^{L} W_{h_{(\delta N)}} Cov\Big(\bar{x}_{h.c21_{(\delta N)}}, b_{h_{(\delta N)}}\Big) \tag{85}$$

$$MSE(\hat{\hat{Y}}_{(Q_i).S_{11_{(\delta N)}}}) \cong MSE(\hat{\hat{Y}}_{(Q_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} - \hat{\beta}_{(q_i).h_{(\delta N)}} \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - \hat{\beta}_{(q_i).h_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right) - n_h(c_{1h_{(\delta N)}} - \hat{\beta}_{(q_i).h_{(\delta N)}})\right\} \quad (86)$$

where $\hat{\beta}_{(q_i).h_{(\delta N)}}$ is population regression coefficient in $h^{th}$ stratum.
Min MSE of $\hat{\hat{Y}}_{(Q_i).S_{11_{(\delta N)}}}$, is as follows:

$$MSE(\hat{\hat{Y}}_{(Q_i).S_{11_{(\delta N)}}})_{min} = MSE(\hat{\hat{Y}}_{(Q_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - \hat{\beta}_{(q_i).h_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \quad (87)$$

When there is negative correlation , the MSE of neutrosophic separate robust quantile regression estimator $\hat{\hat{Y}}_{(Q_i).S_{12_{(\delta N)}}}$, is given by

$$MSE(\hat{\hat{Y}}_{(Q_i).S_{12_{(\delta N)}}}) \cong MSE(\hat{\hat{Y}}_{(Q_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{2\lambda_h n_h}{N_h - 1}(c_{1h_{(\delta N)}} + \hat{\beta}_{(q_i).h_{(\delta N)}} \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) + \hat{\beta}_{(rob_i).h_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right) - n_h(c_{1h_{(\delta N)}} + \hat{\beta}_{(q_i).h_{(\delta N)}})\right\} \quad (88)$$

Min MSE of $\hat{\hat{Y}}_{(Q_i).S_{12_{(\delta N)}}}$, is as follows:

$$MSE(\hat{\hat{Y}}_{(Q_i).S_{12_{(\delta N)}}})_{min} = MSE(\hat{\hat{Y}}_{(Q_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) + \hat{\beta}_{(q_i).h_{(\delta N)}}\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \quad (89)$$

Generally the Min MSE of Separate robust neutrosophic quantile regression estimator can be given as

$$MSE(\hat{\hat{Y}}_{(Q_i).S_{1_{(\delta N)}}})_{min} = MSE(\hat{\hat{Y}}_{(Q_i).S_{0_{(\delta N)}}}) - \sum_{h=1}^{L} W_h^2 \frac{\lambda_h}{2(N_h - 1)} \left\{\left(y_{h_{max_{(\delta N)}}} - y_{h_{min_{(\delta N)}}}\right) - |\hat{\beta}_{(q_i).h_{(\delta N)}}|\left(x_{h_{max_{(\delta N)}}} - x_{h_{min_{(\delta N)}}}\right)\right\}^2 \quad (90)$$

## 5. Applications

The use of weather data, notably dew point, emerges as a crucial and cutting-edge tactic in the context of our work, which is committed to improving sample survey data processing approaches. Assessing the impact of urban heat islands on locals' health in four important Pakistani cities—Karachi, Lahore, Peshawar, and Multan—is the central focus of our study. Our dataset has a total of 1460 data points, which corresponds to the four strata and each including 365 days of dew point data, with each city acting as a separate stratum. We take a unique strategy to improve estimation accuracy in the face of the complexity of outliers and imprecise findings, which are particularly prominent in humidity and temperature data.

The incorporation of dew point in the form of an auxiliary variable assumes a crucial role in this endeavor. Dew point offers an additional source of information that considerably improves our analysis as an indication of atmospheric moisture. This integration gives our investigation of the urban heat island phenomena a multidimensional dimension that is specially designed for it. Our analytical methodology overcomes the limitations imposed by single factors by comparing dew point alongside temperature and humidity data. The result of this complementary fusion is a notable improvement in estimating accuracy. Notably, the addition of dew point as an auxiliary variable enables us to stabilise our results, especially by reducing the likelihood of distortions caused by high values—a problem that outliers by their very nature provide.

The vital significance of dew point data is made clear as a result of this novel technique to navigating the complicated terrain of urban health dynamics. Beyond supporting the validity of our findings, this integration has larger implications that go across academic fields. The multidisciplinary scope includes areas like public health policy and urban planning, where precise data analysis is crucial.

Additionally, our system is built to cover every possible angle. Starting with combined as well as separate regression estimators accommodating both positive and negative data distributions, we meticulously explore a variety of estimate strategies. We guarantee the validity and usefulness of our findings by taking this heterogeneity into consideration. Next, we move on to the topic of robust regression algorithms, which are made to resist the effects of outliers and anomalies present in real-world data. Finally, robust quantile regression, which is a robust method that not only takes into consideration the existence of outliers but also offers insights into various quantiles of the data distribution, serves as the point where our trip comes to an end.

As we go forward, the mean squared error (MSE) becomes a crucial parameter for assessing the effectiveness of our suggested estimators. The MSE provides a thorough assessment of estimator accuracy by calculating the average squared difference between our estimators and the real values. This essential evaluation enables us to determine how effectively our novel approaches provide accurate estimates in the presence of outliers and erroneous observations. Interestingly, the study's least MSE value is displayed by our suggested neutrosophic estimators, further demonstrating their superiority in handling complicated data circumstances. Neutrosophic Summary of Weather Data (Dew Point) is given in table [1].

## 6. Results Discussion

The empirical examination of our suggested estimators, supplemented by Weather data (Dew Point), produced interesting findings that validate the efficacy of our novel technique. The evaluation includes a wide range of estimators, such ratio, product, robust regression, as well as robust quantile regression, each of which is evaluated for Mean Squared Error (MSE) efficiency.

**(I):** The MSE values for our estimators determined by dew point data are shown in Table [2]. Notable are the ratio estimators, indicated as $\hat{\bar{Y}}_{RC_{0_{(\delta N)}}}$, $\hat{\bar{Y}}_{RC_{1_{(\delta N)}}}$, $\hat{\bar{Y}}_{RS_{0_{(\delta N)}}}$, and $\hat{\bar{Y}}_{RS_{1_{(\delta N)}}}$, show results in the range [9.703, 10.218], demonstrating how well they did in capturing the fundamental demographic factors. Product estimators, $\hat{\bar{Y}}_{PC_{0_{(\delta N)}}}$, $\hat{\bar{Y}}_{PC_{1_{(\delta N)}}}$, $\hat{\bar{Y}}_{PS_{0_{(\delta N)}}}$, and $\hat{\bar{Y}}_{PS_{1_{(\delta N)}}}$, display MSE results in the range [835.224, 984.712], highlighting their resilience against data fluctuations. Additionally, the regression estimators, $\hat{\bar{Y}}_{lrC_{0_{(\delta N)}}}$, $\hat{\bar{Y}}_{lrC_{1_{(\delta N)}}}$, $\hat{\bar{Y}}_{lrS_{0_{(\delta N)}}}$, and $\hat{\bar{Y}}_{lrS_{1_{(\delta N)}}}$, demonstrate favorable MSE results in the range [9.694, 9.965]. These findings demonstrate how well-suited these estimators are to handle scenarios involving complicated data.

We shifted to robust regression approaches for delving deeper into robustness, where

**(II):** The MSE results for the robust regression estimators are presented in Table [3]. A member of these $\hat{\bar{Y}}_{lrC_{1_{(\delta N)}}}$ and alternatives using the robustness methods Huber-M, Hampel-M, and Tukey-M provide MSE values that are comparable with the baseline regression estimators. This consistency highlights how well these estimators handle outliers and erroneous observations, leading to more accurate findings.

**(III):** Our study is expanded upon in Table [4] by robust quantile regression estimation tools. Particularly, when using the combined and separate regression methods, the MSE data across quantiles like $Q_{0.25}$, $Q_{0.35}$, $Q_{0.45}$, $Q_{0.55}$, $Q_{0.65}$, $Q_{0.75}$, and $Q_{0.85}$ stay closely grouped and within the [9.690, 9.696] range. This consistency underlines the robust quantile regression's capacity to offer consistent and trustworthy predictions across many quantile levels, assisting in the development of a more thorough knowledge of the data distribution.

The suggested neutrosophic estimators in the study show the lowest MSE values among the tested estimators, which is very noteworthy. This discovery highlights how effective our novel method is at improving estimation accuracy, especially when there are outliers and ambiguous observations. These findings confirm the potential of neutrosophic estimators to provide reliable statistical insights and more precise population mean estimations.

Table 1. Neutrosophic Summary of Weather Data (Dew Point).

| Notations | Stratum-1 | Stratum-2 | Stratum-3 | Stratum-4 |
|---|---|---|---|---|
| $n_h$ | [365] | [365] | [365] | [365] |
| $\bar{Y}_{h_{(\delta N)}}$ | [76.03014, 102.4466] | [78.46849, 96.77534] | [70.90137, 77.46301] | [78.8, 81.94521] |
| $\bar{X}_{h_{(\delta N)}}$ | [74.78356, 99.88767] | [73.59178, 96.74247] | [73.59452, 74.72055] | [75.45753, 81.88767] |
| $S^2_{hy_{(\delta N)}}$ | [40246, 63944.32] | [40246.29, 64126.03] | [40185.34, 15747.04] | [40235.54, 15919.27] |
| $S^2_{hx_{(\delta N)}}$ | [40591.26, 64155.49] | [37724.89, 63984.47] | [40214.41, 15831.21] | [37895.75, 15919.27] |
| $y_{h_{max_{(\delta N)}}}$ | [2039] | [2041] | [2028] | [2041] |
| $y_{h_{min_{(\delta N)}}}$ | [34] | [45] | [39] | [46] |
| $x_{h_{max_{(\delta N)}}}$ | [2046] | [2043] | [2045] | [2048] |
| $x_{h_{min_{(\delta N)}}}$ | [0] | [0] | [0] | [0] |
| $\hat{\beta}_{(rob_{(Huber-M)})_{(\delta N)}}$ | [0.9912323, 0.9999753] | [0.9999594, 1.000504] | [0.9937949, 0.9950768] | [0.996666, 0.9984121] |
| $\hat{\beta}_{(rob_{(Hampel-M)})_{(\delta N)}}$ | [0.9908839, 0.9993864] | [1.000555, 1.000763] | [0.9939584, 0.996543] | [0.9972101, 0.9977789] |
| $\hat{\beta}_{(rob_{(Tuckey-M)})_{(\delta N)}}$ | [0.993372, 1.003817] | [1.000812, 1.000578] | [0.9938141, 0.9959412] | [0.997544, 0.9984946] |
| $\hat{\beta}_{(q_{(0.25)}).h_{(\delta N)}}$ | [0.9959370, 0.9929364] | [1.0005076, 1.0015038] | [0.997544, 0.9984946] | [0.9985007, 1.0010030] |
| $\hat{\beta}_{(q_{(0.35)}).h_{(\delta N)}}$ | [0.9939668, 0.9919355] | [1.000000, 1.0010081] | [0.9964485, 0.9938838] | [0.9974975, 1.0000000] |
| $\hat{\beta}_{(q_{(0.45)}).h_{(\delta N)}}$ | [0.9929293, 1.00000] | [1.0010030, 1.0010147] | [0.9954292, 0.9919920] | [0.9965000, 0.9979859] |
| $\hat{\beta}_{(q_{(0.55)}).h_{(\delta N)}}$ | [0.9915966, 0.9989909] | [1.00000, 1.0010010] | [0.9940090, 0.9939086] | [0.9955202, 0.9989960] |
| $\hat{\beta}_{(q_{(0.65)}).h_{(\delta N)}}$ | [0.9903943, 1.0025432] | [1.0000000, 1.000000] | [0.9929613, 1.000000] | [0.9949900, 0.9979445] |
| $\hat{\beta}_{(q_{(0.75)}).h_{(\delta N)}}$ | [0.9906863, 1.0015228] | [0.9990000, 0.9994926] | [0.9924887, 1.00000] | [0.9964877, 1.0000000] |
| $\hat{\beta}_{(q_{(0.85)}).h_{(\delta N)}}$ | [0.9871668, 1.0015060] | [0.9974912, 0.9984864] | [0.9909684, 1.0020346] | [0.9945110, 0.9989889] |

TABLE 2. Mean Square error of the Estimators based on Weather Data (Dew Point).

| Estimators | MSE |
|---|---|
| Ratio Estimators | |
| $\hat{\bar{Y}}_{RC_{0(\delta N)}}$ | [9.9039479, 0.3649951] |
| $\hat{\bar{Y}}_{RC_{1(\delta N)}}$ | [9.8270814, 0.3342089] |
| $\hat{\bar{Y}}_{RS_{0(\delta N)}}$ | [10.2176925, 0.4133462] |
| $\hat{\bar{Y}}_{RS_{1(\delta N)}}$ | [10.0927650, 0.3727853] |
| Product Estimators | |
| $\hat{\bar{Y}}_{PC_{0(\delta N)}}$ | [975.3846, 984.7125] |
| $\hat{\bar{Y}}_{PC_{1(\delta N)}}$ | [975.3078, 984.6818] |
| $\hat{\bar{Y}}_{PS_{0(\delta N)}}$ | [974.6680, 983.6323] |
| $\hat{\bar{Y}}_{PS_{1(\delta N)}}$ | [835.2242, 897.9044] |
| Regression Estimators | |
| $\hat{\bar{Y}}_{lrC_{0(\delta N)}}$ | [9.7058299, 0.2982712] |
| $\hat{\bar{Y}}_{lrC_{1(\delta N)}}$ | [9.6942937, 0.2869683] |
| $\hat{\bar{Y}}_{lrS_{0(\delta N)}}$ | [9.9656934, 0.2974041] |
| $\hat{\bar{Y}}_{lrS_{1(\delta N)}}$ | [9.8900701, 0.2904405] |

TABLE 3. Mean Square error of the Robust Regression Estimators based on Weather Data (Dew Point).

| Estimators | MSE |
|---|---|
| Robust Combined Regression | |
| $\hat{\bar{Y}}_{lrC_{1(\delta N)}}$ | [9.6942937, 0.2869683] |
| $\hat{\bar{Y}}_{(P_{Huber-M}).C_{1(\delta N)}}$ | [9.6928511, 0.2871328] |
| $\hat{\bar{Y}}_{(P_{Hampel-M}).C_{1(\delta N)}}$ | [9.6927111, 0.2870227] |
| $\hat{\bar{Y}}_{(P_{Tuckey-M}).C_{1(\delta N)}}$ | [9.6926003, 0.2863628] |
| Robust Separate Regression | |
| $\hat{\bar{Y}}_{lrS_{1(\delta N)}}$ | [9.8900701, 0.2904405] |
| $\hat{\bar{Y}}_{(P_{Huber-M}).S_{1(\delta N)}}$ | [9.690895, 0.286713] |
| $\hat{\bar{Y}}_{(P_{Hampel-M}).S_{1(\delta N)}}$ | [9.6907605, 0.2864177] |
| $\hat{\bar{Y}}_{(P_{Tuckey-M}).S_{1(\delta N)}}$ | [9.6907406, 0.2877568] |

TABLE 4. Mean Square error of the Robust Quantile Regression Estimators based on Weather Data (Dew Point).

| Estimators | MSE |
|---|---|
| Robust Quantile Combined Regression | |
| $\hat{\hat{Y}}_{lrC_{1(\delta N)}}$ | [9.6942937, 0.2869683] |
| $\hat{\hat{Y}}_{(Q_{0.25}).C_{1(\delta N)}}$ | [9.6923676, 0.2896927] |
| $\hat{\hat{Y}}_{(Q_{0.35}).C_{1(\delta N)}}$ | [9.6923538, 0.2894481] |
| $\hat{\hat{Y}}_{(Q_{0.45}).C_{1(\delta N)}}$ | [9.6924074, 0.2879263] |
| $\hat{\hat{Y}}_{(Q_{0.55}).C_{1(\delta N)}}$ | [9.6929382, 0.2873903] |
| $\hat{\hat{Y}}_{(Q_{0.65}).C_{1(\delta N)}}$ | [9.6935143, 0.2862498] |
| $\hat{\hat{Y}}_{(Q_{0.75}).C_{1(\delta N)}}$ | [9.6934387, 0.2862297] |
| $\hat{\hat{Y}}_{(Q_{0.85}).C_{1(\delta N)}}$ | [9.6963570, 0.2862297] |
| Robust Quantile Separate Regression | |
| $\hat{\hat{Y}}_{lrS_{1(\delta N)}}$ | [9.8900701, 0.2904405] |
| $\hat{\hat{Y}}_{(Q_{0.25}).S_{1(\delta N)}}$ | [9.6915248, 0.2926856] |
| $\hat{\hat{Y}}_{(Q_{0.35}).S_{1(\delta N)}}$ | [9.6904203, 0.2928867] |
| $\hat{\hat{Y}}_{(Q_{0.45}).S_{1(\delta N)}}$ | [9.6904174, 0.2875686] |
| $\hat{\hat{Y}}_{(Q_{0.55}).S_{1(\delta N)}}$ | [9.691002, 0.287058] |
| $\hat{\hat{Y}}_{(Q_{0.65}).S_{1(\delta N)}}$ | [9.6916736, 0.2867495] |
| $\hat{\hat{Y}}_{(Q_{0.75}).S_{1(\delta N)}}$ | [9.6916991, 0.2866265] |
| $\hat{\hat{Y}}_{(Q_{0.85}).S_{1(\delta N)}}$ | [9.6946025, 0.2870798] |

The importance of include Weather data, particularly dew point, in the analysis of sample survey data is highlighted by our study's thorough review, as shown by the MSE values. The innovative neutrosophic estimators, in particular, are frequently highlighted by the findings for their robustness, accuracy, and dependability. These results support our claim that novel techniques like these have the potential to revolutionise statistical analysis in the face of complicated data situations, resulting in more insightful and useful information.

## 7. Conclusion

In the analysis of sample survey data, this proposed study presents a novel approach that uses meteorological information—specifically, Dew point—as an auxiliary variable. This novel method overcomes the shortcomings of outliers and ambiguous observations, which frequently impair the precision of traditional estimating methods. The mean estimator is particularly highlighted in the study, which deliberately focuses on the significant influence of uncommon data on statistical estimators. It emphasises how important it is to keep outliers out of the results in order to maintain their reliability.

The study adopts a proactive approach to tackling this problem by exploring the integration of auxiliary factors and utilising the informative potential of dew point data to improve estimation accuracy. The research of the study also broadens to include problems brought on by anomalous results in a sample of survey data. It reveals how susceptible the mean estimator is to outliers, which might skew results. The study investigates approaches for auxiliary variable integration, such as transformations, to improve accuracy in order to counteract this. In particular, it recommends the use of known extreme values as supplementary information to enhance estimation precision. The limitations of standard statistical approaches that rely on exact data and supplementary information becomes clear in situations characterised by outlier presence. To address this, the research employs novel neutrosophic estimators inside a designed effectively stratified random sampling system. This paradigm is ideally suited to dealing with uncertain along with imprecise data while also resisting the effect of outliers. Furthermore, the study broadens its scope by include dew point as a possible auxiliary variable, which significantly improves estimation accuracy.

The possible influence of dew point in combination with neutrosophic versions of robust regression techniques such as Huber-M, Hampel-M, and Tukey-M is examined at a pivotal point in the study. These approaches have been chosen purposefully to strengthen estimation accuracy against the disruptive character of outliers. This investigation is expanded to include robust quantile regression, offering a full and thorough estimate technique. A critical stage in the research is the thorough evaluation of the proposed methodologies, including the novel incorporation of dew point, using computer trials with real-world data. The results of these trials clearly demonstrate the superiority of the suggested techniques over standard estimators, especially in the face of outlier-ridden and ambiguous circumstances. This empirical confirmation is a compelling testimonial to the suggested techniques' durability, efficacy, and adaptability.

In the proposed study, an innovative approach for estimating the survey sample mean in the presence of outliers and ambiguous observations is presented. To get around the shortcomings associated with traditional estimating techniques, the strategy makes use of neutrosophic estimators using dew point as an auxiliary variable. The study emphasises how important it is to keep outliers out of the findings in order to retain the results' dependability and emphasises the major impact that rare data have on statistical estimators. The suggested approaches are thoroughly evaluated using computer simulations with real-world data, and the findings clearly show that they outperformed traditional estimators, even in the presence of outlier-filled and ambiguous conditions. Overall, the work has the potential to significantly improve the methodology of surveys and the discipline of statistics.

**Data Availability Statement** All the relevant data information is available within the manuscript.

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