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Fermatean Neutrosophic Fuzzy Graphs: A Study on the Winner Index with Enhancing Election Analysis

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Abstract. In this article, we discuss the fermatean neutrosophic graph of Wiener index, which is an essential topological index formed according to geodesical distance of vertices. The Wiener index is an important factor to describe the structure of a graph and we defined it in relation to fermatean neutrosophic graphs and computed it for some specific fermatean neutrosophic graph structures including complete fermatean neutrosophic graphs, cycles and trees. Subsequently, the Wiener index is compared with the connectivity index, a core-degree based parameter, using a sequence of theorems. As an application the study responds to the difficulties in election analysis in democratic environments where voter choices are often nuanced an unpredictable and the methods of measurement are not sensitive enough to capture these changes. To improve the modeling of election data, this work employs fermatean neutrosophic graphs (FNGs) and the Wiener index, which distinguish nodes that represent leadership qualities, policy suggestions, and public commitment as well as the relationship between these nodes. This approach manages uncertainty and indeterminacy well and provides a sound method of enhancing the measurability and credibility of analytical techniques in managing complicated events like elections.

Keywords: Fuzzy graph, $\mathcal{FNG}$, Winner Index, Uncertainty Modeling, Decision Making.

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1. Introduction

Rosenfeld first proposed the idea of fuzzy graphs in 1975 [18], where Zadeh presented the notion of fuzzy sets in 1965 [27]. Since then, a great deal of study has been done on fuzzy graphs, examining both their features and potential uses. Computing degree-based and distance-based indices in fuzzy graphs has been one topic of interest. These indices provide graphs numerical values, which serve as a helpful standard for comparing graphs with an equal number of vertices.

Arif, W.et al. [6] used some indices of picture fuzzy graphs and their applications are also given. Atanassov first presented the intuitionistic fuzzy set theory [7]. Smarandache [21] developed fuzzy theory in 1995, which resulted in the creation of neutrosophic sets. As a result of this growth, neutrosophic graphs were created, bringing new mathematical ideas to this area. Neutro hyper algebra and anti hyper algebra [22], energy and spectrum analysis [9], supply chain issue resolution [2], and decision-making [1] are just a few of the uses and characteristics of neutrosophic graphs that have been presented by academics in recent years. Ghods and Rostami focus on the application of topological indices in neutrosophic graphs, particularly in the realm of connectivity. Topological indices in neutrosophic graphs, provide an overview of their significance and potential applications, with specific reference to behavioral sciences [13]. In [14] connectivity indices in neutrosophic trees, they present an algorithm to find its maximum size. Finally, [15] shows the importance of summation and the integration index in behavioral sciences while discussing their presentation in the form of a neutrosophic graph. This work contributes to the understanding and use of topological indices in the context of neutersophic diagrams, providing insight into their uses and mathematical methods. Kaviayarasu [[17]] studied regular r-edge neutrosophic graphs. Borzooei and Rashmanlou established new ideas in the interpolation intervals of interest to them [19] and then discussed negative graphs and their applications in linear systems [20]. Sadati, Rashmanlou, and Talebi introduce new notions for m-polar interval-valued intuitionistic fuzzy graphs, advancing this field of research [25]. Omeri, Kaviyarasu, and Rajeshwari use the max product of complement in neutrosophic graphs to discover online streaming services [26], where Al-Omeri W.F. researches mixed b-fuzzy topological spaces [4] and later they discussed virtually e-I-continuous functions [5]. Borzooei and Rashmanlou address cayley interval-valued fuzzy graphs, contributing to the existing corpus of study on the subject [12].

In the context of graph theory, offers an introduction to the theory and applications of fermatean neutrosophic graphs, offering insights into this innovative paradigm [10]. Using the neutrosophic kano technique and extending the idea to fermatean neutrosophic topological spaces, probably investigate the uses of these spaces in symmetry research [16]. The interval-valued fermatean neutrosophic shortest path issue is addressed [11] offers a score function-based solution approach that may have ramifications for optimization and decision-making. A novel interpretation of fermatean neutrosophic dombi fuzzy graphs [23], indicates a step forward in the comprehension and utilization of fuzzy graphs in neutrosophic frameworks. A fermatean fuzzy multi-criteria decision making (MCDM) approach for ranking and selection issues, illustrating real-world uses in the field of decision science [8]. Offering a fresh viewpoint on rough set theory in decision analysis, [24] develop fermatean fuzzy covering-based rough sets and investigate their applications in multi-attribute decision-making. In [3] a significant contribution to transportation planning optimization approaches by creating an enhanced data envelopment analysis (DEA) method for solving multi-objective transportation problems using fermatean fuzzy sets. Broumi et al. [31] presented an original development on the interval-valued Fermatean neutrosophic shortest path problem. In a similar manner, Sasikala and Divya [32] developed a new approach to the Fermatean neutrosophic Dombi fuzzy graphs, increasing the appearance of uncertainty of the graph structure. In medical field, Dhanalakshmi [33] used Rough Fermatean Neutrosophic Sets to enhance the medical diagnosis where handling imprecise and incomprehensive patient information is a common issue.

In section two, we provide preliminary information on the fermatean neutrosophic graphs (FNGs) which models these relationships. In Section 3, we generalize the wiener index to FNGs in order to implement a framework for evaluating the connectivity and characteristics of these graphs under conditions characterized by uncertainty. The innovation in this study is the extension of the wiener index for fermatean neutrosophic graphs to enhance the understanding of systems. In Section 4, we provide description of this extended wiener index in the framework in election results analysis. Section 5 analyzes the new wiener index against the background of other indices presented . Last but not the least, Section 6 presents the conclusion and offers some recommendations for the research that follows.

1.1. *Motivation*

The goal of the research is to fill in the knowledge gaps about the behavior of the well-known topological metric wiener index in the context of fermatean neutrosophic networks. By exploring this uncharted area, the study aims to provide fresh perspectives on the structural characteristics of these graphs and how they affect other fields of study. Meets the requirements of complex network analysis. The need for sophisticated analytical tools that can capture the nuances of network topology increases as real-world networks become more complex. Focusing on the wiener index, which measures the distance between network vertices, the work aims to make a significant addition to the analytical toolbox for fermate neutrosophic networks.

1.2. *Novelty*

This research offers a fresh viewpoint on networks by extending wiener to fermatean neutrosophic networks. wiener extension to neutrosophic networks stratigraphic: Through an examination of the wiener application of the distance-based feature index, this research offers a fresh viewpoint on network analysis and delves into the nature of these intricate systems. The neutrosophic network of Fermat. This study enhances network science and opens up new avenues for future research and applications by expanding the wiener index into previously uncharted terrain. Using the connection index for comparison: Through a comparison of the wiener index and the connectedness index, this study offers a thorough knowledge of the structural features of fermatean neutrosophic network. By clarifying the similarities and differences between the two metrics, this research offers significant insights into the distinctive properties of Fermat neutrosophic networks and their implications for network analysis and design.

1.3. *Goal*

wiener index definition and computation for fermatean neutrosophic graphs: Establishing a framework for calculating the wiener index in fermatean neutrosophic networks throughout a range of graph topologies, such as cycles, trees, and full graphs, is the main goal of the study. The study intends to provide a fuller understanding of the distance-based features of fermatean neutrosophic networks by creating reliable methods for calculating this index. Comparative analysis with connectedness index: Using pertinent theorems, the study not only computes the wiener index but also attempts to compare it with another significant measure, the connectedness index. By comparing these two measures, we want to clarify their unique properties and ramifications in the context of fermatean neutrosophic networks, strengthening the theoretical basis for network research in this field.

1.4. *Key Differences and Contributions of This Manuscript*

This paper rectifies the aforementioned gaps by introducing the wiener index of fermatean neutrosophic graphs (FNGs) and applying it to election analysis. Novel contributions include:

(1) FNGs, which have a higher order of membership than traditional neutrosophic structures, are used to model relations between voters, candidates, and regions.
(2) A new measure referred to as the Winner Index is introduced using the Wiener index to identify both separatist and convert candidate relationships in election graphs.
(3) The applicability of the methodology is demonstrated using election networks with the identification of influential candidates and areas and, as a result, improving decision-making in election strategies.

2. **Preliminary**

In this section, we consider to provide the instruments that made this idea come to our mind. To start, we suppose that (Ω, F, Q) is the probability measure space where Ω is a sample space, F is an event space and Q is a probability function and we work in this space. In the following, we will define the Exchange and Lookback options and show the price equation. To this end, we consider the following two risky assets:

Definition 2.1. [29] Let the universe be $\mathcal{R}$. A truth membership function $(T_{\mathcal{S}})$, an indeterminacy membership function $(I_{\mathcal{S}})$, and a falsity membership function $(F_{\mathcal{S}})$, where $T_{\mathcal{S}}$, $I_{\mathcal{S}}$, and $F_{\mathcal{S}}$ are real standard elements of $[0,1]$, characterize a Neutrosophic set (NS) $\mathcal{S}$ in $\mathcal{R}$. $\mathcal{S} = \{\langle \varkappa, T_{\mathcal{S}}(\varkappa), I_{\mathcal{S}}(\varkappa), F_{\mathcal{S}}(\varkappa)\rangle ; \varkappa \in \mathcal{R}, T_{\mathcal{S}}, I_{\mathcal{S}}, F_{\mathcal{S}} \in]^{-}, 1^{+}[\}$. is one way to write it.

The sum of $T_{\mathcal{S}}(\varkappa), I_{\mathcal{S}}(\varkappa), F_{\mathcal{S}}(\varkappa)$ is unrestricted, and as a result, $0^{-} \leq T_{\mathcal{S}}(\varkappa), I_{\mathcal{S}}(\varkappa), F_{\mathcal{S}}(\varkappa) \leq 3^{+}$.

Definition 2.2. [30] A Fermatean fuzzy set $\mathcal{S}$ on a universe of discourse $\mathcal{R}$ is a structure having the form as: $\mathcal{S} = \{\langle \varkappa, T_{\mathcal{S}}(\varkappa), F_{\mathcal{S}}(\varkappa)\rangle : \varkappa \in \mathcal{R}\}$ where $T_{\mathcal{S}}(\varkappa) : \mathcal{R} \rightarrow [0,1]$ indicates the degree of membership, and $F_{\mathcal{S}}(\varkappa) : \mathcal{R} \rightarrow [0,1]$ indicates the degree of non-membership of the element $\varkappa \in \mathcal{R}$ to the set A, respectively, with the constraints : $0 \leq (T_{\mathcal{S}}(\varkappa))^3 + (F_{\mathcal{S}}(\varkappa))^3 \leq 1$

Definition 2.3. [28] The pair $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ represents $\mathcal{FNG}$ defined on a universal set $\mathcal{U}$, where $\mathcal{W}$ denotes a fermatean neutrosophic set on $\mathcal{R}$ and $\mathcal{H}$ a fermatean neutrosophic relation on $\mathcal{R}$.

$$1. \mathcal{M}_{\mathcal{W}}(\eta, \tau) \leq min\{\mathcal{M}_{\mathcal{H}}(\eta), \mathcal{M}_{\mathcal{H}}(\tau)\}$$

$$2. I\mathcal{M}_{\mathcal{W}}(\eta, \tau) \geq max\{I\mathcal{M}_{\mathcal{H}}(\eta), I\mathcal{M}_{\mathcal{H}}(\tau)\}$$

$$3. N\mathcal{M}_{\mathcal{W}}(\eta, \tau) \geq max\{N\mathcal{M}_{\mathcal{H}}(\eta), N\mathcal{M}_{\mathcal{H}}(\tau)\}$$

and $0 \leq \mathcal{M}^3_{\mathcal{W}}(\eta, \tau) + I\mathcal{M}^3_{\mathcal{W}}(\eta, \tau) + N\mathcal{M}^3_{\mathcal{W}}(\eta, \tau) \leq 2$, for all $\eta, \tau \in \mathcal{R}$, where $\mathcal{M}_{\mathcal{W}} : \mathcal{R} \times \mathcal{R} \rightarrow [0,1], I\mathcal{M}_{\mathcal{W}} : \mathcal{R} \times \mathcal{R} \rightarrow [0,1]$ and $N\mathcal{M}_{\mathcal{W}} : \mathcal{R} \times \mathcal{R} \rightarrow [0,1]$. signifying the degree of membership(M), indeterminacy(IM), and non-membership(NM).

Definition 2.4. [28] The single valued $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ satisfies the following condition

$$1. \mathcal{M}_{\mathcal{W}}(\eta, \tau) \leq min\{\mathcal{M}_{\mathcal{H}}(\eta), \mathcal{M}_{\mathcal{H}}(\tau)\}$$

$$2. I\mathcal{M}_{\mathcal{W}}(\eta, \tau) \leq min\{I\mathcal{M}_{\mathcal{H}}(\eta), I\mathcal{M}_{\mathcal{H}}(\tau)\}$$

$$3. N\mathcal{M}_{\mathcal{W}}(\eta, \tau) \leq max\{N\mathcal{M}_{\mathcal{H}}(\eta), N\mathcal{M}_{\mathcal{H}}(\tau)\}$$

3. Wineer Index of $\mathcal{FNG}$

Definition 3.1. Let $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ be the connected $\mathcal{FNG}$.

$$1. CON_{MG}(\eta, \tau) = max\{min\ \mathcal{M}_{\mathcal{H}}(\eta.\tau)\}$$

$$2. CON_{IMG}(\eta, \tau) = max\{min\ I\mathcal{M}_{\mathcal{H}}(\eta, \tau)\}$$

$$3. CON_{NMG}(\eta, \tau) = min\{max\ N\mathcal{M}_{\mathcal{H}}(\eta, \tau)\}$$

where (η, τ) is the path between them, then the partial connectivity index is denoted as

$$PCI_M(\mathcal{G}) = \sum_{\eta,\tau \in N} \mathcal{M}_{\mathcal{H}}(\eta)\mathcal{M}_{\mathcal{H}}(\tau)CON_{MG}(\eta,\tau)$$
$$PCI_{IM}(\mathcal{G}) = \sum_{\eta,\tau \in N} I\mathcal{M}_{\mathcal{H}}(\eta)I\mathcal{M}_{\mathcal{H}}(\tau)CON_{IMG}(\eta,\tau)$$
$$PCI_{NM}(\mathcal{G}) = \sum_{\eta,\tau \in N} N\mathcal{M}_{\mathcal{H}}(\eta)N\mathcal{M}_{\mathcal{H}}(\tau)CON_{NMG}(\eta,\tau)$$

The total connectivity index is denoted as,
$TWI(\mathcal{G}) = \frac{8+4PCI_M(\mathcal{G})-4PCI_{NM}(\mathcal{G})-3PCI_{IM}(\mathcal{G})-4PCI_{NM}(\mathcal{G})}{12}$

Definition 3.2. The $\mathcal{FNG}$ is represented as $\mathcal{G} = (\mathcal{W}, \mathcal{H})$, where W denotes the set of vertices and H denotes the set of edges. Let vertices in W be τ_1 and τ_2. If there is not a stronger path with a shorter length connecting τ_1 and τ_2 is referred to as a fermatean neutrosophic geodesic. It is worth that, according to the definition provided, the shortest strong path for each of the M, NM, and IM components of the $\mathcal{FNG}$ should be found separately.

Definition 3.3. The total of the weighted geodesic distance between vertices for each aspect of M, IM, and NM yields the partial wineer index of $\mathcal{FNG}$.

$$PWI_M(\mathcal{G}) = \sum_{\eta,\tau \in N} \mathcal{M}_{\mathcal{H}}(\eta)\mathcal{M}_{\mathcal{H}}(\tau)ds_M(\eta,\tau)$$
$$PWI_{NM}(\mathcal{G}) = \sum_{\eta,\tau \in N} IN_{\mathcal{H}}(\eta)I\mathcal{M}_{\mathcal{H}}(\tau)ds_{NM}(\eta,\tau)$$
$$PWI_{IM}(\mathcal{G}) = \sum_{\eta,\tau \in N} N\mathcal{M}_{\mathcal{H}}(\eta)N\mathcal{M}_{\mathcal{H}}(\tau)ds_{IM}(\eta,\tau)$$

The total winner index of fermatean neutrosophic fuzzy graph is
$TWI(\mathcal{G}) = \frac{8+4PWI_M(\mathcal{G})-4PWI_{NM}(\mathcal{G})-3PWI_{IM}(\mathcal{G})}{12}$

Example 3.4. In Figure 1, we will analyze the $\mathcal{FNG}$. The collection of vertex set $V = \{a, b, c, d\}$ and their fermatean neutrosophic membership are as follows

$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})(a) = (0.5, 0.3, 0.2)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})(b) = (0.8, 0.1, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})(c) = (0.6, 0.2, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})(d) = (0.7, 0.2, 0.3)$$

These are the fermatean neutrosophic memberships of edge set M.

$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})\,(a,b) = (0.4, 0.1, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})\,(a,c) = (0.5, 0.2, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})\,(a,d) = (0.4, 0.2, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})\,(b,d) = (0.5, 0.1, 0.1)$$
$$(\mathcal{M}_{\mathcal{H}}, I\mathcal{M}_{\mathcal{H}}, N\mathcal{M}_{\mathcal{H}})\,(c,d) = (0.5, 0.2, 0.1)$$

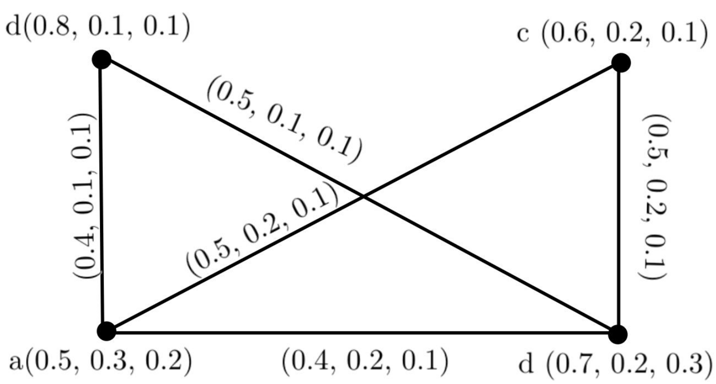

FIGURE 1. Fermatean neutrosophic graph

Edge	$\mathbf{d_{SM}}(\eta,\tau)$	$\mathbf{d_{SIM}}(\eta,\tau)$	$\mathbf{d_{SNM}}(\eta,\tau)$
(a,b)	1.5	0.5	0.1
(a,c)	0.5	0.2	0.1
(a,d)	1	0.2	0.1
(b,c)	1	0.3	0.2
(b,d)	0.5	0.1	0.1
(c,d)	0.5	0.2	0.1

TABLE 1. Total edge weights between η and τ in geodesic paths for $\mathcal{FNG}$

$PWI_M(\mathcal{G}) = 2.07, PWI_{IM}(\mathcal{G}) = 0.055$ and $PWI_{NM}(\mathcal{G}) = 0.018$. $TWI(\mathcal{G}) = 1.3369$.

Theorem 3.5. *Let $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ be the complete $\mathcal{FNG}$ with τ such that*
$m_1 \leq m_2 \leq .... \leq m_n, im_1 \geq im_2 \geq .... \geq m_n$ *and* $nm_1 \geq nm_2 \geq .... \geq nm_n$. *where*

$m_j = \mathcal{M}_{\mathcal{H}}(\tau_j), (im)_j = I\mathcal{M}_{\mathcal{H}}(\tau_j)$ *and* $(nm)_j = N\mathcal{M}_{\mathcal{H}}(\tau_j)$ *for* $1 \leq j \leq n$. *Then*

$$PWI_M(\mathcal{G}) = \sum_{j=1}^{n-1} m_j^4 \sum_{k=j+1}^{n} m_k,$$
$$PWI_{NM}(\mathcal{G}) = \sum_{j=1}^{n-1} nm_j^4 \sum_{k=j+1}^{n} nm_k,$$
$$and PWI_{IM}(\mathcal{G}) = \sum_{j=1}^{n-1} im_j^4 \sum_{k=j+1}^{n} im_k.$$

proof. Consider the $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ under the constraints stated in the theorem. According to the definition, the Wiener Index.

$$PWI_M(\mathcal{G}) = \sum_{\eta,\tau \in N} T_N(\eta) T_N(\tau) ds_M(\eta, \tau) \tag{1}$$

In a frematean neutrosophic graph, any two vertices can have a direct relationship. We want to show that this straight line is a geodesic path. Let $u = \tau_1$. Then for any $2 \leq i \leq n$,we have $m_1 \leq m_i$. This insight ensures

$$d_{SM}(\tau_1, \tau_i) = m_1^3, \quad 2 \leq i \leq n,$$
$$d_{SM}(\tau_2, \tau_i) = m_2^3, \quad 3 \leq i \leq n,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$d_{SM}(\tau_{n-1}, \tau_n) = m_{n-1}^3, \quad 2 \leq i \leq n.$$

Now, we obtain by putting the above values in equ 1.

$$\begin{aligned}
PWI_M(\mathcal{G}) &= \mathcal{M}_{\mathcal{H}}(\tau_1)\mathcal{M}_{\mathcal{H}}(\tau_2)M_1^3 + ... + \mathcal{M}_{\mathcal{H}}(\tau_1)\mathcal{M}_{\mathcal{H}}(\tau_n)M_1^3 + \mathcal{M}_{\mathcal{H}}(\tau_2)\mathcal{M}_{\mathcal{H}}(\tau_3)M_2^3 + ... \\
&+ \mathcal{M}_{\mathcal{H}}(\tau_2)\mathcal{M}_{\mathcal{H}}(\tau_n)M_2^3 + ... + \mathcal{M}_{\mathcal{H}}(\tau_k)\mathcal{M}_{\mathcal{H}}(\tau_n)M_k^3 + ... + \mathcal{M}_{\mathcal{H}}(\tau_{n-1})\mathcal{M}_{\mathcal{H}}(\tau_n)M_{n-1}^3 \\
&= m_1 m_2 m_1^3 + ... + m_1 m_n m_1^3 + m_2 m_3 m_2^3 + ... + m_2 m_n m_2^3 + ... + m_k m_n m_k^3 + ... + m_{n-1} m_n m_{n-1}^3 \\
&= m_1^4(m_2 + ... + m_n) + m_2^4(m_3 + ... + m_n) + ... + m_k^4(m_{k+1} + ... + m_n) + ... + m_{n-1}^4 m_n
\end{aligned}$$
$$PWI_M(\mathcal{G}) = \sum_{j=1}^{n-1} m_j^4 \sum_{k=j+1}^{n} m_k$$

In a similar way, $PWI_{IM}(\mathcal{G})$ and $PWI_{NM}(\mathcal{G})$ are provable.

Corollary 3.6. *The complete $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ meeting the criteria mentioned in the previous theorem, the next connections become clear;*

$$\begin{aligned} PWI_M(\mathcal{G}) &= PCI_M(\mathcal{G}) \\ PWI_{IM}(\mathcal{G}) &= PCI_{IM}(\mathcal{G}) \\ PWI_{NM}(\mathcal{G}) &= PCI_{NM}(\mathcal{G}) \\ Also, TWI(\mathcal{G}) &= TCI(\mathcal{G}) \end{aligned}$$

proof. The above theorem is evident based on this first theorem.

Theorem 3.7. *Let $\mathcal{G}$ be the $\mathcal{FNG}$ where G^* is a tree and $|N^*| = n$. If every $\eta\tau \in M, G - \eta\tau$ has two connected components, w_1&w_2 then it has l and k vertices, respectively, so that $l + k = n$.*

$$PWI_M(\mathcal{G}) = \sum_{\eta\tau \in G} \mathcal{M}_{\mathcal{W}}(\eta) \sum_{i=1}^{l} \mathcal{M}_{\mathcal{H}}(\eta_i) \sum_{j=1}^{k} \mathcal{M}_{\mathcal{H}}(\tau_j)$$

$$PWI_{IM}(\mathcal{G}) = \sum_{\eta\tau \in G} (IM)_{\mathcal{W}}(\eta\tau) \sum_{i=1}^{l} (IM)_{\mathcal{H}}(\eta_i) \sum_{j=1}^{k} (IM)_{\mathcal{H}}(\tau_j)$$

$$PWI_{NM}(\mathcal{G}) = \sum_{\eta\tau \in G} (NM)_{\mathcal{W}}(\eta\tau) \sum_{i=1}^{l} (NM)_{\mathcal{H}}(\eta_i) \sum_{j=1}^{k} (NM)_{\mathcal{H}}(\tau_j)$$

proof.A $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$, where $|N^*| = n$. and G^* is a tree. upon the elimination the chosen edge $\eta\tau$ from $\mathcal{G}$, where $\eta\tau \in M$, the graph is divided into two linked component, w_1 and w_2. The graph w_1 has vertices with a length of l, where w_2 contains vertices with a length of $k = n - 1$. In the event where $l = 1, k = n - 1$ and $\tau_1 \in w_1$, then

$$\begin{aligned} PWI_M(\eta) = &\sum_{\eta,\tau \in N} \mathcal{M}_{\mathcal{H}}(\eta)\mathcal{M}_{\mathcal{H}}(\tau)d_{SM}(\eta,\tau) \\ &= \mathcal{M}_{\mathcal{H}}(\tau_1)\mathcal{M}_{\mathcal{H}}(\tau_2)\mathcal{M}_{\mathcal{W}}(\eta\tau) + \mathcal{M}_{\mathcal{H}}(\tau_1)\mathcal{M}_{\mathcal{H}}(\tau_3)(\mathcal{M}_{\mathcal{W}}(\eta\tau) + e_1) + ...+ \\ &\mathcal{M}_{\mathcal{H}}(\tau_1)\mathcal{M}_{\mathcal{H}}(\tau_n)(\mathcal{M}_{\mathcal{W}}(\eta\tau) + ... + e_m) + \sum_{\eta,\tau \in N - \tau_1} \mathcal{M}_{\mathcal{H}}(\eta)\mathcal{M}_{\mathcal{H}}(\tau)d_{SM}(\eta,\tau) \end{aligned}$$

Let us examine the scenario in which $e_i \in M$ and $e_i \neq M$. Repeat the same for $\sum_{\eta,\tau \in N - \tau_1} \mathcal{M}_{\mathcal{H}}(\eta)\mathcal{M}_{\mathcal{H}}(\tau)d_{SM}(\eta,\tau)$. This technique should be respect until w_2 has just one vertex left. The indented outcome can then be obtained by factoring and adding the two components total number of vertices. Likewise, it is possible to demonstrate $PWI_{IM}(\mathcal{G})$ and $PWI_{NM}(\mathcal{G})$ by employing comparable techniques.

Theorem 3.8. *The $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ has a unique strong spanning tree. Then $PWI_M(\mathcal{G}) = PWI_M(T), PWI_{IM}(\mathcal{G}) = PWI_{IM}(T), PWI_{NM}(\mathcal{G}) = PWI_{NM}(T)$ and $TWI(\mathcal{G}) = TWI(T)$.*

proof. Suppose connected $\mathcal{FNG}$ $\mathcal{G}$, and it is a unique tree strong spanning tree is T. A strong spanning tree is defined as, for any two vertices u and τ in graph $\mathcal{G}$, the distance measure

in graph $\mathcal{G}$ are equal to those in strong spanning tree T.
The partial winner index of the $\mathcal{FNG}$ $\mathcal{G}$ is equivalent to the corresponding values in the strong spanning tree T, as can be seen from the aforementioned relations. Therefore, it is possible to deduced that $TWI(\mathcal{G}) = IWI(T)$.

Theorem 3.9. *Let $\mathcal{G}$ be the $\mathcal{FNG}$ with $G^* = G$. Then. For $n = 2m, m \in N$.*

$$PWI_M(\mathcal{G}) = \sum_{k=1}^{\frac{n}{2}-1} k(m)(\sum_{j=1}^{n} \mathcal{M}_{\mathcal{H}}(\eta_j) * \mathcal{M}_{\mathcal{H}}(\eta_{j+k})) + \frac{n}{2}m \sum_{j=1}^{\frac{n}{2}}(\mathcal{M}_{\mathcal{H}}(\eta_i) * \mathcal{M}_{\mathcal{H}}(\eta_{i+\frac{n}{2}}))$$

$$PWI_{IM}(\mathcal{G}) = \sum_{k=1}^{\frac{n}{2}-1} k(im)(\sum_{j=1}^{n} I\mathcal{M}_{\mathcal{H}}(\eta_j) * I\mathcal{M}_{\mathcal{H}}(\eta_{j+k})) + \frac{n}{2}(im) \sum_{j=1}^{\frac{n}{2}}(I\mathcal{M}_{\mathcal{H}}(\eta_i) * I\mathcal{M}_{\mathcal{H}}(\eta_{i+\frac{n}{2}}))$$

$$PWI_{NM}(\mathcal{G}) = \sum_{k=1}^{\frac{n}{2}-1} k(nm)(\sum_{j=1}^{n} N\mathcal{M}_{\mathcal{H}}(\eta_j) * N\mathcal{M}_{\mathcal{H}}(\eta_{j+k})) + \frac{n}{2}(nm) \sum_{j=1}^{\frac{n}{2}}(N\mathcal{M}_{\mathcal{H}}(\eta_i) * N\mathcal{M}_{\mathcal{H}}(\eta_{i+\frac{n}{2}}))$$

For $n = 2m+1, m \in N$.

$$PWI_M(\mathcal{G}) = \sum_{k=1}^{\frac{n-1}{2}} k(m)(\sum_{j=1}^{n} \mathcal{M}_{\mathcal{H}}(\eta_j) * \mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

$$PWI_{IM}(\mathcal{G}) = \sum_{k=1}^{\frac{n}{2}-1} k(im)(\sum_{j=1}^{n} I\mathcal{M}_{\mathcal{H}}(\eta_j) * I\mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

$$PWI_{NM}(\mathcal{G}) = \sum_{k=1}^{\frac{n}{2}-1} k(nm)(\sum_{j=1}^{n} N\mathcal{M}_{\mathcal{H}}(\eta_j) * N\mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

For $j + k > n, \eta_{j+k} = \eta d$, which means $j + k = d(mode\ n)$. In addition, for $(G - \eta\tau)$, we have

$$PWI_M(G - \eta\tau) = \sum_{k=1}^{n-1} k(m)(\sum_{j=1}^{n-k} \mathcal{M}_{\mathcal{H}}(\eta_j) * \mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

$$PWI_{IM}(G - \eta\tau) = \sum_{k=1}^{n-1} k(im)(\sum_{j=1}^{n-k} I\mathcal{M}_{\mathcal{H}}(\eta_j) * I\mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

$$PWI_{NM}(G - \eta\tau) = \sum_{k=1}^{n-1} k(nm)(\sum_{j=1}^{n-k} N\mathcal{M}_{\mathcal{H}}(\eta_j) * N\mathcal{M}_{\mathcal{H}}(\eta_{j+k}))$$

The function $M = (m, im, nm)$ is constant.

proof. In $\mathcal{FNG}$ $\mathcal{G} = (\mathcal{W}, \mathcal{H})$, where $\mathcal{W}$ is a constant function, $\mathcal{G}^*$ is an even-length cycle, and $\mathcal{G}$ is neutral for each edge for $\mathcal{G}$, the maximum length of $\mathcal{FNG}$ is $\frac{n}{2}$. Consider a situation where $\frac{n}{2}$ is the distance between two vertices.
Assume that distance between two vertices η and v is equal to k, which is smaller than $\frac{n}{2}$. The geodesic length between two vertices is defined as p_k, where $(\eta, \tau) \in \mathcal{H}^* X \mathcal{H}^*$.
Conversely, there are $\frac{n}{2}$ pairs of vertices (η, τ) such that $\frac{n}{2}(M, IM, NM)$ is the geodesic length

between them. For each of these pairs, we may calculated the product of $\mathcal{M}_{\mathcal{H}}(\eta)$ in $\mathcal{M}_{\mathcal{H}}(\tau)$, and then add up all of these pairs of η and τ to get the desired outcome.

$$\frac{n}{2}m\sum_{i=1}^{\frac{n}{2}}\mathcal{M}_{\mathcal{H}}(\eta_i)\mathcal{M}_{\mathcal{H}}(\eta_{l+\frac{n}{2}}) \tag{2}$$

Let us return to the previous condition where $1 \leq k \leq \frac{n}{2}$. This is a vertex with distance km from every vertex in the cycle c_n, such as η. Assuming $k = 1$, we obtain $\mathcal{M}_{\mathcal{H}}(\eta_1)\mathcal{M}_{\mathcal{H}}(\eta_2) + \mathcal{M}_{\mathcal{H}}(\eta_2)\mathcal{M}_{\mathcal{H}}(\eta_3) + ... + \mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+1}) + ... + \mathcal{M}_{\mathcal{H}}(\eta_n)\mathcal{M}_{\mathcal{H}}(\eta_{n+1})$.
Since $n + 1 = 1(modn)$, hence $\mathcal{M}_{\mathcal{H}}(\eta_n)\mathcal{M}_{\mathcal{H}}(\eta_{n+1}) = \mathcal{M}_{\mathcal{H}}(\eta_n)\mathcal{M}_{\mathcal{H}}(\eta_1)$. Then for $k = 1$, we have

$$1 \times m \times \sum_{j=i}^{n}\mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+1}),$$

for k=2

$$2 \times m \times \sum_{j=i}^{n}\mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+2}),$$

for $k = r, r < \frac{n}{2}$

$$r \times m \times \sum_{j=i}^{n}\mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+r})$$

containing along these lines and adding up k gives us

$$\sum_{k=1}^{\frac{n}{2}-1}km(\sum_{j=i}^{n}\mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+r})) \tag{3}$$

utilizing 2 and 3

$$\begin{aligned} PWI_M(\mathcal{G}) &= (1) + (2) \\ &= \sum_{k=1}^{\frac{n}{2}-1}km(\sum_{j=1}^{n}\mathcal{M}_{\mathcal{H}}(\eta_j)\mathcal{M}_{\mathcal{H}}(\eta_{j+r})) \\ &+ \sum_{l=1}^{\frac{n}{2}}\frac{n}{2}m(\sum_{i=l}^{n}\mathcal{M}_{\mathcal{H}}(\eta_l)\mathcal{M}_{\mathcal{H}}(\eta_{l+r})) \end{aligned}$$

The fact that there is a maximum distance of $\frac{n-1}{2}$ between the vertices η and τ indicates that is odd. The proof continuation is comparable to the scenario in which n is even.

Theorem 3.10. *Let $\mathcal{G}$ be a fermatean neutrosophic tree (FNT) with $|N^*| \geq 3$. Subsequently $PCI_M(\mathcal{G}) < PWI_M(\mathcal{G}), PCI_{IM}(\mathcal{G}) < PWI_{IM}(\mathcal{G})$ and*
$PCI_{NM}(\mathcal{G}) < PWI_{NM}(\mathcal{G})$. Its not necessary for $TCI(\mathcal{G})$ to be smaller than or equal to $TFI(\mathcal{G})$.

Let $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ be a FNT. Single for all vertices η and τ in the FNT, there is a single strong path connecting them. Therefore, this path is the only one that is the strongest from η to τ. For every η and τ. $d_{SM}(\eta, \tau)$ is the true of the membership values on the other hand, the membership values of the weakest edges of the $(\eta - \tau)$-path are represented as $CONN_{MG}(\eta, \tau)$. Consequently, $CONN_{MG}(\eta, \tau) \leq d_{SM}(\eta, \tau)$. Equality arises in the relationship above when $\eta\tau$ is a strong edge. If not $CONN_{MG}(\eta, \tau) < d_{SM}(\eta, \tau)$ then, we have $PCI_M(\mathcal{G}) < PWI_M(\mathcal{G})$. Likewise, it is possible to show $PWI_{IM}(\mathcal{G})$ and $PWI_{NM}(\mathcal{G})$.

Example 3.11. The FNT $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ is shown in Figure 2.

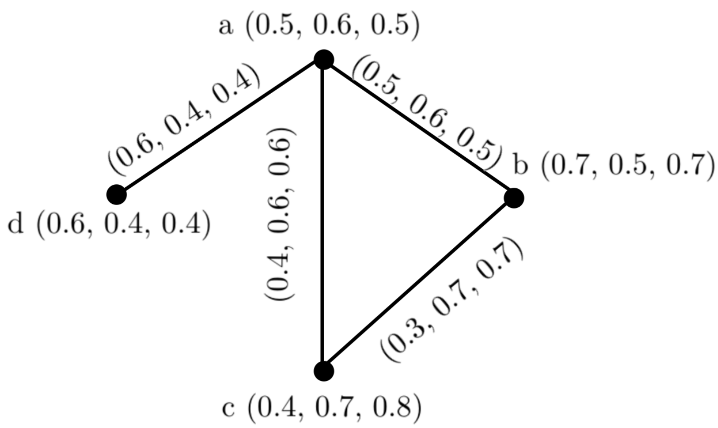

FIGURE 2. Fermatean neutrosophic tree.

It should be observed that bc is a weak edge. $PWI_M(\mathcal{G}) = 1.551, PWI_{IM}(\mathcal{G}) = 1.771$ and

Edges	$\mathbf{d_{SM}}(\eta, \tau)$	$\mathbf{d_{SIM}}(\eta, \tau)$	$\mathbf{d_{SNM}}(\eta, \tau)$
$(\tilde{a}, \tilde{b})$	0.7	1.3	1.3
$(\tilde{a}, \tilde{c})$	0.4	0.6	0.6
$(\tilde{a}, \tilde{d})$	0.6	0.4	0.4
$(\tilde{b}, \tilde{c})$	0.3	0.7	0.7
$(\tilde{b}, \tilde{d})$	1.3	1.7	1.7
$(\tilde{c}, \tilde{d})$	1.4	1.6	1.5

TABLE 2. Total edge weights between η and τ in geodesic paths for fermatean neutrosophic tree.

$PWI_{NM}(\mathcal{G}) = 2.123$. $TWI(\mathcal{G}) = 0.0485$.

$PCI_M(\mathcal{G}) = 0.853, PCI_{IM}(\mathcal{G}) = 0.989$ and $PCI_{NM}(\mathcal{G}) = 1.052$. $TCI(\mathcal{G}) = 0.35308$. As demonstrated by this instance $PCI_M(\mathcal{G}) < PWI_M(\mathcal{G})$,

$PCI_{IM}(\mathcal{G}) < PWI_{IM}(\mathcal{G})$ and $PCI_{NM}(\mathcal{G}) < PWI_{NM}(\mathcal{G})$. But $TCI(\mathcal{G}) > TWI(\mathcal{G})$. In addition, the $\mathcal{FNG}$ depicted in figure 3, below is a tree where $PCI_M(\mathcal{G}) < PWI_M(\mathcal{G}), PCI_{IM}(\mathcal{G}) <$

Edges	$CON_{MG}(\eta, \tau)$	$CON_{IMG}(\eta, \tau)$	$CON_{NMG}(\eta, \tau)$
$(\tilde{a}, \tilde{b})$	0.5	0.6	0.4
$(\tilde{a}, \tilde{c})$	0.4	0.6	0.6
$(\tilde{a}, \tilde{d})$	0.6	0.5	0.4
$(\tilde{b}, \tilde{c})$	0.4	0.7	0.6
$(\tilde{b}, \tilde{d})$	0.5	0.4	0.4
$(\tilde{c}, \tilde{d})$	0.4	0.4	0.6

TABLE 3. Strength of connectedness for for each pair of vertices η,τ

$PWI_{IM}(\mathcal{G})$ and $PCI_{NM}(\mathcal{G}) < PWI_{NM}(\mathcal{G})$.
Moreovr $TCI(\mathcal{G}) < TWI(\mathcal{G})$.

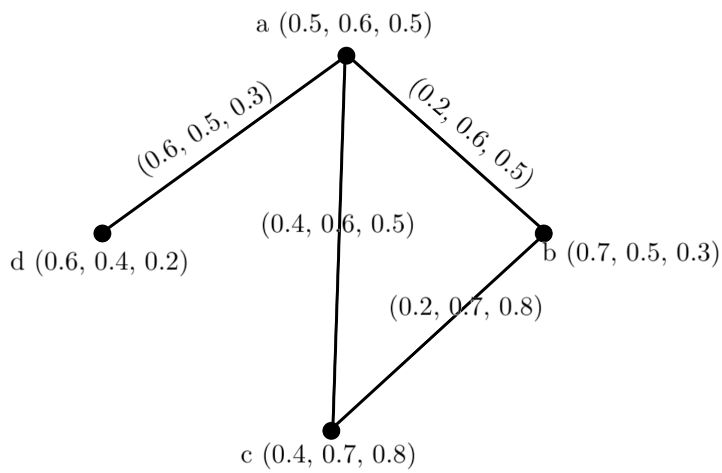

FIGURE 3. Illustration of fermatean neutrosophic tree nodes and relationship

Edges	$d_{SM}\,(\eta, \tau)$	$d_{SIM}\,(\eta, \tau)$	$d_{SNM}\,(\eta, \tau)$
$\tilde{a}, \tilde{b}$	0.2	0.6	0.5
$\tilde{a}, \tilde{c}$	0.5	0.6	0.5
$\tilde{a}, \tilde{d}$	0.6	0.5	0.3
$\tilde{b}, \tilde{c}$	0.2	0.7	0.8
$\tilde{b}, \tilde{d}$	0.9	1.1	0.8
$\tilde{c}, \tilde{d}$	1.4	1.6	1.5

TABLE 4. Total edge weights between η and τ in geodesic paths

$PWI_M(\mathcal{G}) = 1.024, PWI_{IM}(\mathcal{G}) = 1.325$ and $PWI_{NM}(\mathcal{G}) = 1.325.TWI(\mathcal{G}) = 0.4364$. $PCI_M(\mathcal{G}) = 0.5666, PCI_{IM}(\mathcal{G}) = 1.037$ and $PCI_{NM}(\mathcal{G}) = 0.491$. $TCI(\mathcal{G}) = 0.4324$. As demonstrated by this instance $PCI_M(\mathcal{G}) < PWI_M(\mathcal{G}), PCI_{IM}(\mathcal{G}) < PWI_{IM}(\mathcal{G})$ and $PCI_{NM}(\mathcal{G}) < PWI_{NM}(\mathcal{G})$. But $TCI(\mathcal{G}) < TWI(\mathcal{G})$.

Edges	$CON_{MG}(\eta,\tau)$	$CON_{IMG}(\eta,\tau)$	$CON_{NMG}(\eta,\tau)$
$\tilde{a},\tilde{b}$	0.2	0.6	0.5
$\tilde{a},\tilde{c}$	0.4	0.6	0.5
$\tilde{a},\tilde{d}$	0.6	0.5	0.3
$\tilde{b},\tilde{c}$	0.2	0.7	0.5
$\tilde{b},\tilde{d}$	0.2	0.5	0.3
$\tilde{c},\tilde{d}$	0.4	0.5	0.3

TABLE 5. Strength of connectedness

Theorem 3.12. *In a FNT $\mathcal{G}$, where the underlying set is N and the membership function is M, with a constraint that the cardinality of the neutral element set $|N^*| \leq 3$ and G^* is a star, suppose M is a constant function. Assume τ, is the center vertex and $\tau_2, \tau_3, ...\tau_n$ are the vertices connecting to it,*

$$PWI_M(\mathcal{G}) = 2m \sum_{j=1}^{n-1} \mathcal{M}_{\mathcal{H}}(\tau_j) \sum_{k=j+1}^{n} \mathcal{M}_{\mathcal{H}}(\tau_k)$$

$$- m\mathcal{M}_{\mathcal{H}}(\tau_1) \sum_{j=2}^{n} M(\tau_j)$$

$$PWI_{IM}(\mathcal{G}) = 2(im) \sum_{j=1}^{n-1} I\mathcal{M}_{\mathcal{H}}(\tau_j), \sum_{k=j+1}^{n} I\mathcal{M}_{\mathcal{H}}(\tau_k)$$

$$- (im) * (IM)_N(\tau_1) \sum_{j=2}^{n} IM(\tau_j),$$

$$PWI_{NM}(\mathcal{G}) = 2(nm) \sum_{j=1}^{n-1} N\mathcal{M}_{\mathcal{H}}(\tau_j), \sum_{k=j+1}^{n} N\mathcal{M}_{\mathcal{H}}(\tau_k)$$

$$- (nm) * (NM)_N(\tau_1) \sum_{j=2}^{n} NM(\tau_j).$$

proof. Assume that $\mathcal{G} = (\mathcal{W}, \mathcal{H})$ is a FNT with $|N^*| > 3$. A star can be represented by the symbol $G*$. Since $M = (m, im, nm)$ is a constant function and τ_i is the center vertex, for each $\tau_i, 1 \leq i \leq n$ we obtain $d_{SM}(\tau, \tau_i) = m^3, d_{S(IM)}(\tau, \tau_i) = (im)^3$, and $d_{S(NM)}(\tau, \tau_i) = (nm)^3$. where τ_i and τ_j are not equal to one, then $d_{SM}(\tau_j, \tau_i) = 2m^3, d_{S(IM)}(\tau_j, \tau_i) = 2(im)^3$, and

$d_{S(NM)}(\tau_j, \tau_i) = 2(nm)^3$. Then

$$\begin{aligned} PWI_M(\mathcal{G}) &= \sum \tau_i, \tau_j \in N \mathcal{M}_{\mathcal{H}}(\tau_i)\mathcal{M}_{\mathcal{H}}(\tau_j) d_{SM}(\tau_i, \tau_j) \\ &= \sum_{\tau_j \in N} \mathcal{M}_{\mathcal{H}}(\tau_i)\mathcal{M}_{\mathcal{H}}(\tau_j) d_{SM}(\tau_i, \tau_j) \\ &+ \sum_{\tau_i, \tau_j \in N (i \neq 1)} \mathcal{M}_{\mathcal{H}}(\tau_i)\mathcal{M}_{\mathcal{H}}(\tau_j) d_{SM}(\tau_i, \tau_j) \\ &= m^3 \mathcal{M}_{\mathcal{H}}(\tau_1) \sum_{j=2}^{n} \mathcal{M}_{\mathcal{H}}(\tau_j) + (2m)^3 \sum_{j=2}^{n-1} \mathcal{M}_{\mathcal{H}}(\tau_j) \\ &\sum_{k=j+1}^{n} \mathcal{M}_{\mathcal{H}}(\tau_k) - (2m)^3 \mathcal{M}_{\mathcal{H}}(\tau_1) \sum_{j=2}^{n} M(\tau_j) \\ &= (2m)^3 \sum_{j=1}^{n-1} \mathcal{M}_{\mathcal{H}}(\tau_j) \sum_{k=j+1}^{n} \mathcal{M}_{\mathcal{H}}(\tau_k) \\ &- m^3 \mathcal{M}_{\mathcal{H}}(\tau_1) \sum_{j=2}^{n} \mathcal{M}_{\mathcal{H}}(\tau_j). \end{aligned}$$

Likewise it is possible to show that $PWI_{IM}(\mathcal{G})$ and $PWI_{NM}(\mathcal{G})$.

4. **Application of Improving Election Analysis with the Winner Index**

Elections are important processes in democratic countries to make decisions on options through votes and depend on the vague aspirations of the voting population and so on. However, as Mao and Rosen may imply, evaluating the results of the elections can be quite a puzzle. Due to innate indeterminacy and variability of peoples perceptions and their choices. Unfortunately, conventional methods often do not allow for capturing the subtleties of voters emotions sufficiently. Therefore, the utilization of such sophisticated models as FNGs alongside with the winner index can be viewed as a possible way of enhancing the election analysis due to handling uncertainty and indeterminacy in a more effective manner. To solve the problem of modeling and interpreting of connections using data which is fuzzy, FNGs can be used. In this model, nodes represent entities (e. g. , Leadership Traits, Policy Recommendations, Dedication to the Public Interest, while the lines linking them, represent inter connectivity between them (vertices are And Vision and Goals).

They involve the use fermatean neutrosophic numbers to denote the level of $\mathcal{M}$, $\mathcal{NM}$,and the level of $\mathcal{IM}$ about a connection, to make the representation of fuzzy data more realistic. The winner index is one of the essential parameters derived from FNGs namely representing the total favorableness or approval regarding some object/individual inside the graph. In addition to the degree of $\mathcal{M}$, $\mathcal{NM}$ and $\mathcal{IM}$ of each relation is also considered as providing the full degree of the positions of entities. While analyzing the election data, the concepts

of FNGs are beneficial as they can help to model interactions between voters and applicants. Every voter has a baked-in preference for politicians and this choice depends on several factors such as party affiliation, popularity of the politicians, and promises made among others. We may clarify the rather vague and uncertain voter attitudes by representing these preferences as FNGs. The Winner Index may then perhaps be applied to identify which candidate had the most support among the poll-going populace. In contrast with previous methodologies that rely on completely based on precise figures, the Winner Index reflects the vagueness and uncertainty of a voter inputs leading to a kind of study of the election therefore brings out a rather balanced perception of election processes and the choices made regarding them. We define a FNG in the following sequence, depending on each candidate: For first candidate we have G_1, for second candidate G_2, for third candidate G_3 and for the fourth candidate G_4.
$PWI_M(\mathcal{G}) = 1.287, PWI_{IM}(\mathcal{G}) = 0.704$ and $PWI_{NM}(\mathcal{G}) = 0.516$. $TWI(\mathcal{G}) = 0.90606$.

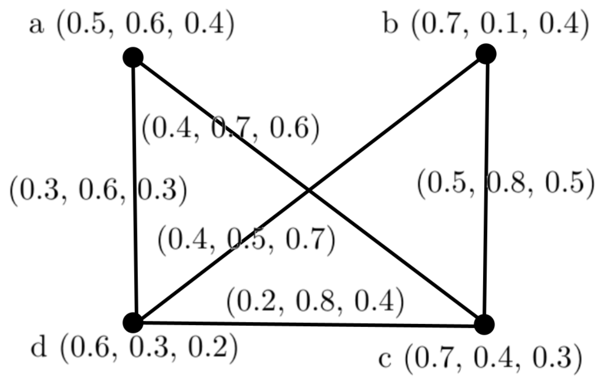

FIGURE 4. Fermatean neutrosophic graph G_1

Edges	$d_{SM}(\eta,\tau)$	$d_{SIM}(\eta,\tau)$	$d_{SNM}(\eta,\tau)$
(a,b)	0.3	1.5	1.0
(a,c)	0.5	0.7	0.6
(a,d)	1	1.5	1.1
(b,c)	0.2	0.8	1.2
(b,d)	0.7	1.6	0.7
(c,d)	0.6	0.8	0.5

TABLE 6. Total Edge weights between η and τ in geodesic paths fermatean neutrosophic G_1

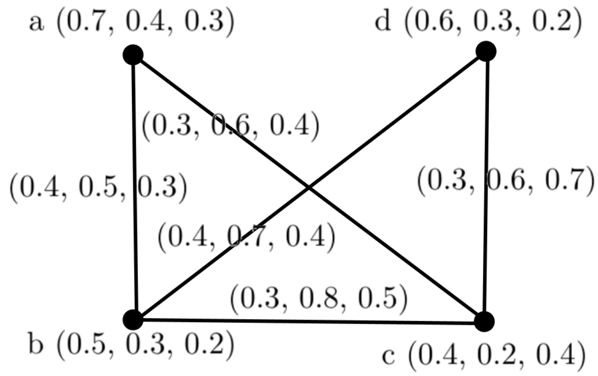

FIGURE 5. Fermatean neutrosophic graph G_2

Edges	$d_{SM}(\eta,\tau)$	$d_{SIM}(\eta,\tau)$	$d_{SNM}(\eta,\tau)$
(a,b)	0.6	1.4	0.9
(a,c)	0.3	0.6	0.4
(a,d)	0.6	1.2	0.8
(b,c)	0.6	0.8	0.5
(b,d)	0.6	1.4	0.9
(c,d)	0.3	1.5	0.9

TABLE 7. Total Edge weights between $\eta$and τ in geodesic paths for fermatean neutrosophic G_2

$PWI_M(\mathcal{G}) = 0.918, PWI_{IM}(\mathcal{G}) = 0.624$ and $PWI_{NM}(\mathcal{G}) = 0.298$. $TWI(\mathcal{G}) = 0.7173$.

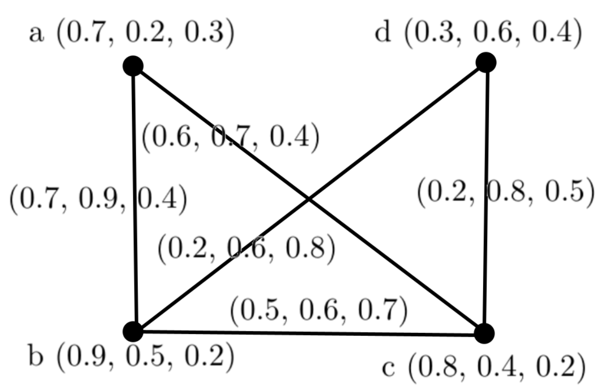

FIGURE 6. Fermatean neutrosophic graph G_3

$PWI_M(\mathcal{G}) = 1.587, PWI_{IM}(\mathcal{G}) = 0.882$ and $PWI_{NM}(\mathcal{G}) = 0.392$. $TWI(\mathcal{G}) = 0.8445$. $PWI_M(\mathcal{G}) = 1.421, PWI_{IM}(\mathcal{G}) = 0.9$ and $PWI_{NM}(\mathcal{G}) = 0.589$. $TWI(\mathcal{G}) = 0.719$.

Edges	$d_{SM}(\eta,\tau)$	$d_{SIM}(\eta,\tau)$	$d_{SNM}(\eta,\tau)$
(a,b)	1.1	0.9	0.4
(a,c)	0.6	1.5	0.4
(a,d)	0.8	1.5	0.9
(b,c)	0.4	0.6	1.3
(b,d)	0.2	0.6	0.8
(c,d)	0.2	0.8	1.5

TABLE 8. Total Edge weights between η and τ in geodesic paths Fermatean neutrosophic graph G_3

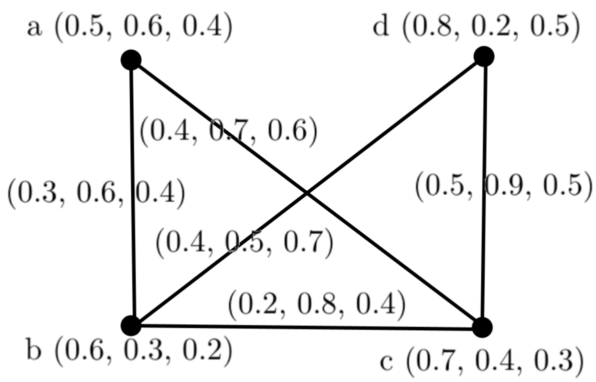

FIGURE 7. Fermatean neutrosophic graph G_4

Edges	$d_{SM}(\eta,\tau)$	$d_{SIM}(\eta,\tau)$	$d_{SNM}(\eta,\tau)$
(a,b)	0.3	1.5	1
(a,c)	0.5	0.7	0.6
(a,d)	1	1.6	1.1
(b,c)	0.2	0.8	1.2
(b,d)	0.7	1.7	0.7
(c,d)	0.6	0.9	0.5

TABLE 9. Total Edge weights between η and τ in geodesic paths fermatean neutrosophic graphG_4

Using the winner index in FNG plots has various advantages in election analysis. This allows policymakers, analysts, and candidates to make informed judgments that take into account the complexity and ambiguity inherent in voters' preferences. It also increases the openness and integrity of the political process, creating a reliable method for objective analysis of election results.

4.1. *Comparative Analysis*

For this reason, the wiener awards are accurate and exhaustive when mapping and illustrating indeterminacy in distance measurement. The current study of the wiener Index in Fermatean Neutrosophic Fuzzy Graphs is found to be more efficient wiener index in neutrosophic graphs. The fermatean neutrosophic components support the FNGs to represent and study the uncertainties in a deeper manner and hence it can be useful to analyze systems which involves highly complex uncertainties. In addition, the wiener index FNGs has captured the direct shortest between vertices and it has also directly dealt with indeterminacy thereby providing a better shot at distance, and a more understanding of the configuration of the network. For academics and practitioners which need to study systems with high level of non-linearity and level of uncertainties, it is the approach of choice because of the its applicability areas as for example biological networks or a complex social system. Hence, wiener index for FNGs is an ideal solution as offers more analytical proficiency to handle with the uncertainty and accurate mesh distances.

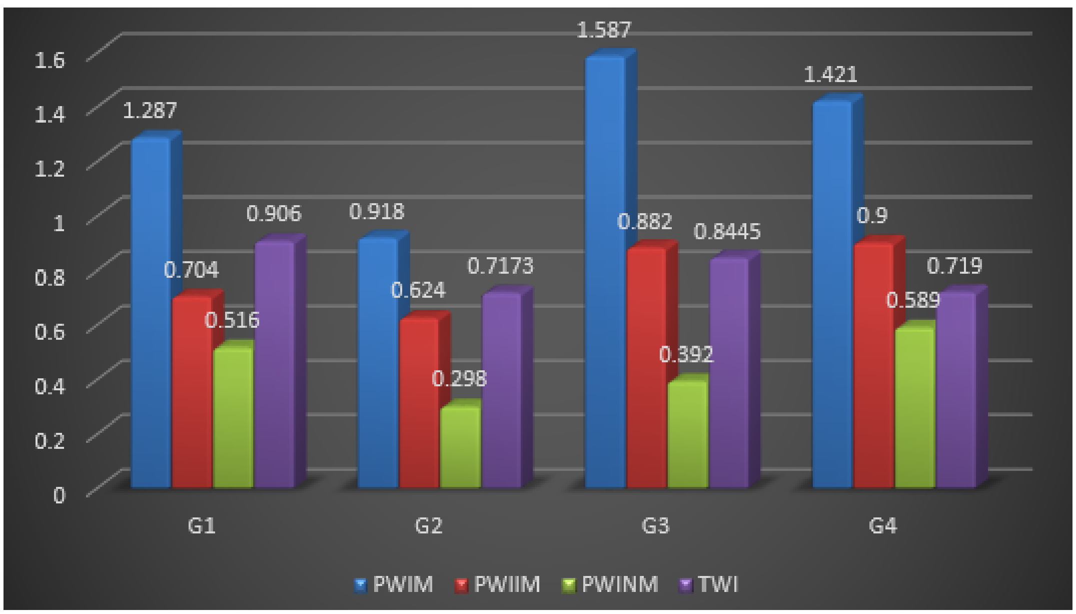

Figure 8. Graphical Representation

5. **Conclusion**

This paper aims to explore the applicability of the wiener index, a basic parameter in the area of graph theory, in this article is set within the context of Fermatean neutrosophic networks. The wiener index offers details on the discrimination of distances separating the vertices in these networks can be undertaken by studying the shortest path between the vertices. This work also aims and provide the definition and computation of the wiener index, making an analysis of several kinds of FNGs including trees, cycles, and full graphs. Using pertinent theories, the paper also looks at the relation of the wiener index to another useful parameter, connectedness. Finally, the wiener index is defined for election analysis using FNGs. In FNGs, the wiener index is an effective way to enhance election analyses in democratic states. It provides an increased understanding of the voters mood in the polling booth through the use of better and truer methodologies.

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