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Different operators via weighted averaging and geometric approach using trigonometric neutrosophic interval-valued set and its extension

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Abstract. This work presents a new method for generating tangent trigonometric neutrosophic interval-valued sets. This article will cover tangent trigonometric neutrosophic interval-valued weighted averaging, geometric, generalized notions. We used an aggregating model to get the weighted average and geometric. Several sets with substantial characteristics will be further studied using the algebraic techniques.

Keywords: weighted averaging, weighted geometric, generalized weighted averaging, generalized weighted geometric.

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1. **Introduction**

To explain uncertainty, a number of theories have been put forth, including fuzzy sets (FS) [1], which have membership grades (MG) that range from 0. Atanassov [2] constructed an intuitionistic FS (IFS) for $\zeta, \omega \in [0, 1]$ using two MGs: $0 \leq \zeta + \omega \leq 1$ and positive ζ and negative ω. Yager [3] developed the Pythagorean FSs (PFS) idea, which is distinguished by its MG and non-MG (NMG) with $\zeta + \omega \geq 1$ to $\zeta^2 + \omega^2 \leq 1$. Numerous studies have examined the use of IFSs and PFSs in various fields. Their ability to communicate information is still restricted. Because of this, the experts were still having trouble interpreting the data in these sets and the associated data. Wang et al. [4] investigated the concept of complex IFS with DOMBI prioritized AOs and its application for trustworthy green supplier selection. Hatamleh et al. [5]- [10] discussed the concept of new algebraic structures such as neutrosophic sets, semigroups and its applications. According to Cuong et al. [11], the three primary ideas of the picture FS are positive MG (ζ), neutral MG ($\propto$), and negative MG (ω). Additionally, it offers greater benefits than PFS and IFS. Since $\zeta, \omega \in [0, 1]$, it has been noted that the picture FS is an upgrade of the IFS that may handle greater inconsistency and $0 \leq \zeta + \omega \leq 1$. According on the picture FS description, expert comments like "yes," "abstain," "no," and "refusal" will be supplied.

Shahzaib et al. [12] defined the SFS for certain AOs using MADM. SFS requires that $0 \leq \zeta^2 + \omega^2 \leq 1$ rather than $0 \leq \zeta + \omega \leq 1$. Hussain et al. [13] first proposed the concept of an intelligent decision support system for SFS. Hatamleh et al. [14]- [18] deals that the different algebraic concepts and its generalization. SFSs and their applications in DM were initially presented by Rafiq et al. [19]. For instance, $\zeta^2 + \omega^2 \geq 1$ is a DM problem with a property. Senapati et al. [20] invented Fermatean FS (FFS) in 2019 with the condition that $0 \leq \zeta^3 + \omega^3 \leq 1$. The concept of generalized orthopair FSs was initially proposed by Yager [21]. In the $\propto$-rung orthogonal pair FS ($\propto$-ROFS), both the MG and the NMG have power $\propto$; however, their sum can never be more than one. Palanikumar et al. [22] introduced the MADM approach for Pythagorean neutrosophic normal interval-valued fuzzy AOs. Aggregation operators are essential to solving MADM problems (AOs). According to Xu et al. [23], there are IFS averaging operators that may be used to average IFS data. Recently many authors discussed the new research and its aggregating operators [24]- [28].

Based on IFSs, Xu et al. [29] developed geometric operators, such as weighted, ordered weighted, and hybrid operators. Li et al. [30] proposed generalized ordered weighted averaging operators (GOWs) in 2002. Using AOs and distance measurements, Zeng et al. [31] described how to calculate ordered weighted distances. Peng et al. [32] examined a basic PFS based on the characteristics of AOs. Fuzzy sphere Dombi AOs were developed by Ashraf et al. [33]. Additionally, Ullah et al. [34, 35] offer more details on SFSs and T-SFSs. Al-Husban [36] introduced the concept of multi-fuzzy rings and its extension. Palanikumar et al. [37–39] investigated a variety of algebraic structures and aggregation methods with applications. The rest of this work will be completed in the manner described below. For an introduction, see section 1. In Section 2, PFS and NS were covered. Section 3 describes a number of techniques on $\propto$ IVNNs. Section 4 discusses the AOs based on IVT $\propto$ NN. The conclusion is covered in section 5.

2. Background

This section has several crucial definitions that we should examine for future learning.

Definition 2.1. Let $\mathcal{A}$ be a universal. The PFS $\Gamma = \left\{\flat, \left\langle \mathcal{L}^T(\flat), \mathcal{L}^F(\flat)\right\rangle \middle| \flat \in \mathcal{A}\right\}$, $\mathcal{L}^T : \mathcal{A} \to (0,1)$ and $\mathcal{L}^F : \mathcal{A} \to (0,1)$ called the MG and NMG of $\flat \in \mathcal{A}$ to Γ, respectively and $0 \preceq (\mathcal{L}^T(\flat))^2 + (\mathcal{L}^F(\flat))^2 \preceq 1$. For $\Gamma = \left\langle \mathcal{L}^T, \mathcal{L}^F \right\rangle$ is called a Pythagorean fuzzy number (PFN).

Definition 2.2. The NS $\Gamma = \left\{\flat, \left\langle \mathcal{L}^T(\flat), \mathcal{L}^I(\flat), \mathcal{L}^F(\flat)\right\rangle \middle| \flat \in \mathcal{A}\right\}$, where $\mathcal{L}^T, \mathcal{L}^I, \mathcal{L}^F : \mathcal{A} \to (0,1)$ is denote the MG, IMG and NMG of $\flat \in \mathcal{A}$, respectively and $0 \leq (\mathcal{L}^T(\flat)) + (\mathcal{L}^I(\flat)) + (\mathcal{L}^F(\flat)) \leq 2$. For $M = \left\langle \mathcal{L}^T, \mathcal{L}^I, \mathcal{L}^F \right\rangle$ is called a neutrosophic number (NN).

Definition 2.3. The Pythagorean NS $\Gamma = \left\{\flat, \left\langle \mathcal{L}^T(\flat), \mathcal{L}^I(\flat), \mathcal{L}^F(\flat)\right\rangle \middle| \flat \in \mathcal{A}\right\}$, where $\mathcal{L}^T, \mathcal{L}^I, \mathcal{L}^F : \mathcal{A} \to (0,1)$ is called the MG, IMG and NMG of $\flat \in \mathcal{A}$, respectively and $0 \leq (\mathcal{L}^T(\flat))^2 + (\mathcal{L}^I(\flat))^2 + (\mathcal{L}^F(\flat))^2 \leq 2$. For $M = \left\langle \mathcal{L}^T, \mathcal{L}^I, \mathcal{L}^F \right\rangle$ is called a Pythagorean neutrosophic number (PyNN).

Definition 2.4. Let $\Gamma_1 = (a_1, b_1) \in N$and $\Gamma_2 = (a_2, b_2) \in N$. Then the distance between Γ_1 and Γ_2 is defined as $\Lambda(\Gamma_1, \Gamma_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

3. Operations for IVT $\propto$ NN

We present the concept of the $\propto$ IVNN, which is a tangent trigonometric. As a consequence, the IVT $\propto$ NN and its operations were established and $\tan \pi/2 = \daleth$

Definition 3.1. The $\propto$ NS $\Gamma = \left\{\flat, \left\langle \left([(\daleth \circ \mathcal{O})(\flat), (\daleth \circ \mathcal{V})(\flat)], [(\daleth \circ \mathcal{P})(\flat), (\daleth \circ \mathcal{X})(\flat)], [(\daleth \circ \mathcal{Q})(\flat), (\daleth \circ \mathcal{Y})(\flat)] \right) \right\rangle \middle| \flat \in \mathcal{A}\right\}$, where $(\daleth \circ \mathcal{O}), (\daleth \circ \mathcal{P}), (\daleth \circ \mathcal{Q}) : \mathcal{A} \to (0,1)$ denote the MG, IMG and NMG of $\flat \in \mathcal{A}$ to Γ, respectively and $0 \leq ((\daleth \circ \mathcal{V})(\flat))^{\propto} + ((\daleth \circ \mathcal{X})(\flat))^{\propto} + ((\daleth \circ \mathcal{Y})(\flat))^{\propto} \leq 1$. For convenience, $\Gamma = \left\langle \left([(\daleth \circ \mathcal{O}), (\daleth \circ \mathcal{V})], [(\daleth \circ \mathcal{P}), (\daleth \circ \mathcal{X})], [(\daleth \circ \mathcal{Q}), (\daleth \circ \mathcal{Y})] \right) \right\rangle$ is represent a IVT $\propto$ NN.

Definition 3.2. Let $\Gamma = \left\langle \left([(\daleth \circ \mathcal{O}), (\daleth \circ \mathcal{V})], [(\daleth \circ \mathcal{P}), (\daleth \circ \mathcal{X})], [(\daleth \circ \mathcal{Q}), (\daleth \circ \mathcal{Y})] \right) \right\rangle, \Gamma_1 = \langle ([(\daleth \circ \mathcal{O}_1), (\daleth \circ \mathcal{V}_1)], [(\daleth \circ \mathcal{P}_1), (\daleth \circ \mathcal{X}_1)], [(\daleth \circ \mathcal{Q}_1), (\daleth \circ \mathcal{Y}_1)]) \rangle, \Gamma_2 = \langle ([(\daleth \circ \mathcal{O}_2), (\daleth \circ \mathcal{V}_2)], [(\daleth \circ \mathcal{P}_2), (\daleth \circ \mathcal{X}_2)], [(\daleth \circ \mathcal{Q}_2), (\daleth \circ \mathcal{Y}_2)]) \rangle$ be any three IVT $\propto$ NNs, and $q > 0$. Then

$$(1)\ \Gamma_1 \curlyvee \Gamma_2 = \begin{bmatrix} \sqrt[\propto]{\begin{array}{c}((\daleth \circ \mathcal{O}_1))^{\propto} + ((\daleth \circ \mathcal{O}_2))^{\propto} \\ -((\daleth \circ \mathcal{O}_1))^{\propto} \cdot ((\daleth \circ \mathcal{O}_2))^{\propto}\end{array}}, \sqrt[\propto]{\begin{array}{c}((\daleth \circ \mathcal{V}_1))^{\propto} + ((\daleth \circ \mathcal{V}_2))^{\propto} \\ -((\daleth \circ \mathcal{V}_1))^{\propto} \cdot ((\daleth \circ \mathcal{V}_2))^{\propto}\end{array}}, \\ \sqrt[\propto]{\begin{array}{c}((\daleth \circ \mathcal{P}_1))^{\propto} + ((\daleth \circ \mathcal{P}_2))^{\propto} \\ -((\daleth \circ \mathcal{P}_1))^{\propto} \cdot ((\daleth \circ \mathcal{P}_2))^{\propto}\end{array}}, \sqrt[\propto]{\begin{array}{c}((\daleth \circ \mathcal{X}_1))^{\propto} + ((\daleth \circ \mathcal{X}_2))^{\propto} \\ -((\daleth \circ \mathcal{X}_1))^{\propto} \cdot ((\daleth \circ \mathcal{X}_2))^{\propto}\end{array}}, \\ ((\daleth \circ \mathcal{Q}_1))^{\propto}((\daleth \circ \mathcal{Q}_2))^{\propto}, ((\daleth \circ \mathcal{Y}_1))^{\propto}((\daleth \circ \mathcal{Y}_2))^{\propto} \end{bmatrix},$$

$$(2)\ \Gamma_1 \oslash \Gamma_2 = \begin{bmatrix} ((\dashv\circ\mathcal{O}_1))^{\propto}((\dashv\circ\mathcal{O}_2))^{\propto}, ((\dashv\circ\mathcal{V}_1))^{\propto}((\dashv\circ\mathcal{V}_2))^{\propto}, \\ \sqrt[\propto]{\begin{matrix} ((\dashv\circ\mathcal{P}_1))^{\propto} + ((\dashv\circ\mathcal{P}_2))^{\propto} \\ -((\dashv\circ\mathcal{P}_1))^{\propto}\cdot((\dashv\circ\mathcal{P}_2))^{\propto} \end{matrix}}, \sqrt[\propto]{\begin{matrix} ((\dashv\circ\mathcal{X}_1))^{\propto} + ((\dashv\circ\mathcal{X}_2))^{\propto} \\ -((\dashv\circ\mathcal{X}_1))^{\propto}\cdot((\dashv\circ\mathcal{X}_2))^{\propto} \end{matrix}} \\ \sqrt[\propto]{\begin{matrix} ((\dashv\circ\mathcal{Q}_1))^{\propto} + ((\dashv\circ\mathcal{Q}_2))^{\propto} \\ -((\dashv\circ\mathcal{Q}_1))^{\propto}\cdot((\dashv\circ\mathcal{Q}_2))^{\propto} \end{matrix}}, \sqrt[\propto]{\begin{matrix} ((\dashv\circ\mathcal{Y}_1))^{\propto} + ((\dashv\circ\mathcal{Y}_2))^{\propto} \\ -((\dashv\circ\mathcal{Y}_1))^{\propto}\cdot((\dashv\circ\mathcal{Y}_2))^{\propto} \end{matrix}} \end{bmatrix}$$

$$(3)\ \partial\cdot\Gamma = \begin{bmatrix} \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{O})^{\propto}\right)^{\partial}}, \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{V})^{\propto}\right)^{\partial}}, \\ \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{P})^{\propto}\right)^{\partial}}, \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{X})^{\propto}\right)^{\partial}}, \\ ((\dashv\circ(\mathcal{Q})^{\propto})^{\partial}, ((\dashv\circ(\mathcal{Y})^{\propto})^{\partial} \end{bmatrix}$$

$$(4)\ \Gamma^{\partial} = \begin{bmatrix} ((\dashv\circ(\mathcal{O})^{\propto})^{\partial}, ((\dashv\circ(\mathcal{V})^{\propto})^{\partial}, \\ \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{P})^{\propto}\right)^{\partial}}, \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{X})^{\propto}\right)^{\partial}}, \\ \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{Q})^{\propto}\right)^{\partial}}, \sqrt[\propto]{1-\left(1-(\dashv\circ(\mathcal{Y})^{\propto}\right)^{\partial}} \end{bmatrix}.$$

We present ED and HD measures for IVT $\propto$ NNs and investigate their mathematical characteristics.

Definition 3.3. For any two IVT $\propto$ NNs $\Gamma_1 = \langle([(\dashv\circ\mathcal{O}_1),(\dashv\circ\mathcal{V}_1)],[(\dashv\circ\mathcal{P}_1),(\dashv\circ\mathcal{X}_1)],[(\dashv\circ\mathcal{Q}_1),(\dashv\circ\mathcal{Y}_1)])\rangle, \Gamma_2 = \langle([(\dashv\circ\mathcal{O}_2),(\dashv\circ\mathcal{V}_2)],[(\dashv\circ\mathcal{P}_2),(\dashv\circ\mathcal{X}_2)],[(\dashv\circ\mathcal{Q}_2),(\dashv\circ\mathcal{Y}_2)])\rangle$. Then

$$\Lambda_E(\Gamma_1,\Gamma_2) = \sqrt{\frac{1}{2}\begin{bmatrix} \begin{bmatrix} 1+((\dashv\circ\mathcal{O}_1))^2-((\dashv\circ\mathcal{P}_1))^2-((\dashv\circ\mathcal{Q}_1))^2 \\ -\left(1+((\dashv\circ\mathcal{O}_2))^2-((\dashv\circ\mathcal{P}_2))^2-((\dashv\circ\mathcal{Q}_2))^2\right) \end{bmatrix}^2 \\ +\begin{bmatrix} ((\dashv\circ\mathcal{V}_1))^2-((\dashv\circ\mathcal{X}_1))^2-((\dashv\circ\mathcal{Y}_1))^2 \\ -\left(((\dashv\circ\mathcal{V}_2))^2-((\dashv\circ\mathcal{X}_2))^2-((\dashv\circ\mathcal{Y}_2))^2\right) \end{bmatrix}^2 \end{bmatrix}}$$

where $\Lambda_E(\Gamma_1,\Gamma_2)$ is called the ED between Γ_1 and Γ_2.

$$\Lambda_H(\Gamma_1,\Gamma_2) = \frac{1}{2}\begin{bmatrix} \begin{vmatrix} 1+((\dashv\circ\mathcal{O}_1))^2-((\dashv\circ\mathcal{P}_1))^2-((\dashv\circ\mathcal{Q}_1))^2 \\ -\left(1+((\dashv\circ\mathcal{O}_2))^2-((\dashv\circ\mathcal{P}_2))^2-((\dashv\circ\mathcal{Q}_2))^2\right) \end{vmatrix} \\ +\begin{vmatrix} ((\dashv\circ\mathcal{V}_1))^2-((\dashv\circ\mathcal{X}_1))^2-((\dashv\circ\mathcal{Y}_1))^2 \\ \left(-((\dashv\circ\mathcal{V}_2))^2-((\dashv\circ\mathcal{X}_2))^2-((\dashv\circ\mathcal{Y}_2))^2\right) \end{vmatrix} \end{bmatrix}$$

where $\Lambda_H(\Gamma_1,\Gamma_2)$ is called the HD between Γ_1 and Γ_2.

4. **Aggregating operators**

We use IVT $\propto$ NWA, IVT $\propto$ NWG, GIVT $\propto$ NWA, and GIVT $\propto$ NWG to describe the AOs.

4.1. *IVT qNWA*

Definition 4.1. Let $\Gamma_i = \langle(([(\exists\circ\mathcal{O}_i),(\exists\circ\mathcal{V}_i)],[(\exists\circ\mathcal{P}_i),(\exists\circ\mathcal{X}_i)],\ [(\exists\circ\mathcal{Q}_i),(\exists\circ\mathcal{Y}_i)]))\rangle$ be the IVT $\propto$ NNs, $W = (\eth_1, \eth_2, ..., \eth_n)$ be the weight of Γ_i, $\eth_i \geq 0$ and $\curlyvee_{i=1}^{n}\eth_i = 1$. Then IVT $\propto$ NWA $(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \curlyvee_{i=1}^{n}\eth_i\Gamma_i$.

Theorem 4.2. *Let* $\Gamma_i = \langle(([(\exists\circ\mathcal{O}_i),(\exists\circ\mathcal{V}_i)],[(\exists\circ\mathcal{P}_i),(\exists\circ\mathcal{X}_i)],\ [(\exists\circ\mathcal{Q}_i),(\exists\circ\mathcal{Y}_i)]))\rangle$ *be the IVT* $\propto$ *NNs. Then IVT* $qNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$= \begin{bmatrix} \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\exists\circ\mathcal{O}_i))^{\propto}\right)^{\eth_i}},\ \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\exists\circ\mathcal{V}_i))^{\propto}\right)^{\eth_i}}, \\ \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\exists\circ\mathcal{P}_i))^{\propto}\right)^{\eth_i}},\ \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\exists\circ\mathcal{X}_i))^{\propto}\right)^{\eth_i}}, \\ \curlywedge_{i=1}^{n}(((\exists\circ\mathcal{Q}_i))^{\propto})^{\eth_i},\ \curlywedge_{i=1}^{n}(((\exists\circ\mathcal{Y}_i))^{\propto})^{\eth_i} \end{bmatrix}.$$

Proof If $n = 2$, then IVT $\propto$ NWA$(\Gamma_1, \Gamma_2) = \eth_1\Gamma_1 \curlyvee \eth_2\Gamma_2$, where

$$\eth_1\Gamma_1 = \begin{bmatrix} \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\eth_1}},\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\eth_1}}, \\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\eth_1}},\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\eth_1}}, \\ (((\exists\circ\mathcal{Q}_1))^{\propto})^{\eth_1},(((\exists\circ\mathcal{Y}_1))^{\propto})^{\eth_1} \end{bmatrix}$$

$$\eth_2\Gamma_2 = \begin{bmatrix} \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{O}_2))^{\propto}\right)^{\eth_2}},\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{V}_2))^{\propto}\right)^{\eth_2}}, \\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{P}_2))^{\propto}\right)^{\eth_2}},\ \sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{X}_2))^{\propto}\right)^{\eth_2}} \\ (((\exists\circ\mathcal{Q}_2))^{\propto})^{\eth_2},(((\exists\circ\mathcal{Y}_2))^{\propto})^{\eth_2} \end{bmatrix}.$$

Now, $\eth_1\Gamma_1 \curlyvee \eth_2\Gamma_2$

$$= \begin{bmatrix} \sqrt[\propto]{\begin{matrix}\left(1-\left(1-((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\eth_1}\right)+ \\ \left(1-\left(1-((\exists\circ\mathcal{O}_2))^{\propto}\right)^{\eth_2}\right) \\ -\left(1-\left(1-((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\eth_1}\right)\cdot \\ \left(1-\left(1-((\exists\circ\mathcal{O}_2))^{\propto}\right)^{\eth_2}\right)\end{matrix}},\ \sqrt[\propto]{\begin{matrix}\left(1-\left(1-((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\eth_1}\right)+ \\ \left(1-\left(1-((\exists\circ\mathcal{V}_2))^{\propto}\right)^{\eth_2}\right) \\ -\left(1-\left(1-((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\eth_1}\right)\cdot \\ \left(1-\left(1-((\exists\circ\mathcal{V}_2))^{\propto}\right)^{\eth_2}\right)\end{matrix}} \\ \sqrt[\propto]{\begin{matrix}\left(1-\left(1-((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\eth_1}\right)+ \\ \left(1-\left(1-((\exists\circ\mathcal{P}_2))^{\propto}\right)^{\eth_2}\right) \\ -\left(1-\left(1-((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\eth_1}\right)\cdot \\ \left(1-\left(1-((\exists\circ\mathcal{P}_2))^{\propto}\right)^{\eth_2}\right)\end{matrix}},\ \sqrt[\propto]{\begin{matrix}\left(1-\left(1-((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\eth_1}\right)+ \\ \left(1-\left(1-((\exists\circ\mathcal{X}_2))^{\propto}\right)^{\eth_2}\right) \\ -\left(1-\left(1-((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\eth_1}\right)\cdot \\ \left(1-\left(1-((\exists\circ\mathcal{X}_2))^{\propto}\right)^{\eth_2}\right),\end{matrix}} \\ (((\exists\circ\mathcal{Q}_1))^{\propto})^{\eth_1},(((\exists\circ\mathcal{Q}_2))^{\propto})^{\eth_2},(((\exists\circ\mathcal{Y}_1))^{\propto})^{\eth_1},(((\exists\circ\mathcal{Y}_2))^{\propto})^{\eth_2} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\propto]{1-\left(1-((\daleth\circ\mathcal{O}_1))^{\propto}\right)^{\eth_1}\left(1-((\daleth\circ\mathcal{O}_2))^{\propto}\right)^{\eth_2}}, \\ \sqrt[\propto]{1-\left(1-((\daleth\circ\mathcal{V}_1))^{\propto}\right)^{\eth_1}\left(1-((\daleth\circ\mathcal{V}_2))^{\propto}\right)^{\eth_2}}, \\ \sqrt[\propto]{1-\left(1-((\daleth\circ\mathcal{P}_1))^{\propto}\right)^{\eth_1}\left(1-((\daleth\circ\mathcal{P}_2))^{\propto}\right)^{\eth_2}}, \\ \sqrt[\propto]{1-\left(1-((\daleth\circ\mathcal{X}_1))^{\propto}\right)^{\eth_1}\left(1-((\daleth\circ\mathcal{X}_2))^{\propto}\right)^{\eth_2}}, \\ (((\daleth\circ\mathcal{Q}_1))^{\propto})^{\eth_1}\cdot(((\daleth\circ\mathcal{Q}_2))^{\propto})^{\eth_2}, \\ (((\daleth\circ\mathcal{Y}_1))^{\propto})^{\eth_1}\cdot(((\daleth\circ\mathcal{Y}_2))^{\propto})^{\eth_2} \end{bmatrix}$$

Hence, IVT $qNWA(\Gamma_1, \Gamma_2)$

$$= \begin{bmatrix} \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-((\daleth\circ\mathcal{O}_i))^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-((\daleth\circ\mathcal{V}_i))^{\propto}\right)^{\eth_i}}, \\ \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-((\daleth\circ\mathcal{P}_i))^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-((\daleth\circ\mathcal{X}_i))^{\propto}\right)^{\eth_i}}, \\ \curlywedge_{i=1}^{2}(((\daleth\circ\mathcal{Q}_i))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{2}(((\daleth\circ\mathcal{Y}_i))^{\propto})^{\eth_i} \end{bmatrix}.$$

It valid for $n \geq 3$. Thus, IVT $qNWA(\Gamma_1, \Gamma_2, ..., \Gamma_l)$

$$= \begin{bmatrix} \sqrt[\propto]{1-\curlywedge_{i=1}^{l}\left(1-((\daleth\circ\mathcal{O}_i))^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1-\curlywedge_{i=1}^{l}\left(1-((\daleth\circ\mathcal{V}_i))^{\propto}\right)^{\eth_i}}, \\ \sqrt[\propto]{1-\curlywedge_{i=1}^{l}\left(1-((\daleth\circ\mathcal{P}_i))^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1-\curlywedge_{i=1}^{l}\left(1-((\daleth\circ\mathcal{X}_i))^{\propto}\right)^{\eth_i}}, \\ \curlywedge_{i=1}^{l}(((\daleth\circ\mathcal{Q}_i))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{l}(((\daleth\circ\mathcal{Y}_i))^{\propto})^{\eth_i} \end{bmatrix}.$$

If $n = l+1$, then IVT $\propto$ NWA $(\Gamma_1, \Gamma_2, ..., \Gamma_l, \Gamma_{l+1})$

$$= \begin{bmatrix} \sqrt[\propto]{\begin{matrix} \curlyvee_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{O}_i))^{\propto}\right)^{\eth_i}\right)+\left(1-\left(1-(\mathcal{O}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \\ -\curlywedge_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{O}_i))^{\propto}\right)^{\eth_i}\right)\cdot\left(1-\left(1-(\mathcal{O}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \end{matrix}}, \\ \sqrt[\propto]{\begin{matrix} \curlyvee_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{V}_i))^{\propto}\right)^{\eth_i}\right)+\left(1-\left(1-(\mathcal{V}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \\ -\curlywedge_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{V}_i))^{\propto}\right)^{\eth_i}\right)\cdot\left(1-\left(1-(\mathcal{V}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \end{matrix}}, \\ \sqrt[\propto]{\begin{matrix} \curlyvee_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{P}_i))^{\propto}\right)^{\eth_i}\right)+\left(1-\left(1-(\mathcal{P}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \\ -\curlywedge_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{P}_i))^{\propto}\right)^{\eth_i}\right)\cdot\left(1-\left(1-(\mathcal{P}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \end{matrix}}, \\ \sqrt[\propto]{\begin{matrix} \curlyvee_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{X}_i))^{\propto}\right)^{\eth_i}\right)+\left(1-\left(1-(\mathcal{X}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \\ -\curlywedge_{i=1}^{l}\left(1-\left(1-((\daleth\circ\mathcal{X}_i))^{\propto}\right)^{\eth_i}\right)\cdot\left(1-\left(1-(\mathcal{X}_{l+1})^{\propto}\right)^{\eth_{l+1}}\right) \end{matrix}}, \\ \curlywedge_{i=1}^{l}(((\daleth\circ\mathcal{Q}_i))^{\propto})^{\eth_i}, ((\mathcal{Q}_{l+1})^{\propto})^{\eth_{l+1}}, \curlywedge_{i=1}^{l}(((\daleth\circ\mathcal{Y}_i))^{\propto})^{\eth_i}, ((\mathcal{Y}_{l+1})^{\propto})^{\eth_{l+1}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\Big(1 - ((\exists \circ \mathcal{O}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\Big(1 - ((\exists \circ \mathcal{V}_i))^{\propto}\Big)^{\eth_i}}, \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\Big(1 - ((\exists \circ \mathcal{P}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\Big(1 - ((\exists \circ \mathcal{X}_i))^{\propto}\Big)^{\eth_i}}, \\ \curlywedge_{i=1}^{l+1}(((\exists \circ \mathcal{Q}_i))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{l+1}(((\exists \circ \mathcal{Y}_i))^{\propto})^{\eth_i} \end{bmatrix}$$

Theorem 4.3. *Let* $\Gamma_i = \langle((([(\exists \circ \mathcal{O}_i), (\exists \circ \mathcal{V}_i)], [(\exists \circ \mathcal{P}_i), (\exists \circ \mathcal{X}_i)], [(\exists \circ \mathcal{Q}_i), (\exists \circ \mathcal{Y}_i)]))\rangle$ *be the* $IVT \propto NNs$*. Then* $IVT \propto NWA\ (\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma$ *(idempotency property).*

Proof Since $(\exists \circ \mathcal{O}_i) = (\exists \circ \mathcal{O})$, $(\exists \circ \mathcal{P}_i) = (\exists \circ \mathcal{P})$ and $(\exists \circ \mathcal{Q}_i) = (\exists \circ \mathcal{Q})$ and $(\exists \circ \mathcal{V}_i) = (\exists \circ \mathcal{V})$, $(\exists \circ \mathcal{X}_i) = (\exists \circ \mathcal{X})$ and $(\exists \circ \mathcal{Y}_i) = (\exists \circ \mathcal{Y})$ and $\curlyvee_{i=1}^{n} \eth_i = 1$. Now, IVT $qNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$\begin{aligned} &= \begin{bmatrix} \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\exists \circ \mathcal{O}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\exists \circ \mathcal{V}_i))^{\propto}\Big)^{\eth_i}}, \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\exists \circ \mathcal{P}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\exists \circ \mathcal{X}_i))^{\propto}\Big)^{\eth_i}}, \\ \curlywedge_{i=1}^{n}(((\exists \circ \mathcal{Q}_i))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{n}(((\exists \circ \mathcal{Y}_i))^{\propto})^{\eth_i} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{O})^{\propto}\Big)^{\curlyvee_{i=1}^{n}\eth_i}}, \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{V})^{\propto}\Big)^{\curlyvee_{i=1}^{n}\eth_i}}, \\ \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{P})^{\propto}\Big)^{\curlyvee_{i=1}^{n}\eth_i}}, \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{X})^{\propto}\Big)^{\curlyvee_{i=1}^{n}\eth_i}}, \\ ((\exists \circ (\mathcal{Q})^{\propto})^{\curlyvee_{i=1}^{n}\eth_i}, ((\exists \circ (\mathcal{Y})^{\propto})^{\curlyvee_{i=1}^{n}\eth_i} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{O})^{\propto}\Big)}, \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{V})^{\propto}\Big)}, \\ \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{P})^{\propto}\Big)}, \sqrt[\propto]{1 - \Big(1 - (\exists \circ (\mathcal{X})^{\propto}\Big)}, \\ (\exists \circ (\mathcal{Q})^{\propto}, (\exists \circ (\mathcal{Y})^{\propto} \end{bmatrix} \\ &= \Gamma. \end{aligned}$$

Theorem 4.4. *Let* $\Gamma_i = \langle((([(\exists \circ \mathcal{O}_i), (\exists \circ \mathcal{V}_i)], [(\exists \circ \mathcal{P}_i), (\exists \circ \mathcal{X}_i)], [(\exists \circ \mathcal{Q}_i), (\exists \circ \mathcal{Y}_i)]))\rangle$ *be the* $IVT \propto NNs$*. Then* $IVT \propto NWA(\Gamma_1, \Gamma_2, ..., \Gamma_n)$
where $\underline{(\exists \circ \mathcal{O})} = \min(\exists \circ \mathcal{O}_{ij})$, $\overline{(\exists \circ \mathcal{O})} = \max(\exists \circ \mathcal{O}_{ij})$, $\underline{(\exists \circ \mathcal{P})} = \min(\exists \circ \mathcal{P}_{ij})$, $\overline{(\exists \circ \mathcal{P})} = \max(\exists \circ \mathcal{P}_{ij})$, $\underline{(\exists \circ \mathcal{Q})} = \min(\exists \circ \mathcal{Q}_{ij})$, $\overline{(\exists \circ \mathcal{Q})} = \max(\exists \circ \mathcal{Q}_{ij})$
and $\underline{(\exists \circ \mathcal{V})} = \min(\exists \circ \mathcal{V}_{ij})$, $\overline{(\exists \circ \mathcal{V})} = \max(\exists \circ \mathcal{V}_{ij})$, $\underline{(\exists \circ \mathcal{X})} = \min(\exists \circ \mathcal{X}_{ij})$, $\overline{(\exists \circ \mathcal{X})} = \max(\exists \circ \mathcal{X}_{ij})$, $\underline{(\exists \circ \mathcal{Y})} = \min(\exists \circ \mathcal{Y}_{ij})$, $\overline{(\exists \circ \mathcal{Y})} = \max(\exists \circ \mathcal{Y}_{ij})$ *and where* $1 \le i \le n,\ j = 1, 2, ..., i_j$*. Then,*

$$\begin{aligned} &\Big\langle \underline{(\exists \circ \mathcal{O}), (\exists \circ \mathcal{V})}, \underline{(\exists \circ \mathcal{P}), (\exists \circ \mathcal{X})}, \overline{(\exists \circ \mathcal{Q}), (\exists \circ \mathcal{Y})} \Big\rangle \\ &\le IVTqNWA(\Gamma_1, \Gamma_2, ..., \Gamma_n) \\ &\le \Big\langle \overline{(\exists \circ \mathcal{O}), (\exists \circ \mathcal{V})}, \overline{(\exists \circ \mathcal{P}), (\exists \circ \mathcal{X})}, \underline{(\exists \circ \mathcal{Q}), (\exists \circ \mathcal{Y})} \Big\rangle. \end{aligned}$$

(Boundedness property).

Proof Since, $\underline{(\daleth \circ \mathcal{O})} = \min(\daleth \circ \mathcal{O}_{ij})$, $\overline{(\daleth \circ \mathcal{O})} = \max(\daleth \circ \mathcal{O}_{ij})$ and $\underline{(\daleth \circ \mathcal{O})} \leq (\daleth \circ \mathcal{O}_{ij}) \leq \overline{(\daleth \circ \mathcal{O})}$ and $\underline{(\daleth \circ \mathcal{V})} = \min(\daleth \circ \mathcal{V}_{ij})$, $\overline{(\daleth \circ \mathcal{V})} = \max(\daleth \circ \mathcal{V}_{ij})$ and $\underline{(\daleth \circ \mathcal{V})} \leq (\daleth \circ \mathcal{V}_{ij}) \leq \overline{(\daleth \circ \mathcal{V})}$.

Now $\underline{(\daleth \circ \mathcal{O})}, \underline{(\daleth \circ \mathcal{V})}$

$$\begin{aligned}
&= \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\underline{(\daleth \circ \mathcal{O})})^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\underline{(\daleth \circ \mathcal{V})})^{\propto}\Big)^{\eth_i}} \\
&\leq \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (((\daleth \circ \mathcal{O}_{ij})))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (((\daleth \circ \mathcal{V}_{ij})))^{\propto}\Big)^{\eth_i}} \\
&\leq \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\overline{(\daleth \circ \mathcal{O})})^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\overline{(\daleth \circ \mathcal{V})})^{\propto}\Big)^{\eth_i}} \\
&= \overline{(\daleth \circ \mathcal{O})}.
\end{aligned}$$

Since, $\underline{(\daleth \circ \mathcal{P})} = \min(\daleth \circ \mathcal{P}_{ij})$, $\overline{(\daleth \circ \mathcal{P})} = \max(\daleth \circ \mathcal{P}_{ij})$ and $\underline{(\daleth \circ \mathcal{P})} \leq (\daleth \circ \mathcal{P}_{ij}) \leq \overline{(\daleth \circ \mathcal{P})}$ and $\underline{(\daleth \circ \mathcal{X})} = \min(\daleth \circ \mathcal{X}_{ij})$, $\overline{(\daleth \circ \mathcal{X})} = \max(\daleth \circ \mathcal{X}_{ij})$ and $\underline{(\daleth \circ \mathcal{X})} \leq (\daleth \circ \mathcal{X}_{ij}) \leq \overline{(\daleth \circ \mathcal{X})}$.

Now, $\underline{(\daleth \circ \mathcal{P})}, \underline{(\daleth \circ \mathcal{X})}$

$$\begin{aligned}
&= \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\underline{(\daleth \circ \mathcal{P})})^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\underline{(\daleth \circ \mathcal{X})})^{\propto}\Big)^{\eth_i}} \\
&\leq \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{P}_{ij}))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{X}_{ij}))^{\propto}\Big)^{\eth_i}} \\
&\leq \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\overline{(\daleth \circ \mathcal{P})})^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - (\overline{(\daleth \circ \mathcal{X})})^{\propto}\Big)^{\eth_i}} \\
&= \overline{(\daleth \circ \mathcal{P})}, \overline{(\daleth \circ \mathcal{X})}.
\end{aligned}$$

Since, $\underline{(\daleth \circ (\mathcal{Q})^{\propto})} = \min((\daleth \circ \mathcal{Q}_{ij}))^{\propto}$, $\overline{(\daleth \circ (\mathcal{Q})^{\propto})} = \max((\daleth \circ \mathcal{Q}_{ij}))^{\propto}$ and $\underline{(\daleth \circ (\mathcal{Q})^{\propto})} \leq ((\daleth \circ \mathcal{Q}_{ij}))^{\propto} \leq \overline{(\daleth \circ (\mathcal{Q})^{\propto})}$ and $\underline{(\daleth \circ (\mathcal{Y})^{\propto})} = \min((\daleth \circ \mathcal{Y}_{ij}))^{\propto}$, $\overline{(\daleth \circ (\mathcal{Y})^{\propto})} = \max((\daleth \circ \mathcal{Y}_{ij}))^{\propto}$ and $\underline{(\daleth \circ (\mathcal{Y})^{\propto})} \leq ((\daleth \circ \mathcal{Y}_{ij}))^{\propto} \leq \overline{(\daleth \circ (\mathcal{Y})^{\propto})}$.

We have,

$$\begin{aligned}
\underline{(\daleth \circ (\mathcal{Q})^{\propto}} &= \curlywedge_{i=1}^{n}\underline{(\daleth \circ (\mathcal{Q})^{\propto})}^{\eth_i}, \curlywedge_{i=1}^{n}\underline{(\daleth \circ (\mathcal{Y})^{\propto})}^{\eth_i} \\
&\leq \curlywedge_{i=1}^{n}(((\daleth \circ \mathcal{Q}_{ij}))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{n}(((\daleth \circ \mathcal{Y}_{ij}))^{\propto})^{\eth_i} \\
&\leq \curlywedge_{i=1}^{n}\overline{(\daleth \circ (\mathcal{Q})^{\propto})}^{\eth_i}, \curlywedge_{i=1}^{n}\overline{(\daleth \circ (\mathcal{Y})^{\propto})}^{\eth_i} \\
&= \overline{(\daleth \circ (\mathcal{Q})^{\propto})}, \overline{(\daleth \circ (\mathcal{Y})^{\propto})}.
\end{aligned}$$

Therefore,

$$\frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\underline{\daleth\circ\mathcal{O}}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\underline{\daleth\circ\mathcal{P}}))^{\propto}\right)^{\eth_i}}\right)^2\\ +1-\left(\curlywedge_{i=1}^{n}((\overline{(\daleth\circ\mathcal{Q})})^{\propto})^{\eth_i}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\underline{\daleth\circ\mathcal{V}}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\underline{\daleth\circ\mathcal{X}}))^{\propto}\right)^{\eth_i}}\right)^2\\ -\left(\curlywedge_{i=1}^{n}((\overline{(\daleth\circ\mathcal{Y})})^{\propto})^{\eth_i}\right)^2\end{array}\right]\end{array}\right]$$

$$\leq\ \frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ(\daleth\circ\mathcal{O}_{ij})))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{P}_{ij}))^{\propto}\right)^{\eth_i}}\right)^2+\\ +1-\left(\curlywedge_{i=1}^{n}(((\daleth\circ\mathcal{Q}_{ij}))^{\propto})^{\eth_i}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ(\daleth\circ\mathcal{V}_{ij})))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{X}_{ij}))^{\propto}\right)^{\eth_i}}\right)^2+\\ -\left(\curlywedge_{i=1}^{n}(((\daleth\circ\mathcal{Y}_{ij}))^{\propto})^{\eth_i}\right)^2\end{array}\right]\end{array}\right]$$

$$\leq\ \frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-(\overline{(\daleth\circ\mathcal{O})})^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-(\overline{(\daleth\circ\mathcal{P})})^{\propto}\right)^{\eth_i}}\right)^2+\\ +1-\left(\curlywedge_{i=1}^{n}((\underline{\daleth\circ\mathcal{Q}}))^{\propto})^{\eth_i}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-(\overline{(\daleth\circ\mathcal{V})})^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-(\overline{(\daleth\circ\mathcal{X})})^{\propto}\right)^{\eth_i}}\right)^2+\\ -\left(\curlywedge_{i=1}^{n}((\underline{\daleth\circ\mathcal{Y}}))^{\propto})^{\eth_i}\right)^2\end{array}\right]\end{array}\right].$$

Hence,

$\left\langle\underline{(\daleth\circ\mathcal{O}),(\daleth\circ\mathcal{V})},\underline{(\daleth\circ\mathcal{P}),(\daleth\circ\mathcal{X})},\overline{(\daleth\circ\mathcal{Q}),(\daleth\circ\mathcal{Y})}\right\rangle$
$\leq IVTqNWA(\Gamma_1,\Gamma_2,...,\Gamma_n)$
$\leq\langle\overline{(\daleth\circ\mathcal{O}),(\daleth\circ\mathcal{V})},\overline{(\daleth\circ\mathcal{P}),(\daleth\circ\mathcal{X})},\underline{(\daleth\circ\mathcal{Q}),(\daleth\circ\mathcal{Y})}\rangle.$

Theorem 4.5. *Let* $\Gamma_i=\langle([(\daleth\circ\mathcal{O}_{t_{ij}}),(\daleth\circ\mathcal{V}_{t_{ij}})],[(\daleth\circ\mathcal{P}_{t_{ij}}),(\daleth\circ\mathcal{X}_{t_{ij}})],[(\daleth\circ\mathcal{Q}_{t_{ij}}),(\daleth\circ\mathcal{Y}_{t_{ij}})])\rangle$ *and* $W_i=\langle([(\daleth\circ\mathcal{O}_{h_{ij}}),(\daleth\circ\mathcal{V}_{h_{ij}})],[(\daleth\circ\mathcal{P}_{h_{ij}}),(\daleth\circ\mathcal{X}_{h_{ij}})],[(\daleth\circ\mathcal{Q}_{h_{ij}}),(\daleth\circ\mathcal{Y}_{h_{ij}})])\rangle$, *be the IVT* $\propto$ *NWAs. For any* i*, if there is* $(\daleth\circ\mathcal{O}_{t_{ij}})^2\leq(\daleth\circ\mathcal{O}_{h_{ij}})^2$ *and* $(\daleth\circ\mathcal{P}_{t_{ij}})^2\leq(\daleth\circ\mathcal{P}_{h_{ij}})^2$ *and* $(\daleth\circ\mathcal{Q}_{t_{ij}})^2\geq(\daleth\circ\mathcal{Q}_{h_{ij}})^2$ *and* $(\daleth\circ\mathcal{V}_{t_{ij}})^2\leq(\daleth\circ\mathcal{V}_{h_{ij}})^2$ *and* $(\daleth\circ\mathcal{X}_{t_{ij}})^2\leq(\daleth\circ\mathcal{X}_{h_{ij}})^2$ *and* $(\daleth\circ\mathcal{Y}_{t_{ij}})^2\geq(\daleth\circ\mathcal{Y}_{h_{ij}})^2$ *or* $\Gamma_i\leq W_i$*. Prove that* $IVT\ qNWA(\Gamma_1,\Gamma_2,...,\Gamma_n)\leq IVTqNWA(W_1,W_2,...,W_n)$*, where* $(i=1,2,...,n);(j=1,2,...,i_j)$ *(monotonicity property).*

Proof For any i, $(\daleth\circ\mathcal{O}_{t_{ij}})^2\leq(\daleth\circ\mathcal{O}_{h_{ij}})^2$.
Therefore, $1-((\daleth\circ\mathcal{O}_{t_i}))^2\geq 1-((\daleth\circ\mathcal{O}_{h_i}))^2$.
Hence, $\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{O}_{t_i}))^2\right)^{\eth_i}\geq\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{O}_{h_i}))^2\right)^{\eth_i}$
and $\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{O}_{t_i}))^{\propto}\right)^{\eth_i}}\leq\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{O}_{h_i}))^{\propto}\right)^{\eth_i}}$.
Similarly, $(\daleth\circ\mathcal{V}_{t_{ij}})^2\leq(\daleth\circ\mathcal{V}_{h_{ij}})^2$.
Therefore, $1-((\daleth\circ\mathcal{V}_{t_i}))^2\geq 1-((\daleth\circ\mathcal{V}_{h_i}))^2$.
Hence, $\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{V}_{t_i}))^2\right)^{\eth_i}\geq\curlywedge_{i=1}^{n}\left(1-((\daleth\circ\mathcal{V}_{h_i}))^2\right)^{\eth_i}$

and $\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{V}_{t_i}))^{\propto}\right)^{\eth_i}} \leq \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{V}_{h_i}))^{\propto}\right)^{\eth_i}}$.
For any i, $(\dashv\circ\mathcal{P}_{t_{ij}})^{\propto} \leq \left((\dashv\circ\mathcal{P}_{h_{ij}})\right)^{\propto}$.
Therefore, $1-((\dashv\circ\mathcal{P}_{t_i}))^{\propto} \geq 1-((\dashv\circ\mathcal{P}_{h_i}))^{\propto}$.
Hence, $\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{t_i}))^{\propto}\right)^{\eth_i} \geq \curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{h_i}))^{\propto}\right)^{\eth_i}$.
This implies that $\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{t_i}))^{\propto}\right)^{\eth_i}} \leq \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{h_i}))^{\propto}\right)^{\eth_i}}$.

Similarly, for any i, $(\dashv\circ\mathcal{X}_{t_{ij}})^{\propto} \leq \left((\dashv\circ\mathcal{X}_{h_{ij}})\right)^{\propto}$.
Therefore, $1-((\dashv\circ\mathcal{X}_{t_i}))^{\propto} \geq 1-((\dashv\circ\mathcal{X}_{h_i}))^{\propto}$.
Hence, $\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{t_i}))^{\propto}\right)^{\eth_i} \geq \curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{h_i}))^{\propto}\right)^{\eth_i}$.
This implies that $\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{t_i}))^{\propto}\right)^{\eth_i}} \leq \sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{h_i}))^{\propto}\right)^{\eth_i}}$.

For any i, $\left((\dashv\circ\mathcal{Q}_{t_{ij}})\right)^2 \geq \left((\dashv\circ\mathcal{Q}_{h_{ij}})\right)^2$ and $\left((\dashv\circ\mathcal{Q}_{t_{ij}})\right)^{\propto} \geq \left((\dashv\circ\mathcal{Q}_{h_{ij}})\right)^{\propto}$.
Therefore, $1-\frac{\left(\curlywedge_{i=1}^{n}(\dashv\circ\mathcal{Q}_{t_{ij}})\right)^{\propto}}{2} \leq 1-\frac{\left(\curlywedge_{i=1}^{n}(\dashv\circ\mathcal{Q}_{h_{ij}})\right)^{\propto}}{2}$.
Similarly, for any i,
$\left((\dashv\circ\mathcal{Y}_{t_{ij}})\right)^2 \geq \left((\dashv\circ\mathcal{Y}_{h_{ij}})\right)^2$ and $\left((\dashv\circ\mathcal{Y}_{t_{ij}})\right)^{\propto} \geq \left((\dashv\circ\mathcal{Y}_{h_{ij}})\right)^{\propto}$.
Therefore, $-\left(\curlywedge_{i=1}^{n}(\dashv\circ\mathcal{Y}_{t_{ij}})\right)^{\propto} \leq -\left(\curlywedge_{i=1}^{n}(\dashv\circ\mathcal{Y}_{h_{ij}})\right)^{\propto}$.

Hence,

$$\frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{O}_{ti}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{t_i}))^{\propto}\right)^{\eth_i}}\right)^2\\ +1-\left(\curlywedge_{i=1}^{n}((\dashv\circ\mathcal{Q}_{t_i}))^{\propto}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{V}_{ti}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{t_i}))^{\propto}\right)^{\eth_i}}\right)^2\\ -\left(\curlywedge_{i=1}^{n}((\dashv\circ\mathcal{Y}_{t_i}))^{\propto}\right)^2\end{array}\right]\end{array}\right]$$

$$\leq\ \frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{O}_{hi}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{P}_{h_i}))^{\propto}\right)^{\eth_i}}\right)^2\\ +1-\left(\curlywedge_{i=1}^{n}((\dashv\circ\mathcal{Q}_{h_i}))^{\propto}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{V}_{hi}))^{\propto}\right)^{\eth_i}}\right)^2-\left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-((\dashv\circ\mathcal{X}_{h_i}))^{\propto}\right)^{\eth_i}}\right)^2\\ -\left(\curlywedge_{i=1}^{n}((\dashv\circ\mathcal{Y}_{h_i}))^{\propto}\right)^2\end{array}\right]\end{array}\right].$$

Hence, IVT $qNWA\,(\Gamma_1,\Gamma_2,...,\Gamma_n) \leq IVTqNWA\,(W_1,W_2,...,W_n)$.

4.2. $IVT \propto NWG$

Definition 4.6. Let $\Gamma_i = \langle((([(\dashv\circ\mathcal{O}_i),(\dashv\circ\mathcal{V}_i)],[(\dashv\circ\mathcal{P}_i),(\dashv\circ\mathcal{X}_i)],\ [(\dashv\circ\mathcal{Q}_i),(\dashv\circ\mathcal{Y}_i)])))\rangle$ be the IVT $\propto$ NNs. Then $\propto$NWG $(\Gamma_1,\Gamma_2,...,\Gamma_n) = \curlywedge_{i=1}^{n}\Gamma_i^{\eth_i}$.

Corollary 4.7. *Let* $\Gamma_i = \langle(([(\daleth \circ \mathcal{O}_i), (\daleth \circ \mathcal{V}_i)], [(\daleth \circ \mathcal{P}_i), (\daleth \circ \mathcal{X}_i)], [(\daleth \circ \mathcal{Q}_i), (\daleth \circ \mathcal{Y}_i)]))\rangle$ *be the* $IVT \propto NNs$*. Then* $IVT \propto NWG\ (\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$= \begin{bmatrix} \curlywedge_{i=1}^{n}(((\daleth \circ \mathcal{O}_i))^{\propto})^{\eth_i}, \curlywedge_{i=1}^{n}(((\daleth \circ \mathcal{V}_i))^{\propto})^{\eth_i} \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{P}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{X}_i))^{\propto}\Big)^{\eth_i}} \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{Q}_i))^{\propto}\Big)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\Big(1 - ((\daleth \circ \mathcal{Y}_i))^{\propto}\Big)^{\eth_i}} \end{bmatrix}.$$

Corollary 4.8. *Let* $\Gamma_i = \langle(([(\daleth \circ \mathcal{O}_i), (\daleth \circ \mathcal{V}_i)], [(\daleth \circ \mathcal{P}_i), (\daleth \circ \mathcal{X}_i)], [(\daleth \circ \mathcal{Q}_i), (\daleth \circ \mathcal{Y}_i)]))\rangle$ *be the* $IVT \propto NNs$ *and all are equal. Then* $\propto NWG(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma$.

Corollary 4.9. *It has other properties, including boundedness and monotonicity, as well as having* $\propto NWG$.

4.3. *Generalized IVT ∝ NWA (GIVT∝NWA)*

Definition 4.10. Let $\Gamma_i = \langle(([(\daleth \circ \mathcal{O}_i), (\daleth \circ \mathcal{V}_i)], [(\daleth \circ \mathcal{P}_i), (\daleth \circ \mathcal{X}_i)], [(\daleth \circ \mathcal{Q}_i), (\daleth \circ \mathcal{Y}_i)]))\rangle$ be the IVT ∝ NN. Then GIVT ∝ NWA $(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Big(\curlyvee_{i=1}^{n} \eth_i \Gamma_i^{\partial}\Big)^{1/\partial}$.

Theorem 4.11. *Let* $\Gamma_i = \langle(([(\daleth \circ \mathcal{O}_i), (\daleth \circ \mathcal{V}_i)], [(\daleth \circ \mathcal{P}_i), (\daleth \circ \mathcal{X}_i)], [(\daleth \circ \mathcal{Q}_i), (\daleth \circ \mathcal{Y}_i)]))\rangle$ *be the* $IVT \propto NNs$*. Then* $GIVT \propto NWA\ (\Gamma_1, \Gamma_2, ..., \Gamma_n)$

$$= \begin{bmatrix} \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{O}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{V}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \\ \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \\ \sqrt[\propto]{1 - \left(1 - \left(\curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1 - \left(1 - ((\daleth \circ \mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{\propto}\right)^{1/\propto}}, \\ \sqrt[\propto]{1 - \left(1 - \left(\curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1 - \left(1 - ((\daleth \circ \mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{\propto}\right)^{1/\propto}} \end{bmatrix}.$$

Proof To illustrate this, we may first show that,

$$\curlyvee_{i=1}^{n}\eth_i\Gamma_i^{\propto} = \begin{bmatrix} \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{O}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{V}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{n}\left(1 - \left(((\daleth \circ \mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1 - \left(1 - ((\daleth \circ \mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}, \curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1 - \left(1 - ((\daleth \circ \mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i} \end{bmatrix}.$$

Put $n = 2$, $\eth_1\Gamma_1 \curlyvee \eth_2\Gamma_2$

$$= \left[\begin{array}{c} \sqrt[\propto]{\begin{array}{c}\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}+\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{O}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\\ -\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\cdot\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{O}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\end{array}} \\ ,\sqrt[\propto]{\begin{array}{c}\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}+\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{V}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\\ -\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\cdot\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{V}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto},\end{array}} \\ \sqrt[\propto]{\begin{array}{c}\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}+\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{P}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\\ -\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\cdot\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{P}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\end{array}} \\ ,\sqrt[\propto]{\begin{array}{c}\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}+\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{X}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\\ -\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\cdot\left(\sqrt[\propto]{1-\left(1-\left(((\exists\circ\mathcal{X}_2))^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto}\end{array}} \\ \left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Q}_1))^{\propto}\right)^{\propto}}\right)^{\eth_1},\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Q}_2))^{\propto}\right)^{\propto}}\right)^{\eth_1} \\ ,\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Y}_1))^{\propto}\right)^{\propto}}\right)^{\eth_1},\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Y}_2))^{\propto}\right)^{\propto}}\right)^{\eth_1} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-\left(((\exists\circ\mathcal{O}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}},\sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-\left(((\exists\circ\mathcal{V}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-\left(((\exists\circ\mathcal{P}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}},\sqrt[\propto]{1-\curlywedge_{i=1}^{2}\left(1-\left(((\exists\circ\mathcal{X}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \curlywedge_{i=1}^{2}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i},\curlywedge_{i=1}^{2}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i} \end{array}\right].$$

Hence,

$$\curlyvee_{i=1}^{l} \eth_i \Gamma_i^{\partial} = \begin{bmatrix} \sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{O}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{V}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{P}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{X}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \curlywedge_{i=1}^{l}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}, \curlywedge_{i=1}^{l}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i} \end{bmatrix}.$$

If $n = l+1$, then $\curlyvee_{i=1}^{l} \eth_i \Gamma_i^{\partial} + \eth_{l+1}\Gamma_{l+1}^{\partial} = \curlyvee_{i=1}^{l+1} \eth_i \Gamma_i^{\partial}$.
Now, $\curlyvee_{i=1}^{l} \eth_i \Gamma_i^{\partial} + \eth_{l+1}\Gamma_{l+1}^{\partial} = \eth_1 \Gamma_1^{\partial} \curlyvee \eth_2 \Gamma_2^{\partial} \curlyvee ... \curlyvee \eth_l \Gamma_l^{\partial} \curlyvee \eth_{l+1}\Gamma_{l+1}^{\partial}$

$$= \begin{bmatrix} \sqrt[\propto]{\begin{array}{c} \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{O}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} + \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{O}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \\ -\left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{O}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} \cdot \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{O}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \end{array}} \\ , \sqrt[\propto]{\begin{array}{c} \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{V}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} + \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{V}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \\ -\left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{V}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} \cdot \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{V}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \end{array}} \\ \sqrt[\propto]{\begin{array}{c} \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} + \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{P}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \\ -\left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} \cdot \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{P}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \end{array}} \\ , \sqrt[\propto]{\begin{array}{c} \left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} + \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{X}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \\ -\left(\sqrt[\propto]{1 - \curlywedge_{i=1}^{l}\left(1 - \left(((\exists \circ \mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{\propto} \cdot \left(\sqrt[\propto]{1 - \left(1 - \left((\mathcal{X}_{l+1})^{\propto}\right)^{\propto}\right)^{\eth_1}}\right)^{\propto} \end{array}} \\ \curlywedge_{i=1}^{l}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}, \left(\sqrt[\propto]{1 - \left(1 - (\mathcal{Q}_{l+1})^{\propto}\right)^{\propto}}\right)^{\eth_1} \\ , \curlywedge_{i=1}^{l}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}, \left(\sqrt[\propto]{1 - \left(1 - (\mathcal{Y}_{l+1})^{\propto}\right)^{\propto}}\right)^{\eth_1} \end{bmatrix}$$

$$\curlyvee_{i=1}^{l+1} \eth_i \Gamma_i^{\propto} = \begin{bmatrix} \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\left(1 - \left(((\exists \circ \mathcal{O}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\left(1 - \left(((\exists \circ \mathcal{V}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\left(1 - \left(((\exists \circ \mathcal{P}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}}, \sqrt[\propto]{1 - \curlywedge_{i=1}^{l+1}\left(1 - \left(((\exists \circ \mathcal{X}_1))^{\propto}\right)^{\propto}\right)^{\eth_i}} \\ \curlywedge_{i=1}^{l+1}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}, \curlywedge_{i=1}^{l+1}\left(\sqrt[\propto]{1 - \left(1 - ((\exists \circ \mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i} \end{bmatrix}.$$

$$\left(\curlyvee_{i=1}^{l+1}\eth_i\Gamma_i^{\partial}\right)^{1/\partial} = \begin{bmatrix} \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{l+1}\left(1-\left(((\exists\circ\mathcal{O}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{l+1}\left(1-\left(((\exists\circ\mathcal{V}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto} \\ \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{l+1}\left(1-\left(((\exists\circ\mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{l+1}\left(1-\left(((\exists\circ\mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto} \\ \sqrt[\propto]{1-\left(1-\left(\curlywedge_{i=1}^{l+1}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Q}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{2}\right)^{1/\propto}}, \\ \sqrt[\propto]{1-\left(1-\left(\curlywedge_{i=1}^{l+1}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{Y}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{2}\right)^{1/\propto}} \end{bmatrix}$$

Corollary 4.12. *If $q = 1$, then $IVT \propto NWA$ operator is used instead of the $GIVT \propto NWA$ operator.*

Theorem 4.13. *If all $\Gamma_i = \langle((([(\exists\circ\mathcal{O}_i),(\exists\circ\mathcal{V}_i)],[(\exists\circ\mathcal{P}_i),(\exists\circ\mathcal{X}_i)],\ [(\exists\circ\mathcal{Q}_i),(\exists\circ\mathcal{Y}_i)]))\rangle$ and all are equal. Then $GIVT \propto NWA(\Gamma_1,\Gamma_2,...,\Gamma_n) = \Gamma$.*

Corollary 4.14. *The $GIVT \propto NWA$ operator meets both boundedness and monotonicity constraints.*

4.4. *Generalized IVT ∝NWG (GIVT ∝NWG)*

Definition 4.15. Let $\Gamma_i = \langle((([(\exists\circ\mathcal{O}_i),(\exists\circ\mathcal{V}_i)],[(\exists\circ\mathcal{P}_i),(\exists\circ\mathcal{X}_i)],\ [(\exists\circ\mathcal{Q}_i),(\exists\circ\mathcal{Y}_i)]))\rangle$ be the IVT ∝ NNs. Then GIVT∝NWG $(\Gamma_1,\Gamma_2,...,\Gamma_n) = \frac{1}{\partial}\left(\curlywedge_{i=1}^{n}(\partial\Gamma_i)^{\eth_i}\right)$.

Corollary 4.16. *Let $\Gamma_i = \langle((([(\exists\circ\mathcal{O}_i),(\exists\circ\mathcal{V}_i)],[(\exists\circ\mathcal{P}_i),(\exists\circ\mathcal{X}_i)],\ [(\exists\circ\mathcal{Q}_i),(\exists\circ\mathcal{Y}_i)]))\rangle$ be the IVT ∝ NNs. Then GIVT∝NWG$(\Gamma_1,\Gamma_2,...,\Gamma_n)$*

$$= \begin{bmatrix} \sqrt[\propto]{1-\left(1-\left(\curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{O}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{\propto}\right)^{1/\propto}}, \\ \sqrt[\propto]{1-\left(1-\left(\curlywedge_{i=1}^{n}\left(\sqrt[\propto]{1-\left(1-((\exists\circ\mathcal{V}_i))^{\propto}\right)^{\propto}}\right)^{\eth_i}\right)^{\propto}\right)^{1/\propto}} \\ \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-\left(((\exists\circ\mathcal{P}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-\left(((\exists\circ\mathcal{X}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto} \\ \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-\left(((\exists\circ\mathcal{Q}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto}, \left(\sqrt[\propto]{1-\curlywedge_{i=1}^{n}\left(1-\left(((\exists\circ\mathcal{Y}_i))^{\propto}\right)^{\propto}\right)^{\eth_i}}\right)^{1/\propto} \end{bmatrix}.$$

Corollary 4.17. *When $\partial = 1$, the $GIVT \propto NWG$ is converted to the ∝NWG.*

Corollary 4.18. *GIVT∝NWG operators satisfy the boundness and monotonicity characteristics.*

Corollary 4.19. *If all* $\Gamma_i = \langle((([(\dashv \circ \mathcal{O}_i), (\dashv \circ \mathcal{V}_i)], [(\dashv \circ \mathcal{P}_i), (\dashv \circ \mathcal{X}_i)], [(\dashv \circ \mathcal{Q}_i), (\dashv \circ \mathcal{Y}_i)])))\rangle$ *are equal. Then* $GIVT{\propto}NWG(\Gamma_1, \Gamma_2, ..., \Gamma_n) = \Gamma$.

5. Conclusion:

This work presents novel weighted operators, such as geometric and averaging operators. Boundedness, idempotency, commutativity, associativity, and monotonicity are some of the characteristics of these operators. To describe the weighted vector, we looked at a number of common metrics. Many aggregation operator criteria have been studied. Some findings have been made after a few aggregating techniques for these IVT $\propto$ NNs have been examined.

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