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Distance Measure for Fermatean Neutrosophic Sets: A Comparative Framework for Qualitative Evaluation

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Abstract: This work introduces a novel distance measure specifically designed for Fermatean Neutrosophic sets (FNS), extending upon existing measures such as the intuitionistic fuzzy distance and single-valued Neutrosophy distance measures. The study proposes a Hypothetical Framework for Fermatean Neutrosophic Sets to evaluate minimum distances and compares six different distance measures. These measures are applied within the contexts of both Fermatean Neutrosophic Sets and Neutrosophic Sets. The primary objective is to identify the shortest distance, enabling precise and appropriate decision-making. The study highlights the importance of false membership in FNS, which alters the ranking of FNS and establishes the Fermatean Neutrosophic set as a more accurate tool for evaluation in specific cases.

Keywords: Fermatean Neutrosophic set; Euclidean Distance (ED), Normalized Euclidean Distance (N-ED), Hamming Distance (HD), Normalized Hamming Distance (N-HD), Sine Metric Single-Valued Neutrosophic Distance Measure (SMSVNDM), Tangent Matric Fermatean Neutrosophic Distance Measure (TMFNDM).

1. Introduction

Distance measures are vital for assessing similarity or dissimilarity between elements, enabling accurate decision-making, ranking, and analysis in uncertain or complex scenarios. They are widely used in fields like medical diagnosis, pattern recognition, and risk analysis to ensure precise evaluations and optimal solutions [1-4].

Zadeh [5] introduced fuzzy set theory, which models vagueness through membership degrees, laying the foundation for advanced uncertainty modelling. Atanassov [6] introduced intuitionistic fuzzy sets, which account for both membership and non-membership degrees, enabling better handling of uncertainty. Distance measures play a key role in assessing similarity or dissimilarity within fuzzy, intuitionistic, and neutrosophic frameworks, which have been widely applied in areas like site selection (Rouyendegh et al. [7]) and education (Citil [8]).

Smarandache [9] introduced neutrosophy, extending intuitionistic fuzzy sets by incorporating indeterminacy, offering greater flexibility in managing incomplete and inconsistent information (Smarandache) [10]. Gong et al. [11] further enhanced decision analysis with spherical distance measures for intuitionistic fuzzy sets, improving similarity assessments. Building upon existing methodologies, this work introduces a novel distance measure tailored specifically to Fermatean Neutrosophic sets [12].

Fermatean Neutrosophic sets extend the framework of Neutrosophic sets [13], introducing additional parameters to capture uncertainty, indeterminacy, and falsity inherent in medical data. By integrating insights from intuitionistic fuzzy distance measures [14] and single-valued Neutrosophy (SVN) distance measures [15], this new distance measure offers a comprehensive approach to evaluating medical conditions within the Fermatean Neutrosophic framework. Recent works on Fermatean neutrosophic sets include foundational matrix operations [21], a framework for green biomedical waste management [22], and a correlation-based measure for electric vehicle selection [23], showcasing their real-world applicability.

Here, we defined distance measures for the Fermatean neutrosophic set to handle higher membership values in a set. Also, we can use this distance measure to handle all the lower case of neutrosophic sets and their extensions like fuzzy, intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, and Fermatean fuzzy.

Throughout this exploration, we have scrutinize six distinct distance measures, including Euclidean Distance, Normalized Euclidean Distance, Hamming Distance, Normalized Hamming Distance, SMSVNDM Distance, and TMFNDM Distance. By comparing and contrasting these measures within the context of Fermatean Neutrosophic sets and traditional Neutrosophic sets, we seek to elucidate the advantages and limitations of each approaches.

2. Preliminaries

Definition 2.1. [19]

The HD between two SVN sets I and J is defined as

$$d_H(I,J)=\frac{1}{3}\sum_{j=1}^{n}\left(\left|\mu_I(x_j)-\mu_J(x_j)\right|+\left|\sigma_I(x_j)-\sigma_J(x_j)\right|+\left|\gamma_I(x_j)-\gamma_J(x_j)\right|\right) \quad (1)$$

Definition 2.2. [19]

The ED between two SVN sets I and J is defined as

$$d_E(I,J)=\left\{\frac{1}{3}\sum_{j=1}^{n}\left(\left(\mu_I(x_j)-\mu_J(x_j)\right)^2+\left(\sigma_I(x_j)-\sigma_J(x_j)\right)^2+\left(\gamma_I(x_j)-\gamma_J(x_j)\right)^2\right)\right\}^{\frac{1}{2}} \quad (2)$$

Definition 2.3. [19]

The N-HD between two SVN sets I and J is defined as

$$d_{n-H}(I,J)=\frac{1}{3n}\sum_{j=1}^{n}\left(\left|\mu_I(x_j)-\mu_J(x_j)\right|+\left|\sigma_I(x_j)-\sigma_J(x_j)\right|+\left|\gamma_I(x_j)-\gamma_J(x_j)\right|\right) \quad (3)$$

Definition 2.4. [19]

The N-ED between two SVN sets I and J is defined as

$$d_{n-E}\left(I,J\right)=\left\{\frac{1}{3n}\sum_{j=1}^{n}\left(\left(\mu_I\left(x_j\right)-\mu_J\left(x_j\right)\right)^2+\left(\sigma_I\left(x_j\right)-\sigma_J\left(x_j\right)\right)^2+\left(\gamma_I\left(x_j\right)-\gamma_J\left(x_j\right)\right)^2\right)\right\}^{\frac{1}{2}} \quad (4)$$

Definition 2.5. [15]

Let $A=\left\{x_1,x_2,\ldots,x_n\right\}$ be a universal set. Let $I=\left\{x_i,\mu_I\left(x_i\right),\sigma_I\left(x_i\right),\gamma_I\left(x_i\right):x_i\in A\right\}$ and $J=\left\{x_i,\mu_J\left(x_i\right),\sigma_J\left(x_i\right),\gamma_J\left(x_i\right):x_i\in A\right\}$ be two SVNSs on A. Then define a mapping $d:SVNS\left(A\right)\times SVNS\left(A\right)\to\left[0,1\right]$ as: $d_{SMSVN}\left(I,J\right)$

$$=\frac{5}{3n}\sum_{i=1}^{n}\frac{\sin\left\{\frac{\pi}{6}\left|\mu_I\left(x_i\right)-\mu_J\left(x_i\right)\right|\right\}+\sin\left\{\frac{\pi}{6}\left|\sigma_I\left(x_j\right)-\sigma_J\left(x_j\right)\right|\right\}+\sin\left\{\frac{\pi}{6}\left|\gamma_I\left(x_j\right)-\gamma_J\left(x_j\right)\right|\right\}}{1+\sin\left\{\frac{\pi}{6}\left|\mu_I\left(x_i\right)-\mu_J\left(x_i\right)\right|\right\}+\sin\left\{\frac{\pi}{6}\left|\sigma_I\left(x_j\right)-\sigma_J\left(x_j\right)\right|\right\}+\sin\left\{\frac{\pi}{6}\left|\gamma_I\left(x_j\right)-\gamma_J\left(x_j\right)\right|\right\}} \quad (5)$$

Definition 2.6 [6]

Let X be a non-empty set (universe). A FNS $\tilde{S}$ on X is an object of the form:

$$\tilde{S}=\{<s,(T_{\tilde{S}}(s),I_{\tilde{S}}(s),F_{\tilde{S}}(s))>|\ s\in X\}$$

where, $T_{\tilde{S}}(s),I_{\tilde{S}}(s),F_{\tilde{S}}(s):X\to[0,1],\quad 0\le T_{\tilde{S}}^{\ 3}(s)+F_{\tilde{S}}^{\ 3}(s)\le 1, 0\le I_{\tilde{S}}^{\ 3}(s)\le 1$ then

$$0\le T_{\tilde{S}}^{\ 3}(s)+I_{\tilde{S}}^{\ 3}(s)+F_{\tilde{S}}^{\ 3}(s)\le 2\quad \forall s\in X$$

$T_{\tilde{S}}(s)$ is the degree of membership, $I_{\tilde{S}}(s)$ is the degree of inderminancy and $F_{\tilde{S}}(s)$ is the degree of non-membership. Here $T_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are dependent components and $I_{\tilde{S}}(s)$ is an independent component.

3. Distance Measure on Fermatean Neutrosophic Sets

The distance measure on Fermatean Neutrosophic Sets defined in this section is a generalization of the single-valued neutrosophic distance measure and intuitionistic fuzzy distance measure. It includes several derived properties.

Definition 3.1.

Let $A=\left\{x_1,x_2,\ldots,x_n\right\}$ be a universal set. Let $I=\left\{x_i,T_I\left(x_i\right),I_I\left(x_i\right),F_I\left(x_i\right):x_i\in A\right\}$

and $J=\{x_i,T_J(x_i),I_J(x_i),F_J(x_i):x_i\in A\}$ be two Fermatean Neutrosophic Sets on A. Then define a mapping $d:FNS(A)\times FNS(A)\to[0,1]$ as:

$$d_{TMFN}(I,J)=\frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\begin{bmatrix}\tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}\end{bmatrix} \tag{6}$$

Theorem 3.2.

The FNSs hold the following properties. I,J,K.

i. $d_{TMFN}(I,J)\geq 0$ for all $I,J\in FNS(A)$.

ii. $d_{TMFN}(I,J)=0$ if and only if $I=J$ for all $I,J\in FNS(A)$.

iii. $d_{TMFN}(I,J)=d_{TMFN}(J,I)$ for all $I,J\in FNS(X)$.

iv. If $I\subseteq J\subseteq K$ for all $I,J,K\in FNS(A)$, then $d_{TMFN}(I,K)\geq d_{TMFN}(I,J)$ and $d_{TMFN}(I,K)\geq d_{TMFN}(J,K)$.

Proof:

Part (i):

If $I,J\in FNS(A)$, then $0\leq T_I(x_i)\leq 1$, $0\leq I_I(x_i)\leq 1, 0\leq F_I(x_i)\leq 1, \forall x_i\in A$

$\Rightarrow 0\leq\left|T_I(x_i)-T_J(x_i)\right|\leq 1, 0\leq\left|I_I(x_i)-I_J(x_i)\right|\leq 1$, and $0\leq\left|F_I(x_i)-F_J(x_i)\right|\leq 1$.

$$\Rightarrow 0\leq\tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}\leq\frac{1}{\sqrt{3}},$$

$$0\leq\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}\leq\frac{1}{\sqrt{3}},$$

$$0\leq\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}\leq\frac{1}{\sqrt{3}}$$

Then

$$0 \le \tan\left\{\frac{\pi}{6}\left|T_I\left(x_i\right)-T_J\left(x_i\right)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_I\left(x_i\right)-I_J\left(x_i\right)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_I\left(x_i\right)-F_J\left(x_i\right)\right|\right\} \le \sqrt{3},$$

$\forall x_i \in X$

Multiple by 2

$$0 \le \frac{2}{\sqrt{3}}\begin{bmatrix}\tan\left\{\frac{\pi}{6}\left|T_I\left(x_i\right)-T_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|I_I\left(x_i\right)-I_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|F_I\left(x_i\right)-F_J\left(x_i\right)\right|\right\}\end{bmatrix} \le 2, \forall x_i \in X.$$

This implies

$$0 \le \frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\begin{bmatrix}\tan\left\{\frac{\pi}{6}\left|T_I\left(x_i\right)-T_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|I_I\left(x_i\right)-I_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|F_I\left(x_i\right)-F_J\left(x_i\right)\right|\right\}\end{bmatrix} \le 2$$

Thus, $0 \le d_{TMNS}\left(I,J\right) \le 2$.

Part (ii):

$$d_{TMNS}\left(I,J\right)=0$$

$$\Leftrightarrow 0 \le \frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\begin{bmatrix}\tan\left\{\frac{\pi}{6}\left|T_I\left(x_i\right)-T_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|I_I\left(x_i\right)-I_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|F_I\left(x_i\right)-F_J\left(x_i\right)\right|\right\}\end{bmatrix} \le 2$$

$$\Leftrightarrow 0 \le \frac{2}{\sqrt{3}}\begin{bmatrix}\tan\left\{\frac{\pi}{6}\left|T_I\left(x_i\right)-T_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|I_I\left(x_i\right)-I_J\left(x_i\right)\right|\right\}\\+\tan\left\{\frac{\pi}{6}\left|F_I\left(x_i\right)-F_J\left(x_i\right)\right|\right\}\end{bmatrix} = 0, \forall x_i \in X$$

$$\Leftrightarrow \begin{bmatrix} \tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\} \\ +\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\} \\ +\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\} \end{bmatrix} = 0\,,\forall x_i \in X$$

$$\Leftrightarrow \tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}=0,$$
$$\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}=0,\,,\forall x_i \in X$$
$$\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}=0$$

$$\Leftrightarrow \left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}=0,$$
$$\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}=0, \quad , \forall x_i \in X$$
$$\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}=0$$

$$\Leftrightarrow \left|T_I(x_i)-T_J(x_i)\right|=0,\left|I_I(x_i)-I_J(x_i)\right|=0,\left|F_I(x_i)-F_J(x_i)\right|=0\,,\forall x_i \in A$$

$$\Leftrightarrow T_I(x_i)=T_J(x_i), I_I(x_i)=I_J(x_i), F_I(x_i)=F_J(x_i), \forall x_i \in A$$

$$\Leftrightarrow A=B.$$

Part (iii):

$$d_{TMNS}(I,J)=\frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\begin{bmatrix} \tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\} \\ +\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\} \\ +\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\} \end{bmatrix}$$

$$=\frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\tan\left\{\frac{\pi}{6}\left|T_J(x_i)-T_I(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_J(x_i)-I_I(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_J(x_i)-F_I(x_i)\right|\right\}$$

$$d_{TMNS}(J,I).$$

Part (iv):

If $I \subseteq J \subseteq K$ then $T_I(x_i) \le T_J(x_i) \le T_K(x_i)$, $I_I(x_i) \le I_J(x_i) \le I_K(x_i)$ and $F_I(x_i) \ge F_J(x_i) \ge F_K(x_i)$, $\forall x_i \in X$. This implies to the following inequalities

$$\left|T_I(x_i)-T_K(x_i)\right| \ge \left|T_I(x_i)-T_J(x_i)\right|, \ \left|T_I(x_i)-T_K(x_i)\right| \ge \left|T_J(x_i)-T_K(x_i)\right|$$

$$\left|I_I(x_i)-I_K(x_i)\right| \ge \left|I_I(x_i)-I_J(x_i)\right|, \ \left|I_I(x_i)-I_K(x_i)\right| \ge \left|I_J(x_i)-I_K(x_i)\right|$$

and $\left|F_I(x_i)-F_K(x_i)\right| \ge \left|F_I(x_i)-F_K(x_i)\right|$, $\left|F_I(x_i)-F_K(x_i)\right| \ge \left|F_J(x_i)-F_K(x_i)\right|$. From these inequalities we have

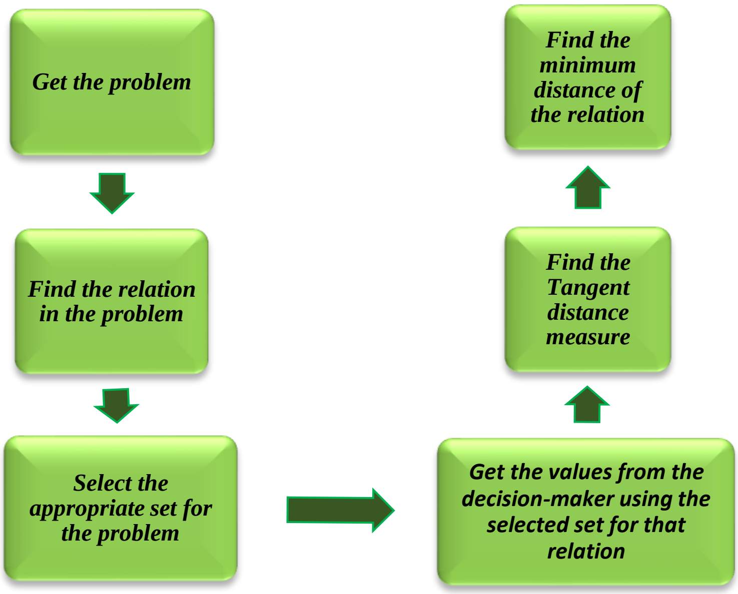
$$\tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_K(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_K(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_K(x_i)\right|\right\}$$
$$\ge \tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}$$

$$\frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_K(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_K(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_K(x_i)\right|\right\}$$
$$\ge \frac{2}{\sqrt{3}n}\sum_{i=1}^{n}\tan\left\{\frac{\pi}{6}\left|T_I(x_i)-T_J(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|I_I(x_i)-I_J(x_i)\right|\right\}+\tan\left\{\frac{\pi}{6}\left|F_I(x_i)-F_J(x_i)\right|\right\}$$

Hence $d_{TMNS}(I,K) \ge d_{TMNS}(I,J)$ is proven. Likewise $d_{TMNS}(I,K) \ge d_{TMNS}(J,K)$ can prove, and hence the theorem.

4. A Comparative Analysis of Distance Measures

In this section, we first construct Distance Measures using Fermatean Neutrosophic sets and Neutrosophic sets. Further, we have given an illustrative example for defined Distance Measures. So, we consider an example of disease, and their associated symptoms and treatment. We examine six different Distance Measures in this context, which include Fermatean Neutrosophic Set and Neutrosophic sets: Euclidean Distance, Normalised Euclidean Distance, Hamming Distance, Normalised Hamming Distance, SMSVNDM Distance, and TMFNDM Distance. We aim to find the minimum distance to ensure that patients receive the right treatment. The flowchart for the Distance Measures is presented below:



Step 1: Problem:

Let $S=\{S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_8,S_9,S_{10}\}$ be the set of patients, $C=\{C_1,C_2,C_3,C_4,C_5\}$ be the set of Treatment Strategies and $R=\{R_1,R_2,R_3,R_4,R_5,R_6\}$ be the set of Symptoms.

Table 4.1. a. shows the information about Treatment Strategies and the Symptoms of patient using Fermatean Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
C_1	(0.4,0.8,0.7)	(0.5,0.5,0.9)	(0.3,0.7,0.4)	(0.65,0.65,0.7)	(0.35,0.95,0.9)	(0.7,0.7,0.65)
C_2	(0.3 0.8 0.9)	(0.55,0.55,0.95)	(0.7,0.8,0.3)	(0.6,0.75,0.75)	(0.9,0.8,0.45)	(0.65,0.65,0.7)
C_3	(0.4,0.7,0.7)	(0.4,0.8,0.7)	(0.9,0.7,0.3)	(0.55,0.8,0.8)	(0.85,0.75,0.5)	(0.6,0.75,0.75)
C_4	(0.4,0.6,0.9)	(0.3,0.8,0.9)	(0.7,0.9,0.2)	(0.5,0.85,0.85)	(0.8,0.7,0.55)	(0.55,0.8,0.8)
C_5	(0.3,0.7,0.4)	(0.4,0.7,0.7)	(0.5,0.5,0.9)	(0.45,0.9,0.9)	(0.75,0.75,0.6)	(0.5,0.85,0.85)

Table 4.1. b. shows the information about Treatment Strategies and the Symptoms of patient using Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
C_1	(0.4,0.8,0.6)	(0.5,0.5,0.5)	(0.3,0.7,0.7)	(0.65,0.65,0.35)	(0.35,0.95,0.65)	(0.7,0.7,0.3)
C_2	(0.3,0.8,0.7)	(0.55,0.55,0.45)	(0.7,0.8,0.3)	(0.6,0.75,0.4)	(0.9,0.8,0.1)	(0.65,0.65,0.35)
C_3	(0.4,0.7,0.6)	(0.4,0.8,0.6)	(0.9,0.7,0.1)	(0.55,0.8,0.45)	(0.85,0.75,0.15)	(0.6,0.75,0.4)
C_4	(0.4,0.6,0.6)	(0.3,0.8,0.7)	(0.7,0.9,0.3)	(0.5,0.85,0.5)	(0.8,0.7,0.2)	(0.55,0.8,0.45)
C_5	(0.3,0.7,0.7)	(0.4,0.7,0.6)	(0.5,0.5,0.5)	(0.45,0.9,0.55)	(0.75,0.75,0.25)	(0.5,0.85,0.5)

Table 4.2.a. Shows the information about the patient's Symptoms using Fermatean Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
S_1	(0.8,0.7,0.55)	(0.7,0.9,0.65)	(0.6,0.75,0.75)	(0.7,0.8,0.6)	(0.95,0.85,0.4)	(0.4,0.95,0.95)
S_2	(0.75,0.75,0.6)	(0.65,0.9,0.7)	(0.55,0.8,0.8)	(0.85,0.9,0.7)	(0.9,0.8,0.45)	(0.35,0.95,0.9)
S_3	(0.7,0.7,0.65)	(0.6,0.9,0.75)	(0.5,0.85,0.85)	(0.9,0.9,0.3)	(0.85,0.75,0.5)	(0.9,0.8,0.45)
S_4	(0.65,0.65,0.7)	(0.55,0.8,0.8)	(0.45,0.9,0.9)	(0.95,0.95,0.4)	(0.8,0.7,0.55)	(0.85,0.75,0.5)
S_5	(0.6,0.75,0.75)	(0.5,0.85,0.85)	(0.4,0.95,0.95)	(0.9,0.9,0.45)	(0.75,0.65,0.6)	(0.8,0.7,0.55)
S_6	(0.55,0.8,0.8)	(0.45,0.9,0.9)	(0.95,0.85,0.4)	(0.85,0.85,0.5)	(0.7,0.9,0.65)	(0.75,0.75,0.6)
S_7	(0.5,0.85,0.85)	(0.4,0.95,0.95)	(0.9,0.8,0.45)	(0.8,0.8,0.55)	(0.65,0.9,0.7)	(0.7,0.7,0.65)
S_8	(0.45,0.9,0.9)	(0.35,0.95,0.9)	(0.85,0.75,0.5)	(0.75,0.75,0.6)	(0.6,0.9,0.75)	(0.65,0.65,0.7)
S_9	(0.4,0.95,0.95)	(0.9,0.8,0.45)	(0.8,0.7,0.55)	(0.7,0.7,0.65)	(0.55,0.8,0.8)	(0.6,0.75,0.75)
S_{10}	(0.95,0.85,0.4)	(0.85,0.75,0.5)	(0.75,0.65,0.6)	(0.65,0.65,0.7)	(0.5,0.85,0.85)	(0.55,0.8,0.8)

Table 4.2.b. Shows the information about the patient's Symptoms using Neutrosophic Set

	R_1	R_2	R_3	R_4	R_5	R_6
S_1	(0.8,0.7,0.2)	(0.7,0.9,0.3)	(0.6,0.75,0.4)	(0.7,0.8,0.3)	(0.95,0.85,0.05)	(0.4,0.95,0.6)
S_2	(0.75,0.75,0.25)	(0.65,0.9,0.35)	(0.55,0.8,0.45)	(0.85,0.9,0.15)	(0.9,0.8,0.1)	(0.35,0.95,0.65)
S_3	(0.7,0.7,0.3)	(0.6,0.9,0.4)	(0.5,0.85,0.5)	(0.9,0.9,0.2)	(0.85,0.75,0.15)	(0.9,0.8,0.1)
S_4	(0.65,0.65,0.35)	(0.55,0.8,0.45)	(0.45,0.9,0.55)	(0.95,0.95,0.05)	(0.8,0.7,0.2)	(0.85,0.75,0.15)
S_5	(0.6,0.75,0.4)	(0.5,0.85,0.5)	(0.4,0.95,0.6)	(0.9,0.9,0.1)	(0.75,0.65,0.25)	(0.8,0.7,0.2)
S_6	(0.55,0.8,0.45)	(0.45,0.9,0.55)	(0.95,0.85,0.05)	(0.85,0.85,0.15)	(0.7,0.9,0.3)	(0.75,0.75,0.25)
S_7	(0.5,0.85,0.5)	(0.4,0.95,0.6)	(0.9,0.8,0.1)	(0.8,0.8,0.2)	(0.65,0.9,0.35)	(0.7,0.7,0.3)
S_8	(0.45,0.9,0.55)	(0.35,0.95,0.65)	(0.85,0.75,0.15)	(0.75,0.75,0.25)	(0.6,0.9,0.4)	(0.65,0.65,0.35)
S_9	(0.4,0.95,0.6)	(0.9.0.8,0.1)	(0.8,0.7,0.2)	(0.7,0.7,0.3)	(0.55,0.8,0.45)	(0.6,0.75,0.4)
S_{10}	(0.95,0.85,0.05)	(0.85,0.75,0.15)	(0.75,0.65,0.25)	(0.65,0.65,0.35)	(0.5,0.85,0.5)	(0.55,0.8,0.45)

Step 2: Distance measure:

The ED measure $d_E(I,J)$ determines the shortest distance between each patient's symptoms (Tables 4.2.a and 4.2.b) and treatment strategies (Tables 4.1.a and 4.1.b), with the results presented in Tables 4.3.a and 4.3.b.Here, 'a' represents Fermatean Neutrosophic Sets and 'b' represents Neutrosophic Sets throughout the paper.

Table 4.3.a. $d_E(I,J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.747774	.543906	.513971	1.814754	.510718
S_2	.736546	.61101	.53929	.6	.500833

S_3	.715309	.653835	.65192	.720532	.691616
S_4	.654472	.607591	.625833	.665207	.62849
S_5	.578072	.597216	.627827	.645497	.606218
S_6	.581664	.423281	.376386	.446281	.677003
S_7	.53929	.414327	.375278	.426224	.643558
S_8	.5058	.422295	.403113	.432049	.623832
S_9	.560506	.527573	.499166	.606905	.639661
S_{10}	.604152	.707696	.6	.728583	.619139

Table 4.3.b. $d_E\left(I,J\right)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.764308	.558271	.543906	1.624038	.593717
S_2	.744983	.580948	.57735	.564948	.602771
S_3	.685565	.588076	.635085	.63705	.70946
S_4	.608961	.538516	.603462	.570088	.640963
S_5	.498331	.51559	.587367	.526783	.595819
S_6	.695821	.465475	.361709	.467262	.630476
S_7	.63901	.44441	.32914	.423281	.570088
S_8	.587367	.438748	.32914	.403113	.521217
S_9	.595819	.469929	.522813	.618466	.602771

S_{10}	.691014	.701189	.680074	.737111	.754431

Tables 4.4.a and 4.4.b present the shortest distances between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b) calculated using the **N-ED Measure** $d_{n-E}\left(I,J\right)$.

Table 4.4.a. $d_{n-E}\left(I,J\right)$

	C_1	C_2	C_3	C_4	C_5
S_1	.431728	.314024	.296742	1.047749	.294863
S_2	.425245	.352767	.311359	.34641	.289156
S_3	.412984	.377492	.376386	.415999	.399305
S_4	.377859	.350793	.361325	.384057	.362859
S_5	.33375	.344803	.362476	.372678	.35
S_6	.335824	.244381	.217307	.25766	.390868
S_7	.311359	.239212	.216667	.24608	.371558
S_8	.292024	.243812	.232737	.249444	.36017
S_9	.323608	.304594	.288194	.350397	.369309
S_{10}	.348807	.408588	.34641	.420648	.35746

Table 4.4.b. $d_{n-E}\left(I,J\right)$

	C_1	C_2	C_3	C_4	C_5
S_1	.441273	.322318	.314024	.937639	.342783
S_2	.430116	.33541	.333333	.326173	.34801

S_3	.395811	.339526	.366667	.367801	.409607
S_4	.351584	.310913	.348409	.32914	.37006
S_5	.287711	.297676	.339116	.304138	.343996
S_6	.401732	.268742	.208833	.269774	.364005
S_7	.368932	.25658	.190029	.244381	.32914
S_8	.339116	.253311	.190029	.232737	.300925
S_9	.343996	.271314	.301846	.357071	.34801
S_{10}	.398957	.404832	.392641	.425572	.435571

HD Measure $d_H(I,J)$ is applied in Tables 4.5.a and 4.5.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.5.a. $d_H(I,J)$

	C_1	C_2	C_3	C_4	C_5
S_1	1.55	1.2	1.016667	4.333333	1.05
S_2	1.55	1.166667	1.05	4.466667	0.983333
S_3	1.533333	1.316667	1.166667	1.35	1.233333
S_4	1.366667	1.25	1.133333	1.15	1.166667
S_5	1.216667	1.2	1.183333	1.166667	1.15
S_6	1	0.883333	0.8	0.916667	1.4
S_7	0.916667	0.8	0.783333	0.933333	1.383333

S_8	0.916667	0.766667	0.85	0.933333	1.383333
S_9	1.05	1	0.883333	1.133333	1.35
S_{10}	1.133333	1.35	1.166667	1.416667	1.2

Table 4.5.b $d_H(I,J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	1.583333	1.066667	1.116667	3.65	1.216667
S_2	1.633333	1.116667	1.233333	3.7	1.2
S_3	1.5	1.183333	1.233333	1.25	1.3
S_4	1.316667	1.133333	1.183333	1.066667	1.216667
S_5	1.1	1.116667	1.166667	1.083333	1.133333
S_6	1.25	1	0.75	1	1.283333
S_7	1.1	0.916667	0.633333	0.95	1.2
S_8	1.033333	0.85	0.666667	0.883333	1.166667
S_9	1.066667	0.916667	0.833333	1.083333	1.266667
S_{10}	1.283333	1.266667	1.283333	1.366667	1.483333

Tables 4.6.a and 4.6.b use the **N-HD Measure** $d_{n-H}(I,J)$ to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.6.a $d_{n-H}(I,J)$					
	C_1	C_2	C_3	C_4	C_5

S_1	.516667	.4	.338889	1.444444	.35
S_2	.516667	.388889	.35	1.488889	.327778
S_3	.511111	.438889	.388889	.45	.411111
S_4	.455556	.416667	.377778	.383333	.388889
S_5	.405556	.4	.394444	.388889	.383333
S_6	.333333	.294444	.266667	.305556	.466667
S_7	.305556	.266667	.261111	.311111	.461111
S_8	.305556	.255556	.283333	.311111	.461111
S_9	.35	.333333	.294444	.377778	.45
S_{10}	.377778	.45	.388889	.472222	.4

Table 4.6.b $d_{n-H}(I,J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.527778	.355556	.372222	1.216667	.405556
S_2	.544444	.372222	.411111	1.233333	.4
S_3	.5	.394444	.411111	.416667	.433333
S_4	.438889	.377778	.394444	.355556	.405556
S_5	.366667	.372222	.388889	.361111	.377778
S_6	.416667	.333333	.25	.333333	.427778
S_7	.366667	.305556	.211111	.316667	.4

S_8	.344444	.283333	.222222	.294444	.388889
S_9	.355556	.305556	.277778	.361111	.422222
S_{10}	.427778	.422222	.427778	.455556	.494444

SMSVNDM $d_{SMSVN}(I,J)$ is applied in Tables 4.7.a and 4.7.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.7.a $d_{SMSVN}(I,J)$

	C_1	C_2	C_3	C_4	C_5
S_1	.464307	.379313	.343275	.874075	.355207
S_2	.467474	.367576	.344268	.885766	.337049
S_3	.464007	.411326	.355037	.415623	.385918
S_4	.421581	.397855	.348495	.356958	.369734
S_5	.379576	.385099	.364244	.367209	.368729
S_6	.330478	.309958	.284596	.317735	.426397
S_7	.304578	.281881	.279889	.323697	.429843
S_8	.310954	.262432	.297704	.322444	.435095
S_9	.3364	.324272	.29669	.364477	.423775
S_{10}	.353776	.410054	.371496	.4165	.386503

Table 4.7.b $d_{SMSVN}(I,J)$

	C_1	C_2	C_3	C_4	C_5

S_1	.469595	.337518	.367908	.803274	.390976
S_2	.490022	.350467	.394929	.808717	.386716
S_3	.458565	.376773	.380913	.393788	.402805
S_4	.411555	.367563	.370281	.342259	.383257
S_5	.354198	.370483	.366593	.356914	.363875
S_6	.378302	.342953	.267461	.342778	.396847
S_7	.33707	.314587	.228695	.329424	.381951
S_8	.327754	.284782	.239657	.308627	.381117
S_9	.330455	.305328	.271538	.345515	.400498
S_{10}	.383377	.382179	.388731	.396423	.437752

The **TMFNDM** $d_{TMFN}(I,J)$ is used in Tables 4.8.a and 4.8.b to calculate the shortest distance between each patient (Tables 4.2.a and 4.2.b) and each treatment strategy (Tables 4.1.a and 4.1.b).

Table 4.8.a $d_{TMFN}(I,J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.95103	.73298	.61939	2.77556	.63946
S_2	.9496	.71299	.64042	2.87073	.59897
S_3	.93827	.80546	.71558	.8302	.75801
S_4	.83509	.76368	.69464	.70746	.71434
S_5	.74384	.73386	.72561	.71758	.70299

S_6	.6133	.53647	.48535	.55705	.8562
S_7	.56139	.48621	.47516	.56657	.84436
S_8	.56002	.46632	.51589	.56665	.84355
S_9	.64143	.61013	.53876	.6938	.82489
S_{10}	.69333	.82934	.71326	.86955	.7348

Table 4.8.b $d_{TMFN}(I,J)$					
	C_1	C_2	C_3	C_4	C_5
S_1	.48593	.32588	.34009	1.16465	.37139
S_2	.49996	.34084	.37563	1.18268	.36643
S_3	.4583	.36104	.37696	.38222	.3992
S_4	.40142	.34494	.36139	.32555	.37245
S_5	.33515	.33955	.35635	.32983	.3463
S_6	.3849	.30378	.2275	.30384	.39198
S_7	.33817	.27852	.19204	.28827	.36557
S_8	.3168	.25841	.20213	.26798	.35468
S_9	.3265	.27896	.25502	.33261	.38657
S_{10}	.39393	.39009	.3938	.42044	.45638

Comparison:

Six different distance measures have been employed to calculate the distance between each patient and each treatment strategy. The comparison of these distance measures is shown in the following table (Table 4.9.a and Table 4.9.b). In this table, the minimum distance for each measure is highlighted with a yellow shade, while the Tangent Metric Fermatean Neutrosophic Distance Measure is highlighted with a red shade.

Table 4.9.a Comparison of distance measures

	C_1	C_2	C_3	C_4	C_5
S_1	0.747774476	0.543906	0.513971	1.814754	0.510718
	0.294863434	0.314024	0.296742	1.047749	0.294863
	1.016666667	1.2	1.016667	4.333333	1.05
	0.338888889	0.4	0.338889	1.444444	0.35
	0.343275464	0.379313	0.343275	0.874075	0.355207
	0.95103165	0.732979	0.619392	2.775565	0.639462
S_2	0.736545993	0.61101	0.53929	0.6	0.500833
	0.28915586	0.352767	0.311359	0.34641	0.289156
	1.166666667	1.166667	1.05	4.466667	0.983333
	0.516666667	0.388889	0.35	1.488889	0.327778
	0.467474217	0.367576	0.344268	0.885766	0.337049
	0.94959805	0.712986	0.640418	2.870727	0.598968
S_3	0.715308791	0.653835	0.65192	0.720532	0.691616
	0.412983723	0.377492	0.376386	0.415999	0.399305
	1.533333333	1.316667	1.166667	1.35	1.233333
	0.511111111	0.438889	0.388889	0.45	0.411111
	0.4640068	0.411326	0.355037	0.415623	0.385918
	0.938269576	0.805462	0.71558	0.830203	0.758013
S_4	0.654471797	0.607591	0.625833	0.665207	0.62849
	0.377859468	0.350793	0.361325	0.384057	0.362859
	1.366666667	1.25	1.133333	1.15	1.166667
	0.455555556	0.416667	0.377778	0.383333	0.388889
	0.421580569	0.397855	0.348495	0.356958	0.369734
	0.835094884	0.763681	0.69464	0.707462	0.714335
S_5	0.578071507	0.597216	0.627827	0.645497	0.606218
	0.33374974	0.344803	0.362476	0.372678	0.35
	1.216666667	1.2	1.183333	1.166667	1.15
	0.405555556	0.4	0.394444	0.388889	0.383333
	0.379576186	0.385099	0.364244	0.367209	0.368729

	0.743837083	0.733858	0.725612	0.717584	0.702993
S_6	0.581664279	0.423281	0.376386	0.446281	0.677003
	0.335824028	0.244381	0.217307	0.25766	0.390868
	1	0.883333	0.8	0.916667	1.4
	0.333333333	0.294444	0.266667	0.305556	0.466667
	0.330477572	0.309958	0.284596	0.317735	0.426397
	0.613302537	0.536471	0.485349	0.55705	0.856196
S_7	0.539289656	0.414327	0.375278	0.426224	0.643558
	0.311359028	0.239212	0.216667	0.24608	0.371558
	0.916666667	0.8	0.783333	0.933333	1.383333
	0.305555556	0.266667	0.261111	0.311111	0.461111
	0.304578182	0.281881	0.279889	0.323697	0.429843
	0.561392765	0.486208	0.475156	0.566565	0.844364
S_8	0.505799697	0.422295	0.403113	0.432049	0.623832
	0.292023591	0.243812	0.232737	0.249444	0.36017
	0.916666667	0.766667	0.85	0.933333	1.383333
	0.305555556	0.255556	0.283333	0.311111	0.461111
	0.310954344	0.262432	0.297704	0.322444	0.435095
	0.560016442	0.466317	0.515891	0.566649	0.843551
S_9	0.560505724	0.527573	0.499166	0.606905	0.639661
	0.323608131	0.304594	0.288194	0.350397	0.369309
	1.05	1	0.883333	1.133333	1.35
	0.35	0.333333	0.294444	0.377778	0.45
	0.336400426	0.324272	0.29669	0.364477	0.423775
	0.64143395	0.61013	0.538757	0.693804	0.824894
S_{10}	0.604152299	0.707696	0.6	0.728583	0.619139
	0.348807492	0.408588	0.34641	0.420648	0.35746
	1.133333333	1.35	1.166667	1.416667	1.2
	0.377777778	0.45	0.388889	0.472222	0.4
	0.353775568	0.410054	0.371496	0.4165	0.386503
	0.693329647	0.82934	0.713259	0.869545	0.734797

Table 4.9.b Comparison of distance measures

	C_1	C_2	C_3	C_4	C_5

	0.764307966	0.558271	0.543906	1.624038	0.593717
	0.441273	0.322318	0.314024	0.937639	0.342783
	1.583333	1.066667	1.116667	3.65	1.216667
	0.527778	0.355556	0.372222	1.216667	0.405556
S_1	0.469595	0.337518	0.367908	0.803274	0.390976
	0.48593	0.32588	0.34009	1.16465	0.37139
	0.744983	0.580948	0.57735	0.564948	0.602771
	0.430116	0.33541	0.333333	0.326173	0.34801
	1.633333	1.116667	1.233333	3.7	1.2
	0.544444	0.372222	0.411111	1.233333	0.4
	0.490022	0.350467	0.394929	0.808717	0.386716
S_2	0.49996	0.34084	0.37563	1.18268	0.36643
	0.685565	0.588076	0.635085	0.63705	0.70946
	0.395811	0.339526	0.366667	0.367801	0.409607
	1.5	1.183333	1.233333	1.25	1.3
S_3	0.5	0.394444	0.411111	0.416667	0.433333
	0.458565	0.376773	0.380913	0.393788	0.402805
	0.4583	0.36104	0.37696	0.38222	0.3992
	0.608961	0.538516	0.603462	0.570088	0.640963
	0.351584	0.310913	0.348409	0.32914	0.37006
	1.316667	1.133333	1.183333	1.066667	1.216667
S_4	0.438889	0.377778	0.394444	0.355556	0.405556
	0.411555	0.367563	0.370281	0.342259	0.383257
	0.40142	0.34494	0.36139	0.32555	0.37245
	0.498331	0.51559	0.587367	0.526783	0.595819
	0.287711	0.297676	0.339116	0.304138	0.343996
	1.1	1.116667	1.166667	1.083333	1.133333
S_5	0.366667	0.372222	0.388889	0.361111	0.377778
	0.354198	0.370483	0.366593	0.356914	0.363875
	0.33515	0.33955	0.35635	0.32983	0.3463
	0.695821	0.465475	0.361709	0.467262	0.630476
	0.401732	0.268742	0.208833	0.269774	0.364005
S_6	1.25	1	0.75	1	1.283333
	0.416667	0.333333	0.25	0.333333	0.427778
	0.378302	0.342953	0.267461	0.342778	0.396847

	0.3849	0.30378	0.2275	0.30384	0.39198
S_7	0.63901	0.44441	0.32914	0.423281	0.570088
	0.368932	0.25658	0.190029	0.244381	0.32914
	1.1	0.916667	0.633333	0.95	1.2
	0.366667	0.305556	0.211111	0.316667	0.4
	0.33707	0.314587	0.228695	0.329424	0.381951
	0.33817	0.27852	0.19204	0.28827	0.36557
S_8	0.587367	0.438748	0.32914	0.403113	0.521217
	0.339116	0.253311	0.190029	0.232737	0.300925
	1.033333	0.85	0.666667	0.883333	1.166667
	0.344444	0.283333	0.222222	0.294444	0.388889
	0.327754	0.284782	0.239657	0.308627	0.381117
	0.3168	0.25841	0.20213	0.26798	0.35468
S_9	0.595819	0.469929	0.522813	0.618466	0.602771
	0.343996	0.271314	0.301846	0.357071	0.34801
	1.066667	0.916667	0.833333	1.083333	1.266667
	0.355556	0.305556	0.277778	0.361111	0.422222
	0.330455	0.305328	0.271538	0.345515	0.400498
	0.3265	0.27896	0.25502	0.33261	0.38657
S_{10}	0.691014	0.701189	0.680074	0.737111	0.754431
	0.398957	0.404832	0.392641	0.425572	0.435571
	1.283333	1.266667	1.283333	1.366667	1.483333
	0.427778	0.422222	0.427778	0.455556	0.494444
	0.383377	0.382179	0.388731	0.396423	0.437752
	0.39393	0.39009	0.3938	0.42044	0.45638

Step 3: Comparative result

The consolidated results are presented in Table 4.9.a and Table 4.9.b, and further summarized in Table 4.10.a and Table 4.10.b. The shortest distance values for each patient and treatment strategy are shown in Table 4.10 below. Therefore, the $d_{TMFN}(I,J)$ offers the most consistent and straightforward solution for measuring distance using Fermatean neutrosophic set values and neutrosophic set values.

Table 4.10.a Comparison (Fermatean neutrosophic set)

	$d_E(I,J)$	$d_{n-E}(I,J)$	$d_H(I,J)$	$d_{n-H}(I,J)$	$d_{SMSVN}(I,J)$	$d_{TMFN}(I,J)$
S_1	C5	C5	C3	C1&C3	C1&C3	C3
S_2	C5	C1&C5	C5	C5	C5	C5
S_3	C3	C3	C3	C3	C3	C3
S_4	C2	C2	C3	C3	C3	C3
S_5	C1	C1	C5	C5	C3	C5
S_6	C3	C3	C3	C3	C3	C3
S_7	C3	C3	C3	C3	C3	C3
S_8	C3	C3	C2	C2	C2	C2
S_9	C3	C3	C3	C3	C3	C3
S_{10}	C3	C3	C5	C1	C1	C1

Table 4.10.b Comparison (Neutrosophic set)

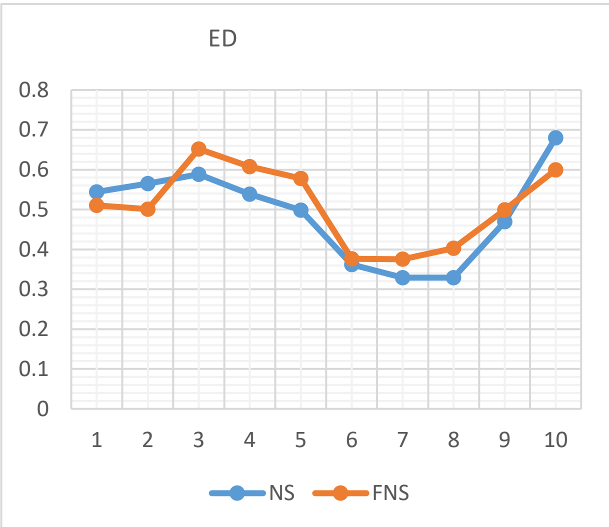
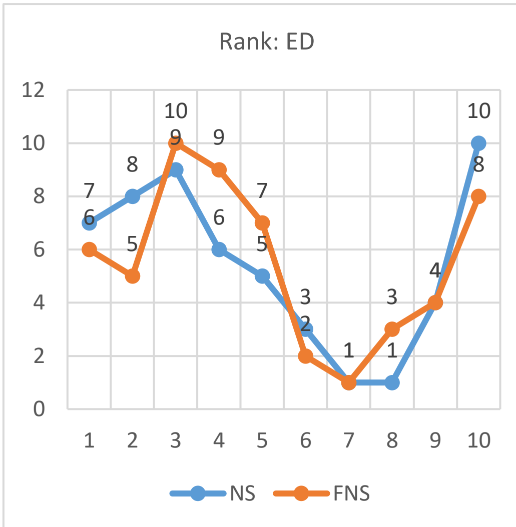
	$d_E(I,J)$	$d_{n-E}(I,J)$	$d_H(I,J)$	$d_{n-H}(I,J)$	$d_{SMSVN}(I,J)$	$d_{TMFN}(I,J)$
S_1	C3	C3	C2	C2	C2	C2
S_2	C4	C4	C2	C2	C2	C2
S_3	C2	C2	C2	C2	C2	C2
S_4	C2	C2	C4	C4	C4	C4
S_5	C2	C1	C4	C4	C1	C4
S_6	C3	C3	C3	C3	C3	C3
S_7	C3	C3	C3	C3	C3	C3

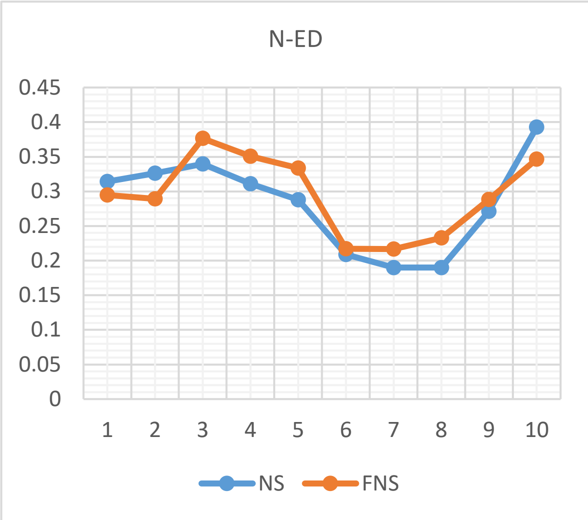
S_8	C3	C3	C3	C3	C3	C3
S_9	C2	C2	C3	C3	C3	C3
S_{10}	C3	C3	C2	C2	C2	C2

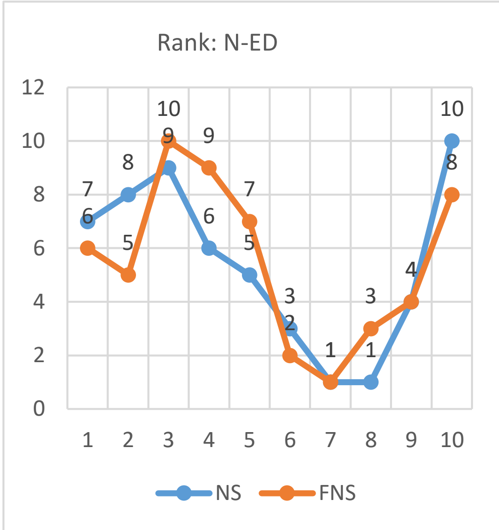
The above table shows that the most frequently occurring shortest distance is found in the TMFNDM. Hence TMFNDM generalized case of other six distance measure.

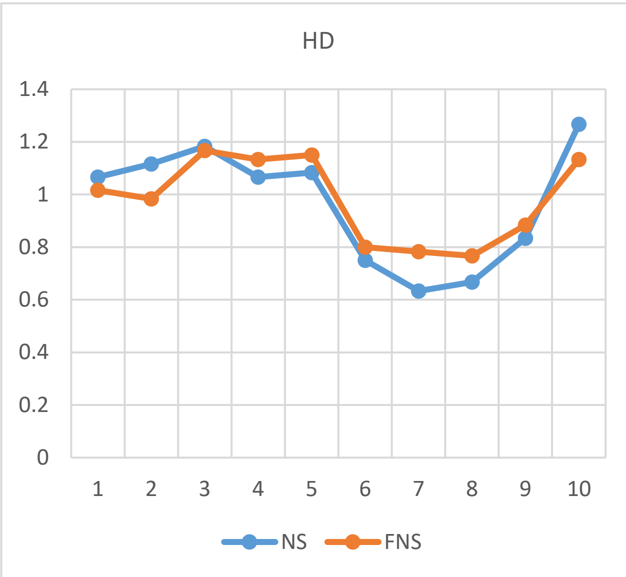
Step 4: Graphical result

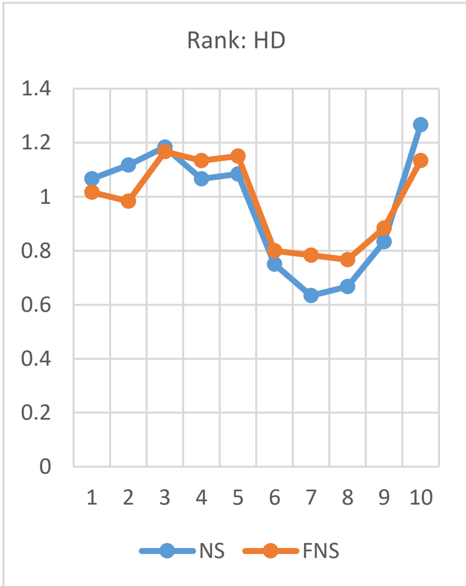
In this step, we compare the results using the FNS and NS values. Also, we present graphical representations of six distinct distance measurements comparing each patient to each Treatment Strategy given in the model example. Additionally, we provide graphs below illustrating rankings using FNS and NS. Top of Form

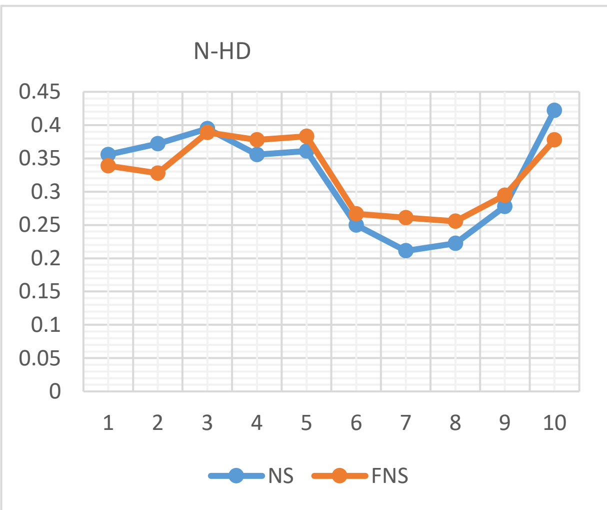
Distance Measure	**Rank of the Distance Measure**
	

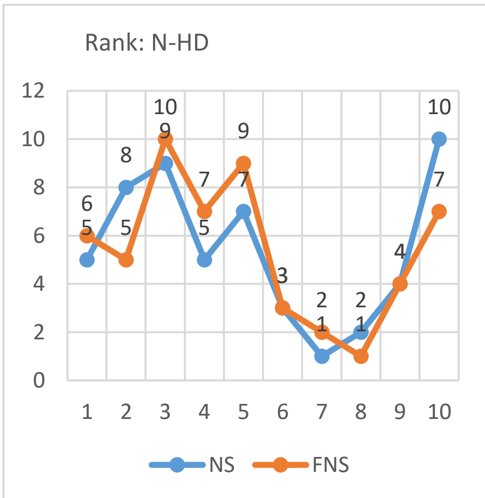

N-ED
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1 2 3 4 5 6 7 8 9 10
NS FNS

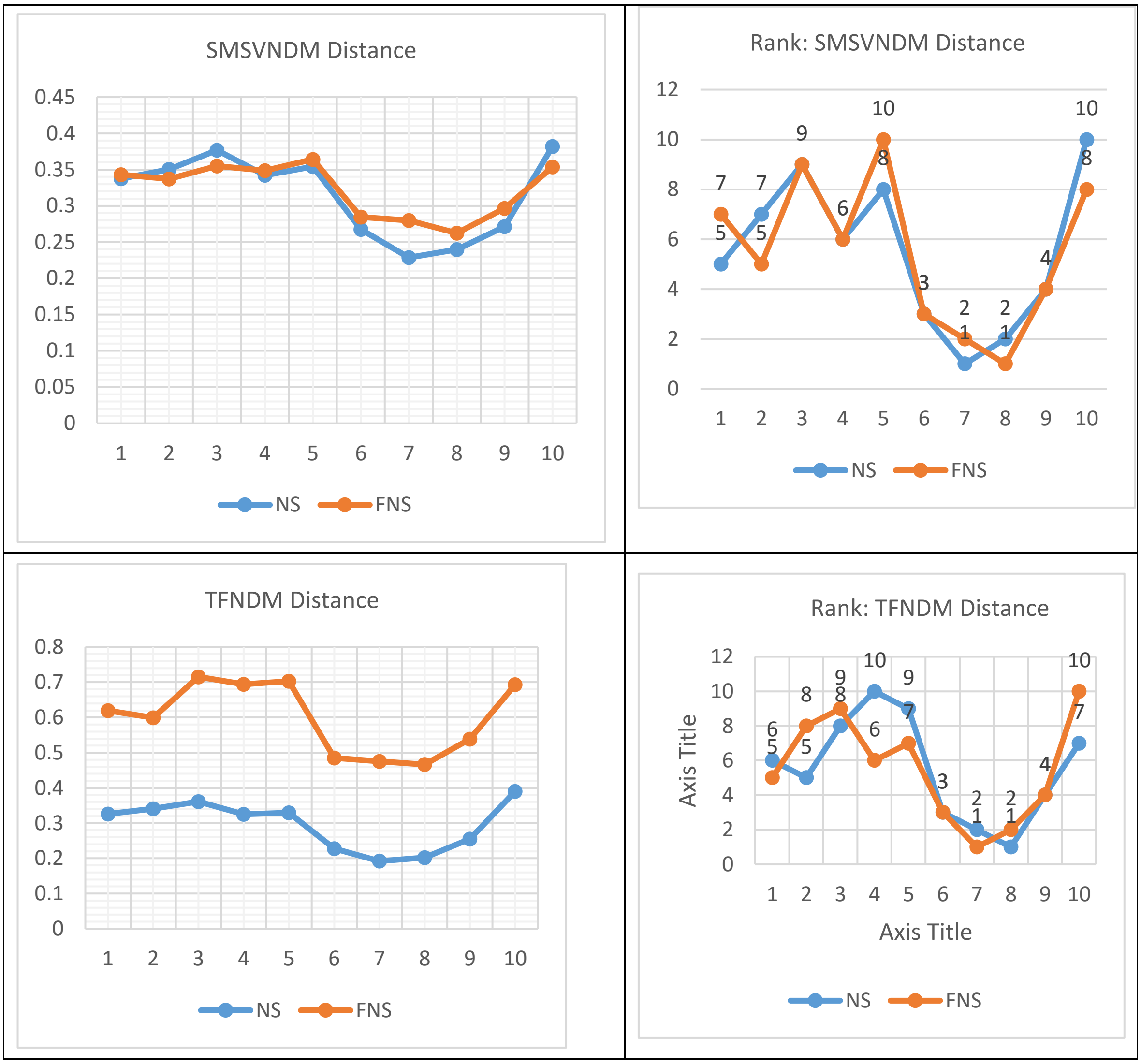

Rank: N-ED
12
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10
NS FNS


HD
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
NS FNS


Rank: HD
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
NS FNS


N-HD
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1 2 3 4 5 6 7 8 9 10
NS FNS


Rank: N-HD
12
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10
NS FNS



In the graphical representation, we observed a difference in ranking between the Neutrosophic and Fermatean Neutrosophic Sets. The Fermatean Neutrosophic Set permits higher membership degree values, resulting in comparatively higher false values, which influence the outcome.

Conclusion

This study presents a novel distance measure tailored for FNSs, with the TMFNDM serving as a generalized case. A comparative analysis showed that the graphical rankings differ between neutrosophic and Fermatean neutrosophic sets, primarily due to the latter's allowance for higher membership degree values. Fermatean Neutrosophic Set values lead to comparatively higher false values, significantly influencing the results. The findings emphasize that selecting an appropriate set is crucial for achieving optimal solutions in scenarios requiring high membership values. The Tangent Metric Fermatean Neutrosophic Distance Measure provides superior outcomes in such cases. Fields like medicine, research, and disaster management often involve high membership values, making this distance measure particularly relevant and effective.

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