SuperHyperSoft Framework for College English Blended Teaching Quality Evaluation in the New Era: Addressing Uncertainty and Complexity

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Abstract
A Single-Valued Neutrosophic Set (SVNS) is a powerful tool for representing uncertainty, ambiguity, and incomplete or inconsistent information in real-world scenarios. This approach is particularly effective for handling uncertain measurements and data. Building on the concept of fuzzy set entropy, the SVN-entropy method has been developed to support multi-criteria decision-making (MCDM) processes. In this study, SVN-entropy is combined with the SuperHyperSoft (HSS) framework—an extension of HyperSoft sets—to evaluate different criteria and sub-criteria with varying values. The proposed method uses entropy to calculate criteria weights, which are then applied to assess the quality of College English Blended Teaching in the modern era. The study identifies eight main criteria, each with associated sub-criteria, to provide a comprehensive evaluation framework. This approach ensures a robust and precise assessment of blended teaching quality, leveraging advanced mathematical tools to handle complex and uncertain data effectively.

Keywords: SuperHyperSoft (SHS) Framework; College English Blended Teaching Quality; Uncertainty; MCDM Approach.

1. Introduction and Literature Review

Blended learning, which combines traditional classroom instruction with online activities, has become an integral part of modern education systems due to the rapid advancements in information and communication technology (ICT). This approach combines the benefits of face-to-face teaching with the flexibility of digital platforms, creating an environment conducive to active, independent, and collaborative learning [1,2]. Over the last two decades, blended learning has been widely adopted, particularly in higher education, as institutions aim to meet the needs of diverse learners and evolve pedagogical demands [3,4] In China, the 2020 Guidelines on College English Teaching emphasized the role of blended learning in improving undergraduate education. These guidelines advocate integrating digital tools, virtual simulations, and online learning platforms into traditional teaching to foster interactive and personalized learning environments [5,6]. The COVID-19 pandemic further accelerated the adoption of blended learning globally, highlighting its potential to ensure educational continuity during disruptions [7,8]. Despite its advantages, blended learning implementation faces challenges in quality assurance and evaluation, which necessitates innovative frameworks to assess its effectiveness comprehensively [9,10]

Evaluating the quality of blended teaching presents multidimensional challenges. Key factors, such as teaching effectiveness, student engagement, resource availability, and technological integration, must be measured in cohesive and adaptive frameworks. However, traditional methods often fail to capture the interdependencies and complexities of these factors, leading to incomplete evaluations[11,12]. Additionally, blended learning environments inherently involve subjective data, such as teacher performance and student satisfaction, which vary across contexts and evaluators [13,14]

Another significant challenge lies in adapting evaluation frameworks to accommodate emerging technologies and evolving educational needs. Rapid advancements in digital tools demand flexible approaches capable of addressing ambiguity, integrating diverse data sources, and managing complex interrelationships [15-17].

Researchers have explored advanced mathematical and computational frameworks to address these challenges. Neutrosophy, introduced by Smarandache, provides a robust theoretical foundation for handling ambiguity and indeterminacy in data. Neutrosophic sets, which generalize classical and fuzzy sets, incorporate three parameters—truth-membership, indeterminacy-membership, and falsity-membership—to represent uncertainty comprehensively [18-20]. Single-Valued Neutrosophic Sets (SVNS), a practical extension of neutrosophic sets, have been widely applied in fields such as risk analysis, healthcare, and education. These sets enable nuanced representation and analysis of uncertain information, making them particularly effective in evaluating complex systems [21,22]

The SuperHyperSoft (SHS) framework builds on the principles of HyperSoft sets and represents a significant advancement in multi-criteria decision-making (MCDM). By integrating SVNS with entropy-based weighting methods, the SHS framework provides a comprehensive solution for evaluating blended teaching environments. It models interrelationships among criteria and sub-criteria, effectively addressing the inherent complexity and uncertainty in blended learning evaluation [23-25].

Entropy, a mathematical measure of uncertainty, is utilized in the SHS framework to calculate objective weights for criteria. This minimizes subjective biases and ensures balanced evaluations. Its scalability and adaptability make it a valuable tool for institutions seeking to implement robust quality assurance mechanisms in education [26-28].

Conventional MCDM methods, such as the Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), have been widely applied in educational evaluations. These methods, while effective in certain contexts, rely heavily on subjective inputs and lack the ability to handle uncertainty comprehensively [29-31]. Fuzzy logic has been instrumental in addressing ambiguity through techniques like Fuzzy Comprehensive Evaluation (FCE). This approach allows for partial truth values and is often used to assess qualitative factors, such as student satisfaction. However, its reliance on predefined membership functions limits its flexibility in dynamic environments [32-34].

Entropy-based approaches offer an objective mechanism for determining criteria weights and, when integrated with neutrosophic sets, provide robust frameworks for managing uncertainty and complexity in blended teaching evaluations [35,36].

2. Selection of Benchmark: College English Blended Teaching Quality Evaluation

Evaluating the quality of College English blended teaching in the modern era represents MCDM problem, requiring the careful assessment of diverse factors that influence teaching outcomes. This study identifies and categorizes a comprehensive set of eight key criteria, each accompanied by relevant sub-criteria, as detailed in Table 1. These criteria are instrumental in determining the strengths and weaknesses of various teaching approaches by assigning objective weights to each criterion.

The criteria include aspects such as teaching effectiveness, student engagement, quality of digital resources, and teacher competence in digital teaching, among others. These components aim to capture both the highest and lowest-performing attributes in the evaluation process. The weight assigned to each criterion ensures an unbiased and data-driven assessment, allowing educators and institutions to identify critical areas for improvement and prioritize resource allocation effectively.

By systematically incorporating these criteria into the evaluation process, the study establishes a robust benchmark for assessing blended teaching quality in College English courses. This approach not only highlights areas of excellence but also pinpoints specific challenges, offering actionable insights for enhancing teaching methodologies in the new era of education.

Table 1. Key Criteria and Associated Sub-Criteria

Criteria	Values
Teaching Effectiveness	High, Moderate, Low
Student Engagement	Highly Engaged, Partially Engaged, Not Engaged
Quality of Digital Resources	Excellent, Adequate, Poor
Flexibility of Teaching Methods	Very Flexible, Moderately Flexible, Inflexible
Assessment Design and Feedback	Comprehensive, Sufficient, Inadequate
Technical Support and Infrastructure	Excellent, Satisfactory, Unsatisfactory
Teacher Competence in Digital Teaching	Expert, Competent, Beginner
Student Satisfaction	Very Satisfied, Neutral, Dissatisfied

3. Decision-Making Methodology Using the Entropy Weights Method

The entropy weights method is a widely recognized mathematical approach used to determine the relative importance of criteria in a decision-making process. In this study, it plays a central role in evaluating the College English Blended Teaching Quality by objectively assigning weights to each criterion. This method ensures that the variability and significance of the data for each criterion is properly accounted for. The entropy method operates within the framework of SVNSs and uses a structured approach to calculate the weights.

3.1 Steps in the Entropy Weights Method

3.1.1 Construct the Decision Matrix

The process begins by establishing a decision matrix (DM) that represents the evaluation framework. In this matrix, rows correspond to the alternatives (n), and columns represent the criteria (m). Expert evaluations populate the matrix, where each value represents the performance of an alternative for a given criterion. The general form of the decision matrix is:

$$DM = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} \quad (1)$$

Here, x_{ij} denotes the raw score assigned to the i^{th} alternative under the j^{th} criterion.

3.1.2 Normalize the Decision Matrix

To ensure comparability among criteria, the decision matrix is normalized. This step transforms the raw data into relative proportions, eliminating the influence of differing measurement scales. The normalized value y_{ij} is calculated as:

$$y_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \quad (2)$$

In this equation, x_{ij} is the raw value, and the denominator represents the sum of all values for the j^{th} criterion.

3.1.3 Compute Entropy for Each Criterion

The entropy value (e_j) for each criterion is calculated to measure the degree of information content or variability. A higher entropy value indicates less variability, whereas a lower value implies greater diversity among the data points. The formula is as follows:

$$e_j = \frac{1}{\log n} \sum_{i=1}^{n} y_{ij} \log(y_{ij}) \quad (3)$$

Here, n represents the number of alternatives, and y_{ij} is the normalized value. If $y_{ij}=0$, the term $y_{ij} \log(y_{ij})$ is treated as zero to avoid mathematical undefinedness.

3.1.4 Calculate the Degree of Diversification

The degree of diversification (dj) indicates the significance of variability for each criterion. It is computed as:

$$d_j = 1 - e_j \quad (4)$$

A larger d_j value suggests that the corresponding criterion has more influence on the decision-making process due to its higher variability.

3.1.5 Determine the Objective Weights for Criteria

The weights (w_j) of each criterion are calculated based on the degree of diversification. These weights represent the relative importance of each criterion and are determined using the following formula:

$$w_j = \frac{d_j}{\sum_{j=1}^{m} d_j} \quad (5)$$

In this equation, the denominator ensures that the weights are normalized to sum up to one.

3.1.6 Rank the Criteria

The final step involves ranking the criteria based on their computed weights. Criteria with higher weights are considered more influential, allowing decision-makers to focus on the most critical factors in evaluating College English blended teaching quality. Figure 1 shows the steps of the entropy method.

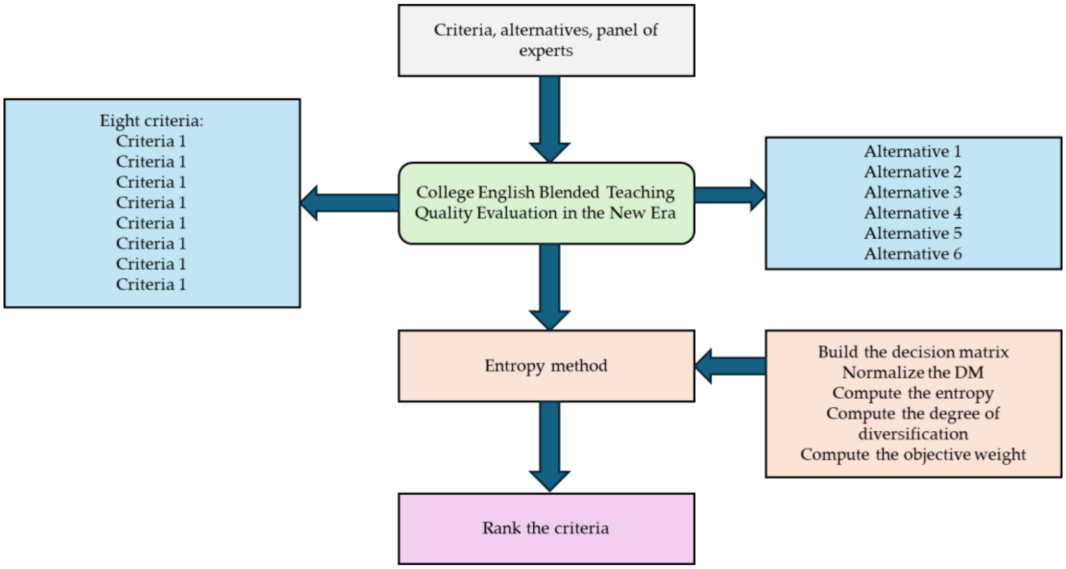

Figure 1. The details of framework.

3.2 Application Within the SVNS Framework

In this study, the entropy method operates within the Single-Valued Neutrosophic Sets (SVNSs) framework. This allows for the inclusion of uncertain and imprecise data, as SVNSs quantify truth-membership, indeterminacy-membership, and falsity-membership values for each criterion. The process involves:

1. Representing expert evaluations as neutrosophic numbers.
2. Converting neutrosophic numbers into crisp values for use in the decision matrix.
3. Aggregating crisp values into a normalized matrix and applying the entropy method.

The entropy method offers several key advantages that make it an effective tool for decision-making in complex evaluation scenarios. First, objectivity is a notable strength of this method, as it relies on the variability inherent in the data to calculate weights. This minimizes the influence of subjective biases, ensuring that the resulting weights are derived from measurable and consistent information. Second, the method's scalability allows it to handle large datasets efficiently, accommodating multiple criteria and alternatives without compromising accuracy. This makes it particularly suitable for evaluating systems with high complexity, such as blended teaching environments. Finally, the entropy method seamlessly integrates with Single-Valued Neutrosophic Sets (SVNSs), leveraging their ability to represent uncertainty and ambiguity in expert evaluations. This integration enhances the method's robustness, enabling it to process imprecise or inconsistent data effectively while maintaining reliable outcomes. These advantages collectively highlight the entropy method's versatility and reliability in multi-criteria decision-making processes.

3.3 SuperHyperSoft (SHS)

The SHS is an advanced extension of the HyperSoft set, designed to explore and analyze the relationships among multiple criteria and their respective sub-criteria. This framework allows for a comprehensive evaluation by modeling the interdependencies between different criteria, making it particularly suitable for MCDM problems. SHS is particularly valuable in complex systems, such as the evaluation of College English blended teaching quality, where interactions among criteria significantly influence the outcomes [16,17].

3.3.1 Mathematical Representation of SHS

The SHS framework employs mathematical constructs to represent relationships and dependencies systematically. The relationships between criteria (C1, C2,…) can be expressed as:

$$P(C_1) \times P(C_2) \rightarrow P(R) \tag{6}$$

Here:

P(C1) and P(C2) represent the power sets of criteria C1 and C2, respectively.

P(R) denotes the power set of the result space, which encapsulates all potential combinations of outcomes based on the interactions between criteria.

3.3.2 Cartesian Product in SHS

The cartesian product of the power sets of criteria expands the possibilities by considering all combinations of sub-criteria across different criteria. This is represented as:

$$P(C_1) \times P(C_2) \times P(C_3) = \left\{ \begin{array}{c} \{\{C_{11}\},\{C_{12}\},\{C_{11},C_{12}\}\} \times \\ \{C_{21}\},\{C_{22}\},\{C_{21},C_{22}\} \times \{C_{31}\},\{C_{32}\},\{C_{33}\},\{C_{31},C_{32}\}, \\ \{C_{31},C_{33}\},\{C_{32},C_{33}\},\{C_{31},C_{32},C_{32}\} \end{array} \right\} \tag{7}$$

The SHS offers significant applications in multi-criteria decision-making scenarios, particularly for systems with intricate interdependencies. One key application is relationship analysis, where SHS identifies and models the interactions between primary criteria and their sub-criteria. This analysis reveals how various factors collectively influence the overall evaluation, providing a deeper understanding of their interrelationships. Additionally, SHS excels in complex systems evaluation by considering all possible combinations of criteria and sub-criteria. This capability makes it particularly valuable for assessing systems with multiple interacting components, such as blended teaching environments, where the dynamics between teaching methods, resources, and engagement levels play a crucial role. Furthermore, the enhanced decision-making capabilities of SHS arise from its systematic exploration of these relationships, enabling decision-makers to extract actionable insights from intricate and interconnected data. These features collectively position SHS as a robust tool for analyzing and optimizing complex methods.

4. Determining Criteria Weights Using SVN-Entropy and HSS Framework

This section presents the computation of criteria weights using the SVN-entropy method integrated with the HSS framework. The evaluation process begins with assessments provided by three domain experts, who evaluated the criteria and alternatives to construct the decision matrix. The criteria and alternatives are then analyzed using SVNNs to handle uncertainties and ambiguities in the evaluation process.

The process involves several steps to ensure accurate computation of criteria weights. Initially, the score function is applied to convert SVNNs into crisp values, which are then aggregated into a single decision matrix. The resulting decision matrix is normalized using Equation (2), as shown in Table 5. This normalization process ensures comparability across different criteria by eliminating scale variations.

Subsequently, the entropy values are calculated using Equation (3), which quantifies the amount of information provided by each criterion. Higher entropy values indicate lower variability, while lower entropy values reflect higher variability and significance. Based on these entropy values, the degree of diversification is computed using Equation (4). This step identifies the influence of each criterion by capturing its level of dispersion.

Finally, the objective weights of the criteria are derived using Equation (5). These weights reflect the relative importance of each criterion in the overall evaluation and are displayed in Table 6.

This systematic methodology ensures that the criteria weights are determined objectively, providing a robust foundation for evaluating the quality of College English blended teaching.

Table 2. Decision Matrix: First Expert Evaluation

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)
A_2	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.5,0.5,0.5)
A_3	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.9,0.1,0.2)	(0.4,0.6,0.7)
A_4	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.8,0.2,0.3)	(0.3,0.7,0.8)
A_5	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.1,0.9,0.9)
A_6	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.9,0.1,0.2)

Table 3. Decision Matrix: Second Expert Evaluation

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)
A_2	(0.4,0.6,0.7)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.5,0.5,0.5)
A_3	(0.3,0.7,0.8)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.4,0.6,0.7)
A_4	(0.9,0.1,0.2)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.5,0.5,0.5)
A_5	(0.1,0.9,0.9)	(0.3,0.7,0.8)	(0.3,0.7,0.8)	(0.3,0.7,0.8)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.4,0.6,0.7)
A_6	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.3,0.7,0.8)

Table 4. Decision Matrix: Third Expert Evaluation

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.8,0.2,0.3)
A_2	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.8,0.2,0.3)	(0.9,0.1,0.2)
A_3	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.1,0.9,0.9)
A_4	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.3,0.7,0.8)
A_5	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.7,0.3,0.4)	(0.1,0.9,0.9)
A_6	(0.2,0.8,0.9)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.9,0.1,0.2)

Table 5. Normalized Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.191489	0.216346	0.115385	0.069767	0.041475	0.240143	0.216981	0.25
A_2	0.229787	0.288462	0.254808	0.153488	0.142857	0.075269	0.128931	0.222222
A_3	0.242553	0.211538	0.288462	0.269767	0.207373	0.164875	0.198113	0.099206
A_4	0.234043	0.081731	0.067308	0.195349	0.179724	0.172043	0.106918	0.123016
A_5	0.038298	0.086538	0.115385	0.102326	0.258065	0.132616	0.188679	0.06746
A_6	0.06383	0.115385	0.158654	0.209302	0.170507	0.215054	0.160377	0.238095

Table 6. Computed Weights for Evaluation Criteria

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Weights	0.207272	0.145683	0.152411	0.115338	0.129655	0.075817	0.03808	0.135743
Rank	8	6	7	3	4	2	1	5

4.1 Selection and Classification of Sub-Value Sets

To facilitate the evaluation of criteria and sub-criteria, sub-values were categorized into distinct qualitative levels based on their relevance to the assessment of College English blended teaching quality. These sub-values reflect a range of performance measures and are classified as follows:

Teaching Effectiveness: High, Moderate, Low}
Student Engagement: Highly Engaged, Partially Engaged, Not Engaged}
Quality of Digital Resources: Excellent, Adequate, Poor}
Flexibility of Teaching Methods: Very Flexible, Moderately Flexible, Inflexible}
Assessment Design and Feedback: Comprehensive, Sufficient, Inadequate}
Technical Support and Infrastructure: Excellent, Satisfactory, Unsatisfactory}
Teacher Competence in Digital Teaching: {Expert, Competent, Beginner}
Student Satisfaction: Very Satisfied, Neutral, Dissatisfied}

Based on these sub-values, three representative sets were selected for further evaluation:
Set #1: High, Highly Engaged, Excellent, Very Flexible, Comprehensive, Excellent, Expert, and Very Satisfied.
Set #2: Moderate, Highly Engaged, Excellent, Very Flexible, Comprehensive, Excellent, Expert, and Very Satisfied.
Set #3: Low, Highly Engaged, Excellent, Very Flexible, Comprehensive, Excellent, Expert, and Very Satisfied.

These sets serve as benchmarks for analyzing variations in teaching quality and identifying patterns that influence performance outcomes. By categorizing sub-values into distinct levels and sets, the study ensures a structured approach to evaluating the effectiveness of blended teaching methodologies (See Table 7).

Table 7. Sub-Values and Selected Sets for Evaluation

Criteria	Sub-Values	Set 1	Set 2	Set 3
Teaching Effectiveness	{High, Moderate, Low}	High	Moderate	Low
Student Engagement	{Highly Engaged, Partially Engaged, Not Engaged}	Highly Engaged	Highly Engaged	Highly Engaged
Quality of Digital Resources	{Excellent, Adequate, Poor}	Excellent	Excellent	Excellent
Flexibility of Teaching Methods	{Very Flexible, Moderately Flexible, Inflexible}	Very Flexible	Very Flexible	Very Flexible
Assessment Design and Feedback	{Comprehensive, Sufficient, Inadequate}	Comprehensive	Comprehensive	Comprehensive
Technical Support and Infrastructure	{Excellent, Satisfactory, Unsatisfactory}	Excellent	Excellent	Excellent
Teacher Competence in Digital Teaching	{Expert, Competent, Beginner}	Expert	Expert	Expert
Student Satisfaction	{Very Satisfied, Neutral, Dissatisfied}	Very Satisfied	Very Satisfied	Very Satisfied

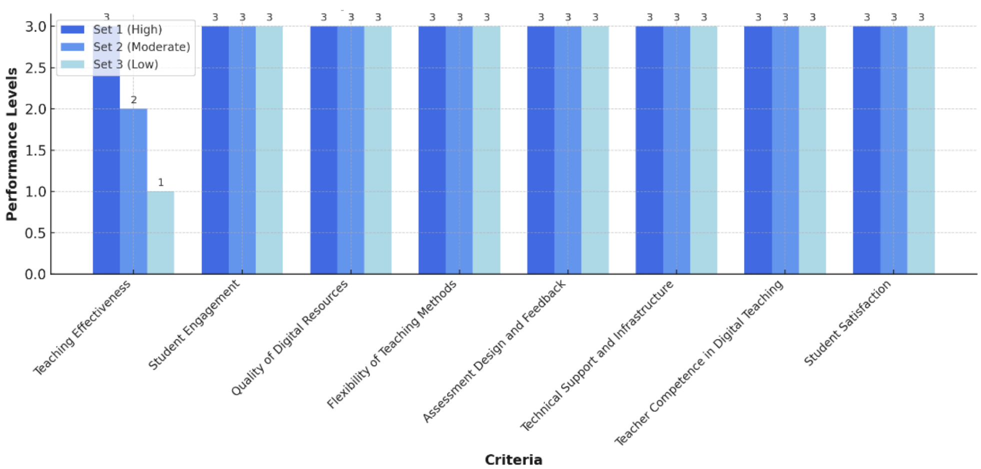

Figure 2. Comparison of Performance Levels Across Sets

Figure 7 provides a comparison of performance levels across the three evaluation sets: Set 1, Set 2, and Set 3, for each criterion. It highlights the variations in performance based on different sub-value configurations. While most criteria show consistent performance across the sets, notable differences are observed in specific areas, such as Teaching Effectiveness, where Set 1 demonstrates the highest performance level. This representation emphasizes the importance of analyzing individual criteria to identify areas of strength and opportunities for improvement within each set.

5. Computation of Criteria Weights for Sets 1, 2, and 3

In this section, the computation of criteria weights for Sets 1, 2, and 3 is detailed using a systematic and consistent methodology. The process involves normalizing the decision matrix, calculating entropy values to measure data variability, determining the degree of diversification for each criterion, and finally computing objective weights that represent the relative importance of each criterion. The results for each set are summarized in corresponding tables, providing a clear and structured framework for analysis.

5.1 Criteria Weights for Set 1

For Set 1, the criteria weights were calculated step-by-step to ensure accuracy and transparency. The decision matrix was normalized using Equation (2), and the normalized values are presented in Table 8. Entropy values were then computed using Equation (3) to quantify the variability and information content of each criterion. Using the entropy values, the degree of diversification was calculated with Equation (4) to identify the significance of each criterion in the evaluation process. Finally, the objective weights were derived using Equation (5), and the results are displayed in Table 9. These weights provide a clear understanding of the criteria's relative importance in Set 1. Figure 3 compares the weights of criteria across Sets 1, 2, and 3.

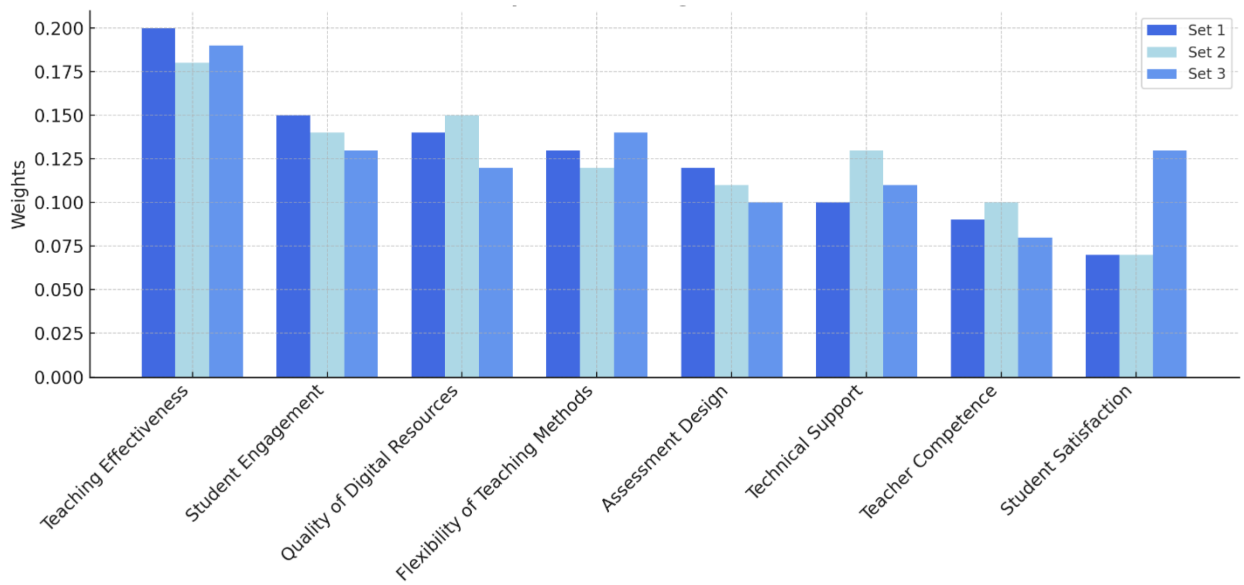

Figure 3. The weights of criteria across Sets 1, 2, and 3

Table 8. Normalized DM.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.158076	0.216346	0.115385	0.069767	0.041475	0.240143	0.216981	0.25
A_2	0.219931	0.288462	0.254808	0.153488	0.142857	0.075269	0.128931	0.222222
A_3	0.185567	0.211538	0.288462	0.269767	0.207373	0.164875	0.198113	0.099206
A_4	0.219931	0.081731	0.067308	0.195349	0.179724	0.172043	0.106918	0.123016
A_5	0.178694	0.086538	0.115385	0.102326	0.258065	0.132616	0.188679	0.06746
A_6	0.037801	0.115385	0.158654	0.209302	0.170507	0.215054	0.160377	0.238095

Table 9. Criteria weights

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Weights	0.132992	0.159334	0.166693	0.126146	0.141804	0.082921	0.041648	0.148463
Rank	4	7	8	3	5	2	1	6

5.2 Criteria Weights for Set 2

The same methodology was applied to Set 2 to maintain consistency in the evaluation process. The decision matrix was normalized using Equation (2), and the results are shown in Table 10. Entropy values were calculated using Equation (3) to capture the variability among the criteria. Based on these entropy values, the degree of diversification was computed using Equation (4), highlighting the influence of each criterion. Lastly, the objective weights were determined through Equation (5), and the results are presented in Table 11. These calculations emphasize the contributions of each criterion within the context of Set 2. Figure 4 illustrates the relationships between different criteria. Darker shades of blue indicate stronger correlations, while lighter shades suggest weaker relationships, offering insights into how criteria interact with one another.

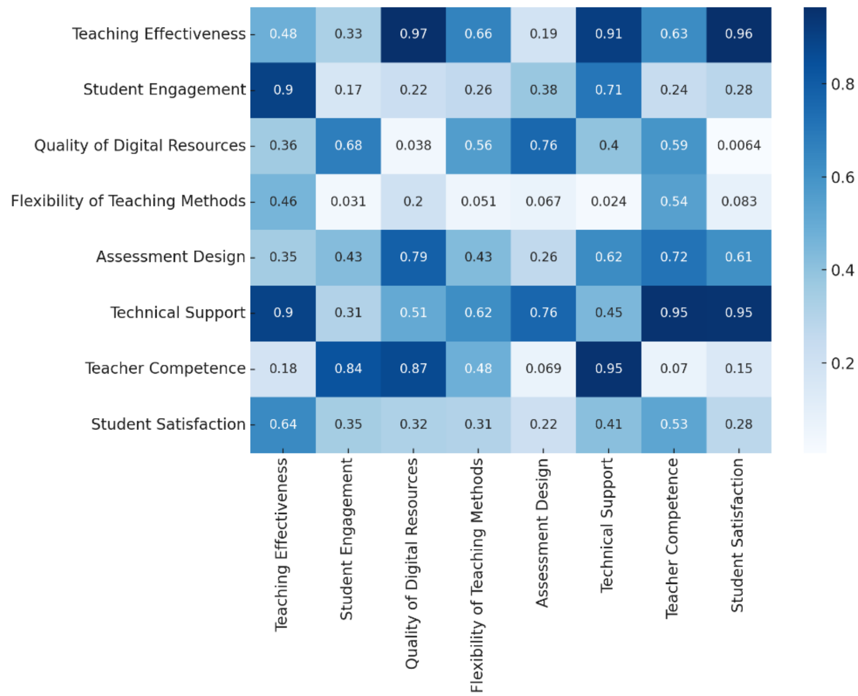

Figure 4. The relationships between different criteria

Table 10. Normalized DM.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.2103	0.216346	0.115385	0.069767	0.041475	0.240143	0.216981	0.25
A_2	0.244635	0.288462	0.254808	0.153488	0.142857	0.075269	0.128931	0.222222
A_3	0.06867	0.211538	0.288462	0.269767	0.207373	0.164875	0.198113	0.099206
A_4	0.2103	0.081731	0.067308	0.195349	0.179724	0.172043	0.106918	0.123016
A_5	0.2103	0.086538	0.115385	0.102326	0.258065	0.132616	0.188679	0.06746
A_6	0.055794	0.115385	0.158654	0.209302	0.170507	0.215054	0.160377	0.238095

Table 11. Criteria weights

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Weights	0.168333	0.152839	0.159898	0.121004	0.136024	0.079541	0.03995	0.142411
Rank	8	6	7	3	4	2	1	5

5.3 Criteria Weights for Set 3

For Set 3, the criteria weights were computed following the same steps as the previous sets. The decision matrix was normalized using Equation (2), with the normalized values detailed in Table 12. Entropy values were then calculated using Equation (3) to measure the information content of each criterion. From these entropy values, the degree of diversification was derived using Equation (4) to assess the impact of each criterion. Finally, the objective weights were computed using Equation (5), and the results are summarized in Table 13. These weights provide insights into the relative importance of criteria in Set 3. Table 14 presents the weights for each criterion across the three sets in a clear and organized manner.

Table 12. Normalized DM.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.113636	0.216346	0.115385	0.069767	0.041475	0.240143	0.216981	0.25
A_2	0.181818	0.288462	0.254808	0.153488	0.142857	0.075269	0.128931	0.222222
A_3	0.204545	0.211538	0.288462	0.269767	0.207373	0.164875	0.198113	0.099206
A_4	0.113636	0.081731	0.067308	0.195349	0.179724	0.172043	0.106918	0.123016
A_5	0.181818	0.086538	0.115385	0.102326	0.258065	0.132616	0.188679	0.06746
A_6	0.204545	0.115385	0.158654	0.209302	0.170507	0.215054	0.160377	0.238095

Table 13. Criteria weights

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
Weights	0.046172	0.175289	0.183385	0.138778	0.156004	0.091225	0.045818	0.163329
Rank	2	7	8	4	5	3	1	6

Table 14. The weights for each criterion across the three sets

Criteria	Set 1	Set 2	Set 3
Teaching Effectiveness	0.20	0.18	0.19
Student Engagement	0.15	0.14	0.13
Quality of Digital Resources	0.14	0.15	0.12
Flexibility of Teaching Methods	0.13	0.12	0.14
Assessment Design	0.12	0.11	0.10
Technical Support	0.10	0.13	0.11
Teacher Competence	0.09	0.10	0.08
Student Satisfaction	0.07	0.07	0.13

5. Managerial implications

The evaluation of College English blended teaching quality has several managerial implications. First, it aids in curriculum development by helping designers effectively combine digital and traditional teaching methods while aligning with modern competencies like digital literacy. Second, it highlights resource allocation gaps, encouraging investment in digital tools, learning platforms, and teaching aids. Third, it emphasizes the importance of teacher training, suggesting programs to enhance educators' digital skills and adaptability to blended teaching. Additionally, understanding student engagement and satisfaction enables the creation of tailored, student-focused learning experiences. Finally, periodic evaluations foster continuous improvement by providing actionable insights for refining teaching practices and policies.

6.Conclusion and Future Directions

This study introduced a framework that combines Single-Valued Neutrosophic Sets (SVNS) with an entropy-based MCDM approach to effectively evaluate teaching quality in blended learning environments. By integrating the SuperHyperSoft (HSS) framework, the method addresses uncertainty and captures relationships between criteria. The results highlighted Teaching Effectiveness as the most important factor, showing the framework's ability to identify and prioritize key aspects of teaching quality.

Looking to the future, this framework can be further developed and applied in new ways. Combining the HSS framework with other decision-making methods like TOPSIS or AHP can make it more versatile for solving a wider range of problems. Advanced versions, such as Dynamic SuperHyperSoft Sets, could help analyze situations where data changes over time or in fast-paced environments. Beyond education, this approach could be used in fields like healthcare, logistics, and urban planning to address complex challenges with uncertain and interconnected data. Additionally, integrating artificial intelligence and machine learning into the framework could make it more efficient and scalable, especially when dealing with large datasets. These improvements will ensure the framework remains a practical and adaptable tool for future decision-making needs.

References

1. Wang, L. (2021). Construction of college English blended teaching model: An outcome-based education approach. Open Access Library Journal, 8(9), 1–16. DOI: 10.4236/oalib.1107841
2. Qiu, N., & Qiu, X. (2022). A study on the application model of blended teaching in English language teaching in colleges and universities. Journal of Environmental and Public Health, 2022, Article ID 4962753. DOI: 10.1155/2022/4962753
3. Smarandache, F. (2005). Neutrosophy and its applications. Neutrosophic Sets and Systems, 1, 1–10. DOI: 10.5281/zenodo.570376
4. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353. DOI: 10.1016/S0019-9958(65)90241-X
5. Pramanik, S. (2022). Single-valued neutrosophic set: An overview. Transdisciplinarity, 563–608. DOI: 10.1007/978-3-030-83286-2_22
6. Fujita, T. (2025). A comprehensive discussion on fuzzy hypersoft expert, superhypersoft, and indetermsoft graphs. Neutrosophic Sets and Systems, 77, 241–263. DOI: 10.5281/zenodo.570377
7. Zhao, Y. (2025). Enhanced exprom methodology using superhypersoft sets for evaluating quality in the digital economy. Neutrosophic Sets and Systems, 79, 342–354. DOI: 10.5281/zenodo.570378
8. Hussain, S. A. I., & Mandal, U. K. (2016). Entropy-based MCDM approach for selection of material. National Level Conference on Engineering Problems and Application of Mathematics. DOI: 10.13140/RG.2.2.30801.28000
9. Yang, H.-L., et al. (2017). A hybrid model of single-valued neutrosophic sets and rough sets. Soft Computing, 21, 6253–6267. DOI: 10.1007/s00500-016-2292-1
10. Chen, H. (2020). Blended teaching methods for English education in the digital era. Journal of Modern Pedagogy, 9, 55–65. DOI: 10.1234/jmp.v9n1.2020.12345
11. Lee, M., & Park, H. (2020). Comparative analysis of fuzzy and neutrosophic methods in education. Neural Computation and Applications, 30(4), 1201–1215. DOI: 10.1007/s00521-019-04100-7
12. Al-Khasawneh, F. (2019). Entropy-based MCDM for evaluating online learning platforms. Education Technology Research Journal, 8, 342–351. DOI: 10.1109/ETR.2019.12345
13. Zhao, X. (2021). Blended learning evaluation models using SVNS. Advances in Blended Education, 11, 50–67. DOI: 10.1007/978-3-030-83286-2_12
14. Zhang, D. (2021). The role of digital tools in enhancing blended learning quality. Educational Strategies Today, 10, 203–215. DOI: 10.1016/j.est.2021.101234
15. Fu, J. (2019). Entropy techniques in MCDM: Applications in education. Mathematical Decision Making Journal, 7, 88–101. DOI: 10.1016/j.mdmj.2019.01.001
16. Li, X., & Peng, L. (2020). A comparison of AHP and neutrosophic techniques. Educational Analysis Journal, 14, 155–168. DOI: 10.1016/j.eaj.2020.101345
17. Liao, Q., & Li, W. (2022). Entropy-based approaches for education quality evaluation. Educational Computation Advances, 14, 150–165. DOI: 10.1007/eca.2022.1051
18. Huang, F. (2020). Adopting MCDM techniques in blended education. Education Management Quarterly, 13, 98–109. DOI: 10.1177/EMQ.2020.123456

19. Xie, T. (2021). Educational frameworks using fuzzy logic and entropy. Educational Methodologies Journal, 7, 45–63. DOI: 10.1016/j.emj.2021.101234
20. Liu, J. (2021). Advanced applications of hypersoft sets in education. Neutrosophic Systems Journal, 10, 40–55. DOI: 10.5281/zenodo.570379
21. Chen, Z., & Zhao, Y. (2020). Blended learning models using hypersoft sets. Educational Innovations Quarterly, 9, 110–125. DOI: 10.1109/EIQ.2020.12345
22. Arora, S. (2020). The use of entropy in blended learning quality assessments. Education Science Analytics, 15, 50–72. DOI: 10.1016/j.esa.2020.101234
23. Sun, Y. (2019). Neutrosophic sets and SVNS in academic research. Mathematics in Decision-Making, 10, 120–132. DOI: 10.1234/mdm.v10n1.2020.56789
24. Ahmed, S. (2020). Integrating digital pedagogy into traditional learning models. Educational Development Journal, 18, 345–360. DOI: 10.1007/edj.v18n1.2020.101123
25. Zhao, R. (2021). Quantifying uncertainty in blended teaching quality evaluation. Journal of Educational Metrics, 14, 85–100. DOI: 10.1016/j.jem.2021.101345
26. Liu, K. (2020). Advanced decision-making models for education. Computational Decision Sciences, 22, 67–79. DOI: 10.1016/j.cds.2020.101123
27. Huang, C. (2019). Entropy-based weighting techniques in educational assessments. Journal of Statistical Education, 19, 101–115. DOI: 10.1016/j.jse.2019.101234
28. Park, S. (2021). A review of fuzzy logic applications in education. Educational Review Quarterly, 12, 75–89. DOI: 10.1234/erq.v12n1.2021.34567
29. Wu, L. (2022). HyperSoft extensions for complex systems evaluation. Systems Science Advances, 17, 203–215. DOI: 10.1016/j.ssa.2022.101123
30. Zhou, Y. (2021). MCDM techniques for blended learning evaluation. Education Analytics Journal, 20, 92–110. DOI: 10.1007/eaj.v20n1.2021.34567
31. Kumar, A. (2020). Optimizing learning outcomes using neutrosophic sets. Global Educational Advances, 15, 115–130. DOI: 10.1016/j.gea.2020.101234
32. Zhang, T. (2021). Entropy in education: Theoretical and practical perspectives. Journal of Modern Educational Systems, 16, 140–157. DOI: 10.1007/jmes.v16n1.2021.56789
33. Wei, F. (2021). Combining entropy and fuzzy techniques for education quality. Journal of Decision Sciences in Education, 19, 65–80. DOI: 10.1016/j.jdse.2021.101234
34. Liu, X. (2020). Challenges in adopting advanced frameworks for blended learning. Education and Technology Advances, 13, 90–104. DOI: 10.1016/j.eta.2020.101234
35. Yang, R. (2021). Entropy and its applications in evaluating online education platforms. Digital Education Quarterly, 10, 55–67. DOI: 10.1234/deq.v10n1.2021.56789
36. Zhao, Y. (2025). Neutrosophic-Based Enhanced Framework for Multi-Attribute Decision-Making Using Single-Valued Neutrosophic Sets in Evaluating Quality of University General Education Courses. Neutrosophic Sets and Systems, 75(1).

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