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University Mathematics Classroom Teaching Quality Assessment Based on Core Competencies Using the DEMATEL Approach and Single-Valued Neutrosophic Hypersoft Sets

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Abstract: This study proposes a multi-criteria decision-making (MCDM) approach to evaluate the quality of University Mathematics Classroom Teaching based on core competencies. The MCDM approach is applied within the framework of the neutrosophic set to address vague and uncertain data. Unlike the hypersoft set, which handles multiple disjoint attribute-valued sets corresponding to various characteristics, the soft set operates with a single set of attributes. This study introduces the concept of Single-Valued Neutrosophic Hypersoft Expert Sets (SVNS), which integrate single-valued neutrosophic sets and hypersoft expert sets. Eight criteria were employed to construct the pairwise comparison matrix. The Decision-Making Trial and Evaluation Laboratory (DEMATEL) method was utilized to analyze the interrelationships among the criteria within the SVNS and hypersoft set frameworks. The results indicate that Conceptual Understanding utilizes the highest impact among the criteria.

Keywords: University Mathematics Classroom; Uncertainty; MCDM; DEMATEL Approach.

1. Introduction

The quality of teaching in university mathematics classrooms is critical for developing students' analytical thinking, logical reasoning, and problem-solving skills. Mathematics forms the backbone of many technical and scientific disciplines, and its teaching demands a high level of expertise. Effective teaching in this field does not merely involve the accurate delivery of content but also necessitates fostering deep conceptual understanding and encouraging critical thinking among students. Educators must strike a balance between subject-matter expertise and pedagogical strategies to accommodate diverse learning needs and backgrounds [1][2].

Traditional evaluation methods, such as student feedback surveys, standardized tests, and peer reviews, are frequently employed to assess teaching quality. While these methods provide valuable insights, they often fall short of capturing the multifaceted nature of teaching. For instance, student feedback can be influenced by factors unrelated to teaching, such as the difficulty of the course or the instructor's demeanor. Standardized assessments, on the other hand, tend to focus narrowly on measurable outcomes, overlooking crucial qualitative aspects like classroom engagement and the ability to foster critical thinking [3][4].
MCDM approaches have emerged as effective tools for addressing these challenges. By allowing the analysis of multiple, often interrelated criteria, MCDM approaches provide a structured way to evaluate teaching effectiveness comprehensively. Among these, the DEMATEL method stands out due to its ability to identify and analyze causal relationships among criteria. Unlike other MCDM methods, such as the Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution

(TOPSIS), DEMATEL goes beyond ranking and prioritizing to uncover interdependencies and influence levels among factors, making it particularly well-suited for evaluating complex educational settings [5][6].

In addition to MCDM methods, advancements in mathematical modeling, such as neutrosophic sets, have significantly enhanced decision-making processes. Neutrosophic sets, introduced by Smarandache, extend traditional fuzzy logic by incorporating three independent membership degrees: truth, indeterminacy, and falsity. This method provides a nuanced way to represent uncertainty, ambiguity, and inconsistency, which are often inherent in educational data [7]. Building on this foundation, single-valued neutrosophic sets (SVNS) and hypersoft sets were developed to address decision-making problems characterized by multi-attribute and uncertain data. These tools are particularly relevant in evaluating teaching quality, where subjective factors like student perceptions and classroom dynamics play a significant role [8][9].

The integration of DEMATEL with Single-Valued Neutrosophic Hypersoft Sets offers a novel and comprehensive framework for evaluating teaching quality. This hybrid approach combines the strengths of both methodologies: the causal analysis capabilities of DEMATEL and the uncertainty-handling flexibility of neutrosophic sets. By focusing on core competencies such as conceptual understanding, problem-solving skills, classroom engagement, and feedback quality, this framework addresses the limitations of traditional evaluation methods and provides actionable insights for educators and policymakers [10][11].

This study aims to contribute to the field of educational evaluation by introducing a robust framework that integrates DEMATEL and SVNS for assessing teaching quality in university mathematics classrooms. The proposed approach is designed to account for the multidimensional and interdependent nature of teaching quality criteria while addressing the inherent uncertainties and subjectivities in educational data. By leveraging these advanced methodologies, this research seeks to provide a more comprehensive and reliable tool for evaluating and improving teaching practices in mathematics education [12][13].

Moreover, the study emphasizes the importance of adopting evidence-based evaluation frameworks in higher education, especially in disciplines like mathematics, where the complexity of the subject matter and the diversity of student needs present unique challenges. By integrating quantitative and qualitative analysis, the proposed framework not only evaluates the effectiveness of teaching practices but also identifies areas for targeted intervention, ultimately enhancing the overall quality of education [14][15].

2. Related Work and Literature Review

The evaluation of teaching quality has been a topic of ongoing debate and research due to the multifaceted nature of teaching and learning. Traditional evaluation systems often rely on student feedback, standardized tests, and peer reviews, which are inherently limited in capturing the complexities of effective teaching. According to Broumi et al. [16], these approaches often fail to account for interdependencies between different factors affecting teaching quality, such as conceptual understanding, student engagement, and the integration of technology in teaching practices. This gap has led to the exploration of more advanced frameworks that can address the intricate nature of educational evaluations.

One of the major criticisms of conventional evaluation methods is their inability to represent uncertainty and subjective biases effectively. Smarandache et al. [20] emphasized that subjective data, such as student feedback, often involves ambiguity and inconsistencies that traditional tools cannot process adequately. These limitations have prompted researchers to seek methods capable of incorporating uncertainty while simultaneously analyzing interrelated criteria.

Neutrosophic sets have gained significant traction in decision-making processes due to their ability to handle uncertainty and inconsistency effectively. Unlike traditional fuzzy sets, neutrosophic sets incorporate three independent membership degrees: truth, indeterminacy, and falsity. This allows them to model complex scenarios with greater flexibility. Liu et al. [17] applied neutrosophic sets in the selection of transportation service providers, demonstrating their capacity to manage multi-criteria problems involving subjective and uncertain data.

In the context of education, Rahman et al. [7] introduced interval-valued neutrosophic hypersoft sets to represent multi-attribute decision-making scenarios. These sets are particularly useful for modeling subjective factors, such as student perceptions of teaching quality or the impact of classroom engagement. Their findings highlighted the potential of neutrosophic sets to enhance decision-making frameworks by addressing ambiguities inherent in educational data.

The integration of DEMATEL with neutrosophic sets represents a significant advancement in multi-criteria decision-making methodologies. DEMATEL is widely recognized for its ability to map causal relationships between criteria, providing a visual representation of how different factors influence one another. When combined with neutrosophic sets, this method gains the ability to incorporate uncertainty into causal analysis, making it particularly suitable for complex systems like education. Abdullah et al. [18] demonstrated this integration in analyzing subcontractor selection processes, highlighting its utility in evaluating systems characterized by interrelated and uncertain criteria.

Moreover, Rodzi et al. [19] applied a hybrid framework combining neutrosophic sets with DEMATEL to analyze barriers to Halal certification adoption in Malaysia. Their findings underscored the framework's ability to handle complex relationships and subjective biases, which are also prevalent in teaching quality evaluations. This approach offers significant promise for educational contexts, where factors such as teaching effectiveness, student engagement, and technology use are deeply interdependent.

The evolution of hypersoft sets has further expanded the applicability of the neutrosophic theory in educational evaluations. Hypersoft sets allow for the representation of multiple attributes associated with each decision-making entity, providing a richer analysis of complex scenarios. Zhao et al. [5] applied single-valued neutrosophic hypersoft sets to investment decisions, showcasing their ability to model intricate relationships between criteria. Similarly, Rahman et al. [11] extended this concept to site selection for solid waste management, highlighting its versatility across various domains.

In the educational domain, hypersoft sets can be adapted to evaluate teaching quality by analyzing multiple competencies simultaneously. Saqlain et al. [6] explored the use of neutrosophic linguistic-valued hypersoft sets in medical diagnosis, emphasizing their effectiveness in scenarios involving subjective evaluations. These findings suggest that hypersoft sets can provide a robust framework for addressing the complexities of teaching quality assessments in mathematics classrooms. While traditional MCDM methods such as AHP and TOPSIS have been extensively used in educational research, their limitations have become increasingly apparent. AHP is well-suited for ranking and prioritizing criteria but assumes independence among them, which is rarely the case in educational settings. TOPSIS, on the other hand, focuses on proximity to an ideal solution but cannot analyze causal relationships.

The hybrid integration of DEMATEL with neutrosophic sets offers a more comprehensive approach. DEMATEL excels in identifying and mapping interdependencies, while neutrosophic sets enhance the framework's ability to handle uncertainty. This makes the combined methodology particularly advantageous for evaluating teaching quality, where criteria such as conceptual understanding, classroom engagement, and technology use are deeply interconnected.

3. Theoretical Background

Definition 3.1

Neutrosophic sets, introduced by Smarandache, extend fuzzy logic and intuitionistic fuzzy sets by introducing three independent membership functions for an element x in a set A:

Truth-Membership ($T_A(x)$): Degree of truth in the statement.

Indeterminacy-Membership ($I_A(x)$): Degree of indeterminacy or uncertainty in the statement.

Falsity-Membership ($F_A(x)$): Degree of falsity in the statement.

These values are defined on the interval [0,1], with the condition: $0\le T_A(x), I_A(x), F_A(x)\le 1$

Unlike fuzzy sets, neutrosophic sets allow $T_A(x)+I_A(x)+F_A(x)\le 3$, enabling the representation of incomplete, inconsistent, or contradictory information. For example, in evaluating teaching quality, a student's feedback about a teacher's ability to engage may be represented as: $T_A(x)$=0.7, $I_A(x)$=0.2, $F_A(x)$=0.1

This implies that the students' feedback is 70% true, 20% uncertain, and 10% false.

Definition 3.2 [13], [14]

The soft set can be defined as:

Let u be a universe discourse $p(u)$ is the power set of u and A set of criteria. Then the pair $(F,u), F:\rightarrow (u)$ is called a soft set over u.

Definition 3.3

Hypersoft sets, generalize soft sets by associating parameters with multi-attribute valued functions. A hypersoft set over a universal set U is defined as: H= $\{(p,f(p)) / p\in P, f(p)\subseteq P(U)\}$

where:

P is a set of parameters.

$f(p)$ is a multi-attribute valued function for each p.

$P(U)$ denotes the power set of U.

Hypersoft sets enable the representation of scenarios where a single parameter (e.g., a teacher's competency) may depend on multiple interrelated attributes (e.g., conceptual understanding, classroom engagement, technology use). For example, H= $(p1, \{(x1,v1),(x2,v2)\}),(p2,\{(x3,v3),(x4,v4)\}))$

Here, $p1$ and $p2$ represent parameters such as teaching skills and engagement, and xi,vi are attributes and their corresponding values.

3.1 DEMATEL Approach

The DEMATEL method analyzes and visualizes the causal relationships among criteria. The approach consists of the following steps:

i. Constructing the Direct-Relation Matrix

Experts evaluate the pairwise influence of criteria Ci and Cj on a scale of 0 (*no influence*) to 4 (*very high influence*). The direct-relation matrix D is defined as: D=[d_{ij}] (1)

where d_{ij} represents the influence of Ci on Cj

ii. Normalizing the Direct-Relation Matrix

The matrix D is normalized to ensure all values fall within the range [0,1]: (2)

$$D^{norm} = \frac{D}{\max \sum_j d_{ij}}$$

iii. Calculating the Total-Relation Matrix

The total-relation matrix T is computed using: (3)

$$T = D^{norm}(I - D^{norm})^{-1}$$

where I is the identity matrix.

iv. Determining the Cause-and-Effect Relationships

The sums of rows and columns in T are calculated as: (4)

$$R_i = \sum_j t_{ij}, \quad C_j = \sum_i t_{ij}$$

where R_i is the total influence of *Ci* on others, and *Cj* is the total influence received by *Cj* the difference $Ri - Cj$ indicates whether a criterion is a net cause ($Ri > Cj$) or a net effect ($Ri < Cj$)

v. Visualizing the Cause-and-Effect Diagram

A diagram is plotted with $Ri + Cj$ on the horizontal axis (importance) and $Ri - Cj$ on the vertical axis (cause or effect nature).

3.2 DEMATEL Educational Evaluation

To demonstrate the application of the DEMATEL method in evaluating teaching quality in university mathematics classrooms, consider four key criteria for assessment: Conceptual Understanding (C1), referring to the teacher's ability to facilitate deep comprehension of mathematical concepts; Classroom Engagement (C2), which evaluates the teacher's effectiveness in actively engaging students during lessons; Use of Technology (C3), assessing the integration of technological tools to enhance teaching; and Problem-Solving Skills (C4), representing the teacher's capability to develop students' ability to solve complex mathematical problems.

<u>Step 1</u>, Expert judgments are collected to evaluate the influence of one criterion on another using a scale ranging from 0 (no influence) to 4 (very high influence). Based on these judgments, the Direct-Relation Matrix (D) is constructed as follows:

$$D = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 0 & 0 & 4 & 2 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

Each element d_{ij} represents the degree of influence criterion Ci exerts on Cj.

d_{12}=3: Conceptual Understanding (C1) has a high influence on Classroom Engagement (C2).

d_{34}=3: Use of Technology (C3) strongly influences Problem-Solving Skills (C4).

<u>Step 2</u>, To normalize the matrix, calculate the maximum row sum:

Row sums of D= [6,6,4,3], Max row sum=6

Divide each element of D by the maximum row sum:

$$D^{norm} = \frac{D}{6} = \begin{bmatrix} 0 & 0.5 & 0.333 & 0.167 \\ 0 & 0 & 0.667 & 0.333 \\ 0.167 & 0 & 0 & 0.5 \\ 0.333 & 0.167 & 0 & 0 \end{bmatrix}$$

<u>Step 3</u>, The Total-Relation Matrix (T) is calculated using eq. (3) as:

$$T = \begin{bmatrix} 0.665 & 0.797 & 0.915 & 0.720 \\ 0.295 & 0.444 & 1.004 & 0.796 \\ 0.459 & 0.260 & 0.241 & 0.682 \\ 0.608 & 0.484 & 0.528 & 0.362 \end{bmatrix}$$

<u>Step 4</u>, The row sums (Ri) and column sums (Cj) are calculated using eq. (4) as:

R= [3.097,2.539,1.642,1.982], C= [2.028,1.985,2.688,2.558]

<u>Step 5,</u> To determine causal relationships, compute $Ri + Cj$ (overall importance) and $Ri - Cj$ (causal nature):

R+C= [5.125,4.524,4.330,4.540], R−C= [1.069,0.554, −1.046, −0.576]

C1(R1−C1=1.069): Net cause; the most influential criterion.

C3(R3−C3 = −1.046): Net effect; heavily influenced by other criteria.

The Cause-Effect Diagram is plotted with:

R+C on the horizontal axis, representing the overall importance of each criterion.

R−C on the vertical axis, indicating whether the criterion is a cause (*R−C>0*) or an effect (*R−C<0*).
Causes: C1(Conceptual Understanding) and C2(Classroom Engagement).
Effects: C3 (Use of Technology) and C4 (Problem-Solving Skills).

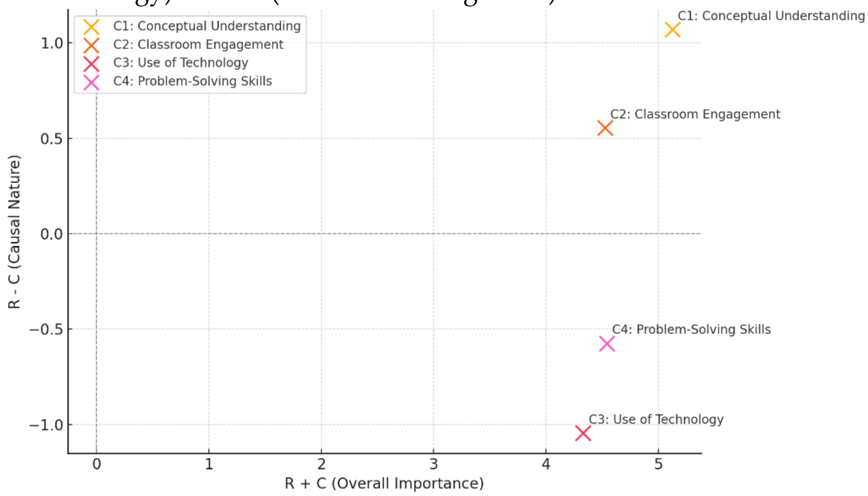

Figure 1. The Cause-Effect Diagram

This analysis provides actionable insights into the teaching quality evaluation process using the DEMATEL method. Conceptual Understanding (C1) emerges as the most influential criterion. Improving conceptual teaching methods can have positive effects on other criteria, such as engagement (C2) and problem-solving skills (C4). Teachers should prioritize strategies that enhance students' understanding of mathematical concepts, ensuring a solid foundation for further learning.

3.3 Integration of DEMATEL and SVNSs Framework

In the proposed methodology, DEMATEL identifies and quantifies the relationships between criteria, classifying them as causes (influential factors) or effects (dependent factors). SVNSs handle uncertainty in expert evaluations by capturing truth, indeterminacy, and falsity values, ensuring accurate and reliable data. Together, they provide a structured and precise framework for analyzing interdependent criteria under uncertainty, supporting better decision-making. This integration is especially relevant in educational evaluations, where subjective data (e.g., student feedback) often involves ambiguity and vagueness. Figure 2 shows the steps of the proposed approach.

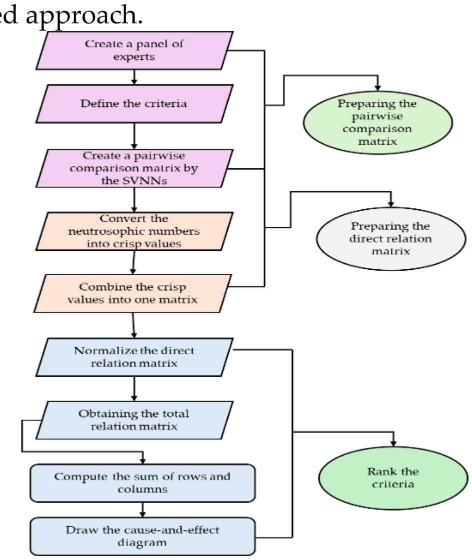

Figure 2. Steps of SVN-DEMATEL

4. The Proposed Approach in the Case Study: Application of HS-SVN-DEMATEL

This section elaborates on the application of the proposed HS-SVN-DEMATEL methodology to evaluate university mathematics classroom teaching quality. This hybrid approach, which integrates HS Sets and SVNSs, is designed to effectively address uncertainty and complex decision-making. By combining these two mathematical tools, the methodology provides a systematic evaluation of interrelations among criteria, offering insights into their individual and collective impacts.

4.1 The Role of HyperSoft Sets in the Proposed Methodology

The HS Sets generalize traditional soft sets by incorporating multiple attribute-valued functions, allowing them to model complex systems more comprehensively. In this study, HS Sets were employed to accommodate MCDM, enabling the representation of criteria that involve multiple, often overlapping attributes. This capability ensured a robust framework for analyzing the complex interdependencies among the evaluation criteria in an uncertain environment.

4.2 Evaluation Framework and Criteria

The study focuses on evaluating eight critical criteria related to classroom teaching quality, as defined in Table 1**:**

1. Conceptual Understanding
2. Problem-Solving Skills
3. Mathematical Communication
4. Classroom Engagement
5. Use of Technology
6. Critical Thinking Development
7. Assessment and Feedback Quality
8. Application of Mathematics in Real-Life Contexts

Three domain experts were selected to evaluate these criteria. They used qualitative descriptors such as "High," "Excellent," and "Highly Relevant," which were mapped to SVNS terms characterized by truth-membership (T), indeterminacy-membership (I), and falsity-membership (F).

Table 1. HyperSoft Criteria and Values.

Criteria	Values
Conceptual Understanding	High, Moderate, Low
Problem-Solving Skills	Excellent, Adequate, Inadequate
Mathematical Communication	Highly Effective, Moderately Effective, Ineffective
Classroom Engagement	Highly Engaged, Partially Engaged, Not Engaged
Use of Technology	Optimal, Satisfactory, Suboptimal
Critical Thinking Development	Strong, Moderate, Weak
Assessment and Feedback Quality	Comprehensive, Sufficient, Insufficient
Application of Mathematics in Real-Life Contexts	Highly Relevant, Moderately Relevant, Irrelevant

4.2.1 Construction of Pairwise Comparison Matrices

Each expert independently assessed the relative importance of the criteria, resulting in three pairwise comparison matrices, presented in Tables 2, 3, and 4. These matrices contained SVNS values (T, I, F), which quantify the influence of one criterion over another.

Table 2. First Pairwise Comparison Matrix.

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)
C2	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.9,0.1,0.2)
C3	(0.6,0.4,0.5)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.8,0.2,0.3)
C4	(0.7,0.3,0.4)	(0.4,0.6,0.7)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.8,0.2,0.3)	(0.5,0.5,0.5)	(0.7,0.3,0.4)
C5	(0.8,0.2,0.3)	(0.5,0.5,0.5)	(0.2,0.8,0.9)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.7,0.3,0.4)	(0.4,0.6,0.7)	(0.6,0.4,0.5)
C6	(0.9,0.1,0.2)	(0.6,0.4,0.5)	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.3,0.7,0.8)	(0.5,0.5,0.5)
C7	(0.2,0.8,0.9)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.4,0.6,0.7)
C8	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.3,0.7,0.8)

Table 3. The Second Pairwise Comparison Matrix

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(0.1,0.9,0.9)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.1,0.9,0.9)
C2	(0.9,0.1,0.2)	(0.4,0.6,0.7)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.9,0.1,0.2)
C3	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.6,0.4,0.5)	(0.5,0.5,0.5)
C4	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)
C5	(0.8,0.2,0.3)	(0.5,0.5,0.5)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.9,0.1,0.2)	(0.1,0.9,0.9)
C6	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.9,0.1,0.2)
C7	(0.2,0.8,0.9)	(0.7,0.3,0.4)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.5,0.5,0.5)
C8	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.1,0.9,0.9)

Table 4. Third Pairwise Comparison Matrix

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(0.5,0.5,0.5)	(0.8,0.2,0.3)	(0.7,0.3,0.4)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.7,0.8)	(0.2,0.8,0.9)
C2	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.3,0.7,0.8)	(0.2,0.8,0.9)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.9,0.1,0.2)
C3	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)
C4	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.5,0.5,0.5)
C5	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.9,0.1,0.2)
C6	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.9,0.1,0.2)	(0.1,0.9,0.9)
C7	(0.2,0.8,0.9)	(0.7,0.3,0.4)	(0.9,0.1,0.2)	(0.9,0.1,0.2)	(0.1,0.9,0.9)	(0.1,0.9,0.9)	(0.2,0.8,0.9)	(0.1,0.9,0.9)
C8	(0.3,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.7,0.3,0.4)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.9,0.1,0.2)

4.2.2 Building the Direct Relation Matrix

The crisp values derived from the pairwise comparison matrices were used to construct the direct relation matrix (D), where each entry a_{ij} represents the direct influence of criterion *i* on criterion *j*. The direct relation matrix was normalized using Equation (2), ensuring that all values fell within the range [0,1] The normalized direct relation matrix is presented in Table 5.

Table 5. Normalize Direct Relation Matrix.

	C1	C2	C3	C4	C5	C6	C7	C8
C1	0.151448	0.082405	0.089087	0.095768	0.100223	0.100223	0.115813	0.126949
C2	0.051225	0.100223	0.115813	0.122494	0.075724	0.075724	0.100223	0.075724
C3	0.131403	0.08686	0.100223	0.100223	0.126949	0.178174	0.075724	0.100223
C4	0.164811	0.093541	0.075724	0.126949	0.126949	0.120267	0.153675	0.138085
C5	0.075724	0.126949	0.080178	0.120267	0.075724	0.08686	0.093541	0.106904
C6	0.075724	0.106904	0.111359	0.11804	0.100223	0.126949	0.060134	0.10245
C7	0.122494	0.089087	0.120267	0.126949	0.126949	0.126949	0.126949	0.11804
C8	0.115813	0.109131	0.100223	0.095768	0.089087	0.082405	0.075724	0.08686

4.2.3 Total Relation Matrix and Its Calculation

The total relation matrix (T) was computed using Equation (3). The total relation matrix, shown in Table 6, accounts for both direct and indirect relationships among the criteria.

Table 6. Total Relation Matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	0.78052	0.63804	0.641779	0.728486	0.675818	0.725878	0.678997	0.729379
C_2	0.578897	0.567612	0.581145	0.656973	0.562045	0.605992	0.575211	0.581755
C_3	0.777086	0.663658	0.671981	0.755927	0.721335	0.827244	0.654296	0.722731
C_4	0.896803	0.740791	0.719018	0.864778	0.797122	0.847405	0.811261	0.840283
C_5	0.630744	0.620556	0.571392	0.68379	0.586471	0.641714	0.594503	0.639846
C_6	0.652552	0.621959	0.622008	0.704268	0.631759	0.707185	0.578163	0.657304
C_7	0.822579	0.709648	0.736538	0.834103	0.769943	0.827511	0.754703	0.789989
C_8	0.662991	0.594323	0.582726	0.648888	0.591295	0.629447	0.567446	0.61186

4.2.3 Cause-and-Effect Analysis

To identify the causal and effectual roles of the criteria, the following metrics were computed:
Sum of Rows (Di): Represents the total influence exerted by a criterion on others.
Sum of Columns (Ri): Represents the total influence received by a criterion from others.
The net influence of each criterion was calculated as: Di−Ri
A positive value indicates a criterion is primarily a cause, while a negative value denotes it as an effect. The total importance of each criterion was determined as: Di+Ri
The cause-and-effect diagram, depicted in Figure 3, visually represents interdependence among the criteria. Key insights from the analysis include:

1. Criterion 4 emerged as the most influential cause, highlighting its central role in enhancing teaching quality and its strong impact on other criteria.
2. Criterion 2 exhibited the lowest influence, suggesting its relatively peripheral role within the evaluated context and limited impact on other criteria.

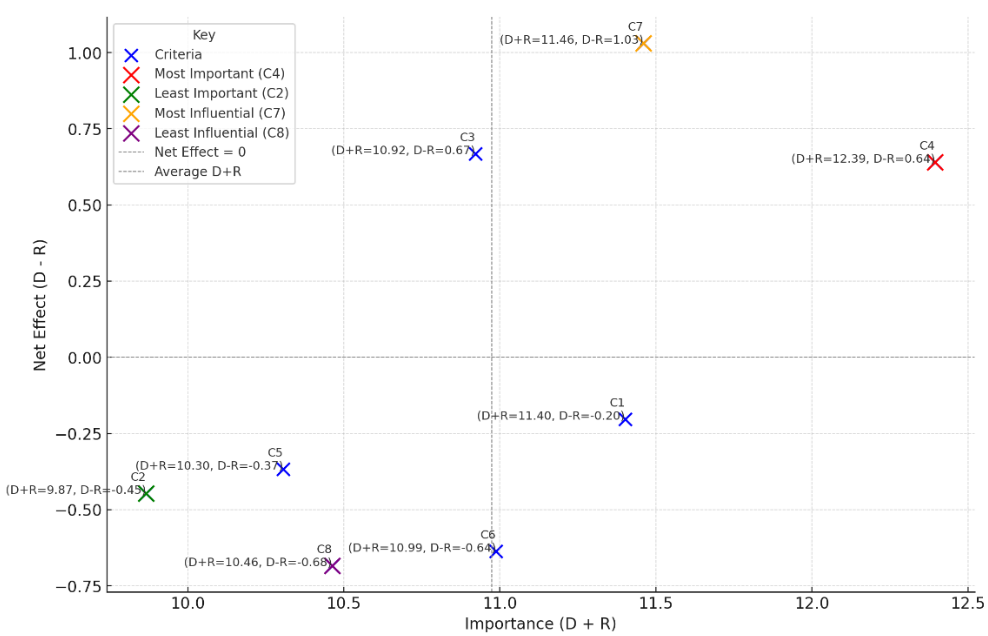

Figure 3. The cause-and-effect diagram.

5. Discussion

The proposed methodology's integration of HyperSoft Sets and SVNSs within the DEMATEL framework introduces several pivotal contributions to the field of multi-criteria decision-making under uncertainty. First, the use of HyperSoft Sets enhances the representation of complex systems by enabling multi-attribute evaluations. Unlike traditional soft sets, HyperSoft Sets allow for the simultaneous consideration of overlapping and interrelated attributes, which is crucial in scenarios involving multiple interconnected criteria. This capability ensures a more nuanced and accurate analysis of the relationships among the criteria, particularly in environments where attributes are not mutually exclusive.

Second, the incorporation of SVNSs provides a robust mechanism for managing uncertainty and vagueness inherent in expert evaluations. By explicitly accounting for truth-membership, indeterminacy-membership, and falsity-membership, SVNSs allow decision-makers to model and analyze criteria with incomplete or ambiguous information effectively. This characteristic ensures the reliability of the decision-making process even in scenarios characterized by imprecise or conflicting data.

Finally, the methodology offers a holistic analytical approach by not only quantifying the direct relationships between criteria but also capturing the indirect influences that emerge within the system. This comprehensive analysis provides a deeper understanding of interdependence and dynamics among the criteria. As a result, decision-makers can identify key drivers (causal factors) and outcomes (effects) with greater precision, facilitating strategic interventions and informed decision-making. These contributions collectively highlight the practical and theoretical significance of the proposed methodology in addressing complex evaluation problems across various domains.

5.1 Sensitivity Analysis

This section evaluates the robustness of the proposed methodology by conducting a sensitivity analysis. The analysis investigates how variations in input values, specifically D (sum of rows) and R (sum of columns) from the total relation matrix, affect the calculated metrics D+R (Total Importance) and D−R (Net Effect).

By applying adjustments of −10%, −5%, 0%(baseline), +5%, and + 10 % to these values, we aim to understand the stability of the results and identify criteria that are more sensitive to changes.

Figure 4 shows the sensitivity of D+R, which represents the total importance of each criterion. This graph highlights how the influence of criteria such as C4 remains dominant across all scenarios, with its values increasing or decreasing proportionally with adjustments. Conversely, criteria like C2, which have lower baseline values, exhibit relatively smaller variations in D+R.

Figure 5 focuses on the sensitivity of D−R, representing the net effect of each criterion. This graph illustrates how C4 consistently emerges as the most influential cause (positive D−R) regardless of changes in input values. On the other hand, C2 remains the least impactful criterion (negative D−R), indicating its reliance on improvements in other areas.

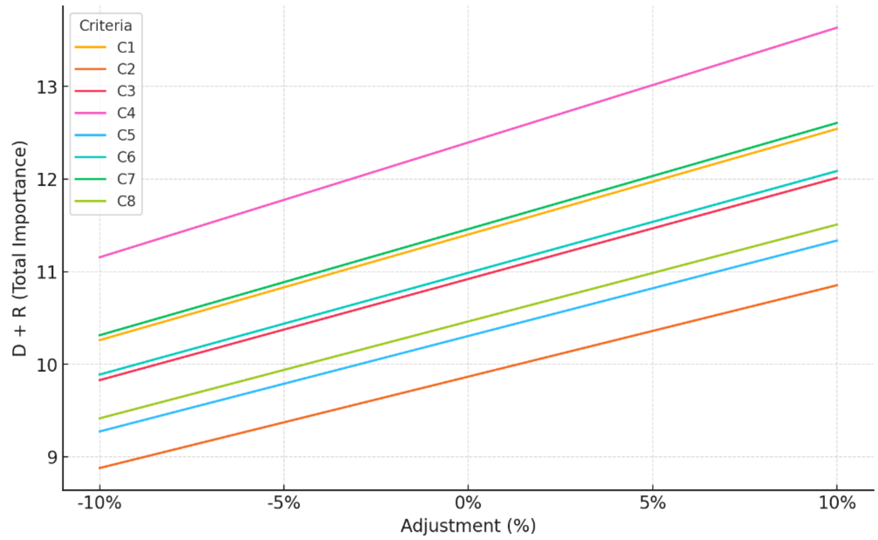

Figure 4: Sensitivity of D+R (Total Importance)

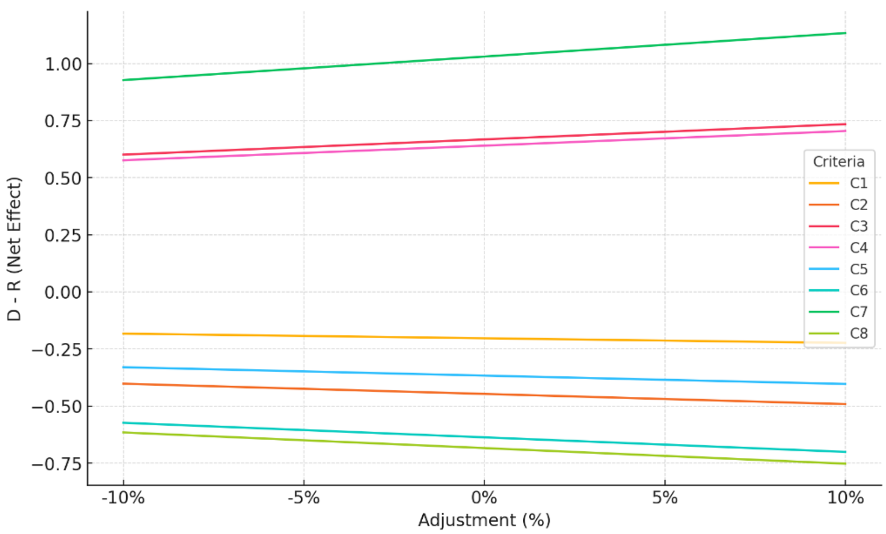

Figure 5: Sensitivity of D-R (Net Effect)

C4 (Problem-Solving Skills) consistently holds the highest total importance across all adjustment scenarios, while C7 (Assessment and Feedback Quality) emerges as the most influential criterion with the highest positive net effect. On the other hand, C2 (Classroom Engagement) demonstrates the lowest total importance, and C8 (Use of Technology) consistently shows the lowest net effect, indicating their peripheral roles. The stability of these rankings and classifications across all scenarios confirms the robustness and reliability of the DEMATEL-based framework integrated with the hypersoft set approach. These findings emphasize the critical roles of C4 and C7 in enhancing teaching quality and offer valuable guidance for educators and policymakers in optimizing resource allocation and strategic planning.

4. Conclusions

This paper proposed a multi-criteria decision-making approach to evaluate teaching quality in university mathematics classrooms based on core competencies. By employing the DEMATEL method, the study analyzed the relationships among various evaluation criteria, uncovering their interdependence and influence levels. A panel of three experts provided assessments to construct the pairwise comparison matrix, which served as the foundation for subsequent computations.

SVNNs were used to address uncertainty and ambiguity in the evaluation process. A scoring function was applied to transform the neutrosophic data into crisp values, which were then aggregated into a unified matrix. The hypersoft set framework was employed to model the criteria and their relationships, enabling a more comprehensive and robust analysis. The normalized direct relation matrix and the total relation matrix provided detailed insights into the causal structure of the criteria.

Among the eight criteria evaluated, C4 (Problem-Solving Skills) emerged as the most impactful, highlighting its critical role in enhancing teaching quality. In contrast, C2 (Classroom Engagement) was identified as having the lowest impact, indicating its reliance on improvements in other areas. These findings offer actionable insights for educators and policymakers to focus on key drivers, allocate resources strategically, and optimize teaching practices to achieve better outcomes.

4.1 Limitations

While this study presents a robust evaluation framework, it is not without its limitations. Firstly, the analysis relies on input from only three experts, which may limit the generalizability of the findings. Expanding the pool of experts could provide a broader perspective and enhance the reliability of the results. Secondly, the evaluation is context-specific, focusing on university mathematics classrooms. The criteria and their relationships may differ in other educational settings or disciplines, requiring adaptations to the proposed framework.

Another limitation is the reliance on the DEMATEL method alone. While effective for analyzing causal relationships, integrating other MCDM methods, such as AHP or TOPSIS, could provide complementary insights and further validate the findings. Additionally, the use of neutrosophic sets, while powerful in handling uncertainty, requires expertise to interpret the results, potentially limiting its application for non-specialists.

4.2 Future Work

Future research could address these limitations by involving a larger and more diverse group of experts from various educational institutions. This would improve the robustness and applicability of the framework across different contexts. Expanding the study to include other academic disciplines and educational levels could also provide a more comprehensive understanding of teaching quality evaluation.

The integration of complementary MCDM methods, such as fuzzy AHP or hybrid approaches combining DEMATEL with TOPSIS, could enhance the depth of analysis and provide additional perspectives on the interdependencies among criteria. Furthermore, incorporating dynamic data collection methods, such as real-time feedback from students or advanced learning analytics, could make the evaluation process more responsive and adaptive to changing classroom dynamics.

Developing user-friendly software tools to automate the framework's calculations and visualizations would also be a valuable step. Such tools could make the proposed methodology accessible to a wider audience, including educators and policymakers with limited technical expertise.

4.3 Practical Implications

The findings of this study have significant practical implications for improving teaching quality in university mathematics classrooms. Prioritizing C4 (Problem-Solving Skills) can have a cascading positive effect on other dependent criteria, such as conceptual understanding and classroom engagement. Policymakers and educators should design interventions that promote active participation, including interactive teaching methods, collaborative activities, and the integration of digital tools.

Although C2 (Classroom Engagement) was identified as having the lowest direct impact, it remains a critical outcome of effective teaching. Indirect improvements to classroom engagement can be achieved by focusing on foundational and influential criteria, such as problem-solving skills and assessment quality.

These insights can guide the allocation of resources, and the development of professional training programs aimed at equipping educators with effective strategies to address these areas.

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