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Finite-precision stabilization for neutrosophic linear systems using a descriptor model approach

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Abstract. This paper focuses on the stabilization problem of neutrosophic linear systems using finite-precision digital controllers. The main challenge lies in decoupling and processing the indeterminacies inherent in such systems. To tackle this, a standardization method that unifies the bounds of parameter indeterminacies is first proposed. Additionally, a fixed-point representation scheme is introduced to define both the precision and representation of the controller parameters. To handle multiple uncertainties in the system, a descriptor model approach is introduced, which transforms the closed-loop system into a singular system, allowing for more effective decoupling of uncertainties. Using singular system theory, a novel solvability condition is derived for system stabilization and a parametric approach for controller design is introduced, accompanied by a numerical example to validate the proposed methods.

Keywords: Neutrosophic linear systems; Finite word length; Descriptor models; System stabilization

1. **Introduction**

Parameter uncertainty is common in model-based analysis and control problems, including military, electricity, medical, etc [1–3]. The presence of uncertainty in system parameters can lead to significant performance degradation and adversely affect the system's stability, as the controller must be designed to operate under the worst-case scenario. Throughout the years, various methods have been developed to tackle uncertainty in control systems, including robust control techniques [4,5], fuzzy logic systems [6,7] and filter-based approach [8,9]. These approaches aim to mitigate the impact of uncertainty on system performance by modeling the uncertainty and ensuring system robustness against it.

A more recent and innovative approach to managing parameter uncertainty is the use of neutrosophic numbers. This method has been applied to both transfer function cases [10] and state-space model cases [11]. Meanwhile, the neutrosophic theory has also been successfully applied to control engineering, e.g., neutrosophic estimation [12], decision making [13], teaching quality evaluation [14] and clustering analysis [15]. Due to the unique structure of neutrosophic numbers, which comprises both determinate and indeterminate components, it is natural appropriate for representing the control systems with parameter uncertainties. With the above-mentioned characteristics, the neutrosophic linear systems is established in [11] to describe parameter uncertain systems and the state feedback stabilization for neutrosophic linear systems is also investigated for the SISO case. However, further research is needed to extend the stabilization problem of MIMO neutrosophic linear systems with more practical constraints being taken into consideration.

In the context of digital controller design, a critical challenge arises due to the finite precision of digital representations. Controller parameters must be quantized to fit within a finite word length (FWL), introducing representation errors that can significantly degrade system performance or even cause instability [16, 17]. The influence of finite word length on system performance has been a subject of extensive study with various stability measures proposed [18, 19]. However, most existing studies focus on FWL effects in isolation, without considering the combined impact of other uncertainties, such as parameter indeterminacies and model inaccuracies. Furthermore, existing approaches are often limited to state-space representations, which are not general enough to address more complex system architectures or the practical constraints of controller realization [20, 21]. Given these challenges, there is a clear need for novel modeling and controller design methods that integrate both parameter uncertainty and finite word length effects. Such methods would provide greater flexibility and accuracy in system analysis and control design, especially for systems with multiple uncertainties and practical implementation constraints.

The stabilization of MIMO neutrosophic linear systems under realistic digital implementation conditions is a complex and underexplored problem in control theory. Existing literature on parameter uncertainty focuses primarily on SISO systems, leaving the stabilization of more complex MIMO systems with parameter uncertainties, especially under finite-precision digital controller constraints, as a critical research gap. This paper proposes an innovative approach that combines neutrosophic modeling with FWL stability analysis, offering a comprehensive solution to the stabilization of MIMO neutrosophic linear systems under realistic digital implementation conditions. To the best of the authors' knowledge, there is a lack of systematic studies in the literature that specifically address the stabilization problem for MIMO neutrosophic linear systems. Although considerable attention has been devoted to the finite word length (FWL) stability analysis of control systems, there is a notable lack of information concerning finite-precision digital controllers design for parameter uncertainty systems.

The aforementioned discussion motivated this study, where the stabilization problem subject to the FWL effect is considered for neutrosophic linear systems. Without requiring additional constraints or complex analysis methods, a novel descriptor model approach is proposed in this paper to describe the closed-loop system with multiple uncertainties. Meanwhile, a systematic procedure is provided on how to separate the coupled uncertainties of plant and controller gains. And the parameterization of the feedback gain is therefore accomplished. This paper presents several novel contributions: 1) a new descriptor model is introduced to represent the closed-loop system with multiple uncertainties, enabling more effective decoupling of plant and controller uncertainties without requiring additional constraints or complex analysis; 2) separation of Coupled Uncertainties: A systematic procedure is proposed to separate the coupled uncertainties in the plant and controller gains, simplifying the stabilization process; 3) a novel method for parameterizing the feedback gain is presented, ensuring more effective and practical digital controller design under finite precision.

Notation

The definitions presented below are used consistently throughout the paper. $\mathbb{R}^n$ represents the n-dimensional Euclidean space, while $\|\cdot\|$ represents the Euclidean norm. For a square matrix, the superscripts-1 and T indicate the inverse and transpose, respectively. Additionally, $\mathbb{Z}$ refers to the set of all neutrosophic numbers, while $\mathbb{Z}^{n\times m}$ denotes the collection of $n\times n$ neutrosophic matrices. $\mathbb{N}$ represents the set of all integers and $\mathbb{N}^+$ denotes the set of all positive integral numbers. The space $l_2[0,\infty)$ consists of square-summable infinite sequences. The symbol $\odot$ and $\otimes$ denote the Hadamard product and Kronecker product, respectively. U_n denotes the identity matrix with $n\times n$ dimensions. The notation 0_n refers to an $n\times n$ zero matrix, and $0_{n\times m}$ denotes a zero matrix of dimensions $n\times m$. $A = diag(a_1,\ a_2,\ \cdots,\ a_n)$ is a diagonal matrix with diagonal elements a_i. The symbol $*$ is used in some matrix expressions to represent the symmetric terms. Moreover, $E\{\cdot\}$, $\rho\{\cdot\}$, $det\{\cdot\}$ and $deg\{\cdot\}$ stand for mathematical expectation, spectral radius, matrix determinant, and polynomial degree, respectively.

2. **Preliminary**

This paper focuses on the stabilization problem for neutrosophic linear systems, where the uncertainty in the system parameters is modeled using neutrosophic numbers. The feedback gain is implemented digitally, with its parameters represented using a floating-point scheme.

To address the coupled constraints inherent in this stabilization problem, a descriptor model-based approach is proposed. This section provides a brief introduction to relevant basic concepts for neutrosophic number, floating-point scheme and descriptor model to facilitate further discussion.

2.1. *Neutrosophic number*

Neutrosophic numbers extend traditional numbers by allowing for more flexibility in representing uncertainty, imprecision, and contradiction. These numbers are often used in areas like decision-making, fuzzy logic, and artificial intelligence, especially when dealing with systems that have conflicting, uncertain, or incomplete information. In uncertain scenarios, a neutrosophic number z is expressed in the standard form as [22]: $z = a + b\mathcal{I} \in \mathbb{Z}$, where a represents the determinate term and $b\mathcal{I}$ represents the indeterminate term. Here, $a,\ b,\ \mathcal{I} \in \mathbb{R}$ with $\mathcal{I}$ lying within the interval $[\mathcal{I}^L,\ \mathcal{I}^U]$. For two neutrosophic numbers with specific forms of $z_1 = a_1 + b_1\mathcal{I}$, $z_2 = a_2 + b_2\mathcal{I}$, the operations are defined as follows [23]:

$$z_1 + z_2 = (a_1 + b_1\mathcal{I}) + (a_2 + b_2\mathcal{I}) = (a_1 + a_2) + (b_1 + b_2)\mathcal{I},$$

$$z_1 - z_2 = (a_1 + b_1\mathcal{I}) - (a_2 + b_2\mathcal{I}) = (a_1 - a_2) + (b_1 - b_2)\mathcal{I},$$

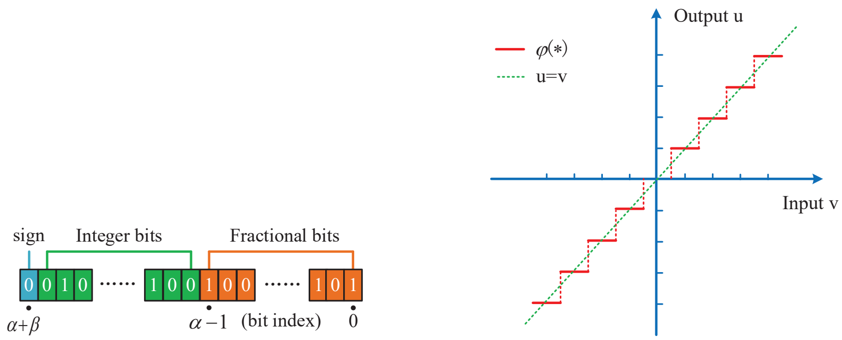
$$z_1 \times z_2 = (a_1 + b_1\mathcal{I}) \times (a_2 + b_2\mathcal{I}) = a_1a_2 + (a_1b_2 + a_2b_1)\mathcal{I} + b_1b_2\mathcal{I}^2,$$

$$\frac{z_1}{z_2} = \frac{a_1 + b_1\mathcal{I}}{a_2 + b_2\mathcal{I}}.$$

2.2. *Parameter representation*

A representation scheme refers to the method or structure used to represent data, information, or numbers in a specific format within a system or framework. Choosing a suitable representation scheme is essential because it determines how information is represented, manipulated, and interpreted by computers, algorithms, or other systems. When approximating a real number $a \in \mathbb{R}$ digitally, the accuracy of its representation is influenced by the chosen structure and word length. This paper introduces a fixed-point representation approach. scheme [25]. to represent the parameters of the feedback gain. The fixed-point representation scheme utilizes a overall bit length of $\gamma = \alpha + \beta + 1$, as shown in Figure 1(a). Here, 1 bit is allocated for the sign, α bits is reserved for the integer part, and β bits for the fractional part. This representation allows for accurate representation of the integral part of a given real number a without overflow, assuming the word length is sufficiently large, specifically $\alpha = \log_2\lfloor a\rfloor$. However, the fractional part cannot be exactly represented, and the error in representation of a is determined only by the word length $\beta \in \mathbb{N}^+$. In particular, after representing a in fixed-point form, it will be perturbed by this error as:

$$\varphi(a) = a + \Delta_a,\ |\Delta_a| < 2^{-(\beta+1)} \in \mathbb{R}, \tag{1}$$



(a) (b)

FIGURE 1. Fixed-point representation: (a) Storage model; (b) Input-output characteristic.

where $\varphi(*)$ represents the mapping to the fixed-point form with the input-output characteristic being shown in Figure 1(b).

2.3. *Descriptor model*

Singular system refers to system of differential equations that include both differential equations and algebraic constraints. These systems are often more complex to solve because they can have singularities that make traditional analysis methods inapplicable. Unlike state-space models, singular systems described by the descriptor models must address regularity and causality, in addition to stability. In a discrete-time singular system:

$$\mathcal{E}\S(k+1) = \mathcal{A}\S(k),$$

where $\S(k) \in \mathbb{R}^{n_\S}$ denotes the state vector, $\mathcal{E},\ \mathcal{A} \in \mathbb{R}^{n_\S \times n_\S}$ are system parameter matrices with known numerical values and $\mathcal{E}$ maybe singular, i.e. $rank(\mathcal{E}) \leq n_\S$, the following definitions are introduced [24]:

- The singular system is regular if $det(z\mathcal{E} - \mathcal{A})$ is not identically zero;
- The singular system is causal if $deg[det(z\mathcal{E} - \mathcal{A})] = rank(\mathcal{E})$;
- The singular system is stable if $\rho(\mathcal{E},\ \mathcal{A}) < 1$;
- The singular system is admissible if it is regular, causal and stable.

3. **Problem Formulation**

Consider systems with unknown parameters described by the following discrete-time neutrosophic state-space model:

$$\begin{cases} x(k+1) = A(I)x(k) + B(I)u(k), \\ \quad y(k) = C(I)x(k), \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^p$ are the state, the control input and the controlled output, respectively. $A(I) \in \mathbb{Z}^{n\times n}$, $B(I) \in \mathbb{Z}^{n\times m}$ and $C(I) \in \mathbb{Z}^{p\times n}$ in (2) are neutrosophic matrices with the specific forms given by:

$$A(I) = \tilde{A} + \breve{A}I,\ B(I) = \tilde{B} + \breve{B}I,\ C(I) = \tilde{C} + \breve{C}I, \tag{3}$$

where $\tilde{A} \in \mathbb{R}^{n\times n}$, $\breve{A} \in \mathbb{R}^{n\times n}$, $\tilde{B} \in \mathbb{R}^{n\times m}$, $\breve{B} \in \mathbb{R}^{n\times m}$, $\tilde{C} \in \mathbb{R}^{p\times n}$, $\breve{C} \in \mathbb{R}^{p\times n}$, $I \in [\breve{I},\ \hat{I}]$ with $\breve{I}$ and $\hat{I}\ \in \mathbb{R}$. In Eq. (3), the indeterminate term I is introduced to express the uncertainty of system parameters caused by manufacturing tolerance, external environment change and other possible factors.

Without loss of generality, assuming the system described by (2) is completely state-controllable [11], and for further stabilization, the state-feedback control law is required to be designed in the following form:

$$v(k) = \begin{bmatrix} v_1(k) & v_2(k) & \cdots & v_m(k) \end{bmatrix}^T = Kx(k), \tag{4}$$

where $K \in \mathbb{R}^{m\times n}$ is the state-feedback gain to be determined. Due to resource limitations and to ensure implementation feasibility, the feedback gain K in (4) must be digitally implemented with finite word length, and the fixed-point representation scheme described in Section 2.2 is adopted. The integer parts of all coefficients in K are assumed to be represented exactly without overflow, whereas the fractional parts are represented with a finite bit length β. Thus, the coefficient representation depends only on β, and the integer word length α is omitted in the rest of this paper.

Based on the discussion in Section 2.2, the representation of a real matrix can be further examined. For a given feedback gain K, let $D^K \in \mathbb{R}^{m\times n}$ be a real matrix with the same dimensions as K, and its elements are determined as follows:

$$D^K_{ij} = \begin{cases} 0,\ if\ K_{ij} \in \mathbb{N}, \\ 1,\ if\ K_{ij} \notin \mathbb{N}, \end{cases}$$

where D^K_{ij} represents the element of D^K in the i-th row and j-th column, while K_{ij} denotes the element in the i-th row and j-th column of K.

In general, controller parameters require careful design and tuning, with their values typically needing to be moderate. High-gain controller parameters can lead to issues such as controller saturation, amplification of measurement errors, and system oscillations. To address these concerns, it is reasonable to assume that the parameters in the feedback gain K are moderate. Specifically, the integer part of these parameters can be accurately represented using a fixed-point representation with a sufficient number of bits α. Therefore, in the remainder of this paper, the discussion will focus exclusively on the effects of finite word length on the fractional part of the controller parameters. Using a fixed-point representation with β bits for the fractional part and considering the analysis of the representation error provided in (1), the feedback gain is perturbed to

$$\varphi(K) = K + \Delta K \in \mathbb{R}^{m\times n}, \tag{5}$$

where

$$\Delta K = \begin{bmatrix} D_{11}^K\Delta_{11}^K & \cdots & D_{1n}^K\Delta_{1n}^K \\ \vdots & \ddots & \vdots \\ D_{m1}^K\Delta_{m1}^K & \cdots & D_{mn}^K\Delta_{mn}^K \end{bmatrix},\ |\Delta_{ij}^K| < 2^{-(\beta+1)} \in \mathbb{R}.$$

Based on the above discussion, the control input $u(k)$ in (2) can be obtained as:

$$u(k) = \varphi(K)x(k) = (K + \Delta K)x(k). \tag{6}$$

Combining the plant (2) and its control input (6), the closed-loop system is obtained as:

$$\begin{cases} x(k+1) = [(\tilde{A} + \check{A}I) + (\tilde{B} + \check{B}I)(K + \Delta K)]x(k), \\ y(k) = (\tilde{C} + \check{C}I)x(k). \end{cases} \tag{7}$$

The control problem addressed in this paper is described as follows.

Problem 1. Design the state-feedback gain K to guarantee the asymptotic stability of the closed-loop system in (7), subject to the unknown indeterminacy I and ΔK.

Noting the unknown indeterminacies I and ΔK present in the closed-loop system (7), Problem 1 is identified as a stabilization problem with multiple coupled unknown coefficients, which makes it challenging to parameterize the feedback gain K. Therefore, a descriptor model approach is proposed in the following section to address these challenges, along with the corresponding solvability conditions for Problem 1.

4. Main Results

In this section, a comprehensive solution method for solving Problem 1 is detailed. More specifically, a descriptor model approach is proposed in subsection 4.1 to rewrite the closed-loop system (7). Based on the obtained equivalent model for (7), a stability analysis condition is deduced in subsection 4.2, along with a method for designing the feedback gain. And Figure 2 illustrates the key steps of the proposed method, providing a clear overview of the entire process from system modeling to stabilization.

To aid in deriving the main results, the following lemmas are presented first.

Lemma 4.1. *For a neutrosophic number $z = a + bI$ with $a,\ b,\ I \in \mathbb{R}$ and $\mathcal{I} \in [\check{I},\ \hat{I}]$, it can be rewritten as:*

$$z = \bar{a} + \bar{b}\bar{I}, \tag{8}$$

where $\bar{a} = a + b\frac{\check{I}+\hat{I}}{2}$, $\bar{b} = b\frac{\check{I}-\hat{I}}{2}$, $\bar{I} \in [-1, 1]$.

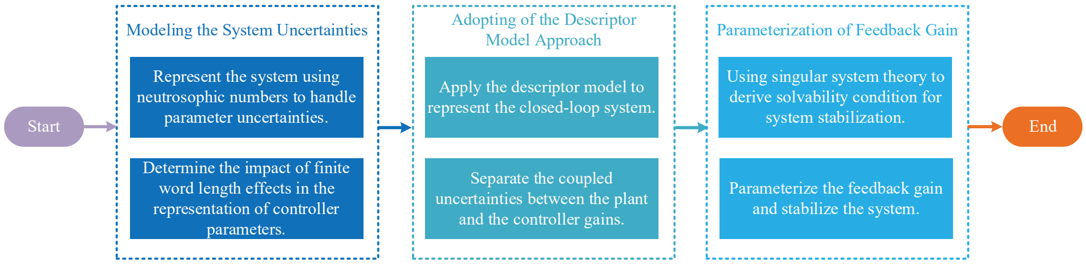


FIGURE 2. Flowchart of the proposed method.

Proof. By submitting $\bar{a} = a + b\frac{\check{I}+\hat{I}}{2}$ and $\bar{b} = b\frac{\check{I}-\hat{I}}{2}$ into Eq. (8), it is obtained that

$$z = a + b\frac{\check{I}+\hat{I}}{2} + b\frac{\check{I}-\hat{I}}{2}\bar{I}.$$

Noting that $\bar{I} = \frac{2}{\check{I}-\hat{I}}(I - \frac{\check{I}+\hat{I}}{2}) \in [-1, 1]$, the above equation can be expressed as:

$$z = a + b\frac{\check{I}+\hat{I}}{2} + b\frac{\check{I}-\hat{I}}{2}\frac{2}{\check{I}-\hat{I}}(I - \frac{\check{I}+\hat{I}}{2}) = a + b\frac{\check{I}+\hat{I}}{2} + b(I - \frac{\check{I}+\hat{I}}{2}).$$

Then it is observed that $z = \bar{a} + \bar{b}\bar{I} = a + bI$. □

Lemma 4.2. *[26] For matrices Φ_1, Φ_2 and Φ_3 with appropriate dimensions with Φ_1 satisfying $\Phi_1 = \Phi_1^T$, then*

$$\Phi_1 + \Phi_2\Phi_4(k)\Phi_3^T + \Phi_3\Phi_4^T(k)\Phi_2^T < 0$$

holds for all $\Phi_4^T(k)\Phi_4(k) < U$ if and only if a scalar $\epsilon > 0$ exists such that

$$\Phi_1 + \epsilon\Phi_2\Phi_2^T + \epsilon^{-1}\Phi_3\Phi_3^T < 0.$$

Lemma 4.3. *[24] For real matrices Φ_{11}, Φ_{12}, Φ_{21} and Φ_{22} with appropriate dimensions, if $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$ is invertible and satisfies $\Phi + \Phi^T < 0$. There holds the following inequality:*

$$\Phi_{11} + \Phi_{11}^T - \Phi_{12}\Phi_{22}^{-1}\Phi_{21} - \Phi_{21}^T\Phi_{22}^{-T}\Phi_{12}^T < 0.$$

4.1. *System remodeling*

This subsection aims to introduce a new approach for remodeling the closed-loop system (7). This framework will enable the proposal of an appropriate analysis and design methodology that can effectively handle the coupled unknown parameters. To this end, the unknown indeterminacy I and the corresponding system parameters in (2) are standardized by employing the method outlined in Lemma 4.1.

By denoting $\bar{I} \in [-1, 1]$, the standardization can be obtained for parameters in (2) as:

$$\begin{aligned} &A_1 = \tilde{A} + \frac{\hat{I} + \check{I}}{2}\breve{A},\ A_2 = \tilde{A} + \frac{\hat{I} - \check{I}}{2}\breve{A},\ B_1 = \tilde{B} + \frac{\hat{I} + \check{I}}{2}\breve{B}, \\ &B_2 = \tilde{B} + \frac{\hat{I} - \check{I}}{2}\breve{B},\ C_1 = \tilde{C} + \frac{\hat{I} + \check{I}}{2}\breve{C},\ C_2 = \tilde{C} + \frac{\hat{I} - \check{I}}{2}\breve{C}, \\ &A_1 \in \mathbb{R}^{n\times n}, A_2 \in \mathbb{R}^{n\times n},\ B_1 \in \mathbb{R}^{n\times m}\ B_2 \in \mathbb{R}^{n\times m},\ C_1 \in \mathbb{R}^{p\times n},\ C_2 \in \mathbb{R}^{p\times n}. \end{aligned}$$

With the above standardization, the closed-loop system described by (7) can be reformulated as:

$$\begin{cases} x(k+1) = [(A_1 + A_2\bar{I}) + (B_1 + B_2\bar{I})(K + \Delta K)]x(k), \\ \quad y(k) = (C_1 + C_2\bar{I})x(k). \end{cases} \tag{9}$$

To further address the coupling between $\bar{I}$ and ΔK, an intermediate variable $\zeta(k) \in \mathbb{R}^m$ is introduced, and the closed-loop system (9) is reformulated in the descriptor model form as follows:

$$\begin{cases} \hat{E}\hat{x}(k+1) = (\hat{A}_1 + \hat{A}_2\bar{I} + \hat{B}\hat{K}_1 + \hat{B}\hat{K}_2)\hat{x}(k), \\ \quad y(k) = (\hat{C}_1 + \hat{C}_2\bar{I})\hat{x}(k), \end{cases} \tag{10}$$

where

$$\hat{x} = \begin{bmatrix} x(k) \\ \zeta(k) \end{bmatrix},\ \hat{E} = \begin{bmatrix} U_n & 0_{n\times m} \\ 0_{m\times n} & 0_m \end{bmatrix},\ \hat{A}_1 = \begin{bmatrix} A_1 & B_1 \\ 0_{m\times n} & -U_m \end{bmatrix},\ \hat{A}_2 = \begin{bmatrix} A_2 & B_2 \\ 0_{m\times n} & 0_m \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0_{n\times m} \\ U_m \end{bmatrix},\ \hat{K}_1 = \begin{bmatrix} K^T \\ 0_m \end{bmatrix}^T,\ \hat{K}_2 = \begin{bmatrix} \Delta K^T \\ 0_m \end{bmatrix}^T \hat{C}_1 = \begin{bmatrix} C_1^T \\ 0_{m\times p} \end{bmatrix}^T,\ \hat{C}_2 = \begin{bmatrix} C_2^T \\ 0_{m\times p} \end{bmatrix}^T,$$

and $\zeta(k) \in \mathbb{R}^m$ is the intermediate vector introduced.

4.2. *Stability analysis and stabilization*

Based on the reformulated descriptor model (10), this subsection derives a stability condition for Problem 1 and proposes a technique for feedback gain design.

Theorem 4.4. *For a given scalar $\beta \in \mathbb{N}^+$, the descriptor system (10) is admissible provided that matrices $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times m}$, $S \in \mathbb{R}^{m\times m}$, $P \in \mathbb{R}^{n\times n}$ (with $P > 0$), and scalars $\epsilon > 0$, $\epsilon_i > 0$ for $i = 1,\ 2,\ \cdots,\ m$ exist, such that the following inequality holds:*

$$\begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & \Theta_{12} & P - Q^T - \frac{1}{2}Q & \Theta_{41} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{42} \\ * & * & -Q - Q^T & \Theta_{43} \\ * & * & * & \Theta_{44} \end{bmatrix} < 0, \tag{11}$$

where

$$\Theta_{12} = \Phi_1(\hat{A}_1 + \hat{B}\hat{K}_1)^T,\ \Theta_{23} = \Theta_{12}^T,\ \Theta_{22} = \Phi_2(\hat{A}_1 + \hat{B}\hat{K}_1)^T + (\hat{A}_1 + \hat{B}\hat{K}_1)\Phi_2^T - \Phi_3 + \Phi_4,$$

$$\Theta_{41} = \Theta_{43} = \underbrace{\begin{bmatrix} \Phi_1 & \Phi_1 & \cdots & \Phi_1 \end{bmatrix}}_{m+1},\ \Theta_{42} = \underbrace{\begin{bmatrix} \Phi_2 & \Phi_2 & \cdots & \Phi_2 \end{bmatrix}}_{m+1},\ \Theta_{44} = E_\epsilon \otimes U_{(m+1)(n+m)},$$

$$\Phi_1 = \begin{bmatrix} Q & R \end{bmatrix},\ \Phi_2 = \begin{bmatrix} 0_n & 0_{n\times m} \\ 0_{m\times n} & S \end{bmatrix},\ \Phi_3 = \begin{bmatrix} P & 0_{n\times m} \\ 0_{m\times n} & 0_m \end{bmatrix},\ \Phi_4 = \begin{bmatrix} \Phi_{41} & 0_{n\times m} \\ 0_{m\times n} & \Phi_{42} \end{bmatrix},$$

$$E_\epsilon = diag([\epsilon\ \epsilon_1\ \epsilon_2\ \cdots\ \epsilon_m]),\ \Delta_\beta = 2^{-(\beta+1)},\ \Phi_{41} = \epsilon(A_2A_2^T + B_2B_2^T),\ \Phi_{42} = \Delta_\beta^2 E_\epsilon \odot (D_K D_K^T),$$

$$D_K = \begin{bmatrix} D_{11}^K & \cdots & D_{1n}^K \\ \vdots & \ddots & \vdots \\ D_{m1}^K & \cdots & D_{mn}^K \end{bmatrix}.$$

Proof of Theorem 4.4. Assuming that the inequality (11) holds, one can apply Schur complement to obtain

$$\bar{\Phi}_4 + \epsilon^{-1}\bar{\Phi}_1\bar{\Phi}_1^T + \epsilon\bar{\Phi}_2\bar{\Phi}_2^T + \sum_{i=1}^{m}(\epsilon_i^{-1}\bar{\Phi}_1\bar{\Phi}_1^T) + \bar{\Phi}_3 < 0,$$

where

$$\bar{\Phi}_4 = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & \Theta_{12} & P - Q^T - \frac{1}{2}Q \\ * & \bar{\Theta}_{22} & \Theta_{23} \\ * & * & -Q - Q^T \end{bmatrix},\ \bar{\Phi}_1 = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_1 \end{bmatrix},\ \bar{\Phi}_2 = \begin{bmatrix} 0_{n\times(n+m)} \\ \hat{A}_2 \\ 0_{n\times(n+m)} \end{bmatrix},$$

$$\bar{\Phi}_3 = \begin{bmatrix} 0_n & 0_n & 0_{n\times m} \\ * & 0_n & 0_{n\times m} \\ * & * & \Phi_4 \end{bmatrix},\ \bar{\Theta}_{22} = \Phi_2(\hat{A}_1 + \hat{B}\hat{K}_1)^T + (\hat{A}_1 + \hat{B}\hat{K}_1)\Phi_2^T - \Phi_3.$$

Noting that

$$\bar{\Phi}_3 = \sum_{i=1}^{m}(\Delta_\beta^2\epsilon_i\bar{\Phi}_{3i}\bar{\Phi}_{3i}^T)$$

where

$$\bar{\Phi}_{3i} = \begin{bmatrix} 0_{n\times(n+m)} \\ \bar{D}_{Ki} \\ 0_{n\times(n+m)} \end{bmatrix},\ \bar{D}_{Ki} = \begin{bmatrix} 0_n & 0_{n\times m} \\ 0_{m\times n} & D_{Ki} \end{bmatrix},\ D_{Ki} = \begin{bmatrix} 0_{(i-1)\times n} \\ [D_{i1}^K\ D_{i2}^K\ \cdots\ D_{in}^K] \\ 0_{(m-i)\times n} \end{bmatrix},$$

there holds

$$\bar{\Phi}_4 + \bar{\Phi}_4 + \epsilon^{-1}\bar{\Phi}_1\bar{\Phi}_1^T + \epsilon\bar{\Phi}_2\bar{\Phi}_2^T + \sum_{i=1}^{m}(\epsilon_i^{-1}\bar{\Phi}_1\bar{\Phi}_1^T + \Delta_\beta^2\epsilon_i\bar{\Phi}_{3i}\bar{\Phi}_{3i}^T) < 0.$$

Based on the above inequality, applying Lemma 4.2 yields:

$$\bar{\Phi}_5 = \bar{\Phi}_4 + \bar{\Phi}_1\bar{I}\bar{\Phi}_2^T + \bar{\Phi}_2\bar{I}\bar{\Phi}_1^T + \sum_{i=1}^{m}(\Delta_\beta\bar{\Phi}_1\frac{\bar{\Delta}_i}{\Delta_\beta}\bar{\Phi}_{3i}^T + \Delta_\beta\bar{\Phi}_{3i}\frac{\bar{\Delta}_i}{\Delta_\beta}\bar{\Phi}_1^T) < 0,$$

where

$$\bar{\Phi}_5 = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & \Phi_1 \hat{A}^T & P - Q^T - \frac{1}{2}Q \\ * & \Phi_2 \hat{A}^T + \hat{A}\Phi_2^T - \Phi_3 & \hat{A}\Phi_1^T \\ * & * & -Q - Q^T \end{bmatrix}, \; \bar{\Delta}_i = \begin{bmatrix} \Delta_i & 0_{n\times m} \\ 0_{m\times n} & 0_m \end{bmatrix},$$

$$\hat{A} = (\hat{A}_1 + \hat{A}_2\bar{I} + \hat{B}\hat{K}_1 + \hat{B}\hat{K}_2) = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} = \begin{bmatrix} A_1 + A_2\bar{I} & B_1 + B_2\bar{I} \\ K + \Delta K & -U_m \end{bmatrix},$$

$$\Delta_i = diag([\Delta_{i1}^K \; \Delta_{i2}^K \; \cdots \; \Delta_{in}^K]).$$

Noting the decompositions of $\hat{A}$, the above inequality can be rewritten as:

$$\bar{\Phi}_5 = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & Q\hat{A}_{11}^T + R\hat{A}_{12}^T & Q\hat{A}_{21}^T + R\hat{A}_{22}^T & P - Q^T - \frac{1}{2}Q \\ * & -P & \hat{A}_{12}S^T & \hat{A}_{11}Q^T + \hat{A}_{12}R^T \\ * & * & S\hat{A}_{22}^T + \hat{A}_{22}S^T & \hat{A}_{21}Q^T + \hat{A}_{22}R^T \\ * & * & * & -Q - Q^T \end{bmatrix} < 0.$$

Multiply the above inequality on the left and right by the following matrix:

$$\begin{bmatrix} U_n & 0_n & 0_{n\times m} & 0_n \\ 0_n & U_n & 0_{n\times m} & 0_n \\ 0_n & 0_n & 0_{n\times m} & U_n \\ 0_{m\times n} & 0_{m\times n} & U_m & 0_{m\times n} \end{bmatrix}$$

and its transpose, respectively, results in

$$\Lambda + \Lambda^T < 0, \tag{12}$$

where

$$\Lambda = \begin{bmatrix} -\frac{1}{2}Q & Q\hat{A}_{11}^T + R\hat{A}_{12}^T & P - Q^T - \frac{1}{2}Q & Q\hat{A}_{21}^T + R\hat{A}_{22}^T \\ 0_n & -\frac{1}{2}P & 0_n & 0_{n\times m} \\ 0_n & Q\hat{A}_{11}^T + R\hat{A}_{12}^T & -Q & Q\hat{A}_{21}^T + R\hat{A}_{22}^T \\ 0_{m\times n} & S\hat{A}_{12}^T & 0_{m\times n} & S\hat{A}_{22}^T \end{bmatrix}.$$

Note that within Λ, the condition $S\hat{A}_{22}^T + \hat{A}_{22}S^T < 0$ holds. Furthermore, based on the matrix norm properties [27], both $\hat{A}_{22}$ and S are nonsingular matrices. Consequently, the singular system (10) is regular and causal [28], and can therefore be reduced to a state-space representation:

$$x(k+1) = \bar{A}x(k),$$

where $\bar{A} = \hat{A}_{11} - \hat{A}_{12}\hat{A}_{22}^{-1}\hat{A}_{21}$.

The above simplified state-space system can be determined to be asymptotic stable if a matrix $P > 0$ can be found such that $\bar{A}^T P\bar{A} - P < 0$. Applying the Schur complement, this condition can be equivalently rewritten as:

$$\begin{bmatrix} -P & P\bar{A} \\ * & -P \end{bmatrix} < 0,$$

which can be reformulated as $\Xi_\Gamma^T \Xi \Xi_\Gamma < 0$, where

$$\Xi = \begin{bmatrix} 0_n & 0_n & P \\ 0_n & -P & 0_n \\ P & 0_n & 0_n \end{bmatrix}, \ \Xi_\Gamma = \begin{bmatrix} U_n & 0_n \\ 0_n & U_n \\ -\frac{1}{2}U_n & \bar{A} \end{bmatrix}.$$

Given that $P > 0$, a direct constraint can be introduced as:

$$\begin{bmatrix} -P & 0_n \\ 0_n & -2P \end{bmatrix} < 0,$$

which can be reformulated as $\Xi_\Psi^T \Xi \Xi_\Psi < 0$, where

$$\Xi_\Psi^T = \begin{bmatrix} 0_n & U_n & 0_n \\ -U_n & 0_n & U_n \end{bmatrix}.$$

Noting that the inequalities $\Xi_\Gamma^T \Xi \Xi_\Gamma < 0$ hold, the application of the Projection Lemma [29] yields

$$\Xi + \Gamma^T Q^T \Psi + \Psi^T Q \Gamma < 0, \tag{13}$$

where

$$\Psi = \begin{bmatrix} U_n & 0_n & U_n \end{bmatrix}, \ \Gamma = \begin{bmatrix} -\frac{1}{2}U_n & \bar{A} & -U_n \end{bmatrix},$$

with $\Psi \Xi_\Psi = 0_{n\times 2n}$, $\Gamma \Xi_\Gamma = 0_{n\times 2n}$.

Applying Lemma 4.3 to inequality (12) results in

$$\begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & Q\bar{A} & P - Q^T - \frac{1}{2}Q \\ * & -P & \bar{A}^T Q^T \\ * & * & -Q - Q^T \end{bmatrix} < 0,$$

which simplifies to inequality (13). Consequently, since $\bar{A}$ is stable, the singular system (10) is admissible.

Remark 4.5. In Theorem 4.4, the variable $\Delta_\beta = 2^{-(\beta+1)}$ represents the maximum error bound for the parameters in the state-feedback gain K in (4) when using fixed-point representation, as discussed in Section 2.2. Here, $\beta \in \mathbb{N}^+$ represents the word length utilized for expressing the fractional component of the parameters. Thus, the parameter representation error can be controlled by selecting an appropriate value for β. Especially, if the feedback matrix K contains only integer parameters, it will be unaffected by representation errors and can be accurately represented. To illustrate this, Theorem 4.4 introduces the matrix D_K. An element of D_K is 0 if the corresponding parameter in K is an integer; otherwise, the element in D_K is 1. For a given constant state-feedback gain K, D_K can be determined according to these rules. Conversely, if K is a matrix to be designed and is not predetermined, D_K can be set to a matrix with all elements equal to 1 if there is no additional available information.

In Theorem 4.4, determining the state-feedback gain K is challenging due to the coupling between the gain parameters in K and the unknown matrix Q. To address this issue, Theorem 4.6 provides a framework that facilitates the practical design of the feedback gain.

Theorem 4.6. *For a given scalar $\beta \in \mathbb{N}^+$, the descriptor system (10) is admissible provided that matrices $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times m}$, $S \in \mathbb{R}^{m\times m}$, $P \in \mathbb{R}^{n\times n}$ (with $P > 0$), and scalars $\epsilon > 0$, $\epsilon_i > 0$ for $i = 1, 2, \cdots, m$ exist, such that the following inequality holds:*

$$\begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^T & \bar{\Theta}_{12} & P - Q^T - \frac{1}{2}Q & \Theta_{41} \\ * & \Theta_{22} & \bar{\Theta}_{23} & \Theta_{42} \\ * & * & -Q - Q^T & \Theta_{43} \\ * & * & * & \Theta_{44} \end{bmatrix} < 0, \tag{14}$$

where $\bar{\Theta}_{12} = \begin{bmatrix} QA_1^T + RB_1^T & Z - R \end{bmatrix}$, $\bar{\Theta}_{23} = \bar{\Theta}_{12}^T$. Furthermore, the determination of the state-feedback gain K is carried out as follows:

$$K = Z^T Q^{-T}. \tag{15}$$

Proof of Theorem 4.6. By substituting $\bar{Q}K^T$ with Z in (11), (14) can be obtained. Therefore, (10) is admissible provided that (14) is satisfied.

5. Example illustration

In the following discussion, a simulation case is given to demonstrate the application of the proposed design method and to validate its effectiveness. In other words, the finite-precision stabilization problem for discrete-time neutrosophic systems, as illustrated in (2), needs to be addressed. As discussed earlier, based on Lemma 4.1, any system of the form shown in (2) can be standardized and ultimately rewritten as the closed-loop system depicted in (9). Based on these results, the standardized system presented in (9) is adopted as the plant, with the known parameters specified as follows:

$$A_1 = \begin{bmatrix} 1.5000 & 0.1000 & 0.3000 \\ 0.7000 & 1.2000 & 0.8000 \\ 0.5000 & 0.8000 & 1.0000 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.5000 & 0.4000 \\ 0.6000 & 0.3000 \\ 0.4000 & 0.2000 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.0300 & 0.0600 & 0.0900 \\ 0.0600 & 0.0900 & 0.0600 \\ 0.0600 & 0.0900 & 0.0300 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.0600 & 0.0300 \\ 0.0900 & 0.0600 \\ 0.0600 & 0.0600 \end{bmatrix}, \ \bar{I} \in [-1, 1].$$

For the system outlined above, the design of the state feedback gain K is required, and the effects of fixed-point representation on its parameters must be considered. The matrix K will be perturbed as shown in (5). The parameters D_{ij}^K, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ for

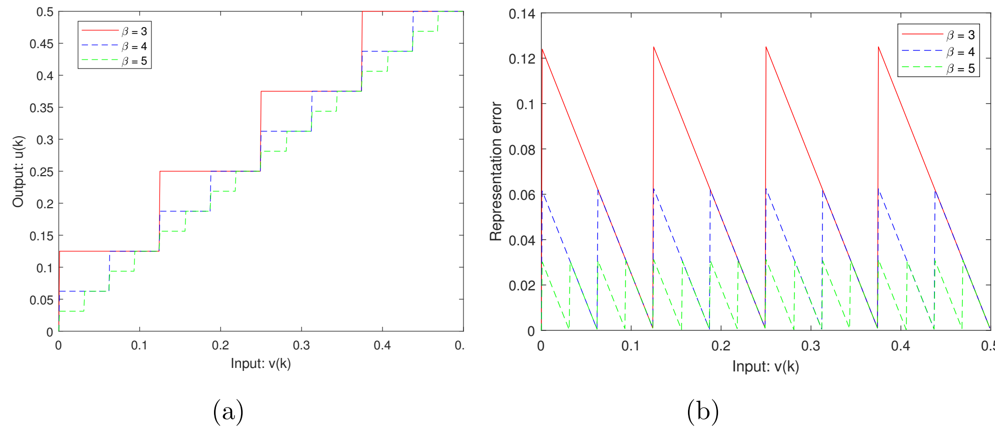


FIGURE 3. (a) Input-output characteristics of the fixed-point representation scheme proposed in Algorithm 1; (b) Representation error of the fixed-point representation scheme.

ΔK in (5) are set as follows:

$$D_K = \begin{bmatrix} D_{11}^K & D_{12}^K & D_{13}^K \\ D_{21}^K & D_{22}^K & D_{23}^K \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Then, the bit length used to represent the fractional part is set to $\beta = 4$, which implies an upper bound on the representation error as $\Delta_\beta = 0.0625$. Based on the parameters of the illustrative example, the coefficient matrix in Theorem 4.6 is constructed, and the inequality (14) is solved to obtain:

$$\begin{aligned} K = Z^T Q^{-T} &= \begin{bmatrix} -3.8027 & -3.7431 \\ -3.3534 & -0.5345 \\ -2.6144 & -0.7285 \end{bmatrix}^T \begin{bmatrix} 2.9995 & 1.1979 & -0.0964 \\ 1.1979 & 3.3472 & -2.5932 \\ -0.0964 & -2.5932 & 4.8143 \end{bmatrix}^{-T} \\ &= \begin{bmatrix} -0.5000 & -2.1875 & -1.7500 \\ -1.4375 & 0.3750 & 0.0625 \end{bmatrix}. \end{aligned} \tag{16}$$

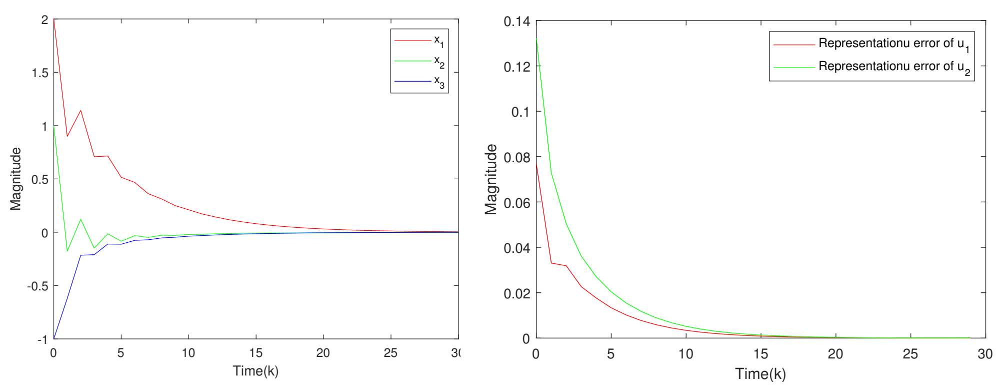
To implement the fixed-point representation scheme proposed in this paper, Algorithm 1 is presented for representing the fractional part of the feedback gain parameters K in (16). The input-output characteristics of this fixed-point representation scheme are illustrated in Figure 3(a) for different bit length values β and the corresponding representation error is shown in Figure 3(b). As observed, with an increase in bit length β, the representation error decreases. The state evolution of the closed-loop system described by (9), using the parameters outlined in this section, is presented in Figure 4(a). Figure 4(b) shows the error introduced in the control signal $u(k)$ by the fixed-point representation scheme. These results validate the effectiveness of the proposed design method.

Algorithm 1 Fixed-point representation scheme

Input: $\beta \in \mathbb{N}^{+}$ and $-1 < v(k) < 1$;

Output: $u(k)$;

if $v(k) >= 0$ **then**
    for all $i = 1,\ 2,\ \cdots,\ 2^{\beta}$ **do**
        if $v(k) >= (k-1) \times 2^{-\beta}\ \&\ v(k) < k \times 2^{-\beta}$ **then**
            $u(k) = k \times 2^{-\beta}$, break;
        end if
    end for
else
    for all $i = 1,\ 2,\ \cdots,\ 2^{\beta}$ **do**
        if $-v(k) >= (k-1) \times 2^{-\beta}\ \&\ -v(k) < k \times 2^{-\beta}$ **then**
            $u(k) = -k \times 2^{-\beta}$, break;
        end if
    end for
end if
return $u(k)$.



(a) (b)

FIGURE 4. (a) State evolution of the closed-loop system; (b) Error introduced in the control signal $u(k)$ by the fixed-point representation scheme.

Table 1 illustrates the feasibility of the inequality (14) under parameter uncertainties of varying magnitudes and different bit lengths. As shown in Table 1, the feasibility of the controller solvability decreases as the system parameter uncertainties increase. Similarly, as the bit length used to represent the fractional part of the control gain decreases, the feasibility of the controller solvability also diminishes.

TABLE 1. Feasibility of the inequality (14) with parameter uncertainties of varying magnitudes and different bit lengths.

	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$
$\bar{I} \in [-0.5, 0.5]$	infeasible	feasible	feasible	feasible
$\bar{I} \in [-1, 1]$	infeasible	infeasible	feasible	feasible
$\bar{I} \in [-2, 2]$	infeasible	infeasible	infeasible	feasible

TABLE 2. Convergence performance of the system under different bit lengths.

Bit lengths for representation	$\beta = 3$	$\beta = 4$	$\beta = 5$
Convergence performance J	8.3064	8.2121	8.0395

To compare the convergence performance of the system under different bit lengths, the following performance index is defined:

$$J = \sum_{k=0}^{\infty} \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) \end{bmatrix}^T .$$

The results are presented in Table 2, where it can be observed that as the bit length increases, the convergence performance of system improves.

6. Discussion

The stabilization of MIMO neutrosophic linear systems under finite-precision digital control is a critical but underexplored area. Existing research on FWL effects and parameter uncertainty in control systems often focuses on simplified models or single-input systems, leaving MIMO systems with complex uncertainties largely unaddressed. This work addresses these challenges by proposing an integrated approach that combines neutrosophic modeling with FWL stability analysis, a unique contribution to the field. Through the proposed descriptor model approach, the closed-loop system is successfully reformulated into a singular system form. This transformation allowed us to apply stability analysis techniques suitable for singular systems, leading to a comprehensive understanding of how parameter uncertainties and fixed-point representations impact the stability and solvability of the controller. While the proposed method effectively addresses the stabilization of MIMO neutrosophic linear systems under finite word length effects, the model does not account for time-varying uncertainties or nonlinearities that may exist in some practical applications. These limitations may limit the applicability of the method in highly dynamic or nonlinear systems, which remains an area for future research.

7. Conclusions

This paper addresses the finite-precision stabilization problem for neutrosophic linear systems, where system parameter uncertainties are represented using neutrosophic numbers and controller parameters are modeled using a fixed-point scheme. A descriptor model approach is proposed to handle the coupling between parameter uncertainties and representation errors, reformulating the closed-loop system into a singular system form. Stability analysis and controller parameterization are achieved through analytical and design methods for singular systems. While the current study focuses on state feedback control, the approach and techniques outlined could be extended to address other types of control, such as dynamic output feedback stabilization, which remains a promising direction for future research. Furthermore, the investigation of distributed control strategies for neutrosophic linear systems under finite-precision conditions presents another exciting avenue for further exploration.

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