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Division of 3-Refined Neutrosophic Numbers by using the Identification Method and the 3-Refined AH-Isometry

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Abstract: This paper aims to study the division of 3-refined neutrosophic numbers, where it was discussed through two methods, the first is the identification method, and the second is the 3-refined AH-isometry. In addition to presenting special cases related to division of 3-refined neutrosophic numbers. The most important idea in this paper is that we were able to move from the formula we obtained in the 3-refined AH-isometry to the formula we found in the identification method and vice versa, hence this proves the validity of both formulas.

Keywords: 3-refined AH-Isometry; 3-refined, neutrosophic numbers; division.

1. Introduction and Preliminaries

Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1I_1, b_2I_2, \ldots, b_nI_n)$ where $a, b_1, b_2, \ldots, b_n \in R \; or \; C$ [1].

Agboola introduced the concept of refined neutrosophic algebraic structures [2]. In addition, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. Abobala presented the papers on some special substructures of refined neutrosophic rings and a study of ah-substructures in n-refined neutrosophic vector spaces [6-7].

Alhasan.Y and Abdulfatah. R also presented a study on the division of refined neutrosophic number. [8].

Finally, there are papers presented in n-Valued Refined Neutrosophic Logic and Its Applications in Physics, Neutrosophic Rings I [4-5].

2. Main Discussion

2.1. Division of 3-refined neutrosophic numbers by using the Identification Method

Let m_1, m_2 are two 3-refined neutrosophic numbers, where:

$$m_1 = a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3 \ , \qquad m_2 = a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3$$

To find $\frac{m_1}{m_2}$, we write:

$$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3} \equiv x + yI_1 + zI_2 + \mathrm{w}I_3$$

where x , y and z are real unknowns.

$$a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3 \equiv (a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3)(x + yI_1 + zI_2 + \mathrm{w}I_3)$$

$$\begin{aligned} a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3 &\equiv a_2 x + a_2 y I_1 + a_2 z I_2 + a_2 \mathrm{w} I_3 + b_2 x I_1 + b_2 y I_1 + b_2 z I_1 + b_2 \mathrm{w} I_1 + c_2 x I_2 + c_2 y I_1 \\ &+ c_2 z I_2 + c_2 \mathrm{w} I_2 + d_2 x I_3 + d_2 y I_1 + d_2 z I_2 + d_2 \mathrm{w} I_3 \end{aligned}$$

$$\begin{aligned} a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3 &\equiv a_2 x + [b_2 x + (a_2 + b_2 + c_2 + d_2) y + b_2 z + b_2 \mathrm{w}] I_1 \\ &+ [c_2 x + (a_2 + c_2 + d_2) z + c_2 \mathrm{w}] I_2 + +[d_2 x + (a_2 + d_2) \mathrm{w}] I_3 \end{aligned}$$

Whereas: $I_1 I_2 = I_2 I_1 = I_1$

By identifying:

$$a_2 x = a_1$$

$$b_2 x + (a_2 + b_2 + c_2 + d_2) y + b_2 z + b_2 \mathrm{w} = b_1 \quad (1)$$

$$c_2 x + (a_2 + c_2 + d_2) z + c_2 \mathrm{w} = c_1 \quad (2)$$

$$d_2 x + (a_2 + d_2) \mathrm{w} = d_1 \quad (3)$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} a_2 & 0 & 0 & 0 \\ b_2 & (a_2 + b_2 + c_2 + d_2) & b_2 & b_2 \\ c_2 & 0 & (a_2 + c_2 + d_2) & c_2 \\ d_2 & 0 & 0 & (a_2 + d_2) \end{vmatrix} \neq 0$$

$$\Longrightarrow \quad a_2 (a_2 + d_2)(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2) \neq 0$$

where:

$$a_2 \neq 0 \ , a_2 \neq d_2 \ , a_2 \neq -c_2 - d_2 \ , a_2 \neq -b_2 - c_2 - d_2$$

then:

$$x = \frac{a_1}{a_2}$$

by substituting in (3):

$$\frac{a_1 d_2}{a_2} + (a_2 + d_2) \mathrm{w} = d_1$$

$$\mathrm{w} = \frac{a_2 d_1 - a_1 d_2}{a_2(a_2 + d_2)}$$

by substituting in (2):

$$\frac{a_1 c_2}{a_2} + (a_2 + c_2 + d_2)z + \frac{a_2 d_1 c_2 - a_1 d_2 c_2}{a_2(a_2 + d_2)} = c_1$$

$$\frac{a_1 c_2 a_2 + a_1 c_2 d_2}{a_2(a_2 + d_2)} + (a_2 + c_2 + d_2)z + \frac{a_2 d_1 c_2 - a_1 d_2 c_2}{a_2(a_2 + d_2)} = c_1$$

$$(a_2 + c_2 + d_2)z = \frac{c_1 a_2(a_2 + d_2) - a_1 c_2 a_2 - a_2 d_1 c_2}{a_2(a_2 + d_2)}$$

$$(a_2 + c_2 + d_2)z = \frac{c_1(a_2 + d_2) - c_2(a_1 + d_1)}{(a_2 + d_2)}$$

$$z = \frac{c_1(a_2 + d_2) - c_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}$$

by substituting in (1):

$$\frac{a_1 b_2}{a_2} + (a_2 + b_2 + c_2 + d_2)y + b_2\left(\frac{c_1(a_2 + d_2) - c_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right) + b_2\left(\frac{a_2 d_1 - a_1 d_2}{a_2(a_2 + d_2)}\right) = b_1$$

$$\frac{a_1 b_2}{a_2} + (a_2 + b_2 + c_2 + d_2)y + \frac{c_1 b_2(a_2 + d_2) - c_2 b_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)} + \frac{a_2 b_2 d_1 - a_1 b_2 d_2}{a_2(a_2 + d_2)} = b_1$$

$$(a_2 + b_2 + c_2 + d_2)y + \frac{b_2\big(a_1(a_2 + d_2)(a_2 + c_2 + d_2) + c_1 a_2(a_2 + d_2) - c_2 a_2(a_1 + d_1) + (a_2 d_1 - a_1 d_2)(a_2 + c_2 + d_2)\big)}{a_2(a_2 + d_2)(a_2 + c_2 + d_2)} = b_1$$

$$(a_2 + b_2 + c_2 + d_2)y + b_2\left(\frac{(a_2 + d_2)(a_2 a_1 + c_2 a_1 + d_2 a_1)}{a_2(a_2 + d_2)(a_2 + c_2 + d_2)}\right) + b_2\left(\frac{c_1 a_2(a_2 + d_2) - c_2 a_1 a_2 - c_2 d_1 a_2}{a_2(a_2 + d_2)(a_2 + c_2 + d_2)}\right) + b_2\left(\frac{{a_2}^2 d_1 - a_2 a_1 d_2 + c_2 a_2 d_1 - a_1 d_2 c_2 + a_2 d_1 d_2 - a_1 {d_2}^2}{a_2(a_2 + d_2)(a_2 + c_2 + d_2)}\right) = b_1$$

$$(a_2+b_2+c_2+d_2)y+b_2\left(\frac{(a_2+d_2)(a_2a_1+c_2a_1+d_2a_1)}{a_2(a_2+d_2)(a_2+c_2+d_2)}\right)+b_2\left(\frac{c_1a_2(a_2+d_2)-c_2a_1a_2+{a_2}^2d_1-a_2a_1d_2-a_1d_2c_2+a_2d_1d_2-a_1{d_2}^2}{a_2(a_2+d_2)(a_2+c_2+d_2)}\right)=b_1$$

$$(a_2+b_2+c_2+d_2)y+b_2\left(\frac{(a_2+d_2)(a_2a_1+c_2a_1+d_2a_1)}{a_2(a_2+d_2)(a_2+c_2+d_2)}\right)+b_2\left(\frac{c_1a_2(a_2+d_2)-c_2a_1(a_2+d_2)-a_1d_2(a_2+d_2)+a_2d_1(a_2+d_2)}{a_2(a_2+d_2)(a_2+c_2+d_2)}\right)=b_1$$

$$(a_2+b_2+c_2+d_2)y+b_2\left(\frac{a_2a_1+c_2a_1+d_2a_1}{a_2(a_2+c_2+d_2)}\right)+b_2\left(\frac{c_1a_2-c_2a_1-a_1d_2+a_2d_1}{a_2(a_2+c_2+d_2)}\right)=b_1$$

$$(a_2+b_2+c_2+d_2)y=b_1-b_2\left(\frac{a_2a_1+c_2a_1+d_2a_1}{a_2(a_2+c_2+d_2)}\right)-b_2\left(\frac{c_1a_2-c_2a_1-a_1d_2+a_2d_1}{a_2(a_2+c_2+d_2)}\right)$$

$$(a_2+b_2+c_2+d_2)y=\frac{b_1a_2(a_2+c_2+d_2)}{a_2(a_2+c_2+d_2)}-b_2\left(\frac{a_2a_1+c_1a_2+a_2d_1}{a_2(a_2+c_2+d_2)}\right)$$

$$y=\frac{b_1(a_2+c_2+d_2)-b_2(a_1+c_1+d_1)}{(a_2+c_2+d_2)(a_2+b_2+c_2+d_2)}$$

hence:

$$\frac{a_1+b_1I_1+c_1I_2+d_1I_3}{a_2+b_2I_1+c_2I_2+d_2I_3}$$

$$\equiv\frac{a_1}{a_2}+\left[\frac{b_1(a_2+c_2+d_2)-b_2(a_1+c_1+d_1)}{(a_2+c_2+d_2)(a_2+b_2+c_2+d_2)}\right]I_1+\left[\frac{c_1(a_2+d_2)-c_2(a_1+d_1)}{(a_2+d_2)(a_2+c_2+d_2)}\right]I_2+\left[\frac{a_2d_1-a_1d_2}{a_2(a_2+d_2)}\right]I_3$$

Example 1

$$\frac{2+I_1+I_2+I_3}{2+I_1+2I_2+3I_3}=1+\frac{3}{56}I_1-\frac{1}{35}I_2-\frac{2}{5}I_3$$

Let's check the answer:

$$(2+I_1+I_2+I_3)\left(1+\tfrac{3}{56}I_1-\tfrac{1}{35}I_2-\tfrac{2}{5}I_3\right)=2+\left[\tfrac{3}{56}(8)+1-\tfrac{1}{35}-\tfrac{2}{5}\right]I_1+\left[-\tfrac{1}{35}(7)+2-\tfrac{4}{5}\right]I_2+$$

$$\left[-\tfrac{2}{5}(5)+3\right]I_3=2+I_1+2I_2+3I_3 \quad \text{(True)}$$

As consequences, we have:

1) $\frac{a_1+b_1I_1+c_1I_2+d_1I_3}{k(a_1+b_1I_1+c_1I_2+d_1I_3)}=\frac{1}{k}$; $a_1\neq 0$, $a_1\neq d_1$, $a_1\neq -c_1-d_1$, $a_1\neq -b_1-c_1-d_1$

2) $$\frac{I_1}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3} = \frac{b_1}{a_2 + b_2 + c_2 + d_2} I_1$$

Example 2

$$\frac{I_1}{4 - 2I_1 + 5I_2 + 4I_3} = \frac{1}{11} I_1$$

Let's check the answer:

$$(4 - 2I_1 + 5I_2 + 4I_3)\left(\frac{1}{11} I_1\right) = I_1 \quad \text{(True)}$$

3) $$\frac{I_2}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3} = \left[\frac{-b_2 c_1}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{c_1}{a_2 + c_2 + d_2}\right] I_2$$

Example 3

$$\frac{I_2}{1 - I_1 + 3I_2 + 7I_3} = \frac{1}{110} I_1 + \frac{1}{11} I_2$$

Let's check the answer:

$$(1 - I_1 + 3I_2 + 7I_3)\left(\frac{1}{110} I_1 + \frac{1}{11} I_2\right) = I_2 \quad \text{(True)}$$

4) $$\frac{I_3}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3} = \left[\frac{-b_2 d_1}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{-c_2 d_1}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{d_1}{a_2 + d_2}\right] I_3$$

Example 4

$$\frac{I_3}{2 - 3I_1 + 4I_2 + 5I_3} = \frac{3}{88} I_1 - \frac{4}{77} I_2 + \frac{1}{7} I_3$$

Let's check the answer:

$$(2 - 3I_1 + 4I_2 + 5I_3)\left(\frac{3}{88} I_1 - \frac{4}{77} I_2 + \frac{1}{7} I_3\right) = I_3 \quad \text{(True)}$$

5) $$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{k} = \frac{a_1}{k} + \frac{b_1}{k} I_1 + \frac{c_1}{k} I_2 + \frac{d_1}{k} I_3 \,; k \neq 0$$

6) $$\frac{k}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3} \equiv \frac{k}{a_2} + \left[\frac{-k b_2}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{-k c_2}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{-k d_2}{a_2(a_2 + d_2)}\right] I_3$$

In particular:

* $$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_1 + I_2 + I_3} = undefined$$

* $$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_1 + I_2} = undefined$$

* $$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_2 + I_3} = undefined$$

$$* \frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_1 + I_3} = undefined$$

$$* \frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_1} = undefined$$

$$* \frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_2} = undefined$$

$$* \frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{I_3} = undefined$$

$$* \frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{k(I_1 + I_2 + I_3)} = undefined\ ;\ k \neq 0$$

2.2. Division of 3-refined neutrosophic numbers by using the 3-Refined AH-Isometry

Let $f: R[I_1, I_2, I_3] \times R[I_1, I_2, I_3] \longrightarrow R[I_1, I_2, I_3]$ be a literal 3-refined neutrosophic function of three variables, then:

$$f(a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3, a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3)$$

$$= f(a_1, a_2) + [f(a_1 + b_1 + c_1 + d_1, a_2 + b_2 + c_2 + d_2) - f(a_2 + c_2 + d_2, a_1 + c_1 + d_1)]I_1 + [f(a_2 + c_2 + d_2, a_1 + c_1 + d_1) - f(a_2 + d_2, a_1 + d_1)]I_2 + [f(a_2 + d_2, a_1 + d_1) - f(a_1, a_2)]I_3$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2}\right] I_1 + \left[\frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2} - \frac{a_1 + d_1}{a_2 + d_2}\right] I_2 + \left[\frac{a_1 + d_1}{a_2 + d_2} - \frac{a_1}{a_2}\right] I_3$$

hence:

$$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3}$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2}\right] I_1 + \left[\frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2} - \frac{a_1 + d_1}{a_2 + d_2}\right] I_2 + \left[\frac{a_1 + d_1}{a_2 + d_2} - \frac{a_1}{a_2}\right] I_3$$

Whereas: $a_2 \neq 0$, $a_2 \neq d_2$, $a_2 \neq -c_2 - d_2$, $a_2 \neq -b_2 - c_2 - d_2$

Example 6

$$\frac{2 + I_1 + I_2 + I_3}{2 + I_1 + 2I_2 + 3I_3} = 1 + \left[\frac{5}{8} - \frac{4}{7}\right] I_1 + \left[\frac{4}{7} - \frac{3}{5}\right] I_2 + \left[\frac{3}{5} - 1\right] I_3 = 1 + \frac{3}{56} I_1 - \frac{1}{35} I_2 - \frac{2}{5} I_3$$

(The same result that we got using the Identification Method in example 1)

Example 7

$$\frac{-5 + I_1 + 4I_2 - 2I_3}{25 + 2I_1 + I_2 + 4I_3} = \frac{-1}{5} + \left[\frac{-1}{16} + \frac{1}{10}\right] I_1 + \left[\frac{-1}{10} + \frac{7}{29}\right] I_2 + \left[\frac{-7}{29} + \frac{1}{5}\right] I_3 = \frac{-1}{5} + \frac{3}{80} I_1 + \frac{41}{290} I_2 - \frac{6}{145} I_3$$

Let's check the answer:

$$(25 + 2I_1 + I_2 + 4I_3)\left(\frac{-1}{5} + \frac{3}{80} I_1 + \frac{41}{290} I_2 - \frac{6}{145} I_3\right) = -5 + I_1 + 4I_2 - 2I_3 \quad \text{(True)}$$

3. The relationship between the identification method and the 3-refined ah-isometry

3.1 Moving from identification method to 3-refined AH-Isometry

We have:

$$\frac{a_1 + b_1I_1 + c_1I_2 + d_1I_3}{a_2 + b_2I_1 + c_2I_2 + d_2I_3}$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{b_1(a_2 + c_2 + d_2) - b_2(a_1 + c_1 + d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{c_1(a_2 + d_2) - c_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_2d_1 - a_1d_2}{a_2(a_2 + d_2)}\right] I_3$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{(a_2a_1 + a_2b_1 + a_2c_1 + a_2d_1) + (c_2a_1 + c_2b_1 + c_2c_1 + c_2d_1) + (d_2a_1 + d_2b_1 + d_2c_1 + d_2d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)} + \frac{-a_2a_1 - a_2c_1 - a_2d_1 - c_2a_1 - c_2c_1 - c_2d_1 - d_2a_1 - d_2c_1 - d_2d_1 - b_2a_1 - b_2c_1 - b_2d_1}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{a_2a_1 + c_1a_2 + a_2d_1 + a_1d_2 + c_1d_2 + d_1d_2 - a_2a_1 - c_2a_1 - a_1d_2 - a_2d_1 - c_2d_1 - d_1d_2}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_2a_1 + a_2d_1 - a_2a_1 - a_1d_2}{a_2(a_2 + d_2)}\right] I_3$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_2(a_1 + b_1 + c_1 + d_1) + c_2(a_1 + b_1 + c_1 + d_1) + d_2(a_1 + b_1 + c_1 + d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)} + \frac{-a_1(a_2 + b_2 + c_2 + d_2) - c_1(a_2 + b_2 + c_2 + d_2) - d_1(a_2 + b_2 + c_2 + d_2)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{a_2(a_1 + c_1 + d_1) + d_2(a_1 + c_1 + d_1) - a_1(a_2 + c_2 + d_2) - d_1(a_2 + c_2 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_2(a_1 + d_1) - a_1(a_2 + d_2)}{a_2(a_2 + d_2)}\right] I_3$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{(a_2 + c_2 + d_2)(a_1 + b_1 + c_1 + d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)} - \frac{(a_1 + c_1 + d_1)(a_2 + b_2 + c_2 + d_2)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{(a_2 + d_2)(a_1 + c_1 + d_1) - (a_1 + d_1)(a_2 + c_2 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_1 + d_1}{a_2 + d_2} - \frac{a_1}{a_2}\right] I_3$$

Hence, we find the form of the method of 3-Refined AH-Isometry:

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2}\right] I_1 + \left[\frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2} - \frac{a_1 + d_1}{a_2 + d_2}\right] I_2 + \left[\frac{a_1 + d_1}{a_2 + d_2} - \frac{a_1}{a_2}\right] I_3$$

3.2 Moving from 3-refined AH-Isometry to identification method

We have:

$$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3}$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2}\right] I_1 + \left[\frac{a_1 + c_1 + d_1}{a_2 + c_2 + d_2} - \frac{a_1 + d_1}{a_2 + d_2}\right] I_2 + \left[\frac{a_1 + d_1}{a_2 + d_2} - \frac{a_1}{a_2}\right] I_3$$

then:

$$\equiv \frac{a_1}{a_2} + \left[\frac{a_2(a_1 + c_1 + d_1) + c_2(a_1 + c_1 + d_1) + d_2(a_1 + c_1 + d_1) + b_1(a_2 + c_2 + d_2)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)} - \frac{a_2(a_1 + c_1 + d_1) - b_2(a_1 + c_1 + d_1) - c_2(a_1 + c_1 + d_1) - d_2(a_1 + c_1 + d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{a_2(a_1 + d_1) + d_2(a_1 + d_1) + c_1(a_2 + d_2) - a_2(a_1 + d_1) - c_2(a_1 + d_1) - d_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_1 a_2 + d_1 a_2 - a_1 a_2 - a_1 d_2}{a_2(a_2 + d_2)}\right] I_3$$

Hence, we find the form of the Identification method:

$$\frac{a_1 + b_1 I_1 + c_1 I_2 + d_1 I_3}{a_2 + b_2 I_1 + c_2 I_2 + d_2 I_3}$$

$$\equiv \frac{a_1}{a_2} + \left[\frac{b_1(a_2 + c_2 + d_2) - b_2(a_1 + c_1 + d_1)}{(a_2 + c_2 + d_2)(a_2 + b_2 + c_2 + d_2)}\right] I_1 + \left[\frac{c_1(a_2 + d_2) - c_2(a_1 + d_1)}{(a_2 + d_2)(a_2 + c_2 + d_2)}\right] I_2 + \left[\frac{a_2 d_1 - a_1 d_2}{a_2(a_2 + d_2)}\right] I_3$$

4. Conclusions

This paper deals with the division of 3-refined neutrosophic numbers using two methods, the identification method and the 3-refined AH-isometry, and we obtained the same results when using either of the two methods by solving a set of examples in this paper. In addition to introduce the relationship between the identification method and the 3-refined ah-isometry.

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