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The integral of 2- refined rational neutrosophic functions

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Abstract: The main objective of this paper is to study the integral of 2- refined rational neutrosophic functions, by providing direct rules that make it easy for us to find the integral of 2- refined rational neutrosophic functions, where the division of refined neutrosophic number was used to find the integral of 2- refined rational neutrosophic functions. In addition to proving these rules and providing a set of examples that illustrate these ideas.

Keywords: refined rational neutrosophic functions; 2- refined rational; integrals.

1. Introduction and Preliminaries

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1I_1, b_2I_2, \dots, b_nI_n)$ where $a, b_1, b_2, \dots, b_n \in R \text{ or } C$ [1]. Agboola introduced the concept of refined neutrosophic algebraic structures [2]. In addition, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. Abobala presented the papers on some special substructures of refined neutrosophic rings and a study of ah-substructures in n-refined neutrosophic vector spaces [6-7]. Alhasan.Y and Abdulfatah. R also presented a study on the division of refined neutrosophic number [8]:

$$\frac{\dot{a_1} + \dot{b_1}I_1 + \dot{c_1}I_2}{\dot{a_2} + \dot{b_2}I_1 + \dot{c_2}I_2} \equiv \frac{\dot{a_1}}{\dot{a_2}} + \left[\frac{\dot{a_2}\dot{b_1} + \dot{b_1}\dot{c_2} - \dot{a_1}\dot{b_2} - \dot{b_2}\dot{c_1}}{(\dot{a_2} + \dot{c_2})(\dot{a_2} + \dot{b_2} + \dot{c_2})}\right] I_1 + \left[\frac{\dot{a_2}\dot{c_1} - \dot{a_1}\dot{c_2}}{\dot{a_2}(\dot{a_2} + \dot{c_2})}\right] I_2$$

Where: $\dot{a_2} \neq 0$, $\dot{a_2} \neq -\dot{c_2}$ and $\dot{a_2} \neq -\dot{b_2} - \dot{c_2}$

In addition to studying the integral calculus according to the logic of neutrosophic by presenting a set of papers on that [9-10]. The AH-Isometry was extended to n-Refined AH-Isometry by Smarandache & Abobala in 2024 [11]

Finally, there are papers presented in n-Valued Refined Neutrosophic Logic and Its Applications in Physics, Neutrosophic Rings I [4-5].

2. The integral of 2- refined rational neutrosophic functions

2.1 integral of standard 2- refined rational neutrosophic functions

- **Standard 2- refined rational neutrosophic integral I:**

$$\int \frac{a_1 + b_1 I_1 + c_1 I_2}{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2} dx$$

$$= \frac{1}{2}\left(\frac{a_1}{a_2} + \left[\frac{a_2 b_1 + b_1 c_2 - a_1 b_2 - b_2 c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right] I_1 + \left[\frac{a_2 c_1 - a_1 c_2}{a_2(a_2 + c_2)}\right] I_2\right) ln\left|\frac{x - a_2 - b_2 I_1 - c_2 I_2}{x + a_2 + b_2 I_1 + c_2 I_2}\right| + C$$

Whereas $a_2 \neq 0,\ a_2 \neq -c_2,\ a_2 \neq -b_2 - c_2, C = a_0 + b_0 I_1 + c_0 I_2\ and\ a_0,\ b_0,\ c_0$ are real numbers.

Proof:

$$\frac{a_1 + b_1 I_1 + c_1 I_2}{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2} = \frac{a_1 + b_1 I_1 + c_1 I_2}{(x - a_2 - b_2 I_1 - c_2 I_2)(x + a_2 + b_2 I_1 + c_2 I_2)}$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left[\frac{2(a_2 + b_2 I_1 + c_2 I_2)}{(x - a_2 - b_2 I_1 - c_2 I_2)(x + a_2 + b_2 I_1 + c_2 I_2)}\right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left[\frac{x + a_2 + b_2 I_1 + c_2 I_2 - x + a_2 + b_2 I_1 + c_2 I_2}{(x - a_2 - b_2 I_1 - c_2 I_2)(x + a_2 + b_2 I_1 + c_2 I_2)}\right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left[\frac{x + a_2 + b_2 I_1 + c_2 I_2 - (x - a_2 - b_2 I_1 - c_2 I_2)}{(x - a_2 - b_2 I_1 - c_2 I_2)(x + a_2 + b_2 I_1 + c_2 I_2)}\right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left[\frac{1}{x - a_2 - b_2 I_1 - c_2 I_2} - \frac{1}{x + a_2 + b_2 I_1 + c_2 I_2}\right]$$

$$\Rightarrow \int \frac{a_1 + b_1 I_1 + c_1 I_2}{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2} dx$$

$$= \int \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left[\frac{1}{x - a_2 - b_2 I_1 - c_2 I_2} - \frac{1}{x + a_2 + b_2 I_1 + c_2 I_2}\right] dx$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}\left(\int \frac{1}{x - a_2 - b_2 I_1 - c_2 I_2} dx - \int \frac{1}{x + a_2 + b_2 I_1 + c_2 I_2} dx\right)$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)}(ln|x - a_2 - b_2 I_1 - c_2 I_2| - ln|x + a_2 + b_2 I_1 + c_2 I_2|) + C$$

$$= \frac{1}{2}\left(\frac{a_1}{a_2} + \left[\frac{a_2 b_1 + b_1 c_2 - a_1 b_2 - b_2 c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right] I_1 + \left[\frac{a_2 c_1 - a_1 c_2}{a_2(a_2 + c_2)}\right] I_2\right) ln\left|\frac{x - a_2 - b_2 I_1 - c_2 I_2}{x + a_2 + b_2 I_1 + c_2 I_2}\right| + C$$

Example 1

Evaluate:

$$\int \frac{4 - 2I_1 + I_2}{x^2 - 64 - 19I_1 - 17I_2} dx$$

Solution:

$$x^2 - 64 - 19I_1 - 17I_2 = x^2 - \left(\sqrt{64 + 19I_1 + 17I_2}\right)^2$$

$$= x^2 - (8 + I_1 + I_2)^2$$

Whereas: $\sqrt{64 + 19I_1 + 17I_2} = \sqrt{64} + \left[\sqrt{64 + 19 + 17} - \sqrt{64 + 17}\right]I_1 + \left[\sqrt{64 + 17} - \sqrt{64}\right]I_2$

$$\Rightarrow \sqrt{64 + 19I_1 + 17I_2} = 8 + I_1 + I_2$$

$$\int \frac{4 - 2I_1 + I_2}{x^2 - 64 - 19I_1 - 17I_2} dx = \frac{1}{2}\left(\frac{4 - 2I_1 + I_2}{8 + I_1 + I_2}\right) ln \left|\frac{x - 8 - I_1 - I_2}{x + 8 + I_1 + I_2}\right| + C$$

$$= \left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right) ln \left|\frac{x - 8 - I_1 - I_2}{x + 8 + I_1 + I_2}\right| + C$$

Whereas $C = a_0 + b_0I_1 + c_0I_2$ and $a_0,\ b_0,\ c_0$ are real numbers.

Let's check the answer:

$$\frac{d}{dx}\left[\left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right) ln \left|\frac{x - 8 - I_1 - I_2}{x + 8 + I_1 + I_2}\right| + C\right]$$

$$= \left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right)(ln|x - 8 - I_1 - I_2| - ln|x + 8 + I_1 + I_2|)$$

$$= \left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right)\left(\frac{1}{x - 8 - I_1 - I_2} - \frac{1}{x + 8 + I_1 + I_2}\right)$$

$$= \left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right)\left(\frac{x + 8 + I_1 + I_2 - (x - 8 - I_1 - I_2)}{(x - 8 - I_1 - I_2)(x + 8 + I_1 + I_2)}\right)$$

$$= \left(\frac{1}{4} - \frac{23}{180}I_1 + \frac{1}{36}I_2\right)\left(\frac{2(8 + I_1 + I_2)}{x - (64 + 19I_1 + 17I_2)}\right)$$

$$= \left(\frac{1}{2} - \frac{23}{90}I_1 + \frac{1}{18}I_2\right)\left(\frac{8 + I_1 + I_2}{x - 64 - 19I_1 - 17I_2}\right)$$

$$= \frac{4 - 2I_1 + I_2}{x - 64 - 19I_1 - 17I_2}\ (The\ same\ integral\ function)$$

- **Standard 2- refined rational neutrosophic integral II:**

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2)^2 - x^2} dx$$

$$= \frac{1}{2}\left(\frac{a_1}{a_2} + \left[\frac{a_2b_1 + b_1c_2 - a_1b_2 - b_2c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right]I_1 + \left[\frac{a_2c_1 - a_1c_2}{a_2(a_2 + c_2)}\right]I_2\right) ln \left|\frac{a_2 + b_2I_1 + c_2I_2 + x}{a_2 + b_2I_1 + c_2I_2 - x}\right| + C$$

Whereas $a_2 \neq 0,\ a_2 \neq -c_2,\ a_2 \neq -b_2 - c_2, C = a_0 + b_0I_1 + c_0I_2$ and $a_0,\ b_0,\ c_0$ are real numbers.

Proof:

$$\frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2)^2 - x^2} = \frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2 - x)(a_2 + b_2I_1 + c_2I_2 + x)}$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left[\frac{2(a_2 + b_2 I_1 + c_2 I_2)}{(a_2 + b_2 I_1 + c_2 I_2 + x)(a_2 + b_2 I_1 + c_2 I_2 - x)} \right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left[\frac{a_2 + b_2 I_1 + c_2 I_2 + x + a_2 + b_2 I_1 + c_2 I_2 - x}{(a_2 + b_2 I_1 + c_2 I_2 + x)(a_2 + b_2 I_1 + c_2 I_2 - x)} \right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left[\frac{x + a_2 + b_2 I_1 + c_2 I_2 + (a_2 + b_2 I_1 + c_2 I_2 - x)}{(a_2 + b_2 I_1 + c_2 I_2 + x)(a_2 + b_2 I_1 + c_2 I_2 - x)} \right]$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left[\frac{1}{a_1 + b_1 I_1 + c_1 I_2 + x} + \frac{1}{a_2 + b_2 I_1 + c_2 I_2 - x} \right]$$

$$\Rightarrow \int \frac{a_1 + b_1 I_1 + c_1 I_2}{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2} dx$$

$$= \int \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left[\frac{1}{a_1 + b_1 I_1 + c_1 I_2 + x} + \frac{1}{a_2 + b_2 I_1 + c_2 I_2 - x} \right] dx$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} \left(\int \frac{1}{a_1 + b_1 I_1 + c_1 I_2 + x} dx - \int \frac{-1}{a_2 + b_2 I_1 + c_2 I_2 - x} dx \right)$$

$$= \frac{a_1 + b_1 I_1 + c_1 I_2}{2(a_2 + b_2 I_1 + c_2 I_2)} (ln|a_1 + b_1 I_1 + c_1 I_2 + x| - ln|a_2 + b_2 I_1 + c_2 I_2 - x|) + C$$

$$= \frac{1}{2} \left(\frac{a_1}{a_2} + \left[\frac{a_2 b_1 + b_1 c_2 - a_1 b_2 - b_2 c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)} \right] I_1 + \left[\frac{a_2 c_1 - a_1 c_2}{a_2(a_2 + c_2)} \right] I_2 \right) ln \left| \frac{a_2 + b_2 I_1 + c_2 I_2 + x}{a_2 + b_2 I_1 + c_2 I_2 - x} \right| + C$$

Example 2

Evaluate:

$$\int \frac{3I_1 + 5I_2}{1 + 72I_1 + 8I_2 - x^2} dx$$

Solution:

$$1 + 72I_1 + 8I_2 - x^2 = \left(\sqrt{1 + 72I_1 + 8I_2} \right)^2 - x^2$$

$$= \left(\sqrt{1 + 72I_1 + 8I_2} \right)^2 - x^2$$

$$= (1 + 6I_1 + 2I_2)^2 - x^2$$

Whereas: $\sqrt{1 + 72I_1 + 8I_2} = \sqrt{1} + \left[\sqrt{1 + 72 + 8} - \sqrt{1 + 8} \right] I_1 + \left[\sqrt{1 + 8} - \sqrt{1} \right] I_2$

$$\Rightarrow \sqrt{1 + 72I_1 + 8I_2} = 1 + 6I_1 + 2I_2$$

$$\int \frac{3I_1 + 5I_2}{1 + 72I_1 + 8I_2 - x^2} dx = \frac{1}{2} \left(\frac{3I_1 + 5I_2}{1 + 6I_1 + 2I_2} \right) ln \left| \frac{1 + 6I_1 + 2I_2 + x}{1 + 6I_1 + 2I_2 - x} \right| + C$$

$$= \left(-\frac{7}{18} I_1 + \frac{5}{6} I_2 \right) ln \left| \frac{1 + 6I_1 + 2I_2 + x}{1 + 6I_1 + 2I_2 - x} \right| + C$$

Whereas $C = a_0 + b_0 I_1 + c_0 I_2$ *and* a_0, b_0, c_0 are real numbers.

Let's check the answer:

$$\frac{d}{dx}\left[\left(-\frac{7}{18}I_1+\frac{5}{6}I_2\right)ln\left|\frac{1+6I_1+2I_2+x}{1+6I_1+2I_2-x}\right|+C\right]$$

$$=\left(-\frac{7}{18}I_1+\frac{5}{6}I_2\right)(ln|1+6I_1+2I_2+x|-ln|1+6I_1+2I_2-x|)$$

$$=\left(-\frac{7}{18}I_1+\frac{5}{6}I_2\right)\left(\frac{1}{1+6I_1+2I_2+x}-\frac{-1}{1+6I_1+2I_2-x}\right)$$

$$=\left(-\frac{7}{18}I_1+\frac{5}{6}I_2\right)\left(\frac{1+6I_1+2I_2-x+1+6I_1+2I_2+x}{(1+6I_1+2I_2+x)(1+6I_1+2I_2-x)}\right)$$

$$=\left(-\frac{7}{18}I_1+\frac{5}{6}I_2\right)\left(\frac{2(1+6I_1+2I_2)}{(1+6I_1+2I_2)^2-x^2}\right)$$

$$=\left(-\frac{7}{9}I_1+\frac{5}{3}I_2\right)\left(\frac{1+6I_1+2I_2}{1+72I_1+8I_2-x^2}\right)$$

$$=\frac{3I_1+5I_2}{1+72I_1+8I_2-x^2}\quad (The\ same\ integral\ function)$$

- **Standard 2- refined rational neutrosophic integral III:**

$$\int\frac{a_1+b_1I_1+c_1I_2}{x^2+(a_2+b_2I_1+c_2I_2)^2}dx$$

$$=\frac{1}{2}\left(\frac{a_1}{a_2}+\left[\frac{a_2b_1+b_1c_2-a_1b_2-b_2c_1}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1+\left[\frac{a_2c_1-a_1c_2}{a_2(a_2+c_2)}\right]I_2\right)tan^{-1}\left(\left(\frac{1}{a_2}+\left[\frac{-b_2}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1-\left[\frac{c_2}{a_2(a_2+c_2)}\right]I_2\right)x\right)+C$$

Whereas $a_2\neq 0,\ a_2\neq -c_2,\ a_2\neq -b_2-c_2, C=a_0+b_0I_1+c_0I_2\ and\ a_0,\ b_0,\ c_0$ are real numbers.

Proof:
Let's put: $x=(a_2+b_2I_1+c_2I_2)\,tan\,\vartheta \quad \Rightarrow dx=(a_2+b_2I_1+c_2I_2)sec^2\vartheta\ d\vartheta$

Then:

$$x^2+(a_2+b_2I_1+c_2I_2)^2=(a_2+b_2I_1+c_2I_2)^2tan^2\vartheta+(a_2+b_2I_1+c_2I_2)^2$$

$$=(a_2+b_2I_1+c_2I_2)^2(tan^2\vartheta+1)$$

$$=(a_2+b_2I_1+c_2I_2)^2sec^2\vartheta$$

$$\Rightarrow\quad\int\frac{a_1+b_1I_1+c_1I_2}{x^2+(a_2+b_2I_1+c_2I_2)^2}dx=\int\frac{a_1+b_1I_1+c_1I_2}{(a_2+b_2I_1+c_2I_2)^2sec^2\vartheta}(a_2+b_2I_1+c_2I_2)sec^2\vartheta\ d\vartheta$$

$$=\int\frac{a_1+b_1I_1+c_1I_2}{a_2+b_2I_1+c_2I_2}d\vartheta=\frac{a_1+b_1I_1+c_1I_2}{a_2+b_2I_1+c_2I_2}\int d\vartheta$$

$$=\frac{a_1+b_1I_1+c_1I_2}{a_2+b_2I_1+c_2I_2}\vartheta=\frac{a_1+b_1I_1+c_1I_2}{a_2+b_2I_1+c_2I_2}tan^{-1}\left(\frac{x}{a_2+b_2I_1+c_2I_2}\right)+C$$

Where:

$$\vartheta = tan^{-1}\left(\frac{x}{a_2 + b_2I_1 + c_2I_2}\right)$$

Hence:

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}dx$$

$$= \left(\frac{a_1}{a_2} + \left[\frac{a_2b_1 + b_1c_2 - a_1b_2 - b_2c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right]I_1 + \left[\frac{a_2c_1 - a_1c_2}{a_2(a_2 + c_2)}\right]I_2\right)tan^{-1}\left(\left(\frac{1}{a_2} + \left[\frac{-b_2}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right]I_1 - \left[\frac{c_2}{a_2(a_2 + c_2)}\right]I_2\right)x\right) + C$$

Whereas $a_2 \neq 0,\ a_2 \neq -c_2,\ a_2 \neq -b_2 - c_2,\ C = a_0 + b_0I_1 + c_0I_2\ and\ a_0,\ b_0,\ c_0$ are real numbers.

Example 3

Evaluate:

$$\int \frac{-6 + 7I_1 + 8I_2}{x^2 + 25 + 80I_1 + 39I_2}dx$$

Solution:

$$x^2 + 25 + 80I_1 + 39I_2 = x^2 + \left(\sqrt{25 + 80I_1 + 39I_2}\right)^2$$

$$x^2 + 25 + 80I_1 + 39I_2 = x^2 + (5 + 4I_1 + 3I_2)^2$$

Whereas: $\sqrt{25 + 80I_1 + 39I_2} = \sqrt{25} + \left[\sqrt{25 + 80 + 39} - \sqrt{25 + 39}\right]I_1 + \left[\sqrt{25 + 39} - \sqrt{25}\right]I_2$

$\Rightarrow\ \sqrt{1 + 72I_1 + 8I_2} = 5 + 4I_1 + 3I_2$

$$\int \frac{-6 + 7I_1 + 8I_2}{x^2 + 25 + 80I_1 + 39I_2}dx = \int \frac{-6 + 7I_1 + 8I_2}{x^2 + (5 + 4I_1 + 3I_2)^2}dx$$

$$= \left(\frac{-6 + 7I_1 + 8I_2}{5 + 4I_1 + 3I_2}\right)tan^{-1}\left(\frac{x}{5 + 4I_1 + 3I_2}\right) + C$$

$$= \left(\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2\right)tan^{-1}\left(\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right)x\right) + C$$

Let's check the answer:

$$\frac{d}{dx}\left[\left(\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2\right)tan^{-1}\left(\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right)x\right) + C\right]$$

$$= \left(\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2\right)\frac{\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2}{\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right)^2 x^2 + 1}$$

$$= \left(\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2\right) \frac{\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2}{\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right)^2 \left(x^2 + \frac{1}{\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right)^2}\right)}$$

$$= \left(\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2\right) \frac{1}{\left(\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2\right) \left(x^2 + \frac{1}{\frac{1}{25} - \frac{5}{576}I_1 - \frac{39}{1600}I_2}\right)}$$

$$= \left(\frac{\frac{-6}{5} + \frac{1}{2}I_1 + \frac{29}{20}I_2}{\frac{1}{5} - \frac{1}{24}I_1 - \frac{3}{40}I_2}\right) \frac{1}{\left(x^2 + \frac{1}{\frac{1}{25} - \frac{5}{576}I_1 - \frac{39}{1600}I_2}\right)}$$

$$= (-6 + 7I_1 + 8I_2) \frac{1}{(x^2 + 25 + 80I_1 + 39I_2)}$$

$$= \frac{-6 + 7I_1 + 8I_2}{x^2 + 25 + 80I_1 + 39I_2} \quad (The\ same\ integral\ function)$$

3. Conclusions

In this paper, we were able to discuss three rules that made it easy for us to solve and find the integral of 2- refined rational neutrosophic functions directly without following long solutions and methods to find these integrals. In addition, to verify the validity of these rules, we provided a proof for each rule. We were also able to obtain results that were verified through a set of examples.

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References

[1] Smarandache, F., " (T,I,F)- Neutrosophic Structures", Neutrosophic Sets and Systems, vol 8, pp. 3-10, 2015.

[2] Agboola,A.A.A. "On Refined Neutrosophic Algebraic Structures", Neutrosophic Sets and Systems, vol 10, pp. 99-101, 2015.

[3] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[4] Smarandache, F.,"n-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, USA, vol 4, pp. 143-146, 2013.

[5] Agboola, A.A.A.; Akinola, A.D; Oyebola, O.Y., "Neutrosophic Rings I", Int. J. of Math. Comb., vol 4, pp.1-14, 2011.

[6] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[7] Abobala, M., "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[8] Alhasan, Y., and Abdulfatah, R., "Division of refined neutrosophic numbers", Neutrosophic Sets and Systems, Vol. 60, pp. 1-5. 2023.

[9] Alhasan,Y., "The neutrosophic integrals and integration methods", Neutrosophic Sets and Systems, Volume 43, pp. 290-301, 2021.

[10] Alhasan,Y., " The neutrosophic integrals by partial fraction ", Neutrosophic Sets and Systems, Volume 44, pp. 290-301, 2021.

[10] Smarandache, F., and Abobala, M., "Operations with n-Refined Literal Neutrosophic Numbers using the Identification Method and the n-Refined AH-Isometry", Neutrosophic Sets and Systems, Volume 70, pp. 350-358, 2024. https://fs.unm.edu/NSS/RefinedLiteral21.pdf

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