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Neutrosophic normal vague set fitting to trigonometric concept via aggregation operators and its augmentation

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Abstract. Using a cotangent trigonometric neutrosophic normal vague set (CotNNVS), we begin this communication article with several novel techniques. Some trigonometric neutrosophic sets and imprecise neutrosophic sets are extended by the novel idea of CotNNVS. We will discuss the various aggregating processes that interpret Cotangent trigonometric neutrosophic normal vague weighted averaging (CotNNVWA), cotangent trigonometric neutrosophic normal vague weighted geometric (CotNNVWG), cotangent trigonometric generalized neutrosophic normal vague weighted averaging (CotGNNVWA), and cotangent trigonometric generalized neutrosophic normal vague weighted geometric (CotGNNVWG) are the new topics covered in this paper.

Keywords: CotNNVWA, CotNNVWG, CotGNNVWA, CotGNNVWG.

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1. **Introduction**

In the majority of actual issues, uncertainty is present everywhere. Fuzzy set (FS) theory [1], intuitionistic fuzzy set (IFS) theory [2], Pythagorean fuzzy set (PFS) theory [3], and neutrosophic set (NSS) theory [4] are only a few of the uncertain theories that have been presented through to deal with the uncertainties. The membership value (MV) of an element in a FS is the degree to which each element of the universe belongs to the set, but with a grade or degree of belongingness that ranges from zero to one. Clustering algorithms are used in applications of FSs, such as fuzzy c-numbers and regression prediction for fuzzy time series [5] [6]. Afterwards, Atanassov introduced the idea of an IFS logic, which is categorized based on the requirement that the total of its MV and non-membership value (NMV) be less than or equal to one [2]. When the total of the MV and NMVs exceeds one, we may occasionally run into difficulties making decisions. The novel idea of PFS logic, an extension of IFS, was proposed by Yager [3]. It is defined by the square sum of its MV and NMV, for which the value is less than or equal to one. [7–9] Akram et al. spoke about the several applications based on the PFS. We extend the logic for geometric aggregation operators under the concept of an interval-valued PFS to a group DM strategy, as explored by Rahman et al. [10]. An IVPFS with aggregation operator was examined by Peng et al. [11]. Some methods for MGCDM based on an induced IVPFS Einstein aggregation operator were suggested by Rahman et al. [12]. An application for PFS Einstein choquet integral operators was used by Khan [13].

A novel hypothesis called as NSS has just been proposed. The primary difference between FS and IFS are represented by the neutrality of mind, which is what the name "neutosophy" refers to. Florentin Smarandache introduced NSS [4]. Each claim is assessed to have a degree of truth, a degree of indeterminacy, and a degree of falsehood in this logic. NSS is a set that falls between $[0, 1]$ and contains degrees of truth, indeterminacy, and falsehood for every element in the universe. It has been demonstrated from a philosophical perspective that an NSS generalizes a classical set, an FS, an IVFS, etc. The Pythagorean neutrosophic IVS (PNSIVS) was first presented by Florentin Smarandache et al. [14]. The single-valued NSS is used based on context analysis [15] and medical diagnosis [16]. The distance measurements for IFSshamming, euclidean, normalized hamming, and normalized euclidean distancesas well as their similarities to PFSs were expanded by Ejegwa [17] and applied to both MCDM and MADM situations. Based on a review of the literature, we find that the majority of distance functions for PNSNIVSs are presented as PNSIVS generalizations. The concept of MADM for a novel class of NSS aggregation operators is discussed by Palanikumar et al. [18]. Applications of spherical ambiguous normal operators for farmer selection by Palanikumar et al. [19]. This study focuses on the novel idea of CotNNVS. The CotNNVS aggregation operators are where we begin. As stated, the paper is divided into five sections. 1 is referred to as an introduction. It is said that Section 2 refers to the fundamental ideas presented. In Section 3, fundamental algebraic operations are covered. In Section 4, the aggregation operations for CotNNVNs are discussed. The conclusion may be found in the last section 5.

2. Basic concept

In this section, we recall some of the basic definitions required for our additional studies.

Definition 2.1. [4] The NSS $\Bbbk = \Big\{u, \langle \aleph^{\curlywedge}_{\Bbbk}[\varkappa], \aleph^{\beth}_{\Bbbk}[\varkappa], \aleph^{\daleth}_{\Bbbk}[\varkappa] \rangle \Big| \varkappa \in \mathscr{U} \Big\}$, where $\aleph^{\curlywedge}_{\Bbbk}, \aleph^{\beth}_{\Bbbk}, \aleph^{\daleth}_{\Bbbk} : \mathscr{U} \to [0, 1]$ is called the truth MV, indeterminacy MV and falsity MV of $\varkappa \in \mathscr{U}$ to $\Bbbk$, respectively and

$0 \preccurlyeq \aleph^{\curlywedge}_{\Bbbk}[\varkappa] + \aleph^{\gimel}_{\Bbbk}[\varkappa] + \aleph^{\daleth}_{\Bbbk}[\varkappa] \preccurlyeq 3$. For comfortable, $\Bbbk = \left\langle \aleph^{\curlywedge}_{\Bbbk}, \aleph^{\gimel}_{\Bbbk}, \aleph^{\daleth}_{\Bbbk} \right\rangle$ is said to be a neutrosophic number[NSN].

Definition 2.2. [11] Let $\Bbbk = \left\langle [\aleph^{\curlywedge-}, \aleph^{\curlywedge+}], [\aleph^{\daleth-}, \aleph^{\daleth+}] \right\rangle$, $\Bbbk_1 = \left\langle [\aleph^{\curlywedge-}_1, \aleph^{\curlywedge+}_1], [\aleph^{\daleth-}_1, \aleph^{\daleth+}_1] \right\rangle$ and $\Bbbk_2 = \left\langle [\aleph^{\curlywedge-}_2, \aleph^{\curlywedge+}_2], [\aleph^{\daleth-}_2, \aleph^{\daleth+}_2] \right\rangle$ be the PIVFNs, and $\varsigma > 0$. Then, the new basic operations are formed as below:

(1) $\Bbbk_1 \ddagger \Bbbk_2 = \left[\begin{matrix} \left[\sqrt{[\aleph^{\curlywedge-}_1]^2 + [\aleph^{\curlywedge-}_2]^2 - [\aleph^{\curlywedge-}_1]^2 \cdot [\aleph^{\curlywedge-}_2]^2}, \sqrt{[\aleph^{\curlywedge+}_1]^2 + [\aleph^{\curlywedge+}_2]^2 - [\aleph^{\curlywedge+}_1]^2 \cdot [\aleph^{\curlywedge+}_2]^2}\right], \\ \left[\aleph^{\daleth-}_1 \cdot \aleph^{\daleth-}_2, \, \aleph^{\daleth+}_1 \cdot \aleph^{\daleth+}_2\right] \end{matrix}\right]$,

(2) $\Bbbk_1 \amalg \Bbbk_2 = \left[\begin{matrix} \left[\aleph^{\curlywedge-}_1 \cdot \aleph^{\curlywedge-}_2, \, \aleph^{\curlywedge+}_1 \cdot \aleph^{\curlywedge+}_2\right], \\ \left[\sqrt{[\aleph^{\daleth-}_1]^2 + [\aleph^{\daleth-}_2]^2 - [\aleph^{\daleth-}_1]^2 \cdot [\aleph^{\daleth-}_2]^2}, \sqrt{[\aleph^{\daleth+}_1]^2 + [\aleph^{\daleth+}_2]^2 - [\aleph^{\daleth+}_1]^2 \cdot [\aleph^{\daleth+}_2]^2}\right] \end{matrix}\right]$,

(3) $\varsigma \cdot \Bbbk = \left[\left[\sqrt{1 - \left[1 - [\aleph^{\curlywedge-}]^2\right]^{\varsigma}}, \sqrt{1 - \left[1 - [\aleph^{\curlywedge+}]^2\right]^{\varsigma}}\right], \left[[\aleph^{\daleth-}]^{\varsigma}, [\aleph^{\daleth+}]^{\varsigma}\right]\right]$,

(4) $\Bbbk^{\varsigma} = \left[\left[[\aleph^{\curlywedge-}]^{\varsigma}, [\aleph^{\curlywedge+}]^{\varsigma}\right], \left[\sqrt{1 - \left[1 - [\aleph^{\daleth-}]^2\right]^{\varsigma}}, \sqrt{1 - \left[1 - [\aleph^{\daleth+}]^2\right]^{\varsigma}}\right]\right]$.

3. Basic Operation for CotNNVN

The novel concept of CotNNVN and its operations, where $\cot \pi/4 = \mho$, were constructed.

Definition 3.1. Let $[\wp, \ell] \in N$, $\Bbbk = \left\langle [\wp, \ell]; [\aleph^{\curlywedge-}, \aleph^{[1-\daleth]+}], [\aleph^{\gimel-}, \aleph^{\gimel+}], [\aleph^{\daleth-}, \aleph^{[1-\curlywedge]+}] \right\rangle$ be the NSNVN. Then $\cot \Bbbk = \Big\{ \Big[\Big[\mho \cdot [\aleph^{\curlywedge-}_{\Bbbk}[\varkappa]]\Big], \Big[\mho \cdot [\aleph^{[1-\daleth]+}_{\Bbbk}[\varkappa]]\Big] \Big], \Big[1 - \Big[\mho \cdot 1 - [\aleph^{\gimel-}_{\Bbbk}[\varkappa]]\Big], 1 - \Big[\mho \cdot 1 - [\aleph^{\gimel+}_{\Bbbk}[\varkappa]]\Big] \Big], \Big[1 - \Big[\mho \cdot [1 - \aleph^{\daleth-}_{\Bbbk}[\varkappa]]\Big], 1 - \Big[\mho \cdot [1 - \aleph^{[1-\curlywedge]+}_{\Bbbk}[\varkappa]]\Big] \Big] \Big\}$.

Thus, $\cot \Bbbk$ is a CotNNVN and

$\Big[\mho \cdot \aleph^{[1-\daleth]+}_{\Bbbk}[\varkappa]\Big] \in [0,1]$, $\Big[\mho \cdot \aleph^{\gimel+}_{\Bbbk}[\varkappa]\Big] \in [0,1]$ and $1 - \Big[\mho \cdot \Big[1 - \aleph^{[1-\curlywedge]+}_{\Bbbk}[\varkappa]\Big]\Big] \in [0,1]$. Hence, $\cot \Bbbk = \Big\{ \Big[\Big[\mho \cdot [\aleph^{\curlywedge-}_{\Bbbk}[\varkappa]]\Big], \Big[\mho \cdot [\aleph^{[1-\daleth]+}_{\Bbbk}[\varkappa]]\Big] \Big], 1 - \Big[\Big[\mho \cdot [1 - [\aleph^{\gimel-}_{\Bbbk}[\varkappa]]]\Big], 1 - \Big[\mho \cdot [1 - [\aleph^{\gimel+}_{\Bbbk}[\varkappa]]]\Big] \Big], \Big[1 - \Big[\mho \cdot [[1 - \aleph^{\daleth-}_{\Bbbk}[\varkappa]]]\Big], 1 - \Big[\mho \cdot [[1 - \aleph^{[1-\curlywedge]+}_{\Bbbk}[\varkappa]]]\Big] \Big] \Big\}$

is a CotNNVN, put $\Big[\aleph^{\curlywedge-}_{\Bbbk}, \aleph^{[1-\daleth]+}_{\Bbbk}\Big] = \Big[\aleph^{\curlywedge-}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}, \aleph^{[1-\daleth]+}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}\Big]$ and $\Big[\aleph^{\gimel-}_{\Bbbk}, \aleph^{\gimel+}_{\Bbbk}\Big] = \Big[\aleph^{\gimel-}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}, \aleph^{\gimel+}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}\Big]$ and $\Big[\aleph^{\daleth-}_{\Bbbk}, \aleph^{[1-\curlywedge]+}_{\Bbbk}\Big] = \Big[\aleph^{\daleth-}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}, \aleph^{[1-\curlywedge]+}_{\Bbbk} e^{-\left[\frac{y-\wp}{\ell}\right]^2}\Big]$, $y \in Y$, Here Y is a non-empty set.

Definition 3.2. Let $\Bbbk = \left\langle [\wp, \ell]; [\aleph^{\curlywedge-}, \aleph^{[1-\daleth]+}], [\aleph^{\gimel-}, \aleph^{\gimel+}], [\aleph^{\daleth-}, \aleph^{[1-\curlywedge]+}] \right\rangle$ be the CotNNVN. Then the score function of $\Bbbk$ is defined by $S[\Bbbk] =$ $\frac{\wp}{2}\left[\frac{2 + [[\mho^2 \cdot \aleph^{\curlywedge-}]] + [[\mho^2 \cdot \aleph^{[1-\daleth]+}]] - [[\mho^2 \cdot \aleph^{\gimel-}]] - [[\mho^2 \cdot \aleph^{\gimel+}]] - [[\mho^2 \cdot \aleph^{\daleth-}]] - [[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]]}{2}\right]$, where $S[\Bbbk]$ lies between -1 and 1.

Definition 3.3. Let $\Bbbk = \left\langle [\wp, \ell]; [\aleph^{\curlywedge-}, \aleph^{[1-\daleth]+}], [\aleph^{\gimel-}, \aleph^{\gimel+}], [\aleph^{\daleth-}, \aleph^{[1-\curlywedge]+}] \right\rangle$, $\Bbbk_1 = \left\langle [\wp_1, \ell_1]; [\aleph^{\curlywedge-}_1, \aleph^{[1-\daleth]+}_1], [\aleph^{\gimel-}_1, \aleph^{\gimel+}_1], [\aleph^{\daleth-}_1, \aleph^{[1-\curlywedge]+}_1] \right\rangle$ and $\Bbbk_2 = \left\langle [\wp_2, \ell_2]; [\aleph^{\curlywedge-}_2, \aleph^{[1-\daleth]+}_2], [\aleph^{\gimel-}_2, \aleph^{\gimel+}_2], [\aleph^{\daleth-}_2, \aleph^{[1-\curlywedge]+}_2] \right\rangle$ be the CotNNVNs and $\varsigma > 0$.

(1) $\cot \Bbbk_1 \ddagger \cot \Bbbk_2 =$

$$\begin{bmatrix} [\wp_1 \ddagger \wp_2, \ell_1 \ddagger \ell_2]; \\ \begin{bmatrix} [[\mho^2 \cdot \aleph_1^{\leftthreetimes -}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{\leftthreetimes -}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{\leftthreetimes -}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{\leftthreetimes -}]]^{\varsigma}, \\ [[\mho^2 \cdot \aleph_1^{[1-\dashv]+}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{[1-\dashv]+}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{[1-\dashv]+}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{[1-\dashv]+}]]^{\varsigma} \end{bmatrix}, \\ \Big[[\mho^2 \cdot \aleph_1^{\gimel -}] \cdot [\mho^2 \cdot \aleph_2^{\gimel -}], [\mho^2 \cdot \aleph_1^{\gimel +}] \cdot [\mho^2 \cdot \aleph_2^{\gimel +}]\Big], \\ \Big[[\mho^2 \cdot \aleph_1^{\dashv -}] \cdot [\mho^2 \cdot \aleph_2^{\dashv -}], [\mho^2 \cdot \aleph_1^{[1-\leftthreetimes]+}] \cdot [\mho^2 \cdot \aleph_2^{[1-\leftthreetimes]+}]\Big] \end{bmatrix}.$$

(2) $\cot \Bbbk_1 \amalg \cot \Bbbk_2 =$

$$\begin{bmatrix} [\wp_1 \amalg \wp_2, \ell_1 \amalg \ell_2]; \\ \Big[[\mho^2 \cdot \aleph_1^{\leftthreetimes -}] \cdot [\mho^2 \cdot \aleph_2^{\leftthreetimes -}], [\mho^2 \cdot \aleph_1^{[1-\dashv]+}] \cdot [\mho^2 \cdot \aleph_2^{[1-\dashv]+}]\Big], \\ \begin{bmatrix} [[\mho^2 \cdot \aleph_1^{\gimel -}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{\gimel -}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{\gimel -}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{\gimel -}]]^{\varsigma}, \\ [[\mho^2 \cdot \aleph_1^{\gimel +}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{\gimel +}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{\gimel +}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{\gimel +}]]^{\varsigma} \end{bmatrix}, \\ \begin{bmatrix} [[\mho^2 \cdot \aleph_1^{\dashv -}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{\dashv -}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{\dashv -}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{\dashv -}]]^{\varsigma}, \\ [[\mho^2 \cdot \aleph_1^{[1-\leftthreetimes]+}]]^{\varsigma} + [[\mho^2 \cdot \aleph_2^{[1-\leftthreetimes]+}]]^{\varsigma} - [[\mho^2 \cdot \aleph_1^{[1-\leftthreetimes]+}]]^{\varsigma} \cdot [[\mho^2 \cdot \aleph_2^{[1-\leftthreetimes]+}]]^{\varsigma} \end{bmatrix} \end{bmatrix}.$$

(3)
$$\varsigma \cdot \cot \Bbbk = \begin{bmatrix} [\varsigma \cdot \wp, \varsigma \cdot \ell]; \\ \Big[1 - \big[1 - [[\mho^2 \cdot \aleph^{\leftthreetimes -}]]^{\varsigma}\big]^{\varsigma}, 1 - \big[1 - [[\mho^2 \cdot \aleph^{[1-\dashv]+}]]^{\varsigma}\big]^{\varsigma}\Big], \\ \Big[[[\mho^2 \cdot \aleph^{\gimel -}]]^{\varsigma}, [[\mho^2 \cdot \aleph^{\gimel +}]]^{\varsigma}\Big], \\ \Big[[[\mho^2 \cdot \aleph^{\dashv -}]]^{\varsigma}, [[\mho^2 \cdot \aleph^{[1-\leftthreetimes]+}]]^{\varsigma}\Big] \end{bmatrix}.$$

(4)
$$[\cot \Bbbk]^{\varsigma} = \begin{bmatrix} [\wp^{\varsigma}, \ell^{\varsigma}]; \Big[[[\mho^2 \cdot \aleph^{\leftthreetimes -}]]^{\varsigma}, [[\mho^2 \cdot \aleph^{[1-\dashv]+}]]^{\varsigma}\Big], \\ \Big[1 - \big[1 - [[\mho^2 \cdot \aleph^{\gimel -}]]^{\varsigma}\big]^{\varsigma}, 1 - \big[1 - [[\mho^2 \cdot \aleph^{\gimel +}]]^{\varsigma}\big]^{\varsigma}\Big], \\ \Big[1 - \big[1 - [[\mho^2 \cdot \aleph^{\dashv -}]]^{\varsigma}\big]^{\varsigma}, 1 - \big[1 - [[\mho^2 \cdot \aleph^{[1-\leftthreetimes]+}]]^{\varsigma}\big]^{\varsigma}\Big] \end{bmatrix}.$$

4. Aggregation operators for CotNNVNs

The novel operators based on CotNNVWA, CotNNVWG, CotGNNVWA and CotGNNVWG are introduced in this section.

4.1. *CotNNV weighted averaging[CotNNVWA]*

Definition 4.1. Let $\Bbbk_i = \Big\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\dashv]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\dashv -}, \aleph_i^{[1-\leftthreetimes]+}] \Big\rangle$ be a finite collection of CotNNVNs, $H = [\eth_1, \eth_2, ..., \eth_n]$ be the weight of $\Bbbk_i$, $\eth_i \not\succcurlyeq 0$ and $\ddagger_{i=1}^{n} \eth_i = 1$. Then CotNNVWA $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \ddagger_{i=1}^{n} \eth_i \Bbbk_i$.

Theorem 4.2. *Let* $\Bbbk_i = \Big\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\dashv]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\dashv -}, \aleph_i^{[1-\leftthreetimes]+}] \Big\rangle$ *be a finite collection of CotNNVNs. Then prove that* $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] =$

$$\begin{bmatrix} \Big[\ddagger_{i=1}^{n} \eth_i \wp_i, \ddagger_{i=1}^{n} \eth_i \ell_i\Big]; \\ \Big[1 - \amalg_{i=1}^{n}\Big[1 - [[\mho^2 \cdot \aleph_i^{\leftthreetimes -}]]^{\varsigma}\Big]^{\eth_i}, 1 - \amalg_{i=1}^{n}\Big[1 - [[\mho^2 \cdot \aleph_i^{[1-\dashv]+}]]^{\varsigma}\Big]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{n} [[\mho^2 \cdot \aleph_i^{\gimel -}]]^{\eth_i}, \amalg_{i=1}^{n} [[\mho^2 \cdot \aleph_i^{\gimel +}]]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{n} [[\mho^2 \cdot \aleph_i^{\dashv -}]]^{\eth_i}, \amalg_{i=1}^{n} [[\mho^2 \cdot \aleph_i^{[1-\leftthreetimes]+}]]^{\eth_i}\Big] \end{bmatrix}.$$

Proof. If $n = 2$, then $\text{CotNNVWA}[\Bbbk_1, \Bbbk_2] = \eth_1 \cot \Bbbk_1 \ddagger \eth_2 \cot \Bbbk_2$, put

$$\eth_1 \cot \Bbbk_1 = \begin{bmatrix} \Big[\eth_1 \wp_1, \eth_1 \ell_1\Big]; \\ \left[1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_1}, 1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_1}\right], \\ \Big[[[\mho^2 \cdot \aleph_1^{\daleth -}]]^{\eth_1}, [[\mho^2 \cdot \aleph_1^{\daleth +}]]^{\eth_1}\Big], \\ \Big[[[\mho^2 \cdot \aleph_1^{\gimel -}]]^{\eth_1}, [[\mho^2 \cdot \aleph_1^{[1-\curlywedge]+}]]^{\eth_1}\Big] \end{bmatrix}$$

and

$$\eth_2 \cot \Bbbk_2 = \begin{bmatrix} \Big[\eth_2 \wp_2, \eth_2 \ell_2\Big]; \\ \left[1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_2}, 1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_2}\right], \\ \Big[[[\mho^2 \cdot \aleph_2^{\daleth -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_2^{\daleth +}]]^{\eth_2}\Big], \\ \Big[[[\mho^2 \cdot \aleph_2^{\gimel -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_2^{[1-\curlywedge]+}]]^{\eth_2}\Big] \end{bmatrix}.$$

Now, $\eth_1 \cot \Bbbk_1 \ddagger \eth_2 \cot \Bbbk_2$

$$= \begin{bmatrix} \Big[\eth_1 \wp_1 \ddagger \eth_2 \wp_2, \eth_1 \ell_1 \ddagger \eth_2 \ell_2\Big]; \\ \begin{bmatrix} \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_1}\Big] + \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_2}\Big] \\ -\Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_1}\Big] \cdot \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_2}\Big], \\ \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_1}\Big] + \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_2}\Big] \\ -\Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_1}\Big] \cdot \Big[1 - \Big[1 - [[\mho^2 \cdot \aleph_2^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_2}\Big] \end{bmatrix}, \\ \Big[[[\mho^2 \cdot \aleph_1^{\daleth -}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{\daleth -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_1^{\daleth +}]]^{\eth_1} [[\mho^2 \cdot \aleph_2^{\daleth +}]]^{\eth_2}\Big], \\ \Big[[[\mho^2 \cdot \aleph_1^{\gimel -}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{\gimel -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_1^{[1-\curlywedge]+}]]^{\eth_1} [[\mho^2 \cdot \aleph_2^{[1-\curlywedge]+}]]^{\eth_2}\Big] \end{bmatrix}$$

$$= \begin{bmatrix} \Big[\eth_1 \wp_1 \ddagger \eth_2 \wp_2, \eth_1 \ell_1 \ddagger \eth_2 \ell_2\Big]; \\ \begin{bmatrix} 1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_1} \cdot \Big[1 - [[\mho^2 \cdot \aleph_2^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_2}, \\ 1 - \Big[1 - [[\mho^2 \cdot \aleph_1^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_1} \cdot \Big[1 - [[\mho^2 \cdot \aleph_2^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_2} \end{bmatrix}, \\ \Big[[[\mho^2 \cdot \aleph_1^{\daleth -}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{\daleth -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_1^{\daleth +}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{\daleth +}]]^{\eth_2}\Big], \\ \Big[[[\mho^2 \cdot \aleph_1^{\gimel -}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{\gimel -}]]^{\eth_2}, [[\mho^2 \cdot \aleph_1^{[1-\curlywedge]+}]]^{\eth_1} \cdot [[\mho^2 \cdot \aleph_2^{[1-\curlywedge]+}]]^{\eth_2}\Big] \end{bmatrix}.$$

Hence $CotNNVWA[\Bbbk_1, \Bbbk_2]$

$$= \begin{bmatrix} \Big[\ddagger_{i=1}^{2} \eth_i \wp_i, \ddagger_{i=1}^{2} \eth_i \ell_i\Big]; \\ \left[1 - \amalg_{i=1}^{2} \Big[1 - [[\mho^2 \cdot \aleph_i^{\curlywedge -}]]^{\varsigma}\Big]^{\eth_i}, 1 - \amalg_{i=1}^{2} \Big[1 - [[\mho^2 \cdot \aleph_i^{[1-\gimel]+}]]^{\varsigma}\Big]^{\eth_i}\right], \\ \Big[\amalg_{i=1}^{2} [[\mho^2 \cdot \aleph_i^{\daleth -}]]^{\eth_i}, \amalg_{i=1}^{2} [[\mho^2 \cdot \aleph_i^{\daleth +}]]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{2} [[\mho^2 \cdot \aleph_i^{\gimel -}]]^{\eth_i}, \amalg_{i=1}^{2} [[\mho^2 \cdot \aleph_i^{[1-\curlywedge]+}]]^{\eth_i}\Big] \end{bmatrix}.$$

Similarly, $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] =$

$$\begin{bmatrix} \Big[\ddagger_{i=1}^{n}\eth_i\wp_i, \ddagger_{i=1}^{n}\eth_i\ell_i\Big]; \\ \Bigg[1-\amalg_{i=1}^{n}\Big[1-[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_i}, 1-\amalg_{i=1}^{n}\Big[1-[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_i}\Bigg], \\ \Big[\amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\beth-}]]^{\eth_i}, \amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\beth+}]]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\eth_i}, \amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\eth_i}\Big] \end{bmatrix}.$$

If $n = l + 1$, then CotNNVWA $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n, \Bbbk_{n+1}]$

$$= \begin{bmatrix} \Big[\ddagger_{i=1}^{n}\eth_i\wp_i \ddagger \eth_{n+1}\wp_{n+1}, \ddagger_{i=1}^{n}\eth_i\ell_i \ddagger \eth_{n+1}\ell_{n+1}\Big]; \\ \begin{bmatrix} \ddagger_{i=1}^{n}\Big[1-\Big[1-[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_i}\Big] + \Big[1-\Big[1-[[\mho^2\cdot\aleph_{n+1}^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_{n+1}}\Big] \\ -\amalg_{i=1}^{n}\Big[1-\Big[1-[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_i}\Big]\cdot\Big[1-\Big[1-[[\mho^2\cdot\aleph_{n+1}^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_{n+1}}\Big], \\ \ddagger_{i=1}^{n}\Big[1-\Big[1-[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_i}\Big] + \Big[1-\Big[1-[[\mho^2\cdot\aleph_{n+1}^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_{n+1}}\Big] \\ -\amalg_{i=1}^{n}\Big[1-\Big[1-[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_i}\Big]\cdot\Big[1-\Big[1-[[\mho^2\cdot\aleph_{n+1}^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_{n+1}}\Big] \end{bmatrix} \\ \Big[\amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\beth-}]]^{\eth_i}\cdot[[\mho^2\cdot\aleph_{n+1}^{\beth-}]]^{\eth_{n+1}}, \amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\beth+}]]^{\eth_i}\cdot[[\mho^2\cdot\aleph_{n+1}^{\beth+}]]^{\eth_{n+1}}\Big], \\ \Big[\amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\eth_i}\cdot[[\mho^2\cdot\aleph_{n+1}^{\daleth-}]]^{\eth_{n+1}}, \amalg_{i=1}^{n}[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\eth_i}\cdot[[\mho^2\cdot\aleph_{n+1}^{[1-\curlywedge]+}]]^{\eth_{n+1}}\Big] \end{bmatrix},$$

$$= \begin{bmatrix} \Big[\ddagger_{i=1}^{n+1}\eth_i\wp_i, \ddagger_{i=1}^{n+1}\eth_i\ell_i\Big]; \\ \Bigg[1-\amalg_{i=1}^{n+1}\Big[1-[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_i}, 1-\amalg_{i=1}^{n+1}\Big[1-[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_i}\Bigg], \\ \Big[\amalg_{i=1}^{n+1}[[\mho^2\cdot\aleph_i^{\beth-}]]^{\eth_i}, \amalg_{i=1}^{n+1}[[\mho^2\cdot\aleph_i^{\beth+}]]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{n+1}[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\eth_i}, \amalg_{i=1}^{n+1}[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\eth_i}\Big] \end{bmatrix}.$$

Theorem 4.3. *If all* $\Bbbk_i = \Big\langle[\wp_i, \ell_i]; \Big[[[\mho\cdot\aleph_i^{\curlywedge-}]], [[\mho\cdot\aleph_i^{[1-\daleth]+}]]\Big], \Big[[[\mho\cdot\aleph_i^{\beth-}]], [[\mho\cdot\aleph_i^{\beth+}]]\Big], \Big[[[\mho\cdot\aleph_i^{\daleth-}]], [[\mho\cdot\aleph_i^{[1-\curlywedge]+}]]\Big]\Big\rangle$ *are equal and* $\Bbbk_i = \Bbbk$ *with* $\varsigma = 1$. *Prove that* $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \Bbbk$.

Proof. Since, $[\wp_i, \ell_i] = [\wp, \ell]$, $[[\mho\cdot\aleph_i^{\curlywedge-}], [\mho\cdot\aleph_i^{[1-\daleth]+}]] = [[\mho\cdot\aleph^{\curlywedge-}], [\mho\cdot\aleph^{[1-\daleth]+}]]$, $[[\mho\cdot\aleph_i^{\beth-}], [\mho\cdot\aleph_i^{\beth+}]] = [[\mho\cdot\aleph^{\beth-}], [\mho\cdot\aleph^{\beth+}]]$ and $[[\mho\cdot\aleph_i^{\daleth-}], [\mho\cdot\aleph_i^{[1-\curlywedge]+}]] = [[\mho\cdot\aleph^{\daleth-}], [\mho\cdot\aleph^{[1-\curlywedge]+}]]$, for $i = 1, 2, ..., n$ and $\ddagger_{i=1}^{n}\eth_i = 1$. Now, $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n]$

$$= \begin{bmatrix} \Big[\ddagger_{i=1}^{n}\eth_i\wp_i, \ddagger_{i=1}^{n}\eth_i\ell_i\Big]; \\ \Bigg[1-\amalg_{i=1}^{n}\Big[1-[[\mho\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\Big]^{\eth_i}, 1-\amalg_{i=1}^{n}\Big[1-[[\mho\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\Big]^{\eth_i}\Bigg], \\ \Big[\amalg_{i=1}^{n}[[\mho\cdot\aleph_i^{\beth-}]]^{\eth_i}, \amalg_{i=1}^{n}[[\mho\cdot\aleph_i^{\beth+}]]^{\eth_i}\Big], \\ \Big[\amalg_{i=1}^{n}[[\mho\cdot\aleph_i^{\daleth-}]]^{\eth_i}, \amalg_{i=1}^{n}[[\mho\cdot\aleph_i^{[1-\curlywedge]+}]]^{\eth_i}\Big] \end{bmatrix}$$

$$
\begin{aligned}
&= \left[\begin{array}{c}
\left[\wp \ddagger_{i=1}^{n} \eth_i, \ell \ddagger_{i=1}^{n} \eth_i\right];\\
\left[1-\left[1-[[\mho\cdot\aleph^{\leftthreetimes-}]]^{\varsigma}\right]^{\ddagger_{i=1}^{n}\eth_i}, 1\left[1-[[\mho\cdot\aleph^{[1-⅃]+}]]^{\varsigma}\right]^{\ddagger_{i=1}^{n}\eth_i}\right],\\
\left[[[\mho\cdot\aleph^{\beth-}]]^{\ddagger_{i=1}^{n}\eth_i}, [[\mho\cdot\aleph^{\beth+}]]^{\ddagger_{i=1}^{n}\eth_i}\right],\\
\left[[[\mho\cdot\aleph^{⅃-}]]^{\ddagger_{i=1}^{n}\eth_i}, [[\mho\cdot\aleph^{[1-\leftthreetimes]+}]]^{\ddagger_{i=1}^{n}\eth_i}\right]
\end{array}\right]\\
&= \left[\begin{array}{c}
[\wp,\ell];\left[1-\left[1-[[\mho\cdot\aleph^{\leftthreetimes-}]]^{\varsigma}\right], 1-\left[1-[[\mho\cdot\aleph^{[1-⅃]+}]]^{\varsigma}\right]\right],\\
\left[[[\mho\cdot\aleph^{\beth-}]], [[\mho\cdot\aleph^{\beth+}]]\right],\\
\left[[[\mho\cdot\aleph^{⅃-}]], [[\mho\cdot\aleph^{[1-\leftthreetimes]+}]]\right]
\end{array}\right]\\
&= \Bbbk.
\end{aligned}
$$

Theorem 4.4. *Let* $\Bbbk_i = \left\langle [\wp_{ij}, \ell_{ij}]; [\aleph_{ij}^{\leftthreetimes-}, \aleph_{ij}^{[1-⅃]+}], [\aleph_{ij}^{\beth-}, \aleph_{ij}^{\beth+}][\aleph_{ij}^{⅃-}, \aleph_{ij}^{[1-\leftthreetimes]+}]\right\rangle$ *be a finite collection of CotNNVWA, where* $[i = 1,2,...,n]; [j = 1,2,...,i_j]$, $\underbrace{\wp} = \inf_{1\preccurlyeq i\preccurlyeq n,\ j=1,2,...,i_j} \wp_{ij}$, $\overbrace{\wp} = \sup_1 \wp_{ij}$, $\underbrace{\ell} = \sup_1 \ell_{ij}$, $\overbrace{\ell} = \inf_1 \ell_{ij}$, $\underbrace{\aleph^{\leftthreetimes-}} = \inf_1 \aleph_{ij}^{\leftthreetimes^l}$, $\overbrace{\aleph^{\leftthreetimes-}} = \sup_1 \aleph_{ij}^{\leftthreetimes^l}$, $\underbrace{\aleph^{[1-⅃]+}} = \inf_1 \aleph_{ij}^{\leftthreetimes^u}$, $\overbrace{\aleph^{[1-⅃]+}} = \sup_1 \aleph_{ij}^{\leftthreetimes^u}$, $\underbrace{\aleph^{\beth-}} = \inf_1 \aleph_{ij}^{\beth^l}$, $\overbrace{\aleph^{\beth-}} = \sup_1 \aleph_{ij}^{\beth^l}$, $\underbrace{\aleph^{\beth+}} = \inf_1 \aleph_{ij}^{\beth^u}$, $\overbrace{\aleph^{\beth+}} = \sup_1 \aleph_{ij}^{\beth^u}$, $\underbrace{\aleph^{⅃-}} = \inf_1 \aleph_{ij}^{⅃^l}$, $\overbrace{\aleph^{⅃-}} = \sup_1 \aleph_{ij}^{⅃^l}$, $\underbrace{\aleph^{[1-\leftthreetimes]+}} = \inf_1 \aleph_{ij}^{⅃^u}$, $\overbrace{\aleph^{[1-\leftthreetimes]+}} = \sup_1 \aleph_{ij}^{⅃^u}$.
Then, $\left\langle [\underbrace{\wp}, \underbrace{\ell}]; [\underbrace{\aleph^{\leftthreetimes-}}, \underbrace{\aleph^{[1-⅃]+}}], [\overbrace{\aleph^{\beth-}}, \overbrace{\aleph^{\beth+}}], [\overbrace{\aleph^{⅃-}}, \overbrace{\aleph^{[1-\leftthreetimes]+}}]\right\rangle$
$\preccurlyeq CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n]$
$\preccurlyeq \left\langle [\overbrace{\wp}, \overbrace{\ell}]; [\overbrace{\aleph^{\leftthreetimes-}}, \overbrace{\aleph^{[1-⅃]+}}], [\underbrace{\aleph^{\beth-}}, \underbrace{\aleph^{\beth+}}], [\underbrace{\aleph^{⅃-}}, \underbrace{\aleph^{[1-\leftthreetimes]+}}]\right\rangle$.

Proof. Since, $\underbrace{\aleph^{\leftthreetimes-}} = \inf_1 \aleph_{ij}^{\leftthreetimes^l}$, $\overbrace{\aleph^{\leftthreetimes-}} = \sup_1 \aleph_{ij}^{\leftthreetimes^l}$ $\underbrace{\aleph^{[1-⅃]+}} = \inf_1 \aleph_{ij}^{\leftthreetimes^u}$, $\overbrace{\aleph^{[1-⅃]+}} = \sup_1 \aleph_{ij}^{\leftthreetimes^u}$ and $\underbrace{\aleph^{\leftthreetimes-}} \preccurlyeq \aleph_{ij}^{\leftthreetimes^l} \preccurlyeq \overbrace{\aleph^{\leftthreetimes-}}$ and $\underbrace{\aleph^{[1-⅃]+}} \preccurlyeq \aleph_{ij}^{\leftthreetimes^u} \preccurlyeq \overbrace{\aleph^{[1-⅃]+}}$.
We have, $\underbrace{[\mho^2\cdot\aleph^{\leftthreetimes-}]} + \underbrace{[\mho^2\cdot\aleph^{[1-⅃]+}]}$

$$
\begin{aligned}
&= 1-\amalg_{i=1}^{n}\left[1-[\underbrace{[\mho^2\cdot\aleph^{\leftthreetimes-}]}]^{\varsigma}\right]^{\eth_i} + 1-\amalg_{i=1}^{n}\left[1-[\underbrace{[\mho^2\cdot\aleph^{[1-⅃]+}]}]^{\varsigma}\right]^{\eth_i}\\
&\preccurlyeq 1-\amalg_{i=1}^{n}\left[1-[\mho^2\cdot\aleph_{ij}^{\leftthreetimes^l}]^{\varsigma}\right]^{\eth_i} + 1-\amalg_{i=1}^{n}\left[1-[\mho^2\cdot\aleph_{ij}^{\leftthreetimes^u}]^{\varsigma}\right]^{\eth_i}\\
&\preccurlyeq 1-\amalg_{i=1}^{n}\left[1-[\overbrace{[\mho^2\cdot\aleph^{\leftthreetimes-}]}]^{\varsigma}\right]^{\eth_i} + 1-\amalg_{i=1}^{n}\left[1-[\overbrace{[\mho^2\cdot\aleph^{[1-⅃]+}]}]^{\varsigma}\right]^{\eth_i}\\
&= \overbrace{[\mho^2\cdot\aleph^{\leftthreetimes-}]} + \overbrace{[\mho^2\cdot\aleph^{[1-⅃]+}]}.
\end{aligned}
$$

Since, $\underbrace{\aleph^{\beth-}} = \inf_1 \aleph_{ij}^{\beth^l}$, $\overbrace{\aleph^{\beth-}} = \sup_1 \aleph_{ij}^{\beth^l}$ $\underbrace{\aleph^{\beth+}} = \inf_1 \aleph_{ij}^{\beth^u}$, $\overbrace{\aleph^{\beth+}} = \sup_1 \aleph_{ij}^{\beth^u}$ and $\underbrace{\aleph^{\beth-}} \preccurlyeq \aleph_{ij}^{\beth^l} \preccurlyeq \overbrace{\aleph^{\beth-}}$ and $\underbrace{\aleph^{\beth+}} \preccurlyeq \aleph_{ij}^{\beth^u} \preccurlyeq \overbrace{\aleph^{\beth+}}$. We have,

$$
\begin{aligned}
\underbrace{[\mho^2\cdot\aleph^{\beth-}]} + \underbrace{[\mho^2\cdot\aleph^{\beth+}]} &= \amalg_{i=1}^{n}[\underbrace{[\mho^2\cdot\aleph^{\beth-}]}]^{\eth_i} + \amalg_{i=1}^{n}[\underbrace{[\mho^2\cdot\aleph^{\beth+}]}]^{\eth_i}\\
&\preccurlyeq \amalg_{i=1}^{n}[\mho^2\cdot\aleph_{ij}^{\beth^l}]^{\eth_i} + \amalg_{i=1}^{n}[\mho^2\cdot\aleph_{ij}^{\beth^u}]^{\eth_i}\\
&\preccurlyeq \amalg_{i=1}^{n}[\overbrace{[\mho^2\cdot\aleph^{\beth-}]}]^{\eth_i} + \amalg_{i=1}^{n}[\overbrace{[\mho^2\cdot\aleph^{\beth+}]}]^{\eth_i}\\
&= \overbrace{[\mho^2\cdot\aleph^{\beth-}]} + \overbrace{[\mho^2\cdot\aleph^{\beth+}]}.
\end{aligned}
$$

Since, $\underbrace{\aleph^{\Finv-}} = \inf_1 \aleph_{ij}^{\Finv^l}$, $\overbrace{\aleph^{\Finv-}} = \sup_1 \aleph_{ij}^{\Finv^l}$ $\underbrace{\aleph^{[1-\curlywedge]+}} = \inf_1 \aleph_{ij}^{\Finv^u}$, $\overbrace{\aleph^{[1-\curlywedge]+}} = \sup_1 \aleph_{ij}^{\Finv^u}$ and $\underbrace{\aleph^{\Finv-}} \preccurlyeq \aleph_{ij}^{\Finv^l} \preccurlyeq \overbrace{\aleph^{\Finv-}}$ and $\underbrace{\aleph^{[1-\curlywedge]+}} \preccurlyeq \aleph_{ij}^{\Finv^u} \preccurlyeq \overbrace{\aleph^{[1-\curlywedge]+}}$. We have,

$$\begin{aligned}
\underbrace{[\mho^2 \cdot \aleph^{\Finv-}]} + \underbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]} &= \coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{\Finv-}]}]^{\eth_i} + \coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]}]^{\eth_i} \\
&\preccurlyeq \coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\Finv^l}]^{\eth_i} + \coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\Finv^u}]^{\eth_i} \\
&\preccurlyeq \coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{\Finv-}]}]^{\eth_i} + \coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]}]^{\eth_i} \\
&= \overbrace{[\mho^2 \cdot \aleph^{\Finv-}]} + \overbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]}.
\end{aligned}$$

Since, $\underbrace{\wp} = \inf_1 \wp_{ij}$, $\overbrace{\wp} = \sup_1 \wp_{ij}$, $\underbrace{\ell} = \sup_1 \ell_{ij}$, $\overbrace{\ell} = \inf_1 \ell_{ij}$ and $\underbrace{\wp} \preccurlyeq \wp_{ij} \preccurlyeq \overbrace{\wp}$ and $\overbrace{\ell} \preccurlyeq \ell_{ij} \preccurlyeq \underbrace{\ell}$.

Hence, $\ddagger_{i=1}^{n}\eth_i \underbrace{\wp} \preccurlyeq \ddagger_{i=1}^{n}\eth_i \wp_{ij} \preccurlyeq \ddagger_{i=1}^{n}\eth_i \overbrace{\wp}$ and $\ddagger_{i=1}^{n}\eth_i \overbrace{\ell} \preccurlyeq \ddagger_{i=1}^{n}\eth_i \ell_{ij} \preccurlyeq \ddagger_{i=1}^{n}\eth_i \underbrace{\ell}$.

Therefore,

$$\begin{aligned}
&\frac{\ddagger_{i=1}^{n}\eth_i \underbrace{\wp}}{2} \times \left[\frac{2+\left[1-\coprod_{i=1}^{n}\left[1-[\underbrace{[\mho^2 \cdot \aleph^{\curlywedge-}]}]^{\varsigma}\right]^{\eth_i}\right]+\left[1-\coprod_{i=1}^{n}\left[1-[\underbrace{[\mho^2 \cdot \aleph^{[1-\Finv]+}]}]^{\varsigma}\right]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{\beth-}]}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{\beth+}]}]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{\Finv-}]}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\overbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]}]^{\eth_i}\right]}{2} \right] \\
\preccurlyeq\ &\frac{\ddagger_{i=1}^{n}\eth_i \wp_{ij}}{2} \times \left[\frac{2+\left[1-\coprod_{i=1}^{n}\left[1-[\mho^2 \cdot \aleph_{ij}^{\curlywedge^l}]^{\varsigma}\right]^{\eth_i}\right]+\left[1-\coprod_{i=1}^{n}\left[1-[\mho^2 \cdot \aleph_{ij}^{\curlywedge^u}]^{\varsigma}\right]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\beth^l}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\beth^u}]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\Finv^l}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\mho^2 \cdot \aleph_{ij}^{\Finv^u}]^{\eth_i}\right]}{2} \right] \\
\preccurlyeq\ &\frac{\ddagger_{i=1}^{n}\eth_i \overbrace{\wp}}{2} \times \left[\frac{2+\left[1-\coprod_{i=1}^{n}\left[1-[\overbrace{[\mho^2 \cdot \aleph^{\curlywedge-}]}]^{\varsigma}\right]^{\eth_i}\right]+\left[1-\coprod_{i=1}^{n}\left[1-[\overbrace{[\mho^2 \cdot \aleph^{[1-\Finv]+}]}]^{\varsigma}\right]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{\beth-}]}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{\beth+}]}]^{\eth_i}\right]}{2} - \frac{\left[\coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{\Finv-}]}]^{\eth_i}\right]+\left[\coprod_{i=1}^{n}[\underbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]}]^{\eth_i}\right]}{2} \right].
\end{aligned}$$

Hence, $\Big\langle [\underbrace{\wp}, \underbrace{\ell}]; [[\underbrace{\mho^2 \cdot \aleph^{\curlywedge-}]}, \underbrace{[\mho^2 \cdot \aleph^{[1-\Finv]+}]}], [\overbrace{[\mho^2 \cdot \aleph^{\beth-}]}, \overbrace{[\mho^2 \cdot \aleph^{\beth+}]}],$
$\overbrace{[[\mho^2 \cdot \aleph^{\Finv-}]}, \overbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]]} \Big\rangle \preccurlyeq CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n]$
$\preccurlyeq \Big\langle [\overbrace{\wp}, \overbrace{\ell}]; [\overbrace{[\mho^2 \cdot \aleph^{\curlywedge-}]}, \overbrace{[\mho^2 \cdot \aleph^{[1-\Finv]+}]}], [\underbrace{[\mho^2 \cdot \aleph^{\beth-}]}, \underbrace{[\mho^2 \cdot \aleph^{\beth+}]}],$
$\underbrace{[[\mho^2 \cdot \aleph^{\Finv-}]}, \underbrace{[\mho^2 \cdot \aleph^{[1-\curlywedge]+}]]} \Big\rangle.$

Theorem 4.5. *Let* $\Bbbk_i = \left\langle [\wp_{t_{ij}}, \ell_{t_{ij}}]; [\aleph^{\curlywedge-}_{t_{ij}}], \aleph^{[1-\Finv]+}_{t_{ij}}]], [\aleph^{\beth-}_{t_{ij}}], \aleph^{\beth+}_{t_{ij}}]], [\aleph^{\Finv-}_{t_{ij}}], \aleph^{[1-\curlywedge]+}_{t_{ij}}]] \right\rangle$ *and* $H_i = \left\langle [\wp_{h_{ij}}, \ell_{h_{ij}}]; [\aleph^{\curlywedge-}_{h_{ij}}], \aleph^{[1-\Finv]+}_{h_{ij}}]], [\aleph^{\beth-}_{h_{ij}}], \aleph^{\beth+}_{h_{ij}}]], [\aleph^{\Finv-}_{h_{ij}}], \aleph^{[1-\curlywedge]+}_{h_{ij}}]] \right\rangle$ *be the two families of CotNNVWAs. For any* i, *if there is* $\wp_{t_{ij}} \preccurlyeq \ell_{h_{ij}}$, $\left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_{ij}}]\right] \preccurlyeq \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_{ij}}]\right]$ *and* $\left[[\mho^2 \cdot \aleph^{\beth-}_{t_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{\beth+}_{t_{ij}}]\right] \succcurlyeq \left[[\mho^2 \cdot \aleph^{\beth-}_{h_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{\beth+}_{h_{ij}}]\right]$ *and* $\left[[\mho^2 \cdot \aleph^{\Finv-}_{t_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{t_{ij}}]\right] \succcurlyeq \left[[\mho^2 \cdot \aleph^{\Finv-}_{h_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{h_{ij}}]\right]$ *or* $\Bbbk_i \preccurlyeq H_i$, *then* $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] \preccurlyeq CotNNVWA[H_1, H_2, ..., H_n]$.

Proof. For every i, $\wp_{t_{ij}} \preccurlyeq \ell_{h_{ij}}$. Thus, $\ddagger_{i=1}^n \wp_{t_{ij}} \preccurlyeq \ddagger_{i=1}^n \ell_{h_{ij}}$.
For any i, $\left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_{ij}}]\right] \preccurlyeq \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_{ij}}]\right]$.
Therefore, $1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_i}]\right] + 1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_i}]\right] \succcurlyeq 1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_i}]\right] + 1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_i}]\right]$.
Hence, $\Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_i}]\right]\right]^{\eth_i} + \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_i}]\right]\right]^{\eth_i} \succcurlyeq$
$\Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_i}]\right]\right]^{\eth_i} + \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_i}]\right]\right]^{\eth_i}$
and $\left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_i}]\right]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_i}]\right]^{\varsigma}\right]^{\eth_i}\right] \preccurlyeq$
$\left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_i}]\right]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_i}]\right]^{\varsigma}\right]^{\eth_i}\right]$.
and $\frac{2 + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{t_i}]\right]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{t_i}]\right]^{\varsigma}\right]^{\eth_i}\right]}{2} \preccurlyeq$
$\frac{2 + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{\curlywedge-}_{h_i}]\right]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - \left[[\mho^2 \cdot \aleph^{[1-\Finv]+}_{h_i}]\right]^{\varsigma}\right]^{\eth_i}\right]}{2}$.
For every i,
$\left[[\mho^2 \cdot \aleph^{\beth-}_{t_{ij}}]\right]^{\varsigma} + \left[[\mho^2 \cdot \aleph^{\beth+}_{t_{ij}}]\right]^{\varsigma} \succcurlyeq \left[[\mho^2 \cdot \aleph^{\beth-}_{h_{ij}}]\right]^{\varsigma} + \left[[\mho^2 \cdot \aleph^{\beth+}_{h_{ij}}]\right]^{\varsigma}$.
Therefore,
$-\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth-}_{t_{ij}}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth+}_{t_{ij}}]\right]}{2} \preccurlyeq -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth-}_{h_{ij}}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth+}_{h_{ij}}]\right]}{2}$.
For any i, $\left[[\mho^2 \cdot \aleph^{\Finv-}_{t_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{t_{ij}}]\right] \succcurlyeq \left[[\mho^2 \cdot \aleph^{\Finv-}_{h_{ij}}]\right] + \left[[\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{h_{ij}}]\right]$.
Therefore,
$-\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv-}_{t_{ij}}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{t_{ij}}]\right]}{2} \preccurlyeq -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv-}_{h_{ij}}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{[1-\curlywedge]+}_{h_{ij}}]\right]}{2}$.

$$\frac{\ddagger_{i=1}^n \wp tij}{2} \times \begin{bmatrix} \frac{2 + \left[1 - \Pi_{i=1}^n \left[1 - [\mho^2 \cdot \aleph^{\curlywedge^l}_{ti}]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - [\mho^2 \cdot \aleph^{\curlywedge^u}_{ti}]^{\varsigma}\right]^{\eth_i}\right]}{2} \\ -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth^l}_{tij}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth^u}_{tij}]\right]}{2} \\ -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv^l}_{tij}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv^u}_{tij}]\right]}{2} \end{bmatrix}$$
$$\preccurlyeq \frac{\ddagger_{i=1}^n \wp hij}{2} \times \begin{bmatrix} \frac{2 + \left[1 - \Pi_{i=1}^n \left[1 - [\mho^2 \cdot \aleph^{\curlywedge^l}_{hi}]^{\varsigma}\right]^{\eth_i}\right] + \left[1 - \Pi_{i=1}^n \left[1 - [\mho^2 \cdot \aleph^{\curlywedge^u}_{hi}]^{\varsigma}\right]^{\eth_i}\right]}{2} \\ -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth^l}_{hij}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\beth^u}_{hij}]\right]}{2} \\ -\frac{\left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv^l}_{hij}]\right] + \left[\Pi_{i=1}^n [\mho^2 \cdot \aleph^{\Finv^u}_{hij}]\right]}{2} \end{bmatrix}.$$

Hence, $CotNNVWA[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] \preccurlyeq CotNNVWA[H_1, H_2, ..., H_n]$.

4.2. *CotNNV weighted geometric[CotNNVWG]*

Definition 4.6. Let $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\daleth -}, \aleph_i^{[1-\leftthreetimes]+}] \right\rangle$ be the finite collection of CotNNVNs. The CotNNVWG operator is defined as CotNNVWG $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \amalg_{i=1}^n [\cot \Bbbk_i]^{\eth_i}$ $[i = 1, 2, ..., n]$.

Corollary 4.7. *Let* $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\daleth -}, \aleph_i^{[1-\leftthreetimes]+}] \right\rangle$ *be a collection of CotNNVNs. Prove that* $CotNNVWG[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] =$

$$\begin{bmatrix} \left[\amalg_{i=1}^n \wp_i^{\eth_i}, \amalg_{i=1}^n \ell_i^{\eth_i}\right]; \left[\amalg_{i=1}^n [[\mho^2 \cdot \aleph_i^{\leftthreetimes -}]]^{\eth_i}, \amalg_{i=1}^n [[\mho^2 \cdot \aleph_i^{[1-\daleth]+}]]^{\eth_i}\right], \\ \left[1 - \amalg_{i=1}^n \left[1 - [[\mho^2 \cdot \aleph_i^{\gimel -}]]^{\varsigma}\right]^{\eth_i}, 1 - \amalg_{i=1}^n \left[1 - [[\mho^2 \cdot \aleph_i^{\gimel +}]]^{\varsigma}\right]^{\eth_i}\right], \\ \left[1 - \amalg_{i=1}^n \left[1 - [[\mho^2 \cdot \aleph_i^{\daleth -}]]^{\varsigma}\right]^{\eth_i}, 1 - \amalg_{i=1}^n \left[1 - [[\mho^2 \cdot \aleph_i^{[1-\leftthreetimes]+}]]^{\varsigma}\right]^{\eth_i}\right] \end{bmatrix}.$$

Corollary 4.8. *If all* $\Bbbk_i = \left\langle [\wp_i, \ell_i]; \left[[[\mho \cdot \aleph_i^{\leftthreetimes -}]], [[\mho \cdot \aleph_i^{[1-\daleth]+}]]\right], \left[[[\mho \cdot \aleph_i^{\gimel -}]], [[\mho \cdot \aleph_i^{\gimel +}]]\right], \left[[[\mho \cdot \aleph_i^{\daleth -}]], [[\mho \cdot \aleph_i^{[1-\leftthreetimes]+}]]\right] \right\rangle$ *are equal and* $\Bbbk_i = \Bbbk$ *with* $\varsigma = 1$*. Prove that* $CotNNVWG[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \Bbbk$.

4.3. *Generalized CotNNVWA [CotGNNVWA]*

Definition 4.9. Let $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\daleth -}, \aleph_i^{[1-\leftthreetimes]+}] \right\rangle$ be the finite collection of CotNNVN. Then CotGNNVWA $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \left[\ddagger_{i=1}^n \eth_i [\cot \Bbbk_i]^{\varsigma}\right]^{1/\varsigma}$.

Theorem 4.10. *Let* $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\leftthreetimes -}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\gimel -}, \aleph_i^{\gimel +}], [\aleph_i^{\daleth -}, \aleph_i^{[1-\leftthreetimes]+}] \right\rangle$ *be the finite collection of CotNNVNs. Prove that CotGNNVWA* $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] =$

$$\begin{bmatrix} \left[\left[\ddagger_{i=1}^n \eth_i \wp_i^{\varsigma}\right]^{1/\varsigma}, \left[\ddagger_{i=1}^n \eth_i \ell_i^{\varsigma}\right]^{1/\varsigma}\right]; \\ \left[\left[1 - \amalg_{i=1}^n \left[1 - \left[[[\mho^2 \cdot \aleph_i^{\leftthreetimes -}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}, \left[1 - \amalg_{i=1}^n \left[1 - \left[[[\mho^2 \cdot \aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}\right], \\ \begin{bmatrix} 1 - \left[1 - \left[\amalg_{i=1}^n \left[1 - \left[1 - [[\mho^2 \cdot \aleph_i^{\gimel -}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma}, \\ 1 - \left[1 - \left[\amalg_{i=1}^n \left[1 - \left[1 - [[\mho^2 \cdot \aleph_i^{\gimel +}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma} \end{bmatrix}, \\ \begin{bmatrix} 1 - \left[1 - \left[\amalg_{i=1}^n \left[1 - \left[1 - [[\mho^2 \cdot \aleph_i^{\daleth -}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma}, \\ 1 - \left[1 - \left[\amalg_{i=1}^n \left[1 - \left[1 - [[\mho^2 \cdot \aleph_i^{[1-\leftthreetimes]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma} \end{bmatrix} \end{bmatrix}.$$

Proof. To prove that, $\ddagger_{i=1}^{n}\eth_i[\cot \Bbbk_i]^{\varsigma} =$

$$\begin{bmatrix} \left[\left[\ddagger_{i=1}^{n}\eth_i\wp_i^{\varsigma}\right],\left[\ddagger_{i=1}^{n}\eth_i\ell_i^{\varsigma}\right]\right]; \\ \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right] \end{bmatrix}.$$

If $n=2$, then $\eth_1[\cot\Bbbk_1]^{\varsigma}\ddagger\eth_2[\cot\Bbbk_2]^{\varsigma}$

$$= \begin{bmatrix} \left[\eth_1\wp_1^{\varsigma}\ddagger\eth_2\wp_2^{\varsigma},\eth_1\ell_1^{\varsigma}\ddagger\eth_2\ell_2^{\varsigma}\right]; \\ \begin{bmatrix} \left[1-\left[1-\left[[[\mho^2\cdot\aleph_1^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}+\left[1-\left[1-\left[[[\mho^2\cdot\aleph_2^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}, \\ -\left[1-\left[1-\left[[[\mho^2\cdot\aleph_1^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}\cdot\left[1-\left[1-\left[[[\mho^2\cdot\aleph_2^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \\ \left[1-\left[1-\left[[[\mho^2\cdot\aleph_1^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}+\left[1-\left[1-\left[[[\mho^2\cdot\aleph_2^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \\ -\left[1-\left[1-\left[[[\mho^2\cdot\aleph_1^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}\cdot\left[1-\left[1-\left[[[\mho^2\cdot\aleph_2^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \end{bmatrix}, \\ \begin{bmatrix} \left[1-\left[1-[[\mho^2\cdot\aleph_1^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_2^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}, \\ \left[1-\left[1-[[\mho^2\cdot\aleph_1^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_2^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1} \end{bmatrix}, \\ \begin{bmatrix} \left[1-\left[1-[[\mho^2\cdot\aleph_1^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_2^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}, \\ \left[1-\left[1-[[\mho^2\cdot\aleph_1^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_2^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\ddagger_{i=1}^{2}\eth_i\wp_i^{\varsigma},\ddagger_{i=1}^{2}\eth_i\ell_i^{\varsigma}\right]; \\ \left[1-\amalg_{i=1}^{2}\left[1-\left[[[\mho^2\cdot\aleph_1^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},1-\amalg_{i=1}^{2}\left[1-\left[[[\mho^2\cdot\aleph_1^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{2}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},\amalg_{i=1}^{2}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{2}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i},\amalg_{i=1}^{2}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right] \end{bmatrix}.$$

In general, the form $\ddagger_{i=1}^{n}\eth_i[\cot \Bbbk_i]^{\varsigma} =$

$$\begin{bmatrix} \left[\ddagger_{i=1}^{n}\eth_i\wp_i^{\varsigma}, \ddagger_{i=1}^{n}\eth_i\ell_i^{\varsigma}\right]; \\ \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_1^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, 1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_1^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right] \end{bmatrix}.$$

If $n = l+1$, then $\ddagger_{i=1}^{n}\eth_i[\cot \Bbbk_i]^{\varsigma} \ddagger \eth_{n+1}[\cot \Bbbk_{n+1}]^{\varsigma} = \ddagger_{i=1}^{n+1}\eth_i[\cot \Bbbk_i]^{\varsigma}$.

Now, $\ddagger_{i=1}^{n}\eth_i[\cot \Bbbk_i]^{\varsigma} \ddagger \eth_{n+1}[\cot \Bbbk_{n+1}]^{\varsigma} =$

$\eth_1[\cot \Bbbk_1]^{\varsigma} \ddagger \eth_2[\cot \Bbbk_2]^{\varsigma} \ddagger ... \ddagger \eth_l[\cot \Bbbk_n]^{\varsigma} \ddagger \eth_{n+1}[\cot \Bbbk_{n+1}]^{\varsigma}$

$$= \begin{bmatrix} \left[\ddagger_{i=1}^{n}\eth_i\wp_i^{\varsigma} \ddagger \eth_{n+1}\wp_{n+1}^{\varsigma}, \ddagger_{i=1}^{n}\eth_i\ell_i^{\varsigma} \ddagger \eth_{n+1}\ell_{n+1}^{\varsigma}\right]; \\ \begin{bmatrix} \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma} + \left[1-\left[1-\left[[[\mho^2\cdot\aleph_{n+1}^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma}, \\ -\left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\cdot\left[1-\left[1-\left[[[\mho^2\cdot\aleph_{n+1}^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \\ \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma} + \left[1-\left[1-\left[[[\mho^2\cdot\aleph_{n+1}^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \\ -\left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\cdot\left[1-\left[1-\left[[[\mho^2\cdot\aleph_{n+1}^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}\right]^{\varsigma} \end{bmatrix}, \\ \begin{bmatrix} \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_{n+1}^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}, \\ \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_{n+1}^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1} \end{bmatrix}, \\ \begin{bmatrix} \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_{n+1}^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1}, \\ \amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{[1-\leftthreetimes]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\cdot\left[1-\left[1-[[\mho^2\cdot\aleph_{n+1}^{[1-\leftthreetimes]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_1} \end{bmatrix} \end{bmatrix}$$

Thus, $\ddagger_{i=1}^{n+1}\eth_i[\cot \Bbbk_i]^{\varsigma} =$

$$\begin{bmatrix} \left[\ddagger_{i=1}^{n+1}\eth_i\wp_i^{\varsigma}, \ddagger_{i=1}^{n+1}\eth_i\ell_i^{\varsigma}\right]; \\ \left[1-\amalg_{i=1}^{n+1}\left[1-\left[[[\mho^2\cdot\aleph_1^{\leftthreetimes-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, 1-\amalg_{i=1}^{n+1}\left[1-\left[[[\mho^2\cdot\aleph_1^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, \amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right], \\ \left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}, \amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right] \end{bmatrix}.$$

Hence, $\left[\ddagger_{i=1}^{n+1}\eth_i[\cot \Bbbk_i]^{\varsigma}\right]^{1/\varsigma}$

$$\left[\begin{array}{c} \left[\left[\ddagger_{i=1}^{n+1}\eth_i\wp_i^{\varsigma}\right]^{1/\varsigma},\left[\ddagger_{i=1}^{n+1}\eth_i\ell_i^{\varsigma}\right]^{1/\varsigma}\right]; \\ \left[\left[1-\amalg_{i=1}^{n+1}\left[1-\left[[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma},\left[1-\amalg_{i=1}^{n+1}\left[1-\left[[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}\right], \\ \left[\begin{array}{c} 1-\left[1-\left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]\right]^{1/\varsigma}, \\ 1-\left[1-\left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]\right]^{1/\varsigma} \end{array}\right], \\ \left[\begin{array}{c} 1-\left[1-\left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]\right]^{1/\varsigma}, \\ 1-\left[1-\left[\amalg_{i=1}^{n+1}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]\right]^{1/\varsigma} \end{array}\right] \end{array}\right].$$

It is true for any n.

In the situation of $\varsigma = 1$, the CotGNNVWA operator is compressed to the CotNNVWA operator.

Theorem 4.11. *If all* $\Bbbk_i = \left\langle [\wp_i, \ell_i]; \left[[[\mho\cdot\aleph_i^{\curlywedge-}]], [[\mho\cdot\aleph_i^{[1-\daleth]+}]]\right], \left[[[\mho\cdot\aleph_i^{\beth-}]], [[\mho\cdot\aleph_i^{\beth+}]]\right], \left[[[\mho\cdot\aleph_i^{\daleth-}]], [[\mho\cdot\aleph_i^{[1-\curlywedge]+}]]\right]\right\rangle$ *are equal and* $\Bbbk_i = \Bbbk$ *with* $\varsigma = 1$. *Prove that CotGNNVWA*$[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \Bbbk$.

For Theorem 4.4 and Theorem 4.5, the CotGNNVWA operator is used to satisfy the boundedness and monotonicity properties.

4.4. *Generalized CotNNVWG [CotGNNVWG]*

Definition 4.12. Let $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\curlywedge-}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\beth-}, \aleph_i^{\beth+}], [\aleph_i^{\daleth-}, \aleph_i^{[1-\curlywedge]+}]\right\rangle$ be the finite collection of CotNNVNs. Then CotGNNVWG $[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] = \frac{1}{\varsigma}\left[\amalg_{i=1}^{n}[\varsigma\cot\Bbbk_i]^{\eth_i}\right]$, $[i = 1, 2, ..., n]$.

Corollary 4.13. *Let* $\Bbbk_i = \left\langle [\wp_i, \ell_i]; [\aleph_i^{\curlywedge-}, \aleph_i^{[1-\daleth]+}], [\aleph_i^{\beth-}, \aleph_i^{\beth+}], [\aleph_i^{\daleth-}, \aleph_i^{[1-\curlywedge]+}]\right\rangle$ *be the finite collection of CotNNVNs. Prove that CotGNNVWG*$[\Bbbk_1, \Bbbk_2, ..., \Bbbk_n] =$

$$\left[\begin{array}{c} \left[\frac{1}{\varsigma}\amalg_{i=1}^{n}[\varsigma\wp_i]^{\eth_i}, \frac{1}{\varsigma}\amalg_{i=1}^{n}[\varsigma\ell_i]^{\eth_i}\right]; \\ \left[\begin{array}{c} 1-\left[1-\left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{\curlywedge-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma}, \\ 1-\left[1-\left[\amalg_{i=1}^{n}\left[1-\left[1-[[\mho^2\cdot\aleph_i^{[1-\daleth]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{\varsigma}\right]^{1/\varsigma} \end{array}\right], \\ \left[\left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\beth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}, \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\beth+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}\right], \\ \left[\left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{\daleth-}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}, \left[1-\amalg_{i=1}^{n}\left[1-\left[[[\mho^2\cdot\aleph_i^{[1-\curlywedge]+}]]^{\varsigma}\right]^{\varsigma}\right]^{\eth_i}\right]^{1/\varsigma}\right] \end{array}\right].$$

The CotGNNVWG operator is changed to the CotNNVWG operator based on $\varsigma = 1$.

The CotGNNVWG operator is used to satisfy the boundedness and monotonicity properties, which are supported by Theorem 4.4 and Theorem 4.5.

Corollary 4.14. *If all* $\mathbb{k}_i = \left\langle [\wp_i, \ell_i]; \left[[[\mho \cdot \aleph_i^{\leftthreetimes -}]], [[\mho \cdot \aleph_i^{[1-\lrcorner]+}]]\right], \left[[[\mho \cdot \aleph_i^{\beth -}]], [[\mho \cdot \aleph_i^{\beth +}]]\right], \left[[[\mho \cdot \aleph_i^{\dashv -}]], [[\mho \cdot \aleph_i^{[1-\leftthreetimes]+}]]\right] \right\rangle$ *are equal and* $\mathbb{k}_i = \mathbb{k}$ *with* $\varsigma = 1$. *Prove that CotGNNVWG*$[\mathbb{k}_1, \mathbb{k}_2, ..., \mathbb{k}_n] = \mathbb{k}$.

5. Conclusion:

The euclidean and hamming distance measurements for CotNNVSs that we have presented in this research are advantageous due to their mathematical simplicity. Appropriate numerical examples demonstrate the superiority of the euclidean and hamming distance metrics. Examples from everyday life demonstrate the usefulness of the euclidean and hamming distance metrics. Better aggregation operation rules for CotNNVWA, CotNNVWG, CotGNNVWA, and CotGNNVWG have been suggested by us. Additionally, we have spoken about a few attributes and provided some instances of how we create these operators. Since this is a new field of study, the author is sure that future academics who are interested in this topic will find the talks in this article useful.

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