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Similarity measure and sine exponential measure of possibility interval-valued neutrosophic hypersoft sets and their applications

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Abstract. The Hypersoft set (HSS) has been created as an extension of a soft set (SS) in order to address limitations for the thought of disjoint attribute-valued sets corresponding to distinct attributes. The HSSs melted with several fuzziness structures present in literature to handle real-world scenarios with complexity and uncertainty. One such extension is the interval-valued neutrosophic hypersoft set (iv-NHSS), which focuses on the partitioning of each attribute into its attribute-valued set with these attribute-valued sets, three iv-NHS-memberships degree line closed interval [0,1]. However, the iv-NHSS model is short of assessing the vague nature of parameters and sub-parameters that cause some doubt in DM opinions. This work focuses primarily on show the impact the degree of fuzzy possibility on efficient work of iv-NHSS when we introduce novel concepts of possibility interval valued neutrosophic hyper soft set (in short piv-NHSS) by giving all iv-NHSs-values, the fuzzy possibility degree increases the efficiency for the multi-argument approximately. Therefore, the axioms of set theory such as piv-NHS-subset, piv-NHS-null set, piv-NHS-absolute set and piv-NHSS-complement defined on this concept, as well as we investigated the union, intersection, AND, OR of two piv-NHSSs, and relevant laws, with the help of several numerical examples and piv-NHS-matrix representations. Moreover, we discovered both similarities measure and sine exponential measure between two piv-NHSSs and these measures are successfully applied in decision-making to judge choose the best tourist place. In the end, a conclusion of this work is presented with some suggestions for future studies of this work.

Keywords: Interval-valued neutrosophic set; soft set; hypersoft set; interval-valued neutrosophic hypersoft set; possibility interval-valued neutrosophic hypersoft set; similarity measures; decision-making.

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1. **Introduction**

Neutrosophic set (NS) [1] is conceptualized by Smarandache as a new mathematical generalization of fuzzy sets (FSs) [2] and intuitionistic fuzzy sets (IFSs) [3] to tackle many intricate problems involving numerous inconsistent, uncertain, indeterminate, and incomplete information. The NS structures are characterized by three membership functions, i.e. true membership (T-membership), indeterminacy-membership (I-membership), and non-membership (F-membership) respectively. The mechanism of action of these structures (T-membership, I-membership, and F-membership) is characterized by their ability to represent the components of the universal set to a degree ranging between 0 and 1. In life applications, which the universal set can represent, it is necessary to define the three functions of NS; therefore Wang et al. [4] proposed the idea of single value-NS (sv-NS) and show its operations and some algebraic properties. Then, in order to take advantage of the flexibility provided by the interval structure, Wang et al. [5] develop sv-NS to interval-valued -NS (iv-NS) where every NS-structure expresses as an interval value. The research works on sv-NS and iv-NS and their related algebraic structures have been employed in a lot of different real-life fields. For example ahin and Kk [6] introduced a neutrosophic subset hood measure for sv-NS to deal with neutrosophic information. Huang [7] defined new distance measures of sv-NS and checked its applications. Luo et al. [8] explored the VIKOR design-making method on sv-NSs. Hashim et al. [9] proposed two kinds of entropy measures on iv-NSs and showed their application. Al-sharqi et al [10]- [15] presented many works on both sv-NS and iv-NSs and explained the mechanism for using these concepts in solving some life problems related to the mechanism for choosing the best option. Dealing with these tools has shown a barrier in dealing with uncertain data and unclear objects that character parameterization tools to handle this issue, Molodtsov [16] offered a new parameterization tool named soft set (SS). From a mathematical perspective, the SS provides a clear description of the components of the universal set, for example, if we suggest that the universal set X contains three houses x_1, x_2, x_3 then SS provide a description of the characteristics that distinguish these three houses. The SSs and iv-NSs inspired many mathematicians to present many research works. Deli [17] smelted the iv-NS in the SS to form a iv-N-soft set (iv-NSS) and gossiping about its important applications. Ihsan et al. [18] depicted iv-NSS as a hybrid model under an expert system and utilized this model in instruction supply chain management to select a suitable supplier for a construction project.

Al-Sharqi et al. [19]- [20] proposed several ideas on iv-NSS under complex environments and showed how to use these ideas in real-life situations. And many research works of high scientific value that dealt with these concepts with high quality as shown [21]– [25] .

On the other hand, in 2018, Smarandache explained that each attribute has sub-attributes, for example, the attribute of size, which is divided into large, medium, or small. This means that the concept of the soft set is no longer sufficient to cover the attributes of the elements of the universal set. Accordingly, in order to cover this gap, an expanded set of the SS was launched, called the hypersoft set (HSS) [26], which is characterized by containing the sub-elements of the SS. As mathematical works for this HSS: The essential operation of HSSs is defined by Saeed et al. [27]. As a developed of the Musa and Asaad [28] idea Al-Quran et al. [29] introduced the idea of BFHSSs. The distance measures employed between two IVFHSSs [30] and tests in the application of suitable medication. The concept of neutrosophic HSSs proposed by Zulqarnain et al. [31]. In addition, there are a lot successful studies to combine the HSS with NS-structures see [32]- [37] . In addition to all of the above, the preference level for the parameter value is 0, i.e., because there is no function that cares about this procedure. Accordingly, the probability degree, whose value ranges between 0 and 1, is considered a suitable tool to clarify the degree of interest of the user. Accordingly, this idea has received wide attention from researchers, who have provided many fuzzy tools, including: Possibility of polar fuzzy soft sets (PPFSSs) characterized by Khalil et al. [38] and the possibility of IVFS-set (PIVFS-set) investigated by Fu et al. [39]. Similarity measures between two possibility neutrosophic soft expert sets (PNSESs) and some elementary axioms and algebraic operations. Possibility degree of single-valued neutrosophic hypersoft set (psv-NHSS) integrated by Rahman et al. [40] and used resolve the solid waste site selection problem. Recently, Romdhini et al. [41] discussed three decision-making algorithms based on possibility-iv-FSSs (piv-FSSs). Based on what was mentioned above and what was mentioned in these previous studies, the main contributions that were given to this work as following:

1.Construct piv-NHSS as integrating both iv-NSs and HSS with the probability degree of all sub-parameters of iv-NHSS.

2.Define all the basic set operations on piv-NHSS and explain them with numerical examples.

3.Introduce some mathematical properties of all set operations and work on proving them.

4. Define similarity measures between two piv-NHSSs and build a new multi step algorithm based on these measures to handle one real-life application.

5. Define a sine exponential measure between two piv-NHSSs and build a new multi step algorithm based on these measures to handle one real-life application.

All the rest of this article's results are systematized in the following Figure 1:

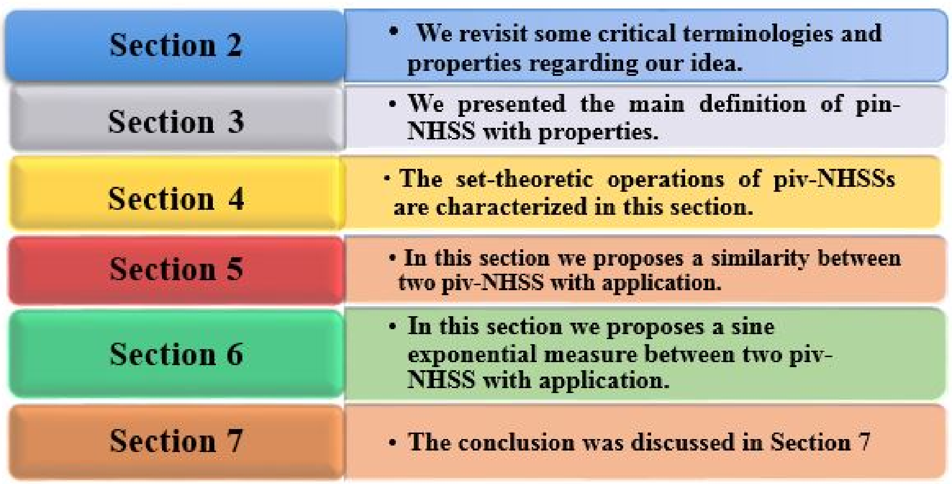

Figure 1: Represents of paper organization.

2. **Preliminaries**

In this part, we revisit some critical definitions and properties for our idea

Definition 2.1. [5] An iv-NS $\overset{\frown}{\Gamma}$ on a fixed set $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$ is stated as $\overset{\frown}{\Gamma}(\ddot{z}^n) = \langle \dagger_{\mathbb{T}}(\ddot{z}^n), \dagger_{\mathbb{I}}(\ddot{z}^n), \dagger_{\mathbb{F}}(\ddot{z}^n)\rangle$ where $\dagger_{\mathbb{T}}(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{T}}(\ddot{z}^n), \dagger^u_{\mathbb{T}}(\ddot{z}^n)\right]\right\rangle$,$\dagger_{\mathbb{I}}(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{I}}(\ddot{z}^n), \dagger^u_{\mathbb{I}}(\ddot{z}^n)\right]\right\rangle$ and $\dagger_{\mathbb{F}}(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{F}}(\ddot{z}^n), \dagger^u_{\mathbb{F}}(\ddot{z}^n)\right]\right\rangle$ respectively, with the condition $0 \leq \dagger^l_{\mathbb{T}}(\ddot{z}^n) + \dagger^l_{\mathbb{I}}(\ddot{z}^n) + \dagger^l_{\mathbb{F}}(\ddot{z}^n) \leq 3$ and $0 \leq \dagger^u_{\mathbb{T}}(\ddot{z}^n) + \dagger^u_{\mathbb{I}}(\ddot{z}^n) + \dagger^u_{\mathbb{F}}(\ddot{z}^n) \leq 3$. Here the terms $\dagger^l_{\mathbb{T}}(\ddot{z}^n), \dagger^l_{\mathbb{I}}(\ddot{z}^n), \dagger^l_{\mathbb{F}}(\ddot{z}^n), \dagger^u_{\mathbb{T}}(\ddot{z}^n), \dagger^u_{\mathbb{I}}(\ddot{z}^n), \dagger^u_{\mathbb{F}}(\ddot{z}^n)$ are declared as lower and upper three NS-memberships.

Definition 2.2. [5] **(Properties of iv-NSs)**

Let $\overset{\frown}{\Gamma}^1 = \left\{\left\langle \dagger^1_{\mathbb{T}}(\ddot{z}^n), \dagger^1_{\mathbb{I}}(\ddot{z}^n), \dagger^1_{\mathbb{F}}(\ddot{z}^n)\right\rangle\right\}$ and $\overset{\frown}{\Gamma}^2 = \left\{\left\langle \dagger^2_{\mathbb{T}}(\ddot{z}^n), \dagger^2_{\mathbb{I}}(\ddot{z}^n), \dagger^2_{\mathbb{F}}(\ddot{z}^n)\right\rangle\right\}$ be two iv-NSs on on a fixed set $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$ as stated in the Definition 1. Then the following basic operations on iv-NSs are given as following:

(i) $\overset{\frown}{\Gamma}^1 \subseteq \overset{\frown}{\Gamma}^2$ iff $\dagger^1_{\mathbb{T}}(\ddot{z}^n) \leq \dagger^2_{\mathbb{T}}(\ddot{z}^n)$, $\dagger^1_{\mathbb{I}}(\ddot{z}^n) \geq \dagger^2_{\mathbb{I}}(\ddot{z}^n)$ and $\dagger^1_{\mathbb{F}}(\ddot{z}^n) \geq \dagger^2_{\mathbb{F}}(\ddot{z}^n)$.

(ii) $\overset{\frown}{\Gamma}^1 = \overset{\frown}{\Gamma}^2$ iff $\dagger^1_{\mathbb{T}}(\ddot{z}^n) = \dagger^2_{\mathbb{T}}(\ddot{z}^n)$, $\dagger^1_{\mathbb{I}}(\ddot{z}^n) = \dagger^2_{\mathbb{I}}(\ddot{z}^n)$ and $\dagger^1_{\mathbb{F}}(\ddot{z}^n) = \dagger^2_{\mathbb{F}}(\ddot{z}^n)$.

(iii) The complement of $\widehat{\Gamma}^1$ denotes $c\left(\widehat{\Gamma}^1\right) = \left\{\left\langle c\,\dagger^1_{\mathbb{T}}(\ddot{z}^n), c\,\dagger^1_{\mathbb{I}}(\ddot{z}^n), c\,\dagger^1_{\mathbb{F}}(\ddot{z}^n)\right\rangle\right\}$, such that $c\,\dagger^1_{\mathbb{T}}(\ddot{z}^n) = \left[\dagger^{1,l}_{\mathbb{F}}(\ddot{z}^n), \dagger^{1,u}_{\mathbb{F}}(\ddot{z}^n)\right]$,$c\,\dagger^1_{\mathbb{I}}(\ddot{z}^n) = \left[1-\dagger^{1,u}_{\mathbb{F}}(\ddot{z}^n), 1-\dagger^{1,i}_{\mathbb{F}}(\ddot{z}^n)\right]$ and $c\,\dagger^1_{\mathbb{F}}(\ddot{z}^n) = \left[\dagger^{1,l}_{\mathbb{T}}(\ddot{z}^n), \dagger^{1,u}_{\mathbb{T}}(\ddot{z}^n)\right]$.

(iv) The union between two iv-NSs is also iv-NS and defined as the following

$\widehat{\Gamma}^3 = \widehat{\Gamma}^1 \cup \widehat{\Gamma}^2$ where $\widehat{\Gamma}^3 = \left\{\left\langle \dagger^3_{\mathbb{T}}(\ddot{z}^n), \dagger^3_{\mathbb{I}}(\ddot{z}^n), \dagger^3_{\mathbb{F}}(\ddot{z}^n)\right\rangle\right\}$ such that

$\dagger^{3,l}_{\mathbb{T}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,l}_{\mathbb{T}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{T}}(\ddot{z}^n)\right]\right)$,$\dagger^{3,u}_{\mathbb{T}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,u}_{\mathbb{T}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{T}}(\ddot{z}^n)\right]\right)$

$\dagger^{3,l}_{\mathbb{I}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,l}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{I}}(\ddot{z}^n)\right]\right)$,$\dagger^{3,u}_{\mathbb{I}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,u}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{I}}(\ddot{z}^n)\right]\right)$

$\dagger^{3,l}_{\mathbb{F}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,l}_{\mathbb{F}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{F}}(\ddot{z}^n)\right]\right)$,$\dagger^{3,u}_{\mathbb{I}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,u}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{F}}(\ddot{z}^n)\right]\right)$

(v) The intersection between two iv-NSs is also iv-NS and defined as the following

$\widehat{\Gamma}^4 = \widehat{\Gamma}^1 \cap \widehat{\Gamma}^2$ where $\widehat{\Gamma}^4 = \left\{\left\langle \dagger^4_{\mathbb{T}}(\ddot{z}^n), \dagger^4_{\mathbb{I}}(\ddot{z}^n), \dagger^4_{\mathbb{F}}(\ddot{z}^n)\right\rangle\right\}$ such that

$\dagger^{3,l}_{\mathbb{T}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,l}_{\mathbb{T}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{T}}(\ddot{z}^n)\right]\right)$,$\dagger^{4,u}_{\mathbb{T}}(\ddot{z}^n) = \left(\min\left[\dagger^{1,u}_{\mathbb{T}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{T}}(\ddot{z}^n)\right]\right)$

$\dagger^{4,l}_{\mathbb{I}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,l}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{I}}(\ddot{z}^n)\right]\right)$,$\dagger^{4,u}_{\mathbb{I}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,u}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{I}}(\ddot{z}^n)\right]\right)$

$\dagger^{4,l}_{\mathbb{F}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,l}_{\mathbb{F}}(\ddot{z}^n), \dagger^{2,l}_{\mathbb{F}}(\ddot{z}^n)\right]\right)$,$\dagger^{4,u}_{\mathbb{I}}(\ddot{z}^n) = \left(\max\left[\dagger^{1,u}_{\mathbb{I}}(\ddot{z}^n), \dagger^{2,u}_{\mathbb{F}}(\ddot{z}^n)\right]\right)$.

Definition 2.3. [16]Assume that $\Lambda = \{\eta_1, \eta_2, \eta_3, ..., \eta_r\}$ for $i = 1, 2, 3, ..., r$ define as a set of attributes (features) and $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$ as a fixed set then a SS $\ddot{\Theta}$ on $\ddot{\mathcal{Z}}$ is declared as $\ddot{\Theta} = \left\{\left(\mathbf{S}_{\ddot{\Theta}}(\eta_r), \Lambda\right), \eta_r \in \Lambda\right\}$ where $\mathbf{S}_{\ddot{\Theta}} : \Lambda \to P^{(\ddot{\mathcal{Z}})}$ and $S_{\ddot{\Theta}}(\eta_r) \subseteq P^{(\ddot{\mathcal{Z}})}$ is denoted as η_r-approximate element of $\ddot{\Theta}$.

Definition 2.4. [26] Assume that $\Lambda = \{\eta_1, \eta_2, \eta_3, ..., \eta_r\}$ for $i = 1, 2, 3, ..., r$ be a set of parameters (attributes (features)) that have relevant sub-parametric values given as Λ_1,Λ_2,Λ_3...,Λ_m then the HSS is declared as $\ddot{\ddot{\Theta}} = \{(\mathbf{S}_{\ddot{\ddot{\Theta}}}(\eta_r), \Lambda), \eta_r \in \Lambda \subseteq \Omega\}$ where $\mathbf{S}_{\ddot{\ddot{\Theta}}} : \Lambda \to P^{(\ddot{\mathcal{Z}})}$ and $S_{\ddot{\ddot{\Theta}}}(\eta_r) \subseteq P^{(\ddot{\mathcal{Z}})}$ is denoted as η_r-approximate element of $\ddot{\ddot{\Theta}}$ and $\Omega = \prod_{i=1}^{n} \Lambda_i$.

3. Possibility interval-valued neutrosophic hypersoft set (piv-NHSS)

This part explores and discusses the elementary characterization of a possibility interval-valued neutrosophic hypersoft set (piv-NHSS) along with the basic set operations of piv-NHSS and some appropriate examples for all these operations.

Definition 3.1. Let $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$be a non-empty softhyper fixed set and $\Lambda_1, \Lambda_2, \Lambda_3, ..., \Lambda_n$ be sets of defferent traits that have $n \geq 1$ sub-value and $\Omega = \prod_{i=1}^{n} \Lambda_i$ such that every $\Lambda_i = \{\eta_1, \eta_2, \eta_3, ..., \eta_r\}$ for $i = 1, 2, 3, ..., r$ Then a piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ define over a non-empty hypersoft fixed set $\left(\ddot{\mathcal{Z}}, \Omega\right)$ as the following form $(\Omega, \dagger_\Omega)$ where

(i) $\dagger_\Omega : \Omega \to iv - NS \times \mathcal{I}_{\hat{Z}}$ such that it define as

$\dagger_\Omega(\eta) = \left\{\left(\frac{\ddot{z}^n}{\widehat{\Gamma}(\eta)(\ddot{z}^n)}, \hat{\mu}(\eta)(\ddot{z}^n)\right) : \ddot{z}^n \in \ddot{Z} \;\&\; \eta \in \Omega\right\}$ and

$\widehat{\Gamma}(\eta)(\ddot{z}^n) = \langle \dagger_{\mathbb{T}}(\eta)(\ddot{z}^n), \dagger_{\mathbb{I}}(\eta)(\ddot{z}^n), \dagger_{\mathbb{F}}(\eta)(\ddot{z}^n)\rangle$

where $\dagger_{\mathbb{T}}(\eta)(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{T}}(\eta)(\ddot{z}^n), \dagger^u_{\mathbb{T}}(\eta)(\ddot{z}^n)\right]\right\rangle$,$\dagger_{\mathbb{I}}(\eta)(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{I}}(\eta)(\ddot{z}^n), \dagger^u_{\mathbb{I}}(\eta)(\ddot{z}^n)\right]\right\rangle$ and $\dagger_{\mathbb{F}}(\eta)(\ddot{z}^n) = \left\langle\left[\dagger^l_{\mathbb{F}}(\eta)(\ddot{z}^n), \dagger^u_{\mathbb{F}}(\eta)(\ddot{z}^n)\right]\right\rangle$ respectively, with the condition $0 \leq \dagger^l_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^l_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^l_{\mathbb{F}}(\eta)(\ddot{z}^n) \leq 3$ and $0 \leq \dagger^u_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^u_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^u_{\mathbb{F}}(\eta)(\ddot{z}^n) \leq 3$.
(ii)$I_{\hat{Z}} : \hat{Z} \to [0,1]$ and $\hat{\mu}(\eta)(\ddot{z}^n) \in I_{\hat{Z}}$ denotes to the degree of possibility of $\ddot{z}^n \in \hat{Z}$ in $\widehat{\Upsilon}(\ddot{z}^n)$.
Based on the above a piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ define over a non-empty hypersoft fixed set $\left(\ddot{\mathcal{Z}}, \Omega\right)$ as the following structure:

$$\widehat{\Upsilon}_{\hat{\mu}} = \left\{\left(\eta, \left\{\left(\frac{\ddot{z}^n}{\Gamma(\eta)(\ddot{z}^n)}, \hat{\mu}(\eta)(\ddot{z}^n)\right) : \ddot{z}^n \in \ddot{Z}\right\}\right) \eta \in \Omega\right\}$$

Example 3.2. The Human Resources (HR) department announces available jobs for applicants to apply for and fill. In this context, a lot number of Contains(He/She) apply for these jobs, and accordingly, a committee of experts is formed to analyze each applicant's skills and qualifications to choose the most suitable one for these positions. Therefore to analyze the information of all candidates using our proposed model, piv-NHSS, we assume that the fixed set contains four candidates, i.e $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, \ddot{z}^4\right\}$ with members of the attributes (Criteria) i.e η_1 = Academic Certificate,η_2 = English language level and η_3 = Computer skills, these attributes (Criteria) having their sub-attribute values given as following: $\Lambda_1 = \{\eta_{1,1} = Diploma, \eta_{1,2} = BA\}$, $\Lambda_2 = \{\eta_{2,1} = intermediate, \eta_{2,2} = High\}$, $\Lambda_2 = \{\eta_{3,1} = High\}$ then $\Omega = \Lambda_1 \times \Lambda_2 \times \Lambda_3$ =$\{\eta_1, \eta_2, \eta_3, \eta_4\}$ where every component η_i has a 3-tuple entity. Then the analyze based on piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ are given as following:
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7\right),\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6\right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8\right)\right\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2\right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5\right),\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3\right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle}, 0.6\right)\right\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_3) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3\right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5\right),\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.9\right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle}, 0.5\right)\right\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_4) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle}, 0.1\right),\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle}, 0.5\right), \left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle}, 0.6\right)\right\}$

The piv-NHSS is formed as

$\widehat{\Upsilon}_{\hat{\mu}} = \left\{ \widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7\right),\right.\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6\right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8\right)\right\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2\right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5\right),\right.$
$\left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3\right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle}, 0.6\right)\right\}$

$$\widehat{\Upsilon}_{\hat{\mu}}(\eta_3)=\Big\{\Big(\tfrac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle},0.3\Big),\Big(\tfrac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle},0.5\Big),$$
$$\Big(\tfrac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle},0.9\Big),\Big(\tfrac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle},0.5\Big)\Big\}$$
$$\widehat{\Upsilon}_{\hat{\mu}}(\eta_4)=\Big\{\Big(\tfrac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle},0.4\Big),\Big(\tfrac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle},0.1\Big),$$
$$\Big(\tfrac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle},0.5\Big),\Big(\tfrac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle},0.6\Big)\Big\}\Bigg\}$$

Here, each part $\widehat{\Upsilon}_{\hat{\mu}}(\eta_{i=1,2,3,4})$ indicates the degree of satisfaction, dissatisfaction, and neutrality for each candidate, in addition to the degree of acceptance of this evaluation, represented by the degree of probability.

Furthermore, the values mentioned above can be displayed as a matrix as follows:

$$\widehat{\Upsilon}_{\hat{\mu}}=\begin{pmatrix} \Omega/\ddot{Z} & \ddot{z}^1 & \ddot{z}^2 & \ddot{z}^3 & \ddot{z}^4 \\ \eta_1 & (\langle[.2,.5],[.1,.3],[.4,.4]\rangle,.4) & (\langle[.5,.6],[.3,.5],[.3,.8]\rangle,.7) & (\langle[.2,.4],[.1,.3],[.5,.6]\rangle,.6) & (\langle[.3,.6],[.4,.5],[.2,.5]\rangle,.8) \\ \eta_2 & (\langle[.3,.6],[.2,.5],[.2,.3]\rangle,.2) & (\langle[.4,.7],[.4,.8],[.1,.5]\rangle,.5) & (\langle[.6,.6],[.3,.7],[.5,.6]\rangle,.3) & (\langle[.5,.7],[.6,.8],[.7,.7]\rangle,.6) \\ \eta_3 & (\langle[.1,.8],[.7,.9],[.6,.8]\rangle,.3) & (\langle[.2,.4],[.6,.9],[.5,.7]\rangle,.5) & (\langle[.4,.7],[.8,.8],[.6,.6]\rangle,.9) & (\langle[.5,.5],[.7,.7],[.3,.3]\rangle,.5) \\ \eta_4 & (\langle[.1,.8],[.5,.7],[.2,.2]\rangle,.4) & (\langle[.1,.1],[.6,.6],[.7,.8]\rangle,.1) & (\langle[.6,.6],[.7,.7],[.9,.9]\rangle,.5) & (\langle[.2,.3],[.4,.8],[.4,.7]\rangle,.6) \end{pmatrix}_{4\times 4}$$

Definition 3.3. (**piv-NHS-nullset**) A piv-NHSS $\widehat{\Upsilon}^1_{\hat{\mu}^1}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ is said to be piv-NHS-nullset and denotes $\widehat{\Phi}^0_{\hat{\mu}^0}$ if
$\widehat{\Upsilon}^1_{\hat{\mu}}(\eta)(\ddot{z}^n)=([0,0],[0,0],[0,0])$ and for possibility membership part $\hat{\mu}^1(\eta)(\ddot{z}^n)=0$.

Example 3.4. If $\widehat{\Phi}^0_{\hat{\mu}^0}$ given as following matrix:

$$\widehat{\Phi}^0_{\mu^0}=\begin{pmatrix}(\langle[0.0],[0.0],[0.0]\rangle,0) & (\langle[0.0],[0.0],[0.0]\rangle,0)\\(\langle[0.0],[0.0],[0.0]\rangle,0) & (\langle[0.0],[0.0],[0.0]\rangle,0)\end{pmatrix}$$

Then here $\widehat{\Phi}^0_{\hat{\mu}^0}$ known piv-NHS-nullset.

Definition 3.5. (**piv-NHS-absolute set**) A piv-NHSS $\widehat{\Upsilon}^1_{\hat{\mu}^1}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ is said to be piv-NHS-nullset and denotes $\widehat{\Phi}^1_{\hat{\mu}^1}$ if
$\widehat{\Upsilon}^1_{\hat{\mu}}(\eta)(\ddot{z}^n)=([1,1],[1,1],[1,1])$ and for possibility membership part $\hat{\mu}^1(\eta)(\ddot{z}^n)=1$.

Example 3.6. If $\widehat{\Phi}^1_{\hat{\mu}^1}$ given as following matrix:

$$\widehat{\Phi}^1_{\mu^1}=\begin{pmatrix}(\langle[1.1],[1.1],[1.1]\rangle,1) & (\langle[1.1],[1.1],[1.1]\rangle,1)\\(\langle[1.1],[1.1],[1.1]\rangle,1) & (\langle[1.1],[1.1],[1.1]\rangle,1)\end{pmatrix}$$

Then here $\widehat{\Phi}^1_{\hat{\mu}^1}$ known piv-NHS-absolute set.

Definition 3.7. (**piv-NHS-subset**) A piv-NHSS $\widehat{\Upsilon}^1_{\hat{\mu}^1}\sqsubseteq\widehat{\Upsilon}^2_{\hat{\mu}^2}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ if
$\widehat{\Upsilon}^1_{\hat{\mu}}(\eta)(\ddot{z}^n)\le\widehat{\Upsilon}^2_{\hat{\mu}}(\eta)(\ddot{z}^n)$ such that $\widehat{\Upsilon}^{1,l}_{\mathbb{T}}(\eta)(\ddot{z}^n)\le\widehat{\Upsilon}^{2,l}_{\mathbb{T}}(\eta)(\ddot{z}^n)$ and $\widehat{\Upsilon}^{1,u}(\eta)(\ddot{z}^n)\le\widehat{\Upsilon}^{2,u}(\eta)(\ddot{z}^n)$, $\widehat{\Upsilon}^{1,l}_{\mathbb{I}}(\eta)(\ddot{z}^n)\ge\widehat{\Upsilon}^{2,l}_{\mathbb{I}}(\eta)(\ddot{z}^n)$, $\widehat{\Upsilon}^{1,u}_{\mathbb{I}}(\eta)(\ddot{z}^n)\ge\widehat{\Upsilon}^{2,u}_{\mathbb{I}}(\eta)(\ddot{z}^n)$, $\widehat{\Upsilon}^{1,l}_{\mathbb{F}}(\eta)(\ddot{z}^n)\ge$

$\overset{\frown}{\Upsilon}_{\mathbb{F}}^{2,l}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}_{\mathbb{F}}^{1,u}(\eta)(\ddot{z}^n) \geq \overset{\frown}{\Upsilon}_{\mathbb{F}}^{2,u}(\eta)(\ddot{z}^n)$ and for possibility membership part $\hat{\mu}^1(\eta)(\ddot{z}^n) \leq \hat{\mu}^2(\eta)(\ddot{z}^n)$.

Example 3.8. Consider the two piv-NHSs $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1}$ $\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$ given as following:

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.2, 0.5], [0.1, 0.3], [0.4, 0.4] \rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle [0.3, 0.6], [0.4, 0.5], [0.2, 0.5] \rangle}, 0.7 \right) \right\}$$

and

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.4, 0.8], [0.1, 0.1], [0.2, 0.3] \rangle}, 0.7 \right), \left(\frac{\ddot{z}^2}{\langle [0.7, 0.7], [0.2, 0.2], [0.1, 0.3] \rangle}, 0.9 \right) \right\}$$

Then its clear here $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} \sqsubseteq \overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$.

Definition 3.9. (**Equality in piv-NHSSs**) A piv-NHSS $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$ on $\left(\ddot{\mathcal{Z}}, \Omega\right)$ if $\overset{\frown}{\Upsilon}^1_{\hat{\mu}}(\eta)(\ddot{z}^n) = \overset{\frown}{\Upsilon}^2_{\hat{\mu}}(\eta)(\ddot{z}^n)$ such that $\overset{\frown}{\Upsilon}^1_{\mathbb{T}}(\eta)(\ddot{z}^n) = \overset{\frown}{\Upsilon}^2_{\mathbb{T}}(\eta)(\ddot{z}^n)$ and $\overset{\frown}{\Upsilon}^{1,u}(\eta)(\ddot{z}^n) = \overset{\frown}{\Upsilon}^{2,u}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^I_{\mathbb{R}}(\eta)(\ddot{z}^n) = \overset{\frown}{\Upsilon}^2_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^F_{\mathbb{R}}(\eta)(\ddot{z}^n) = \overset{\frown}{\Upsilon}^2_{\mathbb{F}}(\eta)(\ddot{z}^n)$ and for possibility membership part $\hat{\mu}^1(\eta)(\ddot{z}^n) = \hat{\mu}^2(\eta)(\ddot{z}^n)$.

Example 3.10. Consider the two piv-NHSs $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1}$ $\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$ given as following:

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.2, 0.5], [0.1, 0.3], [0.4, 0.4] \rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle [0.3, 0.6], [0.4, 0.5], [0.2, 0.5] \rangle}, 0.7 \right) \right\}$$

and

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.2, 0.5], [0.1, 0.3], [0.4, 0.4] \rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle [0.3, 0.6], [0.4, 0.5], [0.2, 0.5] \rangle}, 0.7 \right) \right\}$$

Then its clear here $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$.

4. **The set-theoretic operations of piv-NHSSs**

In this section, we will present the set-theoretic operations of piv-NHSSs as well as some Illustrative examples and some properties.

Definition 4.1. (**Complement in piv-NHSSs**) A piv-NHSS $\overset{\frown}{\Upsilon}_{\hat{\mu}}$ on $\left(\ddot{\mathcal{Z}}, \Omega\right)$ have a complement in the following form $\overset{\frown}{\Upsilon}^c_{\hat{\mu}^c}$ and define as

$$\overset{\frown}{\Upsilon}^c_{\hat{\mu}^c} = \left\{ \left(\eta, \left\{ \left(\frac{\ddot{z}^n}{\dagger^c(\eta)(\ddot{z}^n)}, \hat{\mu}^c(\eta)(\ddot{z}^n) \right) : \ddot{z}^n \in \ddot{Z} \right\} \right) \eta \in \Omega \right\}$$

where

$$\dagger^c_{\mathbb{T}}(\eta)(\ddot{z}^n) = \left\langle \left[\dagger^l_{\mathbb{F}}(\eta)(\ddot{z}^n), \dagger^u_{\mathbb{F}}(\eta)(\ddot{z}^n) \right] \right\rangle, \ \dagger^c_{\mathbb{I}}(\eta)(\ddot{z}^n) = \left\langle \left[1 - \dagger^u_{\mathbb{I}}(\eta)(\ddot{z}^n), 1 - \dagger^l_{\mathbb{I}}(\eta)(\ddot{z}^n) \right] \right\rangle \text{ and}$$
$$\dagger^c_{\mathbb{F}}(\eta)(\ddot{z}^n) = \left\langle \left[\dagger^l_{\mathbb{T}}(\eta)(\ddot{z}^n), \dagger^u_{\mathbb{T}}(\eta)(\ddot{z}^n) \right] \right\rangle$$

and for possibility fuzzy membership part $\hat{\mu}^c(\eta)(\ddot{z}^n) = 1 - \hat{\mu}(\eta)(\ddot{z}^n)$.

Example 4.2. Consider the two piv-NHSs $\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}\right)^c$ $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ given as following:

$$\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.2, 0.5], [0.1, 0.3], [0.4, 0.4] \rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle [0.3, 0.6], [0.4, 0.5], [0.2, 0.5] \rangle}, 0.7 \right) \right\}$$

then the $\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}\right)^c$ is given based on above definition as following:

$$\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}\right)^c = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.4, 0.4], [0.7, 0.9], [0.2, 0.5] \rangle}, 0.6 \right), \left(\frac{\ddot{z}^2}{\langle [0.2, 0.5], [0.5, 0.6], [0.3, 0.6] \rangle}, 0.3 \right) \right\}$$

Definition 4.3. (**Union in piv-NHSSs**) Let $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs. Then the their union denotes $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ is also piv-HSS $\overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3}$ and given as following:

$$\overset{\frown}{\Upsilon}^{3}_{\mathbb{T}}(\eta)(\ddot{z}^n) = \max\left\{ \overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}(\eta)(\ddot{z}^n) \right\}$$
$$= \left\{ \max\left[\overset{\frown}{\Upsilon}^{1,l}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,l}_{\mathbb{T}}(\eta)(\ddot{z}^n) \right], \max\left[\overset{\frown}{\Upsilon}^{1,u}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,u}_{\mathbb{T}}(\eta)(\ddot{z}^n) \right] \right\},$$
$$\overset{\frown}{\Upsilon}^{3}_{\mathbb{I}}(\eta)(\ddot{z}^n) = \min\left\{ \overset{\frown}{\Upsilon}^{1}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}(\eta)(\ddot{z}^n) \right\}$$
$$= \left\{ \min\left[\overset{\frown}{\Upsilon}^{1,l}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,l}_{\mathbb{T}}(\eta)(\ddot{z}^n) \right], \min\left[\overset{\frown}{\Upsilon}^{1,u}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,u}_{\mathbb{I}}(\eta)(\ddot{z}^n) \right] \right\},$$
$$\overset{\frown}{\Upsilon}^{3}_{\mathbb{F}}(\eta)(\ddot{z}^n) = \min\left\{ \overset{\frown}{\Upsilon}^{1}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}(\eta)(\ddot{z}^n) \right\}$$
$$= \left\{ \min\left[\overset{\frown}{\Upsilon}^{1,l}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,l}_{\mathbb{F}}(\eta)(\ddot{z}^n) \right], \min\left[\overset{\frown}{\Upsilon}^{1,u}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2,u}_{\mathbb{F}}(\eta)(\ddot{z}^n) \right] \right\},$$

and for possibility membership part

$$\hat{\mu}^3(\eta)(\ddot{z}^n) = \max\left\{ \hat{\mu}^1(\eta)(\ddot{z}^n), \hat{\mu}^2(\eta)(\ddot{z}^n) \right\}$$

Example 4.4. Consider the two piv-NHSs $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ given as following:

$$\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.2, 0.5], [0.1, 0.3], [0.4, 0.4] \rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle [0.3, 0.6], [0.4, 0.5], [0.2, 0.5] \rangle}, 0.7 \right) \right\}$$

and

$$\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} = \left\{ \left(\frac{\ddot{z}^1}{\langle [0.4, 0.8], [0.1, 0.1], [0.2, 0.3] \rangle}, 0.7 \right), \left(\frac{\ddot{z}^2}{\langle [0.7, 0.7], [0.2, 0.2], [0.1, 0.3] \rangle}, 0.9 \right) \right\}$$

Then the $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$.

$$= \left\{ \left(\frac{\ddot{z}^1}{\langle [0.4, 0.8], [0.1, 0.1], [0.2, 0.3] \rangle}, 0.7 \right), \left(\frac{\ddot{z}^2}{\langle [0.7, 0.7], [0.2, 0.2], [0.1, 0.3] \rangle}, 0.9 \right) \right\}$$

Definition 4.5. (**Intersection in piv-NHSSs**) Let $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs. Then the their intersection denotes $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ is also piv-HSS $\overset{\frown}{\Upsilon}^{4}_{\hat{\mu}^4}$ and given as following:

$$\overset{\frown}{\Upsilon}{}^{4}_{\mathbb{T}}(\eta)(\ddot{z}^n) = \min\left\{\overset{\frown}{\Upsilon}{}^{1}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2}_{\mathbb{T}}(\eta)(\ddot{z}^n)\right\}$$
$$= \left\{\min\left[\overset{\frown}{\Upsilon}{}^{1,l}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,l}_{\mathbb{T}}(\eta)(\ddot{z}^n)\right], \min\left[\overset{\frown}{\Upsilon}{}^{1,u}_{\mathbb{T}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,u}_{\mathbb{T}}(\eta)(\ddot{z}^n)\right]\right\},$$
$$\overset{\frown}{\Upsilon}{}^{4}_{\mathbb{I}}(\eta)(\ddot{z}^n) = \max\left\{\overset{\frown}{\Upsilon}{}^{1}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2}_{\mathbb{I}}(\eta)(\ddot{z}^n)\right\}$$
$$= \left\{\max\left[\overset{\frown}{\Upsilon}{}^{1,l}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,l}_{\mathbb{I}}(\eta)(\ddot{z}^n)\right], \max\left[\overset{\frown}{\Upsilon}{}^{1,u}_{\mathbb{I}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,u}_{\mathbb{I}}(\eta)(\ddot{z}^n)\right]\right\},$$
$$\overset{\frown}{\Upsilon}{}^{4}_{\mathbb{F}}(\eta)(\ddot{z}^n) = \max\left\{\overset{\frown}{\Upsilon}{}^{1}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2}_{\mathbb{F}}(\eta)(\ddot{z}^n)\right\}$$
$$= \left\{\max\left[\overset{\frown}{\Upsilon}{}^{1,l}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,l}_{\mathbb{F}}(\eta)(\ddot{z}^n)\right], \max\left[\overset{\frown}{\Upsilon}{}^{1,u}_{\mathbb{F}}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}{}^{2,u}_{\mathbb{F}}(\eta)(\ddot{z}^n)\right]\right\},$$

and for possibility membership part is given as

$$\hat{\mu}^4(\eta)(\ddot{z}^n) = \min\left\{\hat{\mu}^1(\eta)(\ddot{z}^n), \hat{\mu}^2(\eta)(\ddot{z}^n)\right\}$$

Example 4.6. Consider the two piv-NHSs $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$ $\overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2}$ given as following:

$$\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} = \left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.7\right)\right\}$$

and

$$\overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} = \left\{\left(\frac{\ddot{z}^1}{\langle[0.4,0.8],[0.1,0.1],[0.2,0.3]\rangle}, 0.7\right), \left(\frac{\ddot{z}^2}{\langle[0.7,0.7],[0.2,0.2],[0.1,0.3]\rangle}, 0.9\right)\right\}$$

Then the $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} =$.

$$\left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.7\right)\right\}$$

Proposition 4.7. *Assume that $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$ be a piv-NHSSs on hypersoft fixed set $\left(\ddot{\mathcal{Z}}, \Omega\right)$.Then the next properties are satisfied:*

(i). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

(ii). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

(iii). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Phi}{}^{0}_{\hat{\mu}^0} = \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

(iv). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Phi}{}^{0}_{\hat{\mu}^0} = \overset{\frown}{\Phi}{}^{0}_{\hat{\mu}^0}$.

(v). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Phi}{}^{1}_{\hat{\mu}^1} = \overset{\frown}{\Phi}{}^{1}_{\hat{\mu}^1}$.

(vi). $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Phi}{}^{1}_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

Proposition 4.8. *Assume that $\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$, $\overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2}$ and $\overset{\frown}{\Upsilon}{}^{3}_{\hat{\mu}^3}$ be three piv-NHSSs on hypersoft fixed set $\left(\ddot{\mathcal{Z}}, \Omega\right)$.Then the next properties are satisfied:*

(i).$\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} = \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

(ii).$\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} = \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} \ddot{\sqcap} \overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1}$.

(iii).$\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \left(\overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2} \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{3}_{\hat{\mu}^3}\right) = \left(\overset{\frown}{\Upsilon}{}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{2}_{\hat{\mu}^2}\right) \ddot{\sqcup} \overset{\frown}{\Upsilon}{}^{3}_{\hat{\mu}^3}$.

(iv). $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \left(\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3} \right) = \left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \right) \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3}$.

Proposition 4.9. *Assume that* $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$, $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ *and* $\overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3}$ *be three piv-NHSSs on hypersoft fixed set* $\left(\ddot{\mathcal{Z}}, \Omega \right)$*.Then the next properties are satisfied:*
(i). $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \left(\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3} \right) = \left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \right) \ddot{\sqcap} \left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcup} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3} \right)$.
(ii). $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \left(\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \ddot{\sqcup} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3} \right) = \left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} \right) \ddot{\sqcup} \left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}_{\hat{\mu}^3} \right)$.

Proof.
(i). For all $\eta, \ddot{z}^n \in \Omega$ *and* $\ddot{\mathcal{Z}}$ respectively

$$\chi_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n) \ddot{\sqcup} \left(\overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n) \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}(\eta)(\ddot{z}^n) \right)} = \ddot{\sqcup} \left\{ \chi_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n)}, \chi_{\left(\overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n) \ddot{\sqcap} \overset{\frown}{\Upsilon}^{3}(\eta)(\ddot{z}^n) \right)} \right\}$$
$$= \ddot{\sqcup} \left\{ \chi_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n)}, \ddot{\sqcap} \left(\chi_{\overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)}, \chi_{\overset{\frown}{\Upsilon}^{3}(\eta)(\ddot{z}^n)} \right) \right\}$$
$$= \{ \langle \eta,\ \max \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}(\ddot{z}^n), \min \left(\overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{T}}(\ddot{z}^n) \right) \right),$$
$$\min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}}(\ddot{z}^n), \max \left(\overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{I}}(\ddot{z}^n) \right) \right),$$
$$\min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}}(\ddot{z}^n), \max \left(\overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{F}}(\ddot{z}^n) \right) \right) \Big\}$$
$$\Big\{ \Big\langle \eta, \min \left(\max \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}(\ddot{z}^n) \right) \max \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{T}}(\ddot{z}^n) \right) \right),$$
$$\max \left(\min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}(\ddot{z}^n) \right) \min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{I}}(\ddot{z}^n) \right) \right)$$
$$\max \left(\min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}(\ddot{z}^n) \right) \min \left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}}(\ddot{z}^n), \overset{\frown}{\Upsilon}^{3}_{\mathbb{F}}(\ddot{z}^n) \right) \right) \Big\rangle \Big\}$$
$$= \ddot{\sqcap} \left(\ddot{\sqcup} \left(\chi_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n) \ddot{\sqcup} \overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)}, \chi_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n) \cup \overset{\frown}{\Upsilon}^{3}(\eta)(\ddot{z}^n)} \right) \right)$$
$$= \chi_{\left(\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n) \ddot{\sqcup} \overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n) \right) \ddot{\sqcap} \overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n) \ddot{\sqcup} \overset{\frown}{\Upsilon}^{3}(\eta)(\ddot{z}^n)}.$$

and for possibility membership par

$$\chi_{\hat{\mu}^1(\eta)(\ddot{z}^n) \cup (\hat{\mu}^2(\eta)(\ddot{z}^n) \cap \hat{\mu}^3(\eta)(\ddot{z}^n))} = \max \left\{ \chi_{\hat{\mu}^1(\eta)(\ddot{z}^n)}, \chi_{(\hat{\mu}^2(\eta)(\ddot{z}^n) \cap \hat{\mu}^3(\eta)(\ddot{z}^n))} \right\}$$
$$= \max \left\{ \chi_{\hat{\mu}^1(\eta)(\ddot{z}^n)}, \min \left(\chi_{\hat{\mu}^2(\eta)(\ddot{z}^n)}, \chi_{\hat{\mu}^3(\eta)(\ddot{z}^n)} \right) \right\}$$
$$= \min \left\{ \max \left(\chi_{\hat{\mu}^1(\eta)(\ddot{z}^n)}, \chi_{\hat{\mu}^2(\eta)(\ddot{z}^n)} \right), \max \left(\chi_{\hat{\mu}^1(\eta)(\ddot{z}^n)}, \chi_{\hat{\mu}^3(\eta)(\ddot{z}^n)} \right) \right\}$$
$$= \min \left\{ \chi_{\hat{\mu}^1(\eta)(\ddot{z}^n) \cup \hat{\mu}^2(\eta)(\ddot{z}^n)}, \chi_{\hat{\mu}^1(\eta)(\ddot{z}^n) \cup \hat{\mu}^3(\eta)(\ddot{z}^n)} \right\}$$
$$= \chi_{(\hat{\mu}^1(\eta)(\ddot{z}^n) \ddot{\sqcup} \hat{\mu}^2(\eta)(\ddot{z}^n)) \ddot{\sqcap} (\hat{\mu}^1(\eta)(\ddot{z}^n) \ddot{\sqcup} \hat{\mu}^3(\eta)(\ddot{z}^n))} \ \square$$

(ii) can be proved similar to (i).

5. **Similarity Measure on piv-NHSs**

Similarity techniques play a major role in linking two or more mathematical structures into a mathematical system whose outputs are a fixed degree that can be relied upon to resolve the controversy over many issues of uncertainty. Accordingly, researchers have been interested in linking mathematical concepts through similarity tools. In this section, we will develop some of the previous techniques [42]– [44] to find a mathematical formula that aims to find the similarity ratio between two piv-NHSs.

Definition 5.1. Assume that $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs.Then the Similarity measure between $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ set by $\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right)$ is defined as follows:

$$\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = \dddot{\mathbb{M}}\left(\overset{\frown}{\Upsilon}^{1}, \overset{\frown}{\Upsilon}^{2}\right) \times \dddot{\mathbb{M}}\left(\hat{\mu}^1, \hat{\mu}^2\right),$$

such that

$$\dddot{M}\left(\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)\right) = Max\left(\dddot{M}_i\left(\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)\right)\right)$$

$$\dddot{M}\left(\hat{\mu}^1(\eta)(\ddot{z}^n), \hat{\mu}^2(\eta)(\ddot{z}^n)\right) = Max\left(\dddot{M}_i\left(\hat{\mu}^1(\eta)(\ddot{z}^n), \hat{\mu}^2(\eta)(\ddot{z}^n)\right)\right)$$

where

$$\dddot{M}\left(\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n), \overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)\right) = 1 - \frac{1}{\sqrt{n}} \times \sqrt{\sum_{i=1}^{n}\left(\phi'_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n)} - \phi'_{\overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)}\right)^2}$$

such that,

$$\phi'_{\overset{\frown}{\Upsilon}^{1}(\eta)(\ddot{z}^n)} = \frac{\dagger^{1,l}_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^{1,u}_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^{1,l}_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^{1,u}_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^{1,l}_{\mathbb{F}}(\eta)(\ddot{z}^n) + \dagger^{1,u}_{\mathbb{F}}(\eta)(\ddot{z}^n)}{6}$$

$$\phi'_{\overset{\frown}{\Upsilon}^{2}(\eta)(\ddot{z}^n)} = \frac{\dagger^{2,l}_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^{2,u}_{\mathbb{T}}(\eta)(\ddot{z}^n) + \dagger^{2,l}_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^{2,u}_{\mathbb{I}}(\eta)(\ddot{z}^n) + \dagger^{2,l}_{\mathbb{F}}(\eta)(\ddot{z}^n) + \dagger^{2,u}_{\mathbb{F}}(\eta)(\ddot{z}^n)}{6}$$

and for fuzzy possibility degree

$$\dddot{M}_i\left(\hat{\mu}^1(\eta)(\ddot{z}^n), \hat{\mu}^2(\eta)(\ddot{z}^n)\right) = 1 - \frac{\sum_{i=1}^{m}\left|\hat{\mu}^1_i(\eta)(\ddot{z}^n) - \hat{\mu}^2_i(\eta)(\ddot{z}^n)\right|}{\sum_{i=1}^{m}\left|\hat{\mu}^1_i(\eta)(\ddot{z}^n) + \hat{\mu}^2_i(\eta)(\ddot{z}^n)\right|}$$

Definition 5.2. Assume that $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs.We say that $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ are significantly similar if $\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) \geq \frac{1}{2}$.

Algorithm 1. Engage similarity measures of piv-NHSs to selection the optimal tourist location

Step 1. Build piv-NHSS $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ based on expert opinion.

Step 2. Build piv-NHSS $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ based on external expert opinion.

Step 3. Determine $\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right)$ starting Definition 5.1.

Step 4. Check similarity score and select the maxim value by $\tilde{N} = Max\left\{\hat{\ddot{\mathbb{S}}}_1, \hat{\ddot{\mathbb{S}}}_2, ..., \hat{\ddot{\mathbb{S}}}_m\right\}$ who consider the optimal tourist location.

Step 5. End Algorithm 1.

Algorithm 1 is shown in Figure 2.

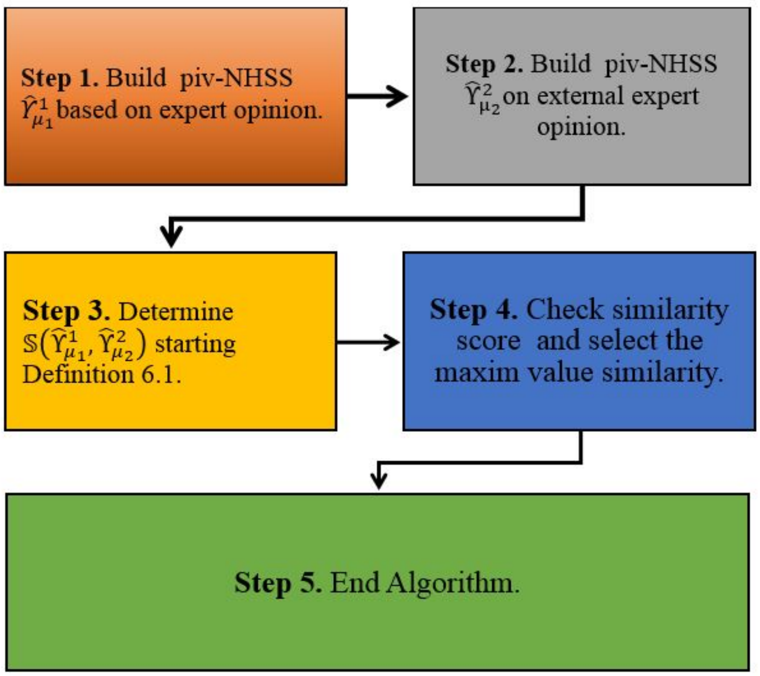

Figure 2: Algorithm 1

5.1. *Application of piv-NHSS in the most qualified employee selection*

In this section, we deal with one of the daily life DM scenarios, specifically the process of selecting a competent employee to fill a position, by presenting a multi-step algorithm, which is an extension or expansion of the algorithm presented in [52].

5.2. *Cuse studty*

The Human Resources (HR) department announces available jobs for applicants to apply for and fill. In this context, a lot number of Contains(He/She) apply for these jobs, and accordingly, a committee of experts is formed to analyze each applicant's skills and qualifications to choose the most suitable one for these positions. Therefore to analyze the information of all candidates using our proposed model, piv-NHSS, we assume that the fixed set contains four candidates, i.e $\ddot{\mathcal{Z}} = \{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, \ddot{z}^4\}$ with members of the attributes (Criteria) i.e η_1 = Academic Certificate,η_2 = English language level and η_3 = Computer skills, these attributes (Criteria) having their sub-attribute values given as following: $\Lambda_1 = \{\eta_{1,1} = Diploma, \eta_{1,2} = BA\}$, $\Lambda_2 = \{\eta_{2,1} = intermediate, \eta_{2,2} = High\}$, $\Lambda_2 = \{\eta_{3,1} = High\}$ then $\Omega = \Lambda_1 \times \Lambda_2 \times \Lambda_3 = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ where every component η_i has a 3-tuple entity. Then accordingly, a committee was formed to interview each candidate individually, and the committee adopted the following steps to accomplish its work:

Step 1. Analyze the information of each candidate based on piv-NHSS $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1}$ are given as following:

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1} = \Bigg\{ \overset{\frown}{\Upsilon}^1_{\hat{\mu}}(\eta_1) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}}(\eta_2) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle}, 0.6 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}}(\eta_3) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.9 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle}, 0.5 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu}}(\eta_4) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle}, 0.1 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle}, 0.5 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle}, 0.6 \Big) \Big\} \Bigg\}$$

Step 2. Build piv-NHSS $\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$ based on external expert opinion.

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2} = \Bigg\{ \overset{\frown}{\Upsilon}^2_{\hat{\mu}}(\eta_1) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.4,0.5],[0.7,0.7],[0.3,0.8]\rangle}, 0.8 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.3,0.5],[0.6,0.7],[0.8,0.9]\rangle}, 0.4 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.7,0.9],[0.1,0.1],[0.5,0.5]\rangle}, 0.8 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.4,0.4],[0.4,0.6],[0.1,0.3]\rangle}, 0.7 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}}(\eta_2) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.4,0.6],[0.2,0.4],[0.3,0.3]\rangle}, 0.1 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.3,0.4],[0.5,0.6],[0.3,0.5]\rangle}, 0.8 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.2,0.5],[0.3,0.7],[0.5,0.7]\rangle}, 0.9 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.6,0.6],[0.4,0.5],[0.6,0.8]\rangle}, 0.9 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}}(\eta_3) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.8 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.7 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.5 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle}, 0.8 \Big) \Big\}$$

$$\overset{\frown}{\Upsilon}^2_{\hat{\mu}}(\eta_4) = \Big\{ \Big(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4 \Big), \Big(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.3,0.5]\rangle}, 0.7 \Big), \Big(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle}, 0.8 \Big), \Big(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.8,0.9],[0.4,0.9]\rangle}, 0.5 \Big) \Big\} \Bigg\}$$

Determine $\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right)$ starting Definition 5.1 as following:

$$\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}}\left(\ddot{z}^1\right), \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}}\left(\ddot{z}^1\right)\right) = 0.547$$
$$\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}}\left(\ddot{z}^2\right), \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}}\left(\ddot{z}^2\right)\right) = 0.703$$
$$\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}}\left(\ddot{z}^3\right), \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}}\left(\ddot{z}^3\right)\right) = 0.549$$
$$\hat{\ddot{\mathbb{S}}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}}\left(\ddot{z}^4\right), \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}}\left(\ddot{z}^4\right)\right) = 0.761$$

Step 4. Check similarity score and select the maxim value by $\tilde{N} = Max\{0.547, 0.703, 0.549, 0.761\}$ who consider the optimal tourist location who $\ddot{z}^4$.

6. Sine Exponential Measure of piv-NHSs

In this section, we reveal sine exponential measure of of piv-NHSSs and employ this type measure in medical diagnosis.

Definition 6.1. Assume that $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs. Then the sine exponential measure is given as follows:

$$\ddot{\mathbb{S}}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = \frac{1}{n}\left[\sum_{i=1}^{n} \sin e^{-\left(\left[\left|\dagger^1_{\mathbb{T}}(\eta)(\ddot{z}^n) - \dagger^2_{\mathbb{T}}(\eta)(\ddot{z}^n)\right| + \left|\dagger^1_{\mathbb{I}}(\eta)(\ddot{z}^n) - \dagger^2_{\mathbb{I}}(\eta)(\ddot{z}^n)\right| + \left|\dagger^1_{\mathbb{F}}(\eta)(\ddot{z}^n) - \dagger^2_{\mathbb{F}}(\eta)(\ddot{z}^n)\right|\right] \times \left(\hat{\mu}^1(\eta)(\ddot{z}^n) \times \hat{\mu}^2(\eta)(\ddot{z}^n)\right)\right)}\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} \sin e^{-\left(\begin{bmatrix}\left|\left(\dagger^{1,l}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,l}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right) - \left(\dagger^{1,u}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,u}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right)\right| + \\ \left|\left(\dagger^{1,l}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,l}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right) - \left(\dagger^{1,u}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,u}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right)\right| + \\ \left|\left|\left(\dagger^{1,l}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,l}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right) - \left(\dagger^{1,u}_{\mathbb{R}}(\eta)(\ddot{z}^n) \times \dagger^{2,u}_{\mathbb{R}}(\eta)(\ddot{z}^n)\right)\right|\right|\end{bmatrix} \times \left(\hat{\mu}^1(\eta)(\ddot{z}^n) \times \hat{\mu}^2(\eta)(\ddot{z}^n)\right)\right)}\right]$$

Here, based on this formula, it can be guaranteed that the value of these measure is greater than 1.

Example 6.2. The new couples planned to spend their New Year's holiday in one of the following tourist destinations: ancient Chinese ruins , the highlands of Penang Island, or Langkawi Island $\ddot{\mathcal{Z}} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, \ddot{z}^4\right\}$ where $\ddot{z}^1 = ancient\ Chinese\ ruins$, $\ddot{z}^2 = highlands\ of\ Penang\ Island$ and $\ddot{z}^3 = Langkawi\ Island$. The couple asked a friend who works for a travel agency to consult them. He asked them to specify the criteria that would determine the selection process. They answered that the criteria were distance and the means of transportation available for travel that can be expressed by i.e η_1 = distance,η_2 = transportation these attributes (Criteria) having their sub-attribute values given as following: $\Lambda_1 = \{\eta_{1,1} = 1000km, \eta_{1,2} = 2000km\}$, $\Lambda_2 = \{\eta_{2,1} = Airplane, \eta_{2,2} = Car\}$, then

$\Omega = \Lambda_1 \times \Lambda_2 = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ where every component η_i has a 2-tuple entity.

In order to solve this problem and help couples choose the best tourist place to spend their vacation, we suggest the following algorithm:

Step 1. Build piv-NHSS $\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}$ based on experts opinion.

Step 2. Determine $\dddot{\mathbb{S}}\left(\overset{\frown}{\Upsilon}^1_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^2_{\hat{\mu}^2}\right)$ starting Definition 6.1.

Step 4. Check Sine Exponential Measure score and select the maxim value by $\tilde{N} = Max\left\{\dddot{\mathbb{S}}_1, \hat{\ddot{\mathbb{S}}}_2, ..., \hat{\ddot{\mathbb{S}}}_m\right\}$ who consider the optimal tourist location.

Step 5. End Algorithm 2.

Algorithm 2 is shown in Figure 3.

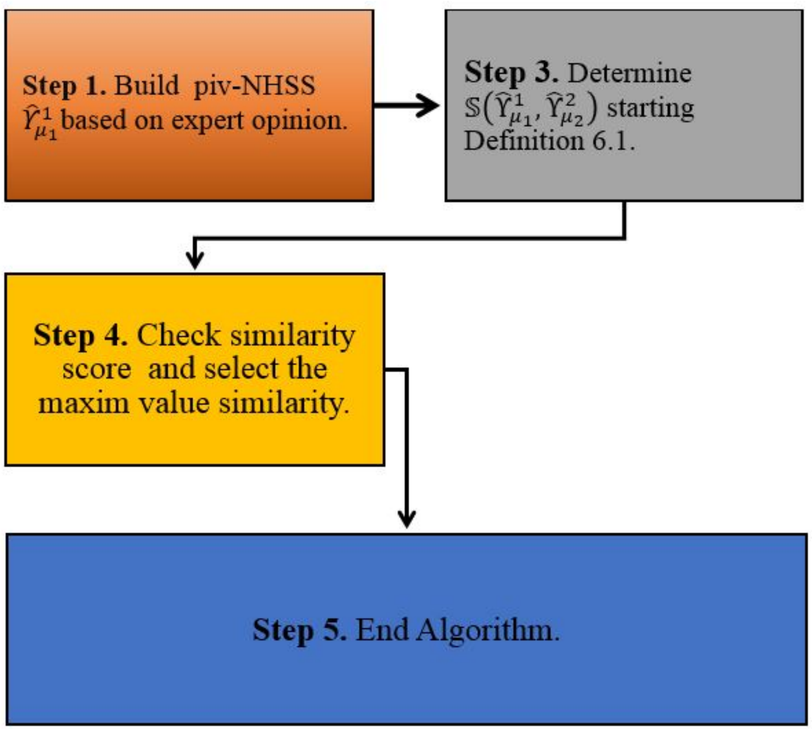

Figure 3: Algorithm 2

Now to implement this algorithm, we start by applying the first step as follows:

$$\overset{\frown}{\Upsilon}^1_{\hat{\mu_1}} = \Bigg\{ \overset{\frown}{\Upsilon}^1{}_{\hat{\mu_1}}(\eta_1) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7 \right), \right.$$

$$\left. \left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6 \right) \right\}$$

$$\overset{\frown}{\Upsilon}^1{}_{\hat{\mu_1}}(\eta_2) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2 \right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5 \right), \right.$$

$$\left. \left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3 \right) \right\}$$

$$\overset{\frown}{\Upsilon}^1{}_{\hat{\mu_1}}(\eta_3) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3 \right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5 \right), \right.$$

$\left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.9\right)\}$

$\widehat{\Upsilon}^1{}_{\hat{\mu_1}}(\eta_4) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle}, 0.1\right),\right.$

$\left.\left(\frac{\ddot{z}^3}{\langle[0.5,0.8],[0.5,0.9],[0.9,0.9]\rangle}, 0.7\right)\right\}\Bigg\}$

$\widehat{\Upsilon}^2_{\hat{\mu_2}} = \Bigg\{ \widehat{\Upsilon}^2_{\hat{\mu_1}}(\eta_1) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.2],[0.4,0.4],[0.2,0.6]\rangle}, 0.5\right), \left(\frac{\ddot{z}^2}{\langle[0.3,0.5],[0.6,0.6],[0.3,0.8]\rangle}, 0.4\right),\right.$

$\left.\left(\frac{\ddot{z}^3}{\langle[0.3,0.6],[0.5,0.5],[0.7,0.8]\rangle}, 0.9\right)\right\}$

$\widehat{\Upsilon}^2_{\hat{\mu_2}}(\eta_2) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.5,0.7],[0.8,0.8],[0.7,0.9]\rangle}, 0.6\right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.7\right),\right.$

$\left.\left(\frac{\ddot{z}^3}{\langle[0.1,0.4],[0.5,0.7],[0.5,0.6]\rangle}, 0.8\right)\right\}$

$\widehat{\Upsilon}^2_{\hat{\mu_2}}(\eta_3) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.5],[0.6,0.6],[0.9,0.9]\rangle}, 0.9\right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5\right),\right.$

$\left.\left(\frac{\ddot{z}^3}{\langle[0.5,0.5],[0.6,0.7],[0.6,0.9]\rangle}, 0.1\right)\right\}$

$\widehat{\Upsilon}^2_{\hat{\mu_2}}(\eta_4) = \left\{\left(\frac{\ddot{z}^1}{\langle[0.5,0.7],[0.4,0.4],[0.1,0.6]\rangle}, 0.5\right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.7],[0.3,0.5],[0.7,0.9]\rangle}, 0.5\right),\right.$

$\left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.8],[0.6,0.9],[0.4,0.6]\rangle}, 0.8\right)\right\}\Bigg\}$

Then the one exponential measures shown in table 1 as following

TABLE 1. Exponential measures values of piv-NHSSs $\widehat{\Upsilon}^1_{\hat{\mu}^1}, \widehat{\Upsilon}^2_{\hat{\mu}^2}$

$\eta_j/\ddot{Z}^i$	z^1	z^2	z^3
η_1	0.0186	0.0241	0.0092
η_2	0.0143	0.0314	0.0157
η_3	0.0261	0.0283	0.01947
η_4	0.0181	0.0381	0.1025
Sum($\ddot{Z}^i$)	0.0771	0.1219	0.1468

From Table 3 we get the best choice which is $\ddot{z}^3$ who is Langkawi Island.

7. Conclusions

In this work, we offer an innovative piv-NHSS by combining both iv-NS and HSS under possibility possible environment.This novel notion is characterized by the ability to deal with complexity and uncertainty issues aptly. The elementary properties and set-theoretical like null piv-NHSS, absolute piv-NHSS, complement of piv-NHSS, union piv-NHSSs, intersection piv-NHSSs, AND-piv-NHSSs, and OR-piv-NHSS was successfully conceptualized as well as support them with examples. Moreover, we discovered both similarities measure and sine exponential measure between two piv-NHSSs and these measures are successfully applied in decision-making to judge choose the best tourist place. As for future studies of this work, the authors recommend developing this concept by integrating it with a number of modern parameters tools [45]– [48] as well as integrating these results with some algebraic [49]– [54] and topological structures [55]– [58].

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