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# Interval-Valued Neutrosophic Framework for Improving the Decision-Making on the Quality Evaluation of English Classics Translation

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**Abstract—**English translations of literary classics are a subject of interest for many scholars and evaluators. However, the evaluation of the translations is a complex and multifaceted task, which necessitates a balance between linguistic accuracy, cultural fidelity, and readability. This makes traditional methods often fall short of catching the uncertainties, ambiguities, and subjective judgments. In an attempt to bridge these gaps between computational linguistics and translation studies, we introduce a mathematically grounded IVNS approach to evaluate the translation of English classics by flexible modeling of the ambiguities inherent in translation decisions. Then, we introduced an extended version of the MULTIMOORA method, called MULTIVNSMOORA, that can simultaneously account for multiple, often conflicting criteria to ensure well-adjusted decision-making about translation quality. Finally, we present a case study on Don Quixote to validate the practical applicability of the proposed IVNS approach, by which we offer a reproducible framework for assessing translations of other English literary works.

**Keywords:** Neutrosophic Logic, Interval-Valued Neutrosophic (IVN), English Classics, Corpus-based evaluation.

## 1. Introduction

The translation of literary classics into English is a complex and nuanced endeavor, requiring not only linguistic proficiency but also a deep understanding of cultural, historical, and contextual subtleties [1]. As these works traverse linguistic boundaries, the fidelity, readability, and aesthetic quality of their translations often become subjects of intense scrutiny. Traditional methods of translation evaluation, while useful, frequently struggle to capture the inherent uncertainties, ambiguities, and subjective judgments involved in the process [2]. This always applies when dealing with corpus-based translations, in which large-scale textual analysis adds additional layers of complexity [3].

In recent years, the development of computational and mathematical approaches has gained traction in translation studies, offering new ways to assess and compare translated texts. Among these, neutrosophic logic—a generalization of fuzzy logic—has emerged as a powerful tool for handling uncertainty, indeterminacy, and imprecision in decision-making processes [4], [5]. The design of NS presented a neutrality degree as a new component that is not predefined fuzzy logic [6], [7]. Thus, neutrosophic logic has three membership components which are falsity, indeterminacy, and truth, allowing more valuable application in different decision-making tasks [8]. Lately, researchers have introduced different types of NS have been proposed in different publications including Intuitionistic NS [9], multi-valued NSs (MVNs) [10], single-valued NSs (SVNSs) [11], [12], and Trapezoidal NS (TNS) [13]. Interval-valued neutrosophic sets (IVNS), in particular, provide a robust framework for modeling multi-dimensional and uncertain data, making them well-suited for the evaluation of translation quality [14].

In this article, we propose a novel IVNSs approach for the evaluation of corpus-based English translations of literary classics through the integration of linguistic, stylistic, and contextual parameters. Our IVNSs approach aims to address the limits that appear in dealing with traditional evaluations of assessment of translation quality. To guarantee well-adjusted decision-making about translation quality, we then presented MULTIVNSMOORA, an expanded version of the MULTIMOORA approach [15] that may concurrently account for several, frequently competing criteria. Lastly, to verify the usefulness of the suggested IVNS technique, we provide a replicable framework for evaluating translations of other English literary works through a case study on Don Quixote.

In subsequent sections, we outline the theoretical foundations of IVNSs, describe the methodology for applying this framework to translation evaluation, and present a case study involving the analysis of English translations of selected literary classics. By doing so, we shed light on the potential of neutrosophic logic as a transformative tool in translation studies.

## 2. Preliminaries

**Definition 1**. Neutrosophic set [4] to extend the fuzzy set for modeling uncertainty, and is defined as:

$$\mathcal{P} = \{\langle c, \mathfrak{t}_{\mathcal{P}}(c), \mathfrak{i}_{\mathcal{P}}(c), \mathfrak{f}_{\mathcal{P}}(c)\rangle : c \in U\} \tag{1}$$

$$0^- \leq \mathfrak{t}_A(x) + \mathfrak{i}_A(x) + \mathfrak{f}_A(x) \leq 3^+ \tag{2}$$

$$\mathfrak{t}_{\mathcal{P}}: X \to ]^-0, 1[^+, \quad \mathfrak{i}_{\mathcal{P}}: X \to ]^-0, 1[^+, \quad \mathfrak{f}_{\mathcal{P}}: X \to ]^-0, 1[^+ \tag{3}$$

**Definition 2.** An IVNS redefines the membership of each component to be interval instead of scalars, which can be expressed as:

$$\mathcal{P} = \{\langle c, [\mathfrak{t}_{\mathfrak{L}}(c), \mathfrak{t}_{\mathfrak{U}}(c)], [\mathfrak{i}_{\mathfrak{L}}(c), \mathfrak{i}_{\mathfrak{U}}(c)], [\mathfrak{f}_{\mathfrak{L}}(c), \mathfrak{f}_{\mathfrak{U}}(c)]\rangle : c \in \mathrm{U}\} \tag{4}$$

where the constituting intervals satisfy the following conditions:

$$\mathfrak{t}_{\mathfrak{L}}(c) + \mathfrak{i}_{\mathfrak{L}}(c) + \mathfrak{f}_{\mathfrak{L}}(c) \leq 3, \; \mathfrak{t}_{\mathfrak{U}}(c) + \mathfrak{i}_{\mathfrak{U}}(c) + \mathfrak{f}_{\mathfrak{U}}(c) \leq 3 \tag{5}$$

In the above formula, $\mathfrak{t}_{\mathfrak{L}}(c)$ and $\mathfrak{t}_{\mathfrak{U}}(c)$ Denote the lower and upper bounds of the truth membership, and similarly for $\mathfrak{i}$ and $\mathfrak{f}$.

**Definition 3**. Given two IVNSs $\mathcal{P}$ and $\mathcal{Q}$, the union $\mathcal{P} \cup \mathcal{Q}$ is defined as:

$$\mathcal{P} \cup \mathcal{Q} = \left\{ \left\langle \begin{array}{c} c, \left[max\left(\mathfrak{t}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{t}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), max\left(\mathfrak{t}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{t}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right], \\ \left[min\left(\mathfrak{i}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{i}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), min\left(\mathfrak{i}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{i}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right], \\ \left[min\left(\mathfrak{f}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{f}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), min\left(\mathfrak{f}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{f}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right] \end{array} \right\rangle : c \in U \right\} \tag{6}$$

**Definition 4**. Given two IVNSs $\mathcal{P}$ and $\mathcal{Q}$, the intersection $\mathcal{P} \cap \mathcal{Q}$ is defined as:

$$\mathcal{P} \cap \mathcal{Q} = \left\{ \left\langle \begin{array}{c} c, \left[min\left(\mathfrak{t}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{t}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), min\left(\mathfrak{t}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{t}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right], \\ \left[max\left(\mathfrak{i}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{i}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), max\left(\mathfrak{i}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{i}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right], \\ \left[max\left(\mathfrak{f}_{\mathfrak{L}}^{\mathcal{P}}(c), \mathfrak{f}_{\mathfrak{L}}^{\mathcal{Q}}(c)\right), max\left(\mathfrak{f}_{\mathfrak{U}}^{\mathcal{P}}(c), \mathfrak{f}_{\mathfrak{U}}^{\mathcal{Q}}(c)\right)\right] \end{array} \right\rangle : c \in U \right\} \tag{7}$$

**Definition 5.** Given an IVNSs $\mathcal{P} = \left\{\left\langle c, \left[\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}\right]\right\rangle : c \in U\right\}$, then the complement $\mathcal{P}^c$ can be expressed as:

$$\mathcal{P}^c = \left\{\left\langle c, \left[\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[1 - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, 1 - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}\right]\right\rangle : c \in U\right\} \tag{8}$$

**Definition 6.** Given two IVNSs $\mathcal{P} = \left\{\left\langle c, \left[\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}\right], \left[\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}\right]\right\rangle : c \in \mathfrak{U}\right\}$, and $\mathcal{Q} = \left\{\left\langle c, \left[\mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}\right], \left[\mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}}\right], \left[\mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}\right]\right\rangle : c \in U\right\}$, the following relations apply as follows:

(9)

$$\mathcal{P} \subseteq \mathcal{Q}\ iff\ \mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} \le \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} \le \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}; \mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}} \ge \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}} \ge \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}}; \mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} \ge \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} \ge \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}$$

$$\mathcal{P} = \mathcal{Q}\ if\ and\ only\ if\ \mathcal{P} \subseteq \mathcal{Q}\ and\ \mathcal{Q} \subseteq \mathcal{P}. \tag{10}$$

**Definition 7.** Given an IVNS $\mathcal{P} = \{\langle c, [\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}]\rangle : c \in \mathfrak{U}\}$, we can compute the score, accuracy, and certainty functions as follows:

$$S_{\mathcal{P}}(c) = \frac{\left(\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} + (1 - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}) + (1 - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}) + (1 - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}) + (1 - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}})\right)}{6}. \tag{11}$$

$$a_{\mathcal{P}}(c) = \frac{(\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}})}{2} \tag{12}$$

**Definition 8.** Given two IVNSs $\mathcal{P} = \{\langle c, [\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}]\rangle : c \in U\}$, and $\mathcal{Q} = \left\{\left\langle c, \left[\mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}\right], \left[\mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}}\right], \left[\mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}\right]\right\rangle : c \in U\right\}$, there are many elementary operations to be applied:

- Scalar multiplications

$$\lambda\mathcal{P} = \left\{\left\langle \begin{matrix} c, \left[1 - (1 - \mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}})^{\lambda}, 1 - (1 - \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}\right], \\ \left[(\mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}, (\mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}})^{\lambda}\right], \\ \left[(\mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}, (\mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}})^{\lambda}\right] \end{matrix} \right\rangle : c \in \mathfrak{U}\right\} \tag{13}$$

- Power

$$\mathcal{P}^{\lambda} = \left\{\left\langle \begin{matrix} c, \left[(\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}})^{\lambda}, (\mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}\right], \\ \left[1 - (1 - \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}, 1 - (1 - \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}})^{\lambda}\right], \\ \left[1 - (1 - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}})^{\lambda}, 1 - (1 - \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}})^{\lambda}\right] \end{matrix} \right\rangle : c \in \mathfrak{U}\right\} \tag{14}$$

- Addition

$$\mathcal{P} \oplus \mathcal{Q} = \left\{\left\langle \begin{matrix} c, \left[\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}} - \mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} + \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}} - \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}\right], \\ \left[\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}}\right], \\ \left[\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}\right] \end{matrix} \right\rangle : c \in \mathfrak{U}\right\} \tag{15}$$

- Subtraction

$$\mathcal{P} \ominus \mathcal{Q} = \left\{\left\langle \begin{matrix} c, \left[\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} - \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} - \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}\right], \\ \left[max(\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}), max(\mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}, \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}})\right], \\ \left[\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} - \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} - \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}\right] \end{matrix} \right\rangle : c \in \mathfrak{U}\right\} \tag{16}$$

- Multiplication

$$\mathcal{P} \otimes \mathcal{Q} = \left\{ \left\langle \begin{matrix} \mathbf{c}, [\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{t}_{\mathcal{Q}}^{\mathfrak{U}}], \\ [\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}} - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}} + \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}} - \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{i}_{\mathcal{Q}}^{\mathfrak{U}}], \\ [\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}} - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} \cdot \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} + \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}} - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} \cdot \mathfrak{f}_{\mathcal{Q}}^{\mathfrak{U}}] \end{matrix} \right\rangle : \mathbf{c} \in \mathfrak{U} \right\} \tag{17}$$

**Definition 9.** Given $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$ representing three IVNSs, where $\lambda, \lambda_1, \lambda_2 > 0$, then, the following condition apply

$$\begin{gathered} \mathcal{P} + \mathcal{Q} = \mathcal{Q} + \mathcal{P} \\ \mathcal{P} \cdot \mathcal{Q} = \mathcal{Q} \cdot \mathcal{P} \\ \lambda(\mathcal{P} + \mathcal{Q}) = \lambda\mathcal{Q} + \lambda\mathcal{P} \\ (\mathcal{P} \cdot \mathcal{Q})^{\lambda} = \mathcal{Q}^{\lambda} \cdot \mathcal{P}^{\lambda} \\ \lambda_1\mathcal{P} + \lambda_2\mathcal{P} = (\lambda_1 + \lambda_2)\mathcal{P} \\ \mathcal{P}^{\lambda_1} \cdot \mathcal{P}^{\lambda_1} = \mathcal{P}^{\lambda_1 + \lambda_2} \\ (\mathcal{P} + \mathcal{Q}) + \mathcal{R} = \mathcal{P} + (\mathcal{Q} + \mathcal{R}) \end{gathered} \tag{18}$$

**Definition 9.** Given an IVN $\mathcal{P} = [\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}}], [\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}}]$, the deneutrosophication can be computed as follows:

$$\mathfrak{D}(\mathcal{P}) = \left( \frac{(\mathfrak{t}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{t}_{\mathcal{P}}^{\mathfrak{U}})}{\mathbf{2}} + \left( \mathbf{1} - \frac{(\mathfrak{i}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}})}{\mathbf{2}} \right) \left( \mathfrak{i}_{\mathcal{P}}^{\mathfrak{U}} \right) - \left( \frac{(\mathfrak{f}_{\mathcal{P}}^{\mathfrak{L}} + \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}})}{\mathbf{2}} \right) \left( \mathbf{1} - \mathfrak{f}_{\mathcal{P}}^{\mathfrak{U}} \right) \right) \tag{19}$$

**Definition 10.** Given a set of IVNSs $\mathcal{P}_j = \left\{ \left\langle c, [\mathfrak{t}_{\mathcal{P}_j}^{\mathfrak{L}}, \mathfrak{t}_{\mathcal{P}_j}^{\mathfrak{U}}], [\mathfrak{i}_{\mathcal{P}_j}^{\mathfrak{L}}, \mathfrak{i}_{\mathcal{P}_j}^{\mathfrak{U}}], [\mathfrak{f}_{\mathcal{P}_j}^{\mathfrak{L}}, \mathfrak{f}_{\mathcal{P}_j}^{\mathfrak{U}}] \right\rangle : c \in U \right\}$, and corresponding weight vector $W = (w_1, w_2, \dots, w_n)^{\mathfrak{t}}$, where $w_j \geq 0$, and $\sum w_j = \mathbf{1}$. Their general weighted aggregation function is formulated as follows:

if $\lambda \to 0$

$$Z = \mathrm{IVNS}_{\mathrm{GWA}}(\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_n) = \prod_{j=1}^{n} A_j^{w_j} = \left\{ \begin{matrix} \left\langle \left[ \prod_{j=1}^{n} (\mathfrak{t}_j^{\mathfrak{L}})^{w_j}, \prod_{j=1}^{n} (\mathfrak{t}_j^{\mathfrak{U}})^{w_j} \right], \right. \\ \left[ 1 - \prod_{j=1}^{n} (1 - \mathfrak{i}_j^{\mathfrak{L}})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \mathfrak{i}_j^{\mathfrak{U}})^{w_j} \right], \\ \left[ 1 - \prod_{j=1}^{n} (1 - \mathfrak{f}_j^{\mathfrak{L}})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \mathfrak{f}_j^{\mathfrak{U}})^{w_j} \right] \end{matrix} \right\} \tag{20}$$

if $\lambda \to 1$

$$Z = \mathrm{IVNS}_{\mathrm{GWA}}(\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_n) = \sum_{j=1}^{n} w_j \mathcal{P}_j = \left\{ \left\langle \begin{array}{c} \left[1 - \prod_{j=1}^{n} (1 - \mathfrak{t}_j^{\mathfrak{L}})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \mathfrak{t}_j^{\mathfrak{U}})^{w_j}\right], \\ \left[\prod_{j=1}^{n} (\mathfrak{i}_j^{\mathfrak{L}})^{w_j}, \prod_{j=1}^{n} (\mathfrak{i}_j^{\mathfrak{U}})^{w_j}\right], \left[\prod_{j=1}^{n} (\mathfrak{f}_j^{\mathfrak{L}})^{w_j}, \prod_{j=1}^{n} (\mathfrak{f}_j^{\mathfrak{U}})^{w_j}\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \begin{array}{c} \left[1 - \prod_{j=1}^{n} (1 - \mathfrak{t}_j^{\mathfrak{L}})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \mathfrak{t}_j^{\mathfrak{U}})^{w_j}\right], \\ \left[\prod_{j=1}^{n} (\mathfrak{i}_j^{\mathfrak{L}})^{w_j}, \prod_{j=1}^{n} (\mathfrak{i}_j^{\mathfrak{U}})^{w_j}\right], \left[\prod_{j=1}^{n} (\mathfrak{f}_j^{\mathfrak{L}})^{w_j}, \prod_{j=1}^{n} (\mathfrak{f}_j^{\mathfrak{U}})^{w_j}\right] \end{array} \right\rangle \right\} \tag{21}$$

3. Research Method

This section provides a holistic explanation of the proposed IVNSMCDM approach for assessing and uncertain process of evaluating the translations of English. Our explanation of the proposed method will go through sequence of steps that are detailed as follows:

Step 1: we construct the Decision Matrix in IVNS form, in which each alternative $A_i$ is evaluated against each criterion $C_j$ using IVNSs:

$$\boldsymbol{D_{n\times m}} = \begin{bmatrix} \langle [\mathfrak{t}_{11}^{\mathfrak{L}}, \mathfrak{t}_{11}^{\mathfrak{U}}], [\mathfrak{i}_{11}^{\mathfrak{L}}, \mathfrak{i}_{11}^{\mathfrak{U}}], [\mathfrak{f}_{11}^{\mathfrak{L}}, \mathfrak{f}_{11}^{\mathfrak{U}}] \rangle & \cdots & \langle [\mathfrak{t}_{1m}^{\mathfrak{L}}, \mathfrak{t}_{1m}^{\mathfrak{U}}], [\mathfrak{i}_{1m}^{\mathfrak{L}}, \mathfrak{i}_{1m}^{\mathfrak{U}}], [\mathfrak{f}_{1m}^{\mathfrak{U}}, \mathfrak{f}_{1m}^{\mathfrak{U}}] \rangle \\ \vdots & \ddots & \vdots \\ \langle [\mathfrak{t}_{n1}^{\mathfrak{L}}, \mathfrak{t}_{n1}^{\mathfrak{U}}], [\mathfrak{i}_{n1}^{\mathfrak{L}}, \mathfrak{i}_{n1}^{\mathfrak{U}}], [\mathfrak{f}_{n1}^{\mathfrak{L}}, \mathfrak{f}_{n1}^{\mathfrak{U}}] \rangle & \cdots & \langle [\mathfrak{t}_{nm}^{\mathfrak{L}}, \mathfrak{t}_{nm}^{\mathfrak{U}}], [\mathfrak{i}_{nm}^{\mathfrak{L}}, \mathfrak{i}_{nm}^{\mathfrak{U}}], [\mathfrak{f}_{nm}^{\mathfrak{U}}, \mathfrak{f}_{nm}^{\mathfrak{U}}] \rangle \end{bmatrix}, \tag{22}$$

where $n$ = number of alternatives, $m$ = number of criteria, and $\left\langle [\mathfrak{t}_{1j}^{\mathfrak{L}}, \mathfrak{t}_{1j}^{\mathfrak{U}}], [\mathfrak{i}_{1j}^{\mathfrak{L}}, \mathfrak{i}_{1j}^{\mathfrak{U}}], [\mathfrak{f}_{1j}^{\mathfrak{U}}, \mathfrak{f}_{1j}^{\mathfrak{U}}] \right\rangle$ represent IVNS evaluation of alternative $A_i$ for criterion $C_j$.

**Step 2**: we conduct normalization of the $\boldsymbol{D_{n\times m}}$ to ensure all criteria are comparable. For IVNSs, the normalized decision matrix $\boldsymbol{R}$ is calculated as:

$$\mathbf{R_{ij}} = \left\langle [\mathfrak{t}_{ij}^{\mathfrak{L}}, \mathfrak{t}_{ij}^{\mathfrak{U}}], [\mathfrak{i}_{ij}^{\mathfrak{L}}, \mathfrak{i}_{ij}^{\mathfrak{U}}], [\mathfrak{f}_{ij}^{\mathfrak{U}}, \mathfrak{f}_{ij}^{\mathfrak{U}}] \right\rangle \tag{23}$$

**Step 3:** we apply a ratio system to calculate the overall score for each alternative by summing the normalized IVNSs for benefit criteria and subtracting those for cost criteria.

Step 3.1. Aggregate the IVNSs for each alternative $\boldsymbol{A_i}$ :

$$\boldsymbol{RS_i} = \sum_{j=1}^{k} \boldsymbol{w_j} \cdot \boldsymbol{R_{ij}} - \sum_{j=k+1}^{m} \boldsymbol{w_j} \cdot \boldsymbol{R_{ij}} \tag{24}$$

where $w_j$ = weight of criterion $C_j$ Obtained using the CRITIC method [16], $k$ = number of benefit criteria, $m$ = total number of criteria.

Step 3.2. calculate the score function for

$$S_{(RS_i)} = \frac{\left\langle \mathfrak{t}_{ij}^{\mathfrak{l}} + \mathfrak{t}_{ij}^{\mathfrak{u}} - \left(\mathfrak{i}_{ij}^{\mathfrak{l}} + \mathfrak{i}_{ij}^{\mathfrak{u}}\right) - \left(\mathfrak{f}_{ij}^{\mathfrak{u}}, \mathfrak{f}_{ij}^{\mathfrak{u}}\right) \right\rangle}{2} \quad (25)$$

Step 3.3. Rank the alternatives based on $S_{(RS_i)}$ in descending order.

Step 4: Apply the Reference Point (RP) Approach

RP identifies the best alternative by minimizing the distance from an ideal reference point.

Step 4.1. Determine the reference point $RP_j$ for each criterion $C_j$ :

$$RP_j = \left\langle \left[\max\left(\mathfrak{t}_{ij}^{\mathfrak{l}}\right), \max\left(\mathfrak{t}_{ij}^{\mathfrak{u}}\right)\right], \left[\min\left(\mathfrak{i}_{ij}^{\mathfrak{l}}\right), \min\left(\mathfrak{i}_{ij}^{\mathfrak{u}}\right)\right], \left[\min\left(\mathfrak{f}_{ij}^{\mathfrak{l}}\right), \min\left(\mathfrak{f}_{ij}^{\mathfrak{u}}\right)\right] \right\rangle \quad (26)$$

Step 4.2. Calculate the distance between each alternative $A_i$ and the reference point $RP_j$ Using a distance measure for IVNSs, such as:

$$d\left(R_{ij}, RP_j\right) = \sqrt{\sum_{k=1}^{n} R_{ij}^{k} - RP_{ij}^{k}} \quad (27)$$

Step 4.3. Compute the overall distance for each alternative:

$$RP_i = \max_j \left(w_j \cdot d\left(R_{ij}, RP_j\right)\right) \quad (28)$$

Step 4.4. Rank the alternatives based on $RP_i$ in ascending order.

Step 5: Apply the Full Multiplicative Form (FMF) by taking the overall score for each alternative by multiplying the normalized IVNSs for benefit criteria and dividing by those for cost criteria.

Step 5.1. Aggregate the IVNSs for each alternative $A_i$ :

$$FMF_i = \frac{\prod_{j=1}^{k} \left(R_{ij}\right)^{w_j}}{\prod_{j=k+1}^{m} \left(R_{ij}\right)^{w_j}} \quad (29)$$

Step 5.2. Calculate the score function for $FMF_i$ :

$$S_{(FMF_i)} = \frac{\left\langle \mathfrak{t}_{ij}^{\mathfrak{l}} + \mathfrak{t}_{ij}^{\mathfrak{u}} - \left(\mathfrak{i}_{ij}^{\mathfrak{l}} + \mathfrak{i}_{ij}^{\mathfrak{u}}\right) - \left(\mathfrak{f}_{ij}^{\mathfrak{u}}, \mathfrak{f}_{ij}^{\mathfrak{u}}\right) \right\rangle}{2} \quad (30)$$

Step 5.3. Rank the alternatives based on $\boldsymbol{S}_{(\boldsymbol{FMF_i})}$ in descending order.

Step 6: we aggregate the ranking of alternatives from the RS, RP, and FMF based on a common aggregation method called the Borda Rule.

## 4. Case Study & Quantitative Results

Herein, we introduce our case study for English translation of classics and analyze the results obtained by the proposed method.

### 4.1. Case Study: English Translations of Don Quixote

Translating literary classics into English involves balancing fidelity to the original text, readability, cultural context, and accessibility. In our study, we used “Don Quixote" by Miguel de Cervantes, as one of the most translated and analyzed classics in literature. Various translations of "Don Quixote" by Miguel de Cervantes have been published over time, each with its strengths and weaknesses. Selecting the most suitable translation requires evaluating different criteria. The assessment of each of the above translations is conducted based on the combination of benefit criteria (higher values are better) as well as cost criteria to assess the translations. First, readability (C1) refers to the degree to which the translation can be easily understood by a contemporary reader. It is always captured by sentence complexity, vocabulary, and flow. Second, cultural Fidelity (C2) measures how well the translation preserves the cultural and historical context of the original. It is always captured by the retention of idioms, cultural references, and tone. Third, literary Style (C3) measures how effectively the translation captures the literary essence of the original. It is always captured by narrative voice, humor, and stylistic devices. Forth, complexity (C4), is the level of linguistic and structural difficulty in the translation. It is always captured by sentence length, archaic language, and syntactic complexity. Fifth, deviation from Source Text (C5) measures the extent to which the translation diverges from the original text. After an internet investigation, we found Below, we found five distinct English translations of the same classic: Translation A1: John Ormsby (1885), Translation A2: Samuel Putnam (1949), Translation A3: Edith Grossman (2003), Translation A4: Tobias Smollett (1755), and Translation A5: Tom Lathrop (2005).

Table 1. Experts-aggregated IVNS-decision matrix for our case study.

| Alternative | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| **A1 (Ormsby)** | {[0.834, 0.955], [0.062, 0.112], [0.144, 0.223]} | {[0.741, 0.846], [0.144, 0.170], [0.244, 0.328]} | {[0.763, 0.860], [0.149, 0.232], [0.167, 0.278]} | {[0.623, 0.737], [0.136, 0.209], [0.353, 0.418]} | {[0.611, 0.724], [0.132, 0.214], [0.222, 0.349]} |

| | | | | | |
|---|---|---|---|---|---|
| **A2 (Putnam)** | {[0.780, 0.900], [0.116, 0.219], [0.156, 0.262]} | {[0.810, 0.902], [0.051, 0.153], [0.122, 0.203]} | {[0.718, 0.807], [0.128, 0.196], [0.203, 0.354]} | {[0.502, 0.619], [0.153, 0.238], [0.437, 0.543]} | {[0.424, 0.738], [0.107, 0.216], [0.348, 0.428]} |
| **A3 (Grossman)** | {[0.929, 1.000], [0.023, 0.053], [0.035, 0.124]} | {[0.872, 0.963], [0.091, 0.149], [0.076, 0.159]} | {[0.831, 0.919], [0.070, 0.141], [0.139, 0.226]} | {[0.726, 0.824], [0.124, 0.204], [0.232, 0.503]} | {[0.402, 0.634], [0.066, 0.139], [0.119, 0.218]} |
| **A4 (Smollett)** | {[0.532, 0.627], [0.239, 0.346], [0.441, 0.504]} | {[0.614, 0.718], [0.154, 0.285], [0.330, 0.421]} | {[0.521, 0.652], [0.210, 0.311], [0.413, 0.540]} | {[0.514, 0.825], [0.125, 0.251], [0.413, 0.742]} | {[0.454, 0.622], [0.130, 0.229], [0.445, 0.525]} |
| **A5 (Lathrop)** | {[0.745, 0.824], [0.145, 0.201], [0.236, 0.342]} | {[0.770, 0.885], [0.142, 0.207], [0.161, 0.274]} | {[0.673, 0.751], [0.200, 0.274], [0.260, 0.377]} | {[0.619, 0.713], [0.105, 0.234], [0.341, 0.525]} | {[0.515, 0.842], [0.129, 0.212], [0.224, 0.339]} |

The weights associated with the evaluation criteria were calculated using the CRITIC method (as mentioned earlier), which determines the importance of each criterion according to the contrast intensity of the data as well as the struggle between criteria. This process was conducted in two distinct ways to ensure robustness and comprehensiveness. First, the weights were derived based on the scores of the IVNS decision matrix, which were computed using the interval components for each alternative-criterion pair (refer to Figure 1). Second, the weights were calculated based on the accuracies of the IVNS

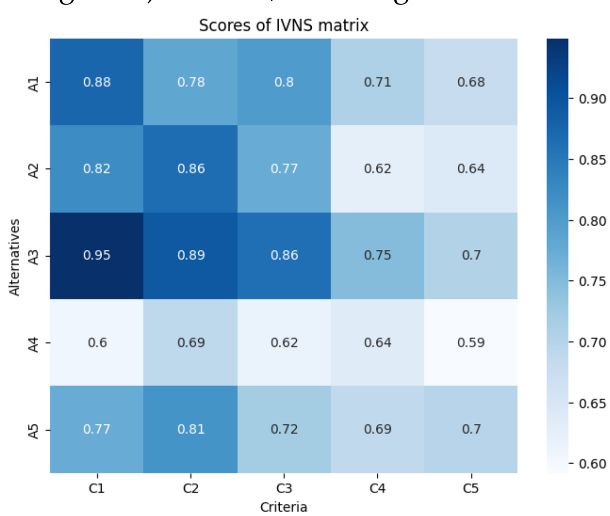


*Figure 1. IVNS score matrix driven from the aggregated decision matrix*

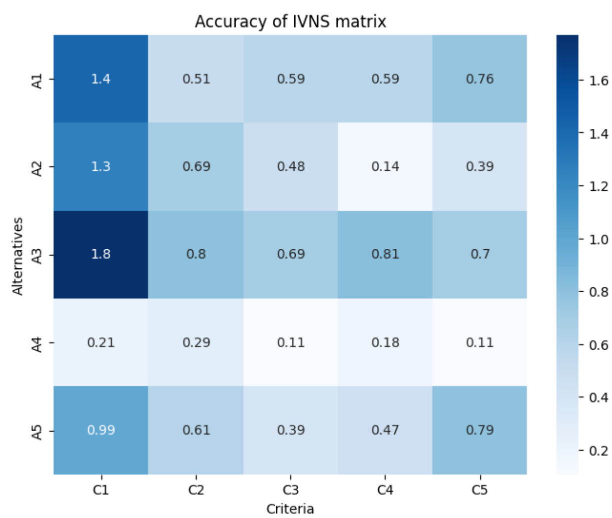


*Figure 2. IVNS accuracy matrix driven from the aggregated decision matrix.*

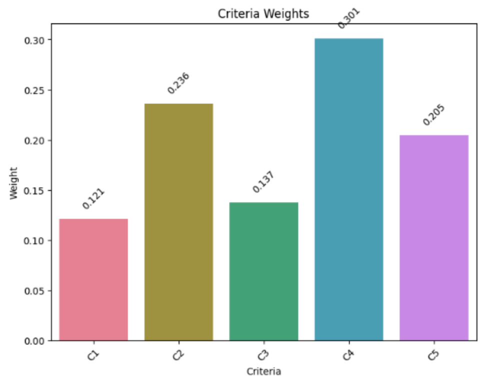


Figure 3. Criterion weights based on IVNS scores matrix.

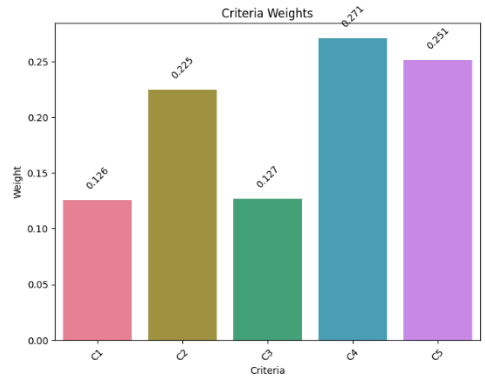


Figure 4. Criterion weights based on IVNS accuracy matrix.

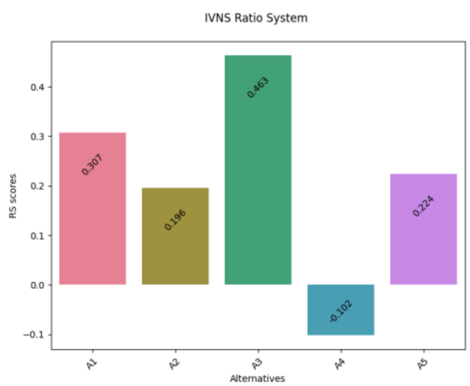


Figure 5. Ratio System Scores for alternatives based on scored weighting scheme.

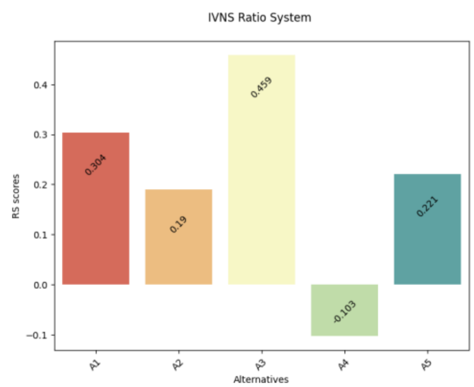


Figure 6: Ratio System Scores for alternatives using the accuracy weighting scheme.

decision matrix, which reflects the degree of certainty or precision in the evaluations (refer to Figure 2). The CRITIC method was applied to both approaches to capture the relative importance of each criterion in the context of the decision-making problem. The resulting weights from the first approach are visualized in Figure 3, which illustrates the distribution of weights across the criteria. Similarly, the weights derived from the second approach are presented in Figure 4, providing a comparative perspective on how the criteria weights vary when calculated based on accuracies. This dual application of the CRITIC method ensures a more nuanced and reliable weighting scheme, enhancing the validity of the evaluation framework.

To visualize the impact of different weighting schemes on the evaluation of alternatives, we plot the Ratio System Scores for both weighting schemes in Figure 5 and Figure 6. In Figure 5, the scores are plotted using the score-based weighting scheme, where all criteria are assigned equal importance. This provides a baseline understanding of how the alternatives perform when no criterion is prioritized over another. In Figure 6, the scores are plotted using the accuracy weighting scheme, where criteria are weighted based on their relative importance as determined by expert judgment or prior analysis.

This allows us to observe how the prioritization of specific criteria influences the overall ranking of alternatives.

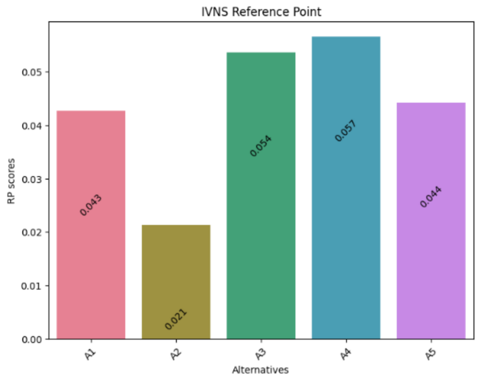


Figure 7: RP Scores for alternatives using the scored weighting scheme.

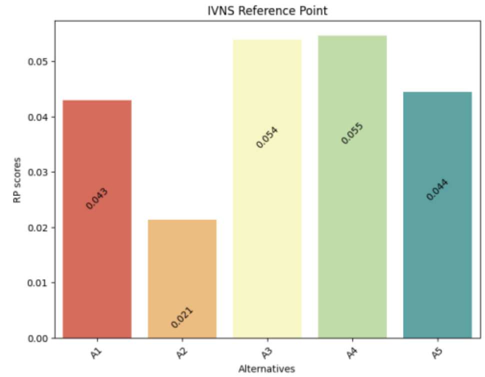


Figure 8: RP Scores for alternatives using the accuracy weighting scheme.

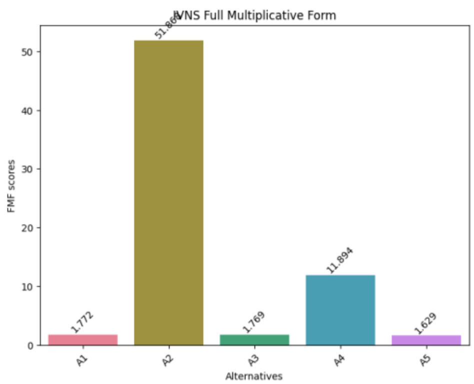


Figure 9: FMF Scores for alternatives using the scored weighting scheme.

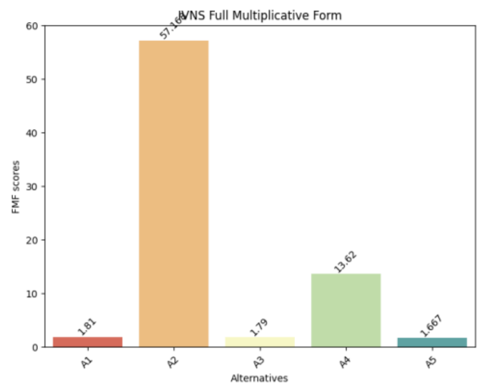


Figure 10: FMF Scores for alternatives using the accuracy weighting scheme.

To further analyzes the robustness of the evaluations processes, we plot the Reference Point Scores for both weighting schemes in Figure 7 and Figure 8. This allow readers to perceive how the arrangement of explicit criteria stimuli the distance of alternatives from the ideal reference point. To ample our analysis, again, we plot the FMF Scores for both weighting schemes in Figure 9 and Figure 10. This enables observing how the ordering of definite criteria can affect the overall multiplicative scores and the subsequent ranking of alternatives.

To validate the effectiveness of the proposed approach, we conduct a comparative analysis against other IVNS-MCDM methods, as summarized in Table 2. The comparison includes methods such as IVNS-TOPSIS [17], MULTIMOORA [15], and IVNS-EDAS [18], which are widely utilized in previous studies for handling uncertainty and imprecision in decision-making. The results demonstrate that the proposed approach consistently outperforms or aligns with these methods in terms of ranking accuracy, robustness, and computational efficiency. Specifically, the proposed approach provides a more balanced evaluation by integrating the strengths of the Ratio System, Reference Point, and Full Multiplicative Form methods, while effectively handling the indeterminacy and ambiguity inherent in IVNSs.

**Table 2**. Comparative analysis of alternative ranking from different IVNS-MCDM methods.

| Rank | IVNS-TOPSIS | MULTIMOORA | IVNS-EDAS | Proposed |
|---|---|---|---|---|
| **1** | A2 | A2 | A2 | A2 |
| **2** | A4 | A3 | A4 | A1 |
| **3** | A3 | A1 | A3 | A3 |
| **4** | A1 | A5 | A1 | A5 |
| **5** | A5 | A4 | A5 | A4 |

## 5. Conclusion

This research article proposes a novel IVNS approach that integrates an extended version of the MULTIMOORA method to deal with uncertainty challenges inherent in the translation of English classics. Our work introduces a proof-of-concept case study based on a corpus of English translations of Don Quixote, to be used to assess the evaluation of our approach for evaluating and ranking translations according to different translation criteria, including readability, cultural fidelity, literary style, complexity, and deviation from the source text. The quantitative and qualitative results demonstrate the versatility and reliability of the MULTIVNSMOORA method in handling the strengths and weaknesses of each translation, meanwhile offering valued intuitions for translators, scholars, and publishers.

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