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Bipolar Neutrosophic Driven Approach for Assessing the Teaching Quality of English Language and Literature Learning Outcomes

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Abstract: The quality of teaching in English Language and Literature is a crucial factor in enhancing student learning outcomes, fostering critical thinking, and promoting linguistic and literary proficiency. Nevertheless, the procedure of assessing the quality of English teaching remains a thought-provoking task since it typically depends on individual judgments, variety in criteria, and inconsistency in human observation, resulting in intrinsic uncertainty. Bipolar Neutrosophic Sets (BNS) offer an expressive way to model ambiguity in a bipolar manner using three positive and three negative memberships. Motivated by that, we propose to explore the novel approach based on BNS to improve the judgmental process of evaluating the quality of teaching of Quality of English Language and related literature. To assess the relative importance of aggregated teaching criteria, we apply the Analytic Hierarchy Process (AHP) to drive representative weights that guide the later decision-making. We drive a new weighting scheme to apply the AHP weights into a bipolar decision matrix. Following, we introduce multi-criteria decision-making (MCDM) to determine the bipolar ideal solutions in a bipolar decision matrix, and then identify the relative closeness teaching techniques. Based on a numerical study, we conduct a holistic analysis of the results of the proposed framework, which demonstrates remarkable power in making appropriate decisions about the best scenario English teaching approach that achieves the optimal benefit for students.

Keywords: Bipolar neutrosophic sets (BNSs), Uncertainty, Neutrosophic Theory, Teaching Quality, English Literature**.**

1. Introduction

Continuous evaluation of the quality of teaching of English language and literature has been gaining much attention due to the important role it plays in shaping learners' linguistic capability, critical thinking, and cultural awareness of students [1]. The practice of Teaching English usually entails improving the students' proficiency in different language skills—listening, speaking, reading, and writing—which is not only a medium of communication but also a allows interpreting the literary traditions, and historical backgrounds, as well as human experiences [2]. Accordingly, judging the English teaching quality within different domains not only necessitates surface-level evaluation practices; instead, it requires a nuanced method that deliberates pedagogical efficiency, student engagement, background, level of education, and the level of interaction between creativity and academic rigor. This, in turn, makes the traditional evaluation techniques are no longer effective and might fail to consider the variation in individual learner experiences and wider pedagogical effects [3].

Florentin Smarandache [4] introduced the Neutrosophic set (NS) as a theoretical extension to the standard and fuzzy sets by integrating three basic elements namely truth (Tr), falsity (F), and indeterminacy (II). A part from these traditional sets, NSs were designed to offer a more elastic representation of data, by simultaneously considering the ambiguity, indistinctness, and illogicalities. Each of these consisting of three elements is self-determining and usually represented using a value between 0 and 1, which makes the NS predominantly operative when it comes to dealing with the complex process of evaluating the teaching quality of English and literature.

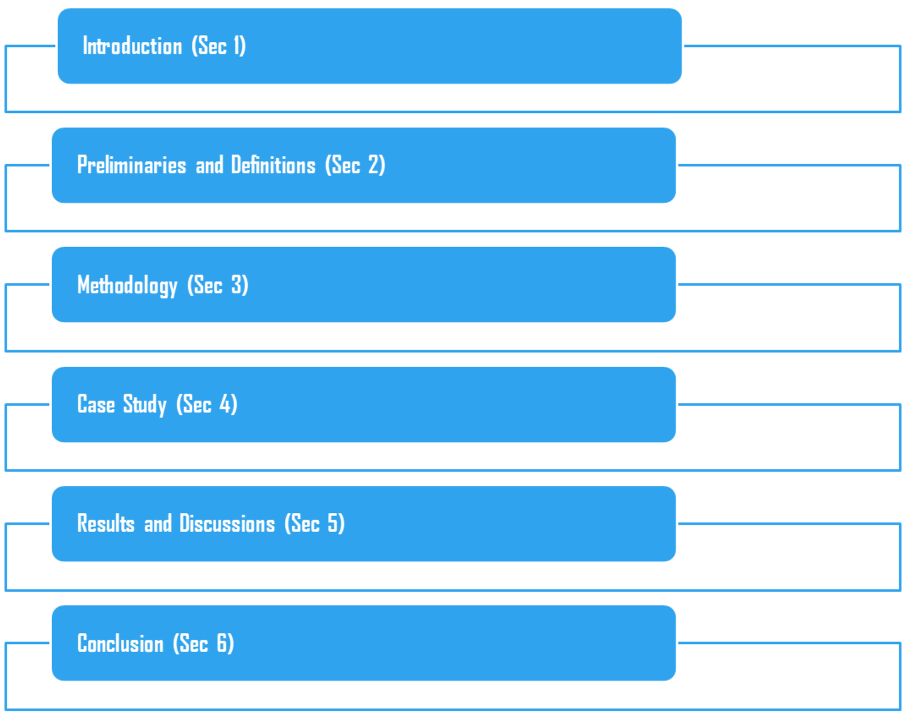

Figure 1. Paper Outline

As an extension of Neutrosophic theory, Bipolar NSs (BNSs) were proposed to include dual degrees for each component of NSs, one in the positive direction, and another in the negative direction [5]. This bipolar nature enables the modeling of information originating from circumstances with bipolar interaction, which provides an expressive representation of multifaceted phenomena [6], [7], [8]. When it comes to assessing teaching superiority, student feedback capacity concurrently replicates optimistic facets (e.g., appealing teaching method) and bad facets (e.g., unclear descriptions). BNS can effectually handle such dualistic problems and help develop a reliable decision-making approach for analyzing the indeterminacy in assessing contradictory feedback about English teaching [5], [9].

To the best of the author's knowledge, the study is the first to explore the potential of BNSs driving a new and fair approach to assessing teaching quality for the English language and literature. The main contributions can be summarized as follows:

- This study introduces a novel logarithmic weighting mechanism based on the AHP to determine the relative importance of criteria within a Bipolar Neutrosophic framework.
- A novel methodology is proposed for defining ideal solutions and computing distances to them within a Bipolar Neutrosophic matrix, which allows for handling uncertainty, contradiction, and hesitation, providing a more precise and comprehensive evaluation framework compared to existing methods.
- We propose a numerical application to prove and analyze the validity of the proposed approach for providing useful insights about the English language and literature being taught to students, and the results demonstrated demonstrate significant advantages over conventional evaluation methods.

The outcomes of the current research project will have significant implications for the administration and management of the teaching process in classes on the English language and related literature. In Figure 1, we present the outline of the remainder of this study to improve the readability of the paper.

2. Preliminaries & Definition

Definition 1. BNSs [5] define dual directed degree for each of the three components of the original neutrosophic theory.

$$\overleftrightarrow{B} = \{e, \langle Tr^{+}(e), II^{+}(e), F^{+}(c), Tr^{-}(e), II^{-}(c), F^{-}(e)\rangle | e \in E\}, \tag{1}$$

Whereas

$$Tr^{+}(e), II^{+}(e), F^{+}(e): E \rightarrow [0,1]$$

$$Tr^{-}(e), II^{-}(e), F^{-}(e): E \rightarrow [-1,0]$$

Definition 2. Assume two BNSs $\mathcal{S}_1 = \{e, \langle {Tr_1}^{+}(e), {II_1}^{+}(e), {F_1}^{+}(e), {Tr_1}^{-}(e), II^{-}_{1}(e), F^{-}_{1}(e)\rangle | e \in E\}$, and

$\mathcal{S}_2 = \{e, \langle Tr^{+}(e), II^{+}(e), F^{+}(e), Tr^{-}(e), II^{-}(e), F^{-}(e)\rangle | \, e \in E\}$, The following are the main operations for BNSs:

1) The complement of $\mathcal{S}_1$ is computed as follows:

$$\mathcal{A}_1^c = \begin{Bmatrix} (\{1^{+}\} - Tr^{+}(e)), (\{1^{+}\} - II^{+}(e)), (\{1^{+}\} - F^{+}(e)), \\ (\{1^{-}\} - Tr^{-}(e)), (\{1^{-}\} - II^{-}(e)), (\{1^{-}\} - F^{-}(e)) \end{Bmatrix} \tag{2}$$

2) $\mathcal{S}_1 \oplus \mathcal{S}_2$ if, and only if,

$$\mathcal{S}_1 \oplus \mathcal{S}_2 = \begin{Bmatrix} (\{1^+\} - Tr^+(e)), (\{1^+\} - II^+(e)), (\{1^+\} - F^+(e)), \\ (\{1^-\} - Tr^-(e)), (\{1^-\} - II^-(e)), (\{1^-\} - F^-(e)) \end{Bmatrix} \tag{3}$$

3) $\mathcal{S}_1 \otimes \mathcal{S}_2$ if, and only if,

$$\mathcal{S}_1 \otimes \mathcal{S}_2 = \begin{Bmatrix} (\{1^+\} - Tr^+(e)), (\{1^+\} - II^+(e)), (\{1^+\} - F^+(e)), \\ (\{1^-\} - Tr^-(e)), (\{1^-\} - II^-(e)), (\{1^-\} - F^-(e)) \end{Bmatrix} \tag{4}$$

4) $\mathcal{S}_1 \subseteq \mathcal{S}_2$ if, and only if,

$$Tr^+(e) < Tr^+(e), F^+(e) < F^+(e), II^+(e) \geq II^+(e) \tag{5}$$

and

$$Tr^+(e) < Tr^+(e), F^+(e) < F^+(e), II^+(e) \geq II^+(e) \tag{6}$$

5) $\mathcal{S}_1 = \mathcal{S}_2$ if, and only if,

$$Tr^+(e) = Tr^+(e), F^+(e) = F^+(e), II^+(e) = II^+(e) > \tag{7}$$

and

$$Tr^+(e) = Tr^+(e), F^+(e) = F^+(e), II^+(e) = II^+(c) > \tag{8}$$

6) The intersection of both $\mathcal{S}_1$ and $\mathcal{S}_2$ is defined as:

$$\mathcal{S}_1 \cap \mathcal{S}_2 = \begin{Bmatrix} e, \min(Tr^+(e), Tr^+(e)), \frac{II^+(e) + II^+(e)}{2}, \max(F^+(e), F^+(e)), \max(Tr^-(e), Tr^-(e)), \\ \frac{II^-(e) + II^-(e)}{2}, \min(F^-(e), F^-(e)) \mid e \in E \end{Bmatrix} \tag{9}$$

7) The union of both $\mathcal{S}_1$ and $\mathcal{S}_2$ is defined as:

$$\mathcal{S}_1 \cup \mathcal{S}_2 = \begin{Bmatrix} c, \max(Tr^+(e), Tr^+(e)), \frac{II^+(e) + II^+(e)}{2}, \min(F^+(e), F^+(e)), \min(Tr^-(c), Tr^-(e)), \\ \frac{II_1^-(e) + II^-(e)}{2}, \max(F^-(e), F^-(e)) \mid e \in E \end{Bmatrix} \tag{10}$$

Definition 3. Given a BNS $\mathcal{S}_1 = \{e, \langle Tr_1{}^+(e), II_1{}^+(e), F_1{}^+(e), Tr_1{}^-(e), II_1^-(e), F_1^-(e)\rangle \mid e \in E\}$,the de-neutrosophication of BNS refers to the act of transforming the BNS into a crisp number. This was originally defined using the following equation [10], [11]:

(11)

$$S(\mathcal{S}_1)=\frac{\left(Tr_1^{+}(e)+1-II_1^{+}(e)+1-F_1^{+}(e)+1+Tr_1^{-}(e)-II_1^{-}(e)-F_1^{-}(e)\right)}{6}.$$

However, this definition can be flowed in many ways. First, it ignores possible synergies or conflicts, such as, how ${Tr_1}^{+}$ interacts with ${II_1}^{+}$, or how $Tr_1^{-}(e)$ affects $F_1^{+}(e)$. Second, by linear aggregation, the summed components deliver a comprehensive representation, which might not reflect the relative changes.

3. Material and Methods

The evaluation of teaching quality in English language and literature focuses on assessing the effectiveness of instructional methods, curriculum design, and student outcomes. Key indicators include the clarity of teaching objectives, the engagement and participation of students, and the development of core skills such as reading, writing, speaking, and critical thinking. A well-structured curriculum should balance language proficiency with literary analysis, encouraging students to appreciate cultural and historical contexts while improving their communication skills. Effective teaching methods utilize interactive activities, discussions, and creative assignments to foster deeper understanding and active learning. Assessment strategies, such as tests, essays, presentations, and class participation, help measure student progress and comprehension. Additionally, teaching quality is influenced by teacher expertise, the use of technology, and the availability of resources like books and multimedia tools. Feedback from students and peer evaluations also play a crucial role in identifying areas for improvement. By maintaining high teaching standards, educators can inspire students to excel in both language mastery and literary appreciation, fostering lifelong skills and intellectual growth. The evaluation of the quality of teaching the English Language and related literature is no longer an easy task, but it is a complex process requiring consideration of multiple pedagogical, technological, and student-centered aspects. According to a recent research study on English teaching [12], we found six main criteria governing teaching quality of English and related literature, namely student feedback (C1), fulfilling workspace expectation (C2), inefficient resource usage (C3), personal traits (C4), classroom engagements(C5), instructional rigidity (C6), English and content knowledge (C7), and complaint rate (C8). These criteria replicate the main dimensions of teaching quality and form the basis for evaluating a set of alternative teaching techniques, such as direct method (A1), task-based learning (A2), communicative language teaching (A3), grammar-translation method (A4), audio-Lingual Method (A5), silent way (A6), content and language integrated learning (A7), and total physical response (A8) [13]. These methods are selected for their exceptional approaches to language education, ranging from traditional to modern, technology-enhanced methods. Each technique offers distinct advantages and challenges, making them perfect applicants for comparative analysis within our BNSs-based decision-making approach. Three English language experts are involved into the process of determining the representative values linguistic variable as defined in Table 1. These experts evaluated the linguistic variables of different criteria for different teaching alternatives across different study semesters, guaranteeing an inclusive and representative analysis of teaching quality. The process of aggregating the expert opinions and semester-wise evaluations enabled achieving balanced and nuanced interpretation, taking into account both the positive and negative facet for each criterion.

Table 1. BNS-based definition of linguistic variables for criteria in assessing teaching quality.

Criteria	Linguistic Variable	BNS Representation	Criteria	Linguistic Variable	BNS Representation
C1	Excellent	⟨0.9, 0.07, 0.06, -0.06, -0.06, -0.91⟩	C5	Highly Engaging	⟨0.92, 0.07, 0.07, -0.09, -0.08, -0.91⟩
	Good	⟨0.80, 0.14, 0.13, -0.10, -0.11, -0.81⟩		Engaging	⟨0.84, 0.12, 0.12, -0.13, -0.11, -0.82⟩
	Fair	⟨0.60, 0.22, 0.24, -0.21, -0.23, -0.65⟩		Neutral	⟨0.51, 0.33, 0.21, -0.24, -0.31, -0.52⟩
	Poor	⟨0.32, 0.23, 0.54, -0.53, -0.25, -0.30⟩		Disengaging	⟨0.35, 0.23, 0.53, -0.50, -0.25, -0.31⟩
	Very Poor	⟨0.11, 0.11, 0.85, -0.81, -0.11, -0.11⟩		Highly Disengaging	⟨0.13, 0.15, 0.83, -0.81, -0.12, -0.15⟩
C2	Highly Relevant	⟨0.85, 0.13, 0.08, -0.05, -0.14, -0.90⟩	C6	Flexible	⟨0.89, 0.11, 0.08, -0.06, -0.11, -0.89⟩
	Relevant	⟨0.78, 0.15, 0.14, -0.12, -0.19, -0.77⟩		Moderately Flexible	⟨0.74, 0.19, 0.15, -0.16, -0.18, -0.71⟩
	Moderately Relevant	⟨0.61, 0.22, 0.25, -0.22, -0.24, -0.64⟩		Rigid	⟨0.32, 0.24, 0.53, -0.51, -0.21, -0.34⟩
	Irrelevant	⟨0.33, 0.25, 0.52, -0.51, -0.25, -0.34⟩		Highly Rigid	⟨0.11, 0.11, 0.82, -0.82, -0.10, -0.10⟩
	Highly Irrelevant	⟨0.10, 0.11, 0.83, -0.81, -0.11, -0.12⟩			
C3	Highly Efficient	⟨0.94, 0.07, 0.05, -0.06, -0.08, -0.94⟩	C7	Advanced	{0.90, 0.05, 0.05, -0.05, -0.05, -0.90}
	Efficient	⟨0.84, 0.11, 0.13, -0.13, -0.11, -0.84⟩		Proficient	⟨0.93, 0.05, 0.09, -0.06, -0.09, -0.92⟩
	Moderately Efficient	⟨0.64, 0.23, 0.22, -0.22, -0.20, -0.65⟩		Competent	⟨0.84, 0.11, 0.15, -0.14, -0.13, -0.82⟩
	Inefficient	⟨0.31, 0.23, 0.53, -0.53, -0.21, -0.31⟩		Inadequate	⟨0.63, 0.22, 0.23, -0.25, -0.22, -0.62⟩
	Highly Inefficient	⟨0.14, 0.15, 0.82, -0.82, -0.15, -0.14⟩		Very Poor	⟨0.34, 0.20, 0.55, -0.52, -0.22, -0.35⟩
C4	Exemplary	⟨0.98, 0.07, 0.03, -0.06, -0.07, -0.96⟩	C8	Very Low	⟨0.92, 0.08, 0.08, -0.09, -0.06, -0.94⟩
	Very Positive	⟨0.82, 0.15, 0.11, -0.11, -0.15, -0.82⟩		Low	⟨0.82, 0.14, 0.11, -0.14, -0.11, -0.83⟩
	Neutral	⟨0.50, 0.31, 0.21, -0.25, -0.34, -0.52⟩		Neutral	⟨0.54, 0.31, 0.22, -0.24, -0.32, -0.54⟩
	Negative	⟨0.31, 0.22, 0.54, -0.50, -0.24, -0.34⟩		High	⟨0.35, 0.24, 0.55, -0.51, -0.20, -0.32⟩
	Highly Negative	⟨0.11, 0.10, 0.80, -0.81, -0.15, -0.14⟩		Very Highly	⟨0.15, 0.11, 0.81, -0.83, -0.11, -0.10⟩

In the following, we provide our proposed approach to BNSs for making appropriate decisions about the quality of teaching of English language and related literature.

Step 1: we define BNSs linguistic variables. In other words, Firstly, we formulate the linguistic variables and determine the corresponding BNSs that better characterize their degree of interaction among different aspects, as shown in Figure 2. Then, the definition of linguistic variables follows the definitions in Section 2, and the corresponding memberships should satisfy the requirements in Definition 1.

Step 2**:** Choose the teaching scenarios and aggregate the bipolar valuations assessed by English experts. In particular, we specify the alternative and ask experts to evaluate different teaching criteria based on the abovementioned linguistic variables. Assume that we have K experts, $\mathrm{E_e} = \{\mathrm{E_1}, \mathrm{E_2}, \dots, \mathrm{E_k}\}(\mathrm{k} = 1,2,\dots,\mathrm{K})$, assessing a number, $\mathrm{N_c}$, of teaching criteria, such that $\Omega_\mathrm{i} = \{\Omega_1, \Omega_2, \dots, \Omega_{\mathrm{N_c}}$. The expert evaluations can be expressed as follows:

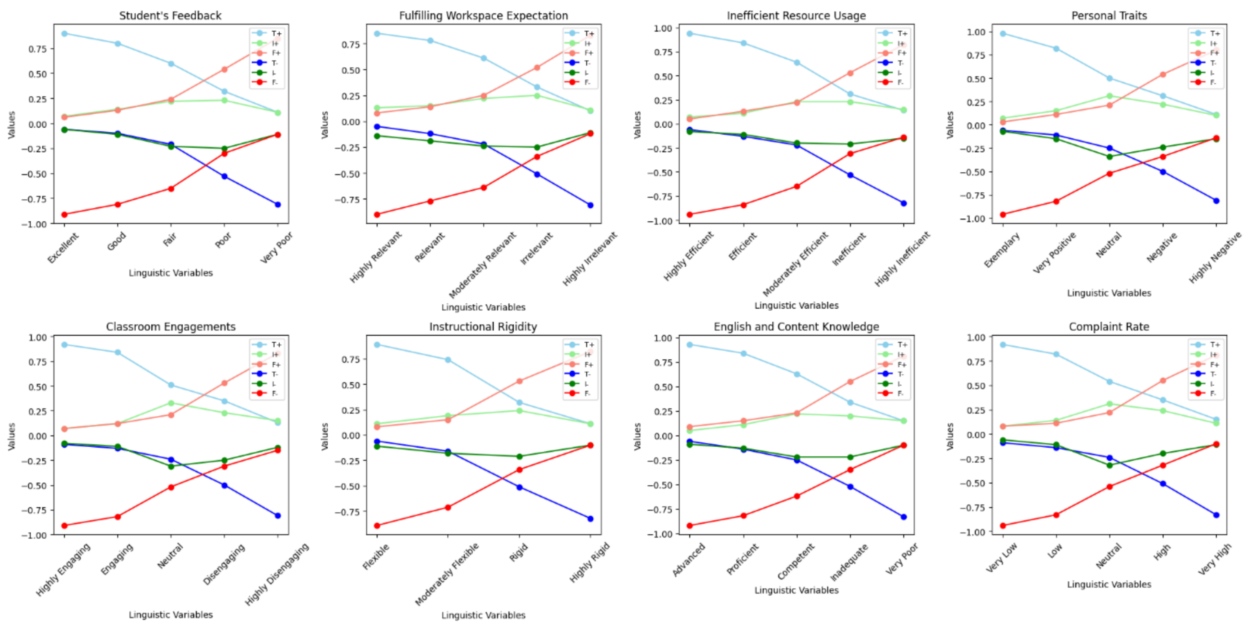

Figure 2. visualization of BNS representation of linguistic variables.

$$D^k_{N_A \times N_c} = d^k_{ij} = \begin{bmatrix} d^k_{11} & \cdots & d^k_{1N_c} \\ \vdots & \ddots & \vdots \\ d^k_{N_A 1} & \cdots & d^k_{N_A N_c} \end{bmatrix}, k \in K, \tag{12}$$

where $\mathrm{D^k_{N_A \times N_c}}$ denotes the BNS decision matrix of the $\mathrm{K-th}$ experts. $d^k_{ij} = \langle (Tr^+)^k_{ij}, (II^+)^k_{ij}, (F^+)^k_{ij}, (Tr^-)^k_{ij}, (II^-)^k_{ij}, (F^-)^k_{ij} \rangle, k = 1, \cdots, K; i = 1, \cdots, N_A; j = 1, \cdots, N_c$.

Step 3, we aggregate evaluations from different experts, then, the aggregated decision matrix can be expressed as follows:

$$D_{N_A \times N_c} = d_{ij} = \begin{bmatrix} d_{11} & \cdots & d_{1N_c} \\ \vdots & \ddots & \vdots \\ d_{N_A 1} & \cdots & d_{N_A N_c} \end{bmatrix}, \tag{13}$$

where $d_{ij} = \langle (Tr^+)_{ij}, (II^+)_{ij}, (F^+)_{ij}, (Tr^-)_{ij}, (II^-)_{ij}, (F^-)_{ij} \rangle, i = 1, \cdots, N_A; j = 1, \cdots, N_c$. The language expert evaluations are collected based on majority voting between linguistic variables.

In step 4, we transform the aggregated BNSs decision matrix into the crisp valued matrix by applying the de-neutrosophication operation as previously described in Definition 4. However, this definition can be flowed in many ways. First, it ignores possible synergies or conflicts, such as, how ${Tr_1}^+$ interacts with ${II_1}^+$, or how $Tr_1^-(e)$ affects $F_1^+(e)$. Second, by linear aggregation, the summed components deliver a comprehensive representation, which might not reflect the relative changes. To avoid these limitations, we propose the de-neutrosophication method based on logarithms to emphasize diminishing returns and highlight the relative changes instead of absolute magnitudes. This can be expressed with the following de-neutrosophication equation:

$$S(d_{ij}) = \Big(\ln\big(1 + Tr^+(d_{ij})\big) + \ln\Big(1 + \big(1 - II^+(d_{ij})\big)\Big) + \ln\Big(1 + \big(1 - F^+(d_{ij})\big)\Big) - \ln\big(1 + Tr^-(d_{ij})\big) - \ln\big(1 + II^-(d_{ij})\big) - \ln\big(1 + F^-(d_{ij})\big)\Big)/6 \tag{14}$$

By applying this formula, we obtain a representative decision matrix consisting of crisp values.

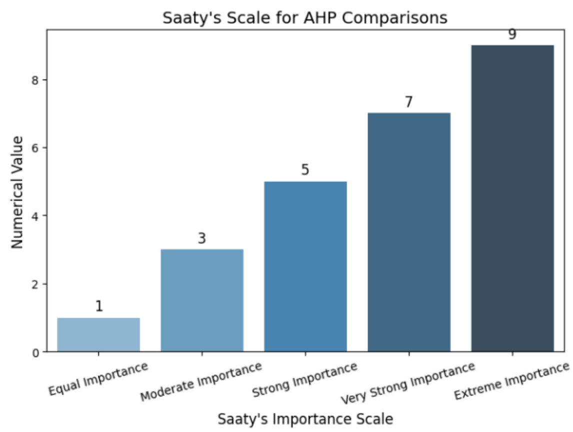

Figure 2. visualization of Saaty's scale

Let B_{ij} be a crisp number matrix, then the matrix is denoted as:

$$B_{N_A \times N_c} = b_{ij} = \begin{bmatrix} b_{11} & \cdots & b_{1N_c} \\ \vdots & \ddots & \vdots \\ b_{N_A 1} & \cdots & b_{N_A N_c} \end{bmatrix}, \tag{15}$$

Step 5: we apply the AHP to calculate the relative importance of each criterion.

Step 5.1, we create a pairwise comparison matrix of size. $N_c \times N_c$, where N_c is the number of criteria, and $a_{i,j}$ Symbolize the relative importance of $i - th$ criterion and $j - th$ criterion. If $i = j$ then $a_{i,j} = 1$. In building our pairwise comparison, we used Saaty's scale [14], [15] displayed in Figure 2.

Reciprocal values ($a_{i,j} = 1$) for inverse comparisons.

Step 5.2: we normalize the pairwise comparison matrix, and calculate the sum for each column in $B_{N_A \times N_c}$:

$$P_{N_c \times N_c} = p_{ij} = \begin{bmatrix} b_{11} & \cdots & b_{1N_c} \\ \vdots & \ddots & \vdots \\ b_{N_c 1} & \cdots & b_{N_c N_c} \end{bmatrix}, \tag{16}$$

$$\mathrm{Sum}_j = \sum_{i=1}^{N_c} p_{ij}, \ \forall j \in \{1, \dots, N_c\} \tag{17}$$

Then, we divide each element, p_{ij}, by the corresponding column sum to get the normalized matrix P_{norm} :

$$p_{\text{norm},ij} = \frac{p_{ij}}{\mathrm{Sum}_j}, \ \forall i, j \tag{18}$$

In step 5.3, we calculate the relative Weights based on the priority vector $w = [w_1, w_2, \dots, w_{N_c}]^T$ representing the average of the rows of P_{norm} :

$$w_i = \frac{\sum_{j=1}^{N_c} p_{\text{norm},ij}}{N_c}, \ \forall i \in \{1, \cdots, N_c\} \tag{19}$$

Step 5.4, we can check consistency by computing the weighted sum vector $\lambda = P \cdot w$, then find the largest eigenvalue. $\lambda_{\max}$:

$$\lambda_{\max} = \frac{\sum_{i=1}^{N_c} \left(\frac{\lambda_i}{w_i}\right)}{N_c} \tag{20}$$

Next, we calculate the Consistency Index (Cl), and Consistency Ratio (CR) as follows:

$$\mathrm{CI} = \frac{\lambda_{\max} - N_c}{N_c - 1} \tag{21}$$

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \tag{22}$$

Where RI (Random Index) depends on N_c .

Based on the computed weights, we can drive a weighted decision matrix as follows:

$$\begin{aligned} b_{ij}^{w_j} &= \left\{ Tr_{ij}^{w_j+}, II_{ij}^{w_j+}, F_{ij}^{w_j+}, Tr_{ij}^{w_j-}, II_{ij}^{w_j-}, F_{ij}^{w_j-} \right\} \\ &= \left\{ 1 - \left(1 - Tr_{ij}^{+}\right)^{w_j}, \left(II_{ij}^{+}\right)^{w_j}, \left(F_{ij}^{+}\right)^{w_j}, -\left(-Tr_{ij}^{-}\right)^{w_j}, -\left(-II_{ij}^{-}\right)^{w_j}, -\left(\left(1 - \left(-F_{ij}^{-}\right)\right)^{w_j}\right) \right\}, \end{aligned} \tag{23}$$

Step 6, we compute BNS relative positive ideal solution ($\mathfrak{P}$) as formulated below:

$$\mathfrak{P} = \left\{\begin{matrix}\langle {}^{+}Tr_1^{w_1+}, {}^{+}II_1^{w_1+}, {}^{+}F_1^{w_1+}, {}^{+}Tr_1^{w_1-}, {}^{+}II_1^{w_1-}, {}^{+}F_1^{w_1-}\rangle, \langle {}^{+}Tr_2^{w_2+}, {}^{+}II_2^{w_2+}, {}^{+}F_2^{w_2+}, {}^{+}Tr_2^{w_2-}, II_2^{w_2-}, {}^{+}F_2^{w_2-}\rangle, \\ \dots, \langle {}^{+}Tr_{N_c}^{w_n+}, {}^{+}II_{N_c}^{w_n+}, {}^{+}F_{N_c}^{w_n+}, {}^{+}Tr_{N_c}^{w_n-}, {}^{+}II_{N_c}^{w_n-}, {}^{+}F_{N_c}^{w_n-}\rangle\end{matrix}\right\} \tag{24}$$

and compute BNS relative negative ideal solution ($\mathfrak{N}$) as formulated below:

$$\mathfrak{N} = \left\{\begin{matrix}\langle {}^{-}Tr_1^{w_1+}, {}^{-}II_1^{w_1+}, {}^{-}F_1^{w_1+}, {}^{-}Tr_1^{w_1-}, {}^{-}II_1^{w_1-}, {}^{-}F_1^{w_1-}\rangle, \langle {}^{-}Tr_2^{w_2+}, {}^{-}II_2^{w_2+}, {}^{-}F_2^{w_2+}, {}^{-}Tr_2^{w_2-}, {}^{-}II_2^{w_2-}, {}^{-}F_2^{w_2-}\rangle, \\ \dots, \langle {}^{-}Tr_n^{w_n+}, {}^{-}II_n^{w_n+}, {}^{-}F_n^{w_n+}, {}^{-}Tr_n^{w_n-}, {}^{-}II_n^{w_n-}, {}^{-}F_n^{w_n-}\rangle\end{matrix}\right\} \tag{25}$$

For benefit criteria, $j = 1,2, \dots, N_c$,

$$\begin{aligned}\langle {}^{+}Tr_j^{w_j+}, {}^{+}II_j^{w_j+}, {}^{+}F_j^{w_j+}, {}^{+}Tr_j^{w_j-}, {}^{+}II_j^{w_j-}, {}^{+}F_j^{w_j-}\rangle &= \langle \max\left(Tr_{ij}^{w_j+}\right), \min\left(II_{ij}^{w_j+}\right), \min\left(F_{ij}^{w_j+}\right), \\ &\quad \min\left(Tr_{ij}^{w_j-}\right), \max\left(II_{ij}^{w_j-}\right), \max\left(F_{ij}^{w_j-}\right)\rangle, \\ \langle {}^{-}Tr_j^{w_j+}, {}^{-}II_j^{w_j+}, {}^{-}F_j^{w_j+}, {}^{-}Tr_j^{w_j-}, {}^{-}II_j^{w_j-}, {}^{-}F_j^{w_j-}\rangle &= \langle \min\left(Tr_{ij}^{w_j+}\right), \max\left(II_{ij}^{w_j+}\right), \max\left(F_{ij}^{w_j+}\right), \\ &\quad \max\left(Tr_{ij}^{w_j-}\right), \min\left(II_{ij}^{w_j-}\right), \min\left(F_{ij}^{w_j-}\right)\rangle.\end{aligned} \tag{26}$$

While cost criteria, $j = 1,2, \dots, N_c$

$$\begin{aligned}\langle {}^{+}Tr_j^{w_j+}, {}^{+}II_j^{w_j+}, {}^{+}F_j^{w_j+}, {}^{+}Tr_j^{w_j-}, {}^{+}II_j^{w_j-}, {}^{+}F_j^{w_j-}\rangle &= \langle \min\left(Tr_{ij}^{w_j+}\right), \max\left(II_{ij}^{w_j+}\right), \max\left(F_{ij}^{w_j+}\right), \\ &\quad \max\left(Tr_{ij}^{w_j-}\right), \min\left(II_{ij}^{w_j-}\right), \min\left(F_{ij}^{w_j-}\right)\rangle, \\ \langle {}^{-}Tr_j^{w_j+}, {}^{-}II_j^{w_j+}, {}^{-}F_j^{w_j+}, {}^{-}Tr_j^{w_j-}, {}^{-}II_j^{w_j-}, {}^{-}F_j^{w_j-}\rangle &= \langle \max\left(Tr_{ij}^{w_j+}\right), \min\left(II_{ij}^{w_j+}\right), \min\left(F_{ij}^{w_j+}\right), \\ &\quad \min\left(Tr_{ij}^{w_j-}\right), \max\left(II_{ij}^{w_j-}\right), \max\left(F_{ij}^{w_j-}\right)\rangle.\end{aligned} \tag{27}$$

Step 7, we compute the distance between each alternative w.r.t $\mathfrak{P}$,

$$\begin{aligned}&D_{mins}\,(\mathcal{S}_i, \mathfrak{P}) \\ &= \left(\frac{1}{6N_c}\sum_{j=1}^{N_c}\left\{\begin{matrix}\left|Tr_{ij}^{w_j+} - {}^{+}Tr_j^{w_j+}\right|^p + \left|II_{ij}^{w_j+} - {}^{+}II_j^{w_j+}\right|^p + \left|F_{ij}^{w_j+} - {}^{+}F_j^{w_j+}\right|^p + \\ \left|Tr_{ij}^{w_j-} - {}^{+}Tr_j^{w_j-}\right|^p + \left|II_{ij}^{w_j-} - {}^{+}II_j^{w_j-}\right|^p + \left|F_{ij}^{w_j-} - {}^{+}F_j^{w_j-}\right|^p\end{matrix}\right\}\right)^{\frac{1}{p}},\end{aligned} \tag{28}$$

and we compute the distance between each alternative w.r.t $\mathfrak{N}$,

$$\begin{aligned}&D_{mins}(\mathcal{S}_i, \mathfrak{N}) \\ &= \left(\frac{1}{6N_c}\sum_{j=1}^{N_c}\left\{\begin{matrix}\left|Tr_{ij}^{w_j+} - {}^{-}Tr_j^{w_j+}\right|^p + \left|II_{ij}^{w_j+} - {}^{-}II_j^{w_j+}\right|^p + \left|F_{ij}^{w_j+} - {}^{-}F_j^{w_j+}\right|^p + \\ \left|Tr_{ij}^{w_j-} - {}^{-}Tr_j^{w_j-}\right|^p + \left|II_{ij}^{w_j-} - {}^{-}II_j^{w_j-}\right|^p + \left|F_{ij}^{w_j-} - {}^{-}F_j^{w_j-}\right|^p\end{matrix}\right\}\right).\end{aligned} \tag{29}$$

Step 8, for each alternative, we calculate the level of nearness, μ_i, w.r.t $\mathfrak{P}$ as formulated below:

(30)

$$\mu_i(S_i) = \frac{D_{mins}(S_i, \mathfrak{N})}{max\{D_{mins}(S_i, \mathfrak{N})\} + D_{mins}(S_i, \mathfrak{P})}, i = 1,2, \dots, N_A.$$

Step 9, for each alternative, we use the above degrees to calculate the inferior ratio, $\vartheta(i)$ as below:

$$\vartheta(i) = \frac{\mu_i(S_i)}{\underset{1 \le i \le m}{min}\left(\mu_i(S_i)\right)}. \quad (31)$$

where the value of $\vartheta(i)$ belong to interval [0,1]. In Last, the alternatives are ranked according to the order of values of $\vartheta(i)$, in which the finest alternative is nominated according to the lowest value of $\vartheta(i)$.

4. Results and Analysis

Result-set 1: according to the definition of linguistic variables in Table 1, the BNS-based decision matrix is driven as shown in Table 2.

Table 2. BNS-based decision matrix.

Alternative \ Criteria	C1	C2	C3	C4
S_1	⟨ 0.9 , 0.07 , 0.06 , -0.06 , -0.06 , -0.91 ⟩	⟨ 0.61 , 0.22 , 0.25 , -0.22 , -0.24 , -0.64 ⟩	⟨ 0.14 , 0.15 , 0.82 , -0.82 , -0.15 , -0.14 ⟩	⟨ 0.98 , 0.07 , 0.03 , -0.06 , -0.07 , -0.96 ⟩
S_2	⟨ 0.6 , 0.22 , 0.24 , -0.21 , -0.23 , -0.65 ⟩	⟨ 0.85 , 0.13 , 0.08 , -0.05 , -0.14 , -0.9 ⟩	⟨ 0.31 , 0.23 , 0.53 , -0.53 , -0.21 , -0.31 ⟩	⟨ 0.82 , 0.15 , 0.11 , -0.11 , -0.15 , -0.82 ⟩
S_3	⟨ 0.8 , 0.14 , 0.13 , -0.1 , -0.11 , -0.81 ⟩	⟨ 0.33 , 0.25 , 0.52 , -0.51 , -0.25 , -0.34 ⟩	⟨ 0.84 , 0.11 , 0.13 , -0.13 , -0.11 , -0.84 ⟩	⟨ 0.5 , 0.31 , 0.21 , -0.25 , -0.34 , -0.52 ⟩
S_4	⟨ 0.9 , 0.07 , 0.06 , -0.06 , -0.06 , -0.91 ⟩	⟨ 0.78 , 0.15 , 0.14 , -0.12 , -0.19 , -0.77 ⟩	⟨ 0.14 , 0.15 , 0.82 , -0.82 , -0.15 , -0.14 ⟩	⟨ 0.31 , 0.22 , 0.54 , -0.5 , -0.24 , -0.34 ⟩
S_5	⟨ 0.11 , 0.11 , 0.85 , -0.81 , -0.11 , -0.11 ⟩	⟨ 0.1 , 0.11 , 0.83 , -0.81 , -0.11 , -0.12 ⟩	⟨ 0.94 , 0.07 , 0.05 , -0.06 , -0.08 , -0.94 ⟩	⟨ 0.11 , 0.1 , 0.8 , -0.81 , -0.15 , -0.14 ⟩
S_6	⟨ 0.32 , 0.23 , 0.54 , -0.53 , -0.25 , -0.3 ⟩	⟨ 0.78 , 0.15 , 0.14 , -0.12 , -0.19 , -0.77 ⟩	⟨ 0.31 , 0.23 , 0.53 , -0.53 , -0.21 , -0.31 ⟩	⟨ 0.82 , 0.15 , 0.11 , -0.11 , -0.15 , -0.82 ⟩
S_7	⟨ 0.6 , 0.22 , 0.24 , -0.21 , -0.23 , -0.65 ⟩	⟨ 0.1 , 0.11 , 0.83 , -0.81 , -0.11 , -0.12 ⟩	⟨ 0.84 , 0.11 , 0.13 , -0.13 , -0.11 , -0.84 ⟩	⟨ 0.31 , 0.22 , 0.54 , -0.5 , -0.24 , -0.34 ⟩
S_8	⟨ 0.8 , 0.14 , 0.13 , -0.1 , -0.11 , -0.81 ⟩	⟨ 0.33, 0.25 , 0.52 , -0.51 , -0.25 , -0.34 ⟩	⟨ 0.64 , 0.23 , 0.22 , -0.22 , -0.2 , -0.65 ⟩	⟨ 0.98 , 0.07 , 0.03 , -0.06 , -0.07 , -0.96 ⟩

Table 2. BNS based decision matrix (cont..).

Alternative \ Criteria	C5	C6	C7	C8
S_1	⟨0.92, 0.07 , 0.07 , -0.09 , -0.08 , -0.91 ⟩	⟨0.11 , 0.11 , 0.82 , -0.82 , -0.1 , -0.1 ⟩	⟨0.93 , 0.05 , 0.09 , -0.06 , -0.09 , -0.92 ⟩	⟨0.92, 0.08 , 0.08 , -0.09 , -0.06 , -0.94⟩
S_2	⟨0.84, 0.12 , 0.12 , -0.13 , -0.11 , -0.82 ⟩	⟨0.74 , 0.19 , 0.15 , -0.16 , -0.18 , -0.71 ⟩	⟨0.84 , 0.11 , 0.15 , -0.14 , -0.13 , -0.82 ⟩	⟨0.82, 0.14 , 0.11 , -0.14 , -0.11 , -0.83⟩
S_3	⟨0.51, 0.33 , 0.21 , -0.24 , -0.31 , -0.52 ⟩	⟨0.32 , 0.24 , 0.53 , -0.51 , -0.21 , -0.34 ⟩	⟨0.15 , 0.15 , 0.81 , -0.83 , -0.1 , -0.1 ⟩	⟨0.54, 0.31 , 0.22 , -0.24 , -0.32 , -0.54⟩
S_4	⟨0.35, 0.23 , 0.53 , -0.5 , -0.25 , -0.31 ⟩	⟨0.11 , 0.11 , 0.82 , -0.82 , -0.1 , -0.1 ⟩	⟨0.34 , 0.2 , 0.55 , -0.52 , -0.22 , -0.35 ⟩	⟨0.35, 0.24 , 0.55 , -0.51 , -0.2 , -0.32⟩
S_5	⟨0.13, 0.15 , 0.83 , -0.81 , -0.12 , -0.15 ⟩	⟨0.74 , 0.19 , 0.15 , -0.16 , -0.18 , -0.71 ⟩	⟨0.15 , 0.15 , 0.81 , -0.83 , -0.1 , -0.1 ⟩	⟨0.15, 0.11 , 0.81 , -0.83 , -0.11 , -0.1⟩
S_6	⟨0.35, 0.23 , 0.53 , -0.5 , -0.25 , -0.31 ⟩	⟨0.89, 0.11 , 0.08 , -0.06 , -0.11 , -0.89 ⟩	⟨0.93 , 0.05 , 0.09 , -0.06 , -0.09 , -0.92 ⟩	⟨0.54, 0.31 , 0.22 , -0.24 , -0.32 , -0.54⟩

S_7	⟨0.51, 0.33 , 0.21 , -0.24 , -0.31 , -0.52 ⟩	⟨0.32 , 0.24 , 0.53 , -0.51 , -0.21 , -0.34 ⟩	⟨0.84 , 0.11 , 0.15 , -0.14 , -0.13 , -0.82 ⟩	⟨0.82, 0.14 , 0.11 , -0.14 , -0.11 , -0.83⟩
S_8	⟨0.84, 0.12 , 0.12 , -0.13 , -0.11 , -0.82 ⟩	⟨0.74 , 0.19 , 0.15 , -0.16 , -0.18 , -0.71 ⟩	⟨0.63 , 0.22 , 0.23 , -0.25 , -0.22 , -0.62 ⟩	⟨0.35, 0.24 , 0.55 , -0.51 , -0.2 , -0.32⟩

Result-set 2, the pairwise comparison matrix is given in Table 3.

Table 3. Aggregated pairwise comparison matrix for different criteria

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
C1	1	3	5	4	2	6	3	7
C2	$\frac{1}{3}$	1	3	2	$\frac{1}{2}$	4	2	5
C3	$\frac{1}{5}$	$\frac{1}{3}$	1	$\frac{1}{2}$	$\frac{1}{3}$	3	$\frac{1}{2}$	4
C4	$\frac{1}{4}$	$\frac{1}{2}$	2	1	$\frac{1}{3}$	3	$\frac{1}{2}$	4
C5	$\frac{1}{2}$	2	3	3	1	5	2	6
C6	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	1	$\frac{1}{2}$	3
C7	$\frac{1}{3}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	2	1	4
C8	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	1

Result-set 3, the numerical results representing weights from the AHP method are given in Table 3.

Table 3. AHP weights for various English teaching criteria.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
Weights	0.315	0.15	0.067	0.087	0.208	0.042	0.107	0.25

Result-set 4, the decision matrix of weighted BNS is shown in Table 4.

Table 4. AHP-weighted decision matrix.

Criteria / Alternative	C1	C2	C3	C4
S_1	(0.5157, 0.4329, 0.4124, -0.4124, -0.4124, -0.5315)	(0.1317, 0.7969, 0.8123, -0.7969, -0.8074, -0.142)	(0.0101, 0.88, 0.9867, -0.9867, -0.88, -0.0101)	(0.2876, 0.7941, 0.7378, -0.7835, -0.7941, -0.2435)
S_2	(0.2506, 0.6208, 0.6381, -0.6118, -0.6296, -0.2815)	(0.2476, 0.7365, 0.6848, -0.6382, -0.7447, -0.292)	(0.0247, 0.9057, 0.9581, -0.9581, -0.9002, -0.0247)	(0.1381, 0.8483, 0.8258, -0.8258, -0.8483, -0.1381)
S_3	(0.3975, 0.5385, 0.5261, -0.4843, -0.4991, -0.4072)	(0.0583, 0.8123, 0.9066, -0.904, -0.8123, -0.0604)	(0.1162, 0.8618, 0.8715, -0.8715, -0.8618, -0.1162)	(0.0583, 0.9034, 0.8734, -0.8868, -0.9107, -0.0617)
S_4	(0.5157, 0.4329, 0.4124, -0.4124, -0.4124, -0.5315)	(0.2031, 0.7524, 0.7447, -0.7277, -0.7796, -0.1978)	(0.0101, 0.88, 0.9867, -0.9867, -0.88, -0.0101)	(0.0317, 0.877, 0.948, -0.9417, -0.8836, -0.0354)
S_5	(0.036, 0.4991, 0.9501, -0.9358, -0.4991, -0.036)	(0.0157, 0.7182, 0.9724, -0.9689, -0.7182, -0.019)	(0.1727, 0.8359, 0.8172, -0.8273, -0.8435, -0.1727)	(0.0101, 0.819, 0.9808, -0.9819, -0.8483, -0.013)
S_6	(0.1143, 0.6296, 0.8237, -0.8188, -0.6463, -0.1062)	(0.2031, 0.7524, 0.7447, -0.7277, -0.7796, -0.1978)	(0.0247, 0.9057, 0.9581, -0.9581, -0.9002, -0.0247)	(0.1381, 0.8483, 0.8258, -0.8258, -0.8483, -0.1381)
S_7	(0.2506, 0.6208, 0.6381, -0.6118, -0.6296, -0.2815)	(0.0157, 0.7182, 0.9724, -0.9689, -0.7182, -0.019)	(0.1162, 0.8618, 0.8715, -0.8715, -0.8618, -0.1162)	(0.0317, 0.877, 0.948, -0.9417, -0.8836, -0.0354)

S_8	(0.3975, 0.5385, 0.5261, -0.4843, -0.4991, -0.4072)	(0.0583, 0.8123, 0.9066, -0.904, -0.8123, -0.0604)	(0.0665, 0.9057, 0.903, -0.903, -0.8972, -0.0683)	(0.2876, 0.7941, 0.7378, -0.7835, -0.7941, -0.2435)

Table 4. AHP-weighted decision matrix (cont..).

Alternative \ Criteria	C5	C6	C7	C8
S_1	(0.408, 0.5758, 0.5758, -0.6066, -0.592, -0.3934)	(0.0048, 0.9124, 0.9918, -0.9918, -0.9088, -0.0044)	(0.2474, 0.726, 0.7731, -0.7403, -0.7731, -0.2366)	(0.0615, 0.9385, 0.9385, -0.9413, -0.9318, -0.0682)
S_2	(0.3164, 0.6439, 0.6439, -0.6547, -0.6324, -0.2995)	(0.0544, 0.9333, 0.9242, -0.9267, -0.9312, -0.0501)	(0.1779, 0.7898, 0.8165, -0.8105, -0.8041, -0.1675)	(0.0422, 0.9518, 0.9461, -0.9518, -0.9461, -0.0435)
S_3	(0.1376, 0.7944, 0.7233, -0.7436, -0.7842, -0.1413)	(0.0159, 0.9424, 0.974, -0.9724, -0.9372, -0.0171)	(0.0172, 0.8165, 0.9777, -0.9803, -0.7818, -0.0112)	(0.0193, 0.971, 0.9627, -0.9648, -0.9718, -0.0193)
S_4	(0.0855, 0.7371, 0.8765, -0.866, -0.7499, -0.0741)	(0.0048, 0.9124, 0.9918, -0.9918, -0.9088, -0.0044)	(0.0434, 0.842, 0.9381, -0.9325, -0.8506, -0.045)	(0.0108, 0.9648, 0.9851, -0.9832, -0.9604, -0.0096)
S_5	(0.0285, 0.6745, 0.9621, -0.9572, -0.6439, -0.0332)	(0.0544, 0.9333, 0.9242, -0.9267, -0.9312, -0.0501)	(0.0172, 0.8165, 0.9777, -0.9803, -0.7818, -0.0112)	(0.0041, 0.9461, 0.9947, -0.9953, -0.9461, -0.0026)
S_6	(0.0855, 0.7371, 0.8765, -0.866, -0.7499, -0.0741)	(0.0876, 0.9124, 0.9004, -0.8897, -0.9124, -0.0876)	(0.2474, 0.726, 0.7731, -0.7403, -0.7731, -0.2366)	(0.0193, 0.971, 0.9627, -0.9648, -0.9718, -0.0193)
S_7	(0.1376, 0.7944, 0.7233, -0.7436, -0.7842, -0.1413)	(0.0159, 0.9424, 0.974, -0.9724, -0.9372, -0.0171)	(0.1779, 0.7898, 0.8165, -0.8105, -0.8041, -0.1675)	(0.0422, 0.9518, 0.9461, -0.9518, -0.9461, -0.0435)
S_8	(0.3164, 0.6439, 0.6439, -0.6547, -0.6324, -0.2995)	(0.0544, 0.9333, 0.9242, -0.9267, -0.9312, -0.0501)	(0.1008, 0.8506, 0.8546, -0.8623, -0.8506, -0.0982)	(0.0108, 0.9648, 0.9851, -0.9832, -0.9604, -0.0096)

Result-set 4, the bipolar ideal solutions are shown in Table 5.

Table 5. The ideal solutions are based on a weighted BNS decision matrix.

$\mathfrak{P}$	$\mathfrak{N}$
(0.5158, 0.0464, 0.0429, -0.9358, -0.0334, -0.1709)	(0.0360, 0.6294, 0.9501, -0.0305, -0.6462, -0.9640)
(0.2477, 0.0360, 0.2064, -0.9689, -0.0334, -0.0363)	(0.0157, 0.8123, 0.9724, -0.1386, -0.8123, -0.9810)
(0.0101, 0.9062, 0.9868, -0.0392, -0.9007, -0.9899)	(0.4386, 0.0360, 0.0429, -0.9868, -0.0334, -0.1709)
(0.7084, 0.0226, 0.0095, -0.9818, -0.0215, -0.0966)	(0.0101, 0.9031, 0.9808, -0.0185, -0.9104, -0.9870)
(0.4386, 0.0395, 0.0395, -0.9571, -0.0334, -0.1410)	(0.0286, 0.7941, 0.9620, -0.0392, -0.7838, -0.9668)
(0.0239, 0.7432, 0.9596, -0.0479, -0.7228, -0.9783)	(0.3682, 0.0642, 0.0499, -0.9596, -0.0535, -0.0966)
(0.4386, 0.0360, 0.0499, -0.9620, -0.0392, -0.1641)	(0.0332, 0.7298, 0.9571, -0.0421, -0.7298, -0.9783)
(0.0332, 0.7838, 0.9571, -0.0421, -0.7890, -0.9783)	(0.4173, 0.0464, 0.0360, -0.9620, -0.0334, -0.0914)

Result-set 4, in Table 6, the quantitative distances w.r.t $\mathfrak{P}$ and $\mathfrak{N}$ are computed for each for each alternative.

Table 6. The distances of alternative w.r.t ideal solutions.

Alternatives	$D_{mins}\ (\mathcal{S}_i, \mathfrak{P})$	$D_{mins}\ (\mathcal{S}_i, \mathfrak{N})$
S_1	0.387762	0.378713
S_2	0.406064	0.360411
S_3	0.418849	0.347626
S_4	0.403471	0.363004
S_5	0.445369	0.321106
S_6	0.427612	0.338863
S_7	0.354894	0.41158
S_8	0.327378	0.439097

Result-set 5, the calculated inferior ratio, and the related ranking are displayed in Table 7.

Table 7. Ranking of different teaching criteria based on Inferior ratio.

	$\vartheta(i)$	Rank
s_1	0.494097	8
s_2	0.470219	7
s_3	0.453538	1
s_4	0.473602	4
s_5	0.418939	2
s_6	0.442106	3
s_7	0.536978	6
s_8	0.572879	5

Result-set 6, we conducted a comparative analysis against different MCDM approaches namely CoCoSo [16], CODAS [17], and TOPSIS [18], [19], then, we compared their ranking outcomes in Table 8.

Table 8. Results of comparative analysis.

	CoCoSo	Proposed	CODAS	TOPSIS
s_1	1	8	1	1
s_2	2	7	3	4
s_3	6	1	6	5
s_4	5	4	5	3
s_5	8	2	8	8
s_6	4	3	4	6
s_7	7	6	7	7
s_8	3	5	2	2

Result-set 7, we conduct sensitivity analysis by randomly varying the weight of criteria with 10% and 20% of increment and decrement, then, we display the results as shown in Figure 4.

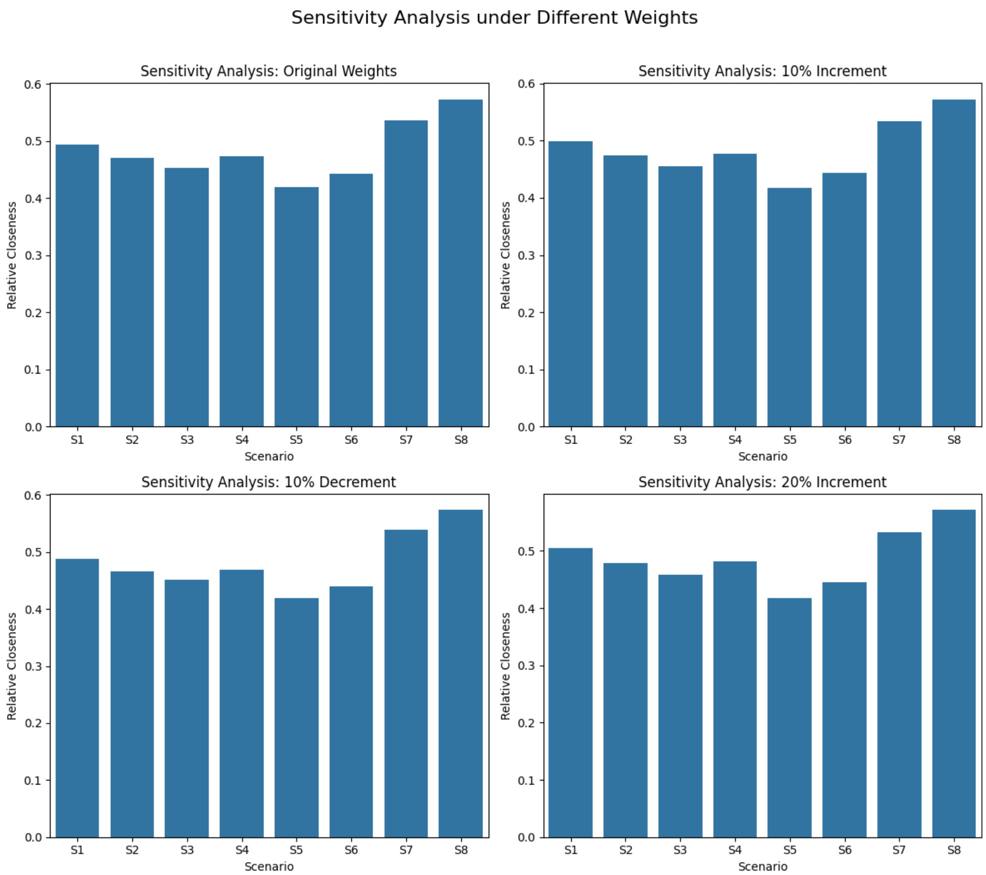

Figure 4. Quantitative results of sensitivity analysis.

5. Conclusions

In conclusion, this study introduces a Bipolar Neutrosophic-driven approach for assessing the teaching quality of English Language and Literature, incorporating a customized AHP-based weighting technique, a novel method for identifying ideal solutions and computing distances within a Bipolar Neutrosophic matrix, and a comprehensive comparative and sensitivity analysis. The proposed model effectively captures uncertainty, hesitation, and contradiction in decision-making, offering a more robust and reliable assessment framework. The results validate the superiority of our approach over traditional methods, highlighting its practical applicability and adaptability in educational evaluation systems.

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