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Inverse fractional function setting sine trigonometric neutrosophic set approach to interaction aggregating operators

Murugan Palanikumar[1], Nasreen kausar[2], Tonguc Cagin[3,*]

[1] Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai-602105, India; palanimaths86@gmail.com

[2] Department of Mathematics, Faculty of Arts and Science, Balikesir University, 10145 Balikesir, Turkey; kausar.nasreen57@gmail.com

[3] College of Business Administration, American University of the Middle East, Kuwait; tonguc.cagin@aum.edu.kw

*Correspondence: tonguc.cagin@aum.edu.kw

Abstract. In this paper we present novel techniques for the interacting aggregating operator of the inverse fractional function sine trigonometric neutrosophic set. Swapping the input and output variables and solving for the original input variable in terms of the original output variable are the steps involved in determining the inverse of a function. The innovative averaging and geometric operations of inverse fractional function sine trigonometric neutrosophic numbers are studied using the universal aggregation function. The inverse fractional function sine trigonometric neutrosophic set is idempotent, boundedness compatible, associative and commutative. Four new aggregating operators are introduced: inverse fractional function sine trigonometric neutrosophic weighted averaging, inverse fractional function sine trigonometric neutrosophic weighted geometric, generalized inverse fractional function sine trigonometric neutrosophic weighted averaging, and generalized inverse fractional function sine trigonometric neutrosophic weighted geometric. The aggregation functions are frequently thought to be represented by the Euclidean distance, Hamming distance and score values.

Keywords: weighted averaging, weighted geometric, generalized weighted averaging, generalized weighted geometric.

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1. Introduction

The uncertainties have led to the development of the fuzzy set (FS) [1], intuitionistic FS (IFS) [2], Pythagorean FS (PFS) [3,4], neutrosophic set (NSS) [5], and Fermatean FS (FFS) [6]. For decision-makers, Zadeh's FS [1] suggests a membership value (MV). An IFS notion was introduced by Atanassov [2] because each object has MV τ and non-membership value (NMV)

ψ and meets $0 \leq \tau + \psi \leq 1$, for $\tau, \psi \in [0,1]$. Yager introduced the condition that $\tau^2 + \psi^2 \leq 1$, which states that PFSs are defined by their MVs and NMVs [3]. Both PFSs and IFSs have been widely utilized and investigated in a wide range of fields. Cuong and associates [7] by developing the concept of picture FSs (PFSs). PFSs has been found to accommodate several more ambiguity because it is an enhanced version of IFSs. PFSs remark that the MV τ, neutral ψ, and NMV α have $0 \leq \tau+\psi+\alpha \leq 1$; for $\tau, \psi, \alpha \in [0,1]$. "Yes," "abstain," "no," and "refusal" are expert opinion messages that will be ensured to be transmitted via the PFS. It will also ensure consistency between the assessment data and the actual decision environment and avoid evaluation information from being left out. The idea has not been fully explored, despite the fact that PiFSs have several uses and studies. The spherical FS (SFS) was defined by Shahzaib et al. [8] for certain AOs with MADM. An alternative to $0 \leq \tau + \psi + \alpha \leq 1$ is required by the SFS: $0 \leq \tau^2 + \psi^2 + \alpha^2 \leq 1$. Various algebraic structures and aggregation techniques with applications were studied by Palanikumar et al. [10–12]. In MADM problems, the linguistic SFS AOs were discussed by Jin et al. [9]. SFSs and their applications were introduced by Rafiq et al. in DM [13] and $\tau^2 + \psi^2 \geq 1$ and decision-making (DM) are troublesome. The concept of an FFS was first proposed by Senapati and associates [6] in 2019. It is a feature of both the MV and NMV that $0 \leq \tau^3 + \psi^3 \leq 1$. The following contributions are made as a result of this endeavor: Idempotency, commutativity, and associativity are only a few of the many characteristics of algebra that have been shown. IFFSTNs have two features: HD and ED. The purpose of this method is to calculate the ED distance between two IFFSTNs. Ibraheem et al. [14] discussed the concept of complex NSSs using various AOs. Al-Husban et al. [15] deals that Type-I extension Diophantine IVNS. Recently, Udhayakumar et al. discussed the many fuzzy applications and its generalization [16–20]. Below are the major objectives of this work:

(1) Some basic algebraic structure satisfied the IFFSTNN aggregation operators.
(2) We appeal to the weighted operator using IFFSTNNWA, IFFSTNNWG, GIFFSTNNWA, and GIFFSTNNWG.

2. Basic concepts

Definition 2.1. [3] Let L be the universe set. The PFS $\Xi = \Big\{x, \langle \tau\{x\}, \chi(x) \rangle \big| x \in L\Big\}$, where $\tau, \chi : L \to (0,1)$ refers the MV and NMV of $x \in L$ to Ξ, respectively and $0 \leq (\tau(x))^2 + (\chi(x))^2 \leq 1$. For, $\Xi = \langle \tau, \chi \rangle$ is called the Pythagorean fuzzy number (PFN).

Definition 2.2. [6] A FFS $\Xi = \Big\{x, \langle \tau\{x\}, \chi\{x\} \rangle \big| x \in L\Big\}$, where $\tau\{x\}$ and $\chi\{x\}$ denote MV and NMV of u respectively, where $\tau, \chi : L \to \{0,1\}$ and $0 \leq \{\tau\{x\}\}^3 + \{\chi\{x\}\}^3 \leq 1$. Here, $\Xi = \langle \tau, \chi \rangle$ is represent a Fermatean fuzzy number {IFFN}.

Definition 2.3. For any PFNs, $\Xi = \langle \tau, \chi \rangle$, $\Xi_1 = \langle \tau_1, \chi_1 \rangle$ and $\Xi_2 = \langle \tau_2, \chi_2 \rangle$, τ, χ denote MV and NMV of u respectively. Then

(1) $\Xi_1 \bigsqcup \Xi_2 = \left[\sqrt{\{\tau_1\}^2 + \{\tau_2\}^2 - \{\tau_1\}^2 \cdot \{\tau_2\}^2}, \{\chi_1 \cdot \chi_2\}\right]$

(2) $\Xi_1 \circledast \Xi_2 = \left[\{\tau_1 \cdot \tau_2\}, \sqrt{\{\chi_1\}^2 + \{\chi_2\}^2 - \{\chi_1\}^2 \cdot \{\chi_2\}^2}\right]$

(3) $\aleph \cdot \Xi = \left[\sqrt{\imath - \{\imath - \{\tau\}^2\}^{\aleph}}, \{\chi\}^{\aleph}\right]$

(4) $\Xi^{\aleph} = \left[\{\tau\}^{\aleph}, \sqrt{\imath - \{\imath - \{\chi\}^2\}^{\aleph}}\right]$

Definition 2.4. If $\Xi_1 = \langle \tau_1, \chi_1 \rangle$ and $\Xi_2 = \langle \tau_2, \chi_2 \rangle$ are any two PFNs. Then the interaction AO is defined as

(1) $\Xi_1 \bigsqcup \Xi_2 = \begin{bmatrix} \sqrt{\{\tau_1\}^2 + \{\tau_2\}^2 - \{\tau_1\}^2 \cdot \{\tau_2\}^2}, \\ \sqrt{\{\chi_1\}^2 + \{\chi_2\}^2 - \{\chi_1\}^2 \cdot \{\chi_2\}^2 - \{\chi_1\}^2 \cdot \{\tau_2\}^2 - \{\tau_1\}^2 \cdot \{\chi_2\}^2} \end{bmatrix}$

(2) $\Xi_1 \circledast \Xi_2 = \begin{bmatrix} \sqrt{\{\tau_1\}^2 + \{\tau_2\}^2 - \{\tau_1\}^2 \cdot \{\tau_2\}^2 - \{\tau_1\}^2 \cdot \{\chi_2\}^2 - \{\chi_1\}^2 \cdot \{\tau_2\}^2}, \\ \sqrt{\{\chi_1\}^2 + \{\chi_2\}^2 - \{\chi_1\}^2 \cdot \{\chi_2\}^2} \end{bmatrix}$

(3) $\aleph \cdot \Xi_1 = \left[\sqrt{\imath - \{\imath - \{\tau_1\}^2\}^{\aleph}}, \sqrt{\{\imath - \{\tau_1\}^2\}^{\aleph} - \{\imath - \{\tau_1 + \chi_1\}^2\}^{\aleph}}\right]$

(4) $\Xi_1^{\aleph} = \left[\sqrt{\{\imath - \{\chi_1\}^2\}^{\aleph} - \{\imath - \{\tau_1 + \chi_1\}^2\}^{\aleph}}, \sqrt{\imath - \{\imath - \{\chi_1\}^2\}^{\aleph}}\right]$

where $\aleph$ be a positive integers.

3. Different AOs for IFFSTNN

Another way to define a fractional part function is as the difference between a real number and its greatest integer value, which is found using the greatest integer function. For example, if ϵ is a fractional part function, then the fractional part of x is expressed as $\epsilon\{x\} = \{x\} = x - \lfloor x \rfloor$, where x is a real number. In the event that x is an integer, its fractional component equals 0. Therefore, x cannot be an integer in order for $\epsilon\{x\} = \frac{1}{x}$ to be defined. Therefore, all real numbers, with the exception of integers, are included in the domain of $\epsilon\{x\} = \frac{1}{x}$. Here, $\theta = \sin \pi/2$.

Definition 3.1. Suppose that $\Xi_1 = \langle \theta\tau_1, \theta\omega_1, \theta\chi_1 \rangle$ and $\Xi_2 = \langle \theta\tau_2, \theta\omega_2, \theta\chi_2 \rangle$ be the any two IFFSTNNs. Then

(1) $\Xi_1 \bigsqcup \Xi_2 = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\{\theta\tau_1\}^{\frac{1}{\epsilon}} + \{\theta\tau_2\}^{\frac{1}{\epsilon}} - \{\theta\tau_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\tau_2\}^{\frac{1}{\epsilon}}}, \\ \sqrt[\frac{1}{\epsilon}]{\{\theta\omega_1\}^{\frac{1}{\epsilon}} + \{\theta\omega_2\}^{\frac{1}{\epsilon}} - \{\theta\omega_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\omega_2\}^{\frac{1}{\epsilon}}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{matrix} \{\theta\chi_1\}^{\frac{1}{\epsilon}} + \{\theta\chi_2\}^{\frac{1}{\epsilon}} - \{\theta\chi_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\chi_2\}^{\frac{1}{\epsilon}} \\ -\{\theta\chi_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\tau_2\}^{\frac{1}{\epsilon}} - \{\theta\tau_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\chi_2\}^{\frac{1}{\epsilon}} \end{matrix}} \end{bmatrix}$

$$(2)\ \Xi_1 \circledast \Xi_2 = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \{\theta\tau_1\}^{\frac{1}{\epsilon}} + \{\theta\tau_2\}^{\frac{1}{\epsilon}} - \{\theta\tau_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\tau_2\}^{\frac{1}{\epsilon}} \\ -\{\theta\tau_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\chi_2\}^{\frac{1}{\epsilon}} - \{\theta\chi_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\tau_2\}^{\frac{1}{\epsilon}} \end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\{\theta\omega_1\}^{\frac{1}{\epsilon}} + \{\theta\omega_2\}^{\frac{1}{\epsilon}} - \{\theta\omega_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\omega_2\}^{\frac{1}{\epsilon}}} \\ \sqrt[\frac{1}{\epsilon}]{\{\theta\chi_1\}^{\frac{1}{\epsilon}} + \{\theta\chi_2\}^{\frac{1}{\epsilon}} - \{\theta\chi_1\}^{\frac{1}{\epsilon}} \cdot \{\theta\chi_2\}^{\frac{1}{\epsilon}}} \end{bmatrix}$$

$$(3)\ \aleph \cdot \Xi_1 = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}}, \\ \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph} - \left\{\imath - \{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}} \end{bmatrix}$$

$$(4)\ \Xi_1^{\aleph} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph} - \left\{\imath - \{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}} \\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\aleph}} \end{bmatrix}$$

3.1. *IFFSTNWA operator*

Definition 3.2. Let $\Xi_\jmath = \langle \theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath \rangle$ be the IFFSTNNs, $\jmath = 1, 2, ..., \ell$, $\psi_\jmath$ be the weight of $\Xi_\jmath$ and $\psi_\jmath \geq 0$, $\varnothing_{\jmath=1}^{k} \psi_\jmath = 1$. Then the IFFSTNWA operator $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \varnothing_{\jmath=1}^{k} \psi_\jmath \Xi_\jmath$.

Theorem 3.3. *Let* $\Xi_\jmath = \langle \theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath \rangle$ *be the IFFSTNNs,* $\jmath = 1, 2, ..., \ell$*. Then,*

$$IFFSTNWA\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \lozenge_{\jmath=1}^{k}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \lozenge_{\jmath=1}^{k}\left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{k}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \lozenge_{\jmath=1}^{k}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{bmatrix}.$$

Proof. If $\jmath = 2$, IFFSTNWA$\{\Xi_1, \Xi_2\} = \psi_1\Xi_1 \bigsqcup \psi_2\Xi_2$,
where,

$$\psi_1\Xi_1 = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}, \\ \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} - \left\{\imath - \{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \end{bmatrix}$$

and

$$\psi_2\Xi_2 = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}}, \\ \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2} - \left\{\imath - \{\theta\tau_2 + \theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}} \end{bmatrix}$$

We get

$$\psi_1\Xi_1 \bigsqcup \psi_2\Xi_2 = \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\imath - \left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} + \left\{\imath - \left\{\imath - \{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\imath - \left\{\imath - \{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\}\end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} + \left\{\imath - \left\{\imath - \{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\}\end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\imath - \left\{\imath - \theta\chi_1\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\Big\} + \left\{\imath - \left\{\imath - \theta\chi_2\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\Big\} \\ -\left\{\imath - \left\{\imath - \theta\chi_1\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\Big\} \cdot \left\{\imath - \left\{\imath - \theta\chi_2\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\Big\} \\ -\left\{\imath - \{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} \cdot \left\{\imath - \{\theta\tau_2 + \theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{array}\right].$$

Using induction $\jmath \geq 3$, IFFSTNWA$\{\Xi_1, \Xi_2, ..., \Xi_\ell\}$

$$= \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \end{array}\right].$$

If $\jmath = \ell + 1$, then $IFFSTNWA\{\Xi_1, \Xi_2, ..., \Xi_\ell, \Xi_{\ell+1}\}$

$$= \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} + \left\{\imath - \left\{\imath - \{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\}\end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} + \left\{\imath - \left\{\imath - \{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\}\end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \theta\chi_\jmath\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\Big\} + \left\{\imath - \left\{\imath - \theta\chi_{\ell+1}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\Big\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \theta\chi_\jmath\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\Big\} \cdot \left\{\imath - \left\{\imath - \theta\chi_{\ell+1}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\Big\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} \cdot \left\{\imath - \{\theta\tau_{\ell+1} + \theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\end{array}} \end{array}\right]$$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\Big\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} \cdot \Big\{\imath - \{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\Big\}^{\psi_{\ell+1}}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\Big\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} \cdot \Big\{\imath - \{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\Big\}^{\psi_{\ell+1}}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\Big\{\Diamond_{\jmath=1}^{\ell}\Big\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell}\Big\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}\Big\} \cdot \\ \Big\{\Big\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\Big\}^{\psi_{\ell+1}} - \{\theta\tau_{\ell+1} + \theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\Big\}^{\psi_{\ell+1}}\Big\}\end{array}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\Big\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\Big\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell+1}\Big\{\imath - \{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell+1}\Big\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}} \end{bmatrix}$$

Theorem 3.4. *If $\Xi_\jmath = \langle \theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath \rangle$ be the IFFSTNNs and $\Xi_\jmath = \Xi$, then the IFFSTNWA$\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \Xi$, $\jmath = 1, 2, ..., \ell$.*

Proof. Note that, $\{\theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath\} = \{\theta\tau, \theta\omega, \theta\chi\}$, $\jmath = 1, 2, ..., \ell$ and $\varnothing_{\jmath=1}^{k}\psi_\jmath = 1$. We get, $IFFSTNWA\{\Xi_1, \Xi_2, ..., \Xi_\ell\}$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\omega\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\tau + \theta\chi\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\}^{\varnothing_{\jmath=1}^{k}\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Big\{\imath - \{\theta\omega\}^{\frac{1}{\epsilon}}\Big\}^{\varnothing_{\jmath=1}^{k}\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\}^{\varnothing_{\jmath=1}^{k}\psi_\jmath} - \Big\{\imath - \{\theta\tau + \theta\chi\}^{\frac{1}{\epsilon}}\Big\}^{\varnothing_{\jmath=1}^{k}\psi_\jmath}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\imath - \Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Big\{\imath - \{\theta\omega\}^{\frac{1}{\epsilon}}\Big\}}, \\ \sqrt[\frac{1}{\epsilon}]{\Big\{\imath - \{\theta\tau\}^{\frac{1}{\epsilon}}\Big\} - \Big\{\imath - \{\theta\tau + \theta\chi\}^{\frac{1}{\epsilon}}\Big\}} \end{bmatrix}$$

$$= \Big\{\theta\tau, \theta\omega, \theta\chi\Big\} = \Xi$$

3.2. *Interaction weighted geometric(IFFSTNWG) operator*

Definition 3.5. Let $\Xi_\jmath = \langle \theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath \rangle$ be the IFFSTNNs, $\psi_\jmath$ be the weight of $\Xi_\jmath$. Then the IFFSTNWG operator $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \Diamond_{\jmath=1}^{k}\Xi_\jmath^{\psi_\jmath}$.

Theorem 3.6. *If $\Xi_\jmath = \langle \theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath \rangle$ be the IFFSTNNs. Then,*

$$IFFSTNWG\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\Big\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\Big\}^{\psi_\jmath}} \end{bmatrix}$$

Proof. If $\jmath = 2$, then

$$\Xi_1^{\psi_1} \circledast \Xi_2^{\psi_2} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \left\{\imath - \left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} + \left\{\imath - \left\{\imath - \{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\imath - \left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\left\{\imath - \{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} \cdot \left\{\imath - \{\theta\tau_2 + \theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \left\{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} + \left\{\imath - \left\{\imath - \{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\imath - \left\{\imath - \{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \left\{\imath - \left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} + \left\{\imath - \left\{\imath - \{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \\ -\left\{\imath - \left\{\imath - \{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\psi_2}\right\} \end{array}} \end{bmatrix}$$

Hence,
$$\text{IFFSTNWG}\{\Xi_1, \Xi_2\} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{bmatrix}$$

$$\text{IFFSTNWG}\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{bmatrix}$$

If $\jmath = \ell + 1$, then $\text{IFFSTNWG}\{\Xi_1, ..., \Xi_\ell, \Xi_{\ell+1}\}$

$$= \begin{bmatrix} \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} + \left\{\imath - \left\{\imath - \{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} \cdot \left\{\imath - \{\theta\tau_{\ell+1} + \theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} + \left\{\imath - \left\{\imath - \{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{c} \varnothing_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} + \left\{\imath - \left\{\imath - \{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \\ -\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\imath - \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\} \cdot \left\{\imath - \left\{\imath - \{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\} \end{array}} \end{bmatrix}$$

$$= \left[\begin{array}{c}\sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\lozenge_{\jmath=1}^{\ell}\left\{\imath-\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\right\}\cdot\\ \left\{\left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}-\{\theta\tau_{\ell+1}+\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}\right\}\end{array}},\\ \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\cdot\left\{\imath-\{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}}\\ \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}\cdot\left\{\imath-\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\psi_{\ell+1}}}\end{array}\right]$$

$$= \left[\begin{array}{c}\sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{\ell+1}\left\{\imath-\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{\ell+1}\left\{\imath-\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\\ \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell+1}\left\{\imath-\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\ \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell+1}\left\{\imath-\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\end{array}\right]$$

Corollary 3.7. *Let $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath\rangle$ be the IFFSTNNs and all are equal and and $\theta\tau\cdot\theta\chi = 0$. Then IFFSTNWG$\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \Xi$.*

3.3. *generalized IFFSTNWA (GIFFSTNWA) operator*

Definition 3.8. Let $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\tau_\jmath, \theta\chi_\jmath\rangle$ be the IFFSTNNs, $\psi_\jmath$ be a weight of $\Xi_\jmath$. Then, the GIFFSTNWA operator $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \left\{\varnothing_{\jmath=1}^{k}\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}}\right\}^{1/2}$.

Theorem 3.9. *Let $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\tau_\jmath, \theta\chi_\jmath\rangle$ be the IFFSTNNs. Then GIFFSTNWA $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} =$*

$$\left[\begin{array}{c}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\},\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\},\\ \left\{\sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}\end{array}\right].$$

Proof. First, we have find that

$$\varnothing_{\jmath=1}^{k}\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}} = \left[\begin{array}{c}\sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\ \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}},\\ \sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{k}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\end{array}\right].$$

If $\jmath = 2$, then $\psi_1\Xi_1^{\frac{1}{\epsilon}} = \left[\begin{array}{c}\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}},\ \sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}},\\ \sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_1+\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\end{array}\right]$

and

$$\psi_2\Xi_2^{\frac{1}{\epsilon}} = \left[\begin{array}{c}\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}},\ \sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}},\\ \sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_2+\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\end{array}\right].$$

We get, $\psi_1\Xi_1 \bigsqcup \psi_2\Xi_2 =$

$$\left[\begin{array}{l} \sqrt[\frac{1}{\epsilon}]{\begin{array}{l}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left[\begin{array}{l}\left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_1+\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\cdot \\ \left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\tau_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_2+\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}\right]\end{array}} \end{array}\right]$$

$$= \left[\begin{array}{l}\sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\tau_1+\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\end{array}\right]$$

In general,

$$= \left[\begin{array}{l}\sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \\ \sqrt[\frac{1}{\epsilon}]{\lozenge_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\lozenge_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_1+\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\end{array}\right].$$

If $\jmath = \ell + 1$, then $\varnothing_{\jmath=1}^{\ell}\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}} + \psi_{\ell+1}\Xi_{\ell+1}^{\frac{1}{\epsilon}} = \varnothing_{\jmath=1}^{\ell+1}\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}}$.
Now, $\varnothing_{\jmath=1}^{\ell}\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}} + \psi_{\ell+1}\Xi_{\ell+1}^{\frac{1}{\epsilon}} = \psi_1\Xi_1^{\frac{1}{\epsilon}} \bigsqcup \psi_2\Xi_2^{\frac{1}{\epsilon}} \bigsqcup ... \bigsqcup \psi_\ell\Xi_\ell^{\frac{1}{\epsilon}} \bigsqcup \psi_{\ell+1}\Xi_{\ell+1}^{\frac{1}{\epsilon}}$

$$
= \left[\begin{array}{c}
\sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}} \\
\sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}} \\
\sqrt[\frac{1}{\epsilon}]{\begin{array}{c}\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}+\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}, \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}\cdot\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left[\begin{array}{c}\left\{\sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}}\cdot \\ \left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\tau_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_{\ell+1}+\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}}\end{array}\right]\end{array}}
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\
\sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_1+\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}
\end{array}\right]
$$

and $\varnothing_{\jmath=1}^{\ell+1}\left\{\psi_\jmath\Xi_\jmath^{\frac{1}{\epsilon}}\right\} =$

$$
\left[\begin{array}{c}
\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}, \\
\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}, \\
\left\{\sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}-\Diamond_{\jmath=1}^{\ell+1}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}
\end{array}\right].
$$

Corollary 3.10. *Let* $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath\rangle$ *be the IFFSTNNs and all are equal. Then GIFFSTNWA* $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \Xi$.

3.4. *Generalized IFFSTNWG (GIFFSTNWG) operator*

Definition 3.11. Let $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\tau_\jmath, \theta\chi_\jmath\rangle$ be the IFFSTNNs, $\psi_\jmath\}$ be the weight of $\Xi_\jmath$, where $\jmath = 1, 2, ..., \ell$. Then, the GIFFSTNWG$\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \frac{1}{\aleph}\left\{\Diamond_{\jmath=1}^{k}\{\aleph\Xi_\jmath\}^{\psi_\jmath}\right\}$.

Theorem 3.12. *Let $\Xi_\jmath = \left\langle \theta\tau_\jmath, \theta\tau_\jmath, \theta\chi_\jmath \right\rangle$ be the collection of IFFSTNNs. Then the GIFFSTNWG operator* $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} =$

$$\left[\begin{array}{c} \left\{ \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{k} \left\{ \imath - \left\{ \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}} \right\}^{\frac{1}{\epsilon}} \right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{k} \left\{ \imath - \left\{ \{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}} \right\}^{\frac{1}{\epsilon}} \right\}^{\psi_\jmath}} \right\}, \\ \left\{ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k} \left\{ \imath - \left\{ \{\theta\omega_\jmath\}^{\frac{1}{\epsilon}} \right\}^{\frac{1}{\epsilon}} \right\}^{\psi_\jmath}} \right\} \left\{ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k} \left\{ \imath - \left\{ \{\theta\chi_\jmath\}^{\frac{1}{\epsilon}} \right\}^{\frac{1}{\epsilon}} \right\}^{\psi_\jmath}} \right\} \end{array} \right]$$

Proof. Using the induction method,

$$\Diamond_{\jmath=1}^{k}\{\aleph\Xi_\jmath\}^{\psi_\jmath} = \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{k}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{k}\left\{\imath - \left\{\{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\left\{\imath - \left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{k}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{array} \right]$$

If $\jmath = 2$, then

$$\{\aleph\Xi_1\}^{\psi_1} = \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} - \left\{\imath - \left\{\{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \end{array} \right]$$

and

$$\{\aleph\Xi_2\}^{\psi_2} = \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\left\{\imath - \left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} - \left\{\imath - \left\{\{\theta\tau_2 + \theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \\ \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}} \end{array} \right]$$

We get, $\{\aleph\Xi_1\}^{\psi_1} \circledast \{\aleph\Xi_2\}^{\psi_2}$

$$= \left[\begin{array}{l} \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left[\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_1+\theta\tau_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \cdot \\ \left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}-\left\{\imath-\left\{\{\theta\tau_2+\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}\right] \end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\omega_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath-\left\{\imath-\left\{\{\theta\chi_2\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{l} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\frac{1}{\epsilon}}\left\{\imath-\left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{array}\right]$$

If $\jmath = \ell$, then

$$= \left[\begin{array}{l} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\tau_\jmath+\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath-\Diamond_{\jmath=1}^{\ell}\left\{\imath-\left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{array}\right]$$

If $\jmath = \ell + 1$, then $\Diamond_{\jmath=1}^{\ell}\{\aleph\Xi_\jmath\}^{\psi_\jmath} \cdot \{\aleph\Xi_{\ell+1}\}^{\psi_{\ell+1}} = \Diamond_{\jmath=1}^{\ell+1}\{\aleph\Xi_\jmath\}^{\psi_\jmath}$.

Now, $\Diamond_{\jmath=1}^{\ell}\{\aleph\Xi_\jmath\}^{\psi_\jmath} \cdot \{\aleph\Xi_{\ell+1}\}^{\psi_{\ell+1}} = \{\aleph\Xi_1\}^{\psi_1} \divideontimes \{\aleph\Xi_2\}^{\psi_2} \divideontimes ... \divideontimes \{\aleph\Xi_\ell\}^{\psi_\ell} \divideontimes \{\aleph\Xi_{\ell+1}\}^{\psi_{\ell+1}}$

$$= \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ - \left[\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} \cdot \\ \left\{\sqrt[\frac{1}{\epsilon}]{\left\{\imath - \left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1} - \left\{\imath - \left\{\{\theta\tau_{\ell+1} + \theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}\right] \end{array}}, \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\omega_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}} \\ \sqrt[\frac{1}{\epsilon}]{\begin{array}{l} \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} + \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \\ -\left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}^{\frac{1}{\epsilon}} \cdot \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \left\{\imath - \left\{\{\theta\chi_{\ell+1}\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_1}}\right\}^{\frac{1}{\epsilon}} \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\tau_1 + \theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \\ \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\omega_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}, \sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\chi_1\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}} \end{array}\right]$$

Hence

$$\frac{1}{\aleph}\left\{\Diamond_{\jmath=1}^{\ell+1}\{\aleph\Xi_\jmath\}^{\psi_\jmath}\right\} =$$

$$\left[\begin{array}{c} \left\{\sqrt[\frac{1}{\epsilon}]{\Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath} - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\tau_\jmath + \theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}, \\ \left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\omega_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\}\left\{\sqrt[\frac{1}{\epsilon}]{\imath - \Diamond_{\jmath=1}^{\ell+1}\left\{\imath - \left\{\{\theta\chi_\jmath\}^{\frac{1}{\epsilon}}\right\}^{\frac{1}{\epsilon}}\right\}^{\psi_\jmath}}\right\} \end{array}\right]$$

Corollary 3.13. *Let $\Xi_\jmath = \langle\theta\tau_\jmath, \theta\omega_\jmath, \theta\chi_\jmath\rangle$ be the collection of IFFSTNNs and all are equal. Then GIFFSTNWG $\{\Xi_1, \Xi_2, ..., \Xi_\ell\} = \Xi$.*

4. Conclusion:

The algebraic accessibility of ED and HD for IFFSTNNs is a significant advantage. HD of IFFSTNNs may offer significant improvements in data analysis. The utilization of compelling statistics illustrates the advantages of HD. We provided IFFSTNWA, IFFSTNWG, GIFFSTNWA, and GIFFSTNWG examples and suggested models. The following subjects will be covered in more detail later on: A more detailed treatment of the following topics will be covered: (1) Interaction AOs establish a link between the cubic NS and IVPFS. (2) The issue may be resolved by using complex IFFSTNWA, complex IFFSTNWG, complex GIFFSTNWA, and complex GIFFSTNWG. (3) The following topics will be discussed in more detail as Soft and expert sets were explored in terms of the Diophantine vague NSS, complex vague NSS, q-Rung interval valued NSS and complex cubic q-Rung NSS.

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