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Characterization of interaction aggregating operators setting interval-valued Pythogorean neutrosophic set

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Abstract. In this work, we present novel techniques for the interval-valued Pythogorean neutrosophic interaction aggregating operator. A hybrid of the neutrosophic set and the interval-valued Pythogorean fuzzy set. The innovative averaging and geometric operations of interval-valued Pythogorean neutrosophic interaction numbers are studied using aggregation operator. The interval-valued Pythogorean neutrosophic interaction is boundedness compatible, idempotent, associative, and commutative. Four new aggregating operators are introduced: IPNI weighted averaging operator, interval-valued Pythogorean neutrosophic interaction weighted geometric operator, generalized interval-valued Pythogorean neutrosophic interaction weighted averaging and generalized interval-valued Pythogorean neutrosophic interaction weighted geometric.

Keywords: weighted averaging, weighted geometric, generalized weighted averaging, generalized weighted geometric.

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1. Introduction

The uncertainties led to the development of the fuzzy set (FS) [1], intuitionistic FS (IFS) [2], Pythagorean FS (PFS) [3, 4], neutrosophic set (NS) [5], Fuzzy extension and its application [6–14] and Fermatean FS (FFS) [15]. FS suggests that decision-makers use Zadeh's [1] with the membership degree (MD). Each object has MD $\measuredangle$ and non-membership degree (NMD)

$\backepsilon$, and it fulfills $0 \leq \measuredangle + \backepsilon \leq 1$, for $\measuredangle, \backepsilon \in (0,1)$. Atanassov developed an IFS idea [2]. PFSs are defined by their MDs and NMDs under the condition that $\measuredangle^2 + \backepsilon^2 \leq 1$, which was devised by Yager [3]. IFSs and PFSs are widely utilized and have been explored in a wide range of fields. Cuong and associates [16] by developing the concept of picture FSs (PiFSs). PiFSs has been found to accommodate certain more ambiguity because it is an expanded version of IFSs. For $\measuredangle, \backepsilon, \varrho \in (0,1)$, it is observed in PiFSs that the MD $\measuredangle$, the neutral $\backepsilon$, and the NMD ϱ have $0 \leq \measuredangle + \backepsilon + \varrho \leq 1$. "Yes," "abstain," "no," and "refusal" are expert opinion messages that will be ensured to be transmitted via the PiFS. Furthermore, it will ensure consistency between the assessment data and the actual decision environment and avoid evaluation information from being left out. Despite the fact that PiFSs have been the subject of several applications and studies, the concept has not been fully explored. Raed Hatamleh et al. discussed the various applications such as Uryson's Operator, Sine-Gordon System and Traces Class Pertubations [17–22]. Shahzaib and colleagues [23] defined the spherical FS (SFS) for certain AOs with MADM. An alternative to $0 \leq \measuredangle + \backepsilon + \varrho \leq 1$ is required by the SFS: $0 \leq \measuredangle^2 + \backepsilon^2 + \varrho^2 \leq 1$. Linguistic SFS AOs were introduced by Jin et al. [24] and discussed in MADM problems. Rafiq and colleagues introduced SFSs and their applications in DM [25] with the condition that $\measuredangle^2 + \backepsilon^2 \geq 1$ and decision-making (DM) are troublesome. The concept of an FFS was first proposed by Senapati and associates [15] in 2019. When $0 \leq \measuredangle^3 + \backepsilon^3 \leq 1$, both the MD and NMD have this characteristic. Al-Husban et al. discussed the algebraic structures via complex Fuzzy Hyperring, Fuzzy Soft Groups based on Fuzzy Space and Structures of fibers of groups actions on graphs [26–33].

2. **Different AOs for IPNN**

Definition 2.1. Suppose that $\daleth_1 = \left\langle (\measuredangle_1^{ta}, \measuredangle_1^{tb}), (\sigma_1^{ia}, \sigma_1^{ib}), (\varrho_1^{fa}, \varrho_1^{fb}) \right\rangle$ and $\daleth_2 = \left\langle (\measuredangle_2^{ta}, \measuredangle_2^{tb}), (\sigma_2^{ia}, \sigma_2^{ib}), (\varrho_2^{fa}, \varrho_2^{fb}) \right\rangle$ be the any two IPNNs. Then

$$(1)\ \daleth_1 \veebar \daleth_2 = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{(\measuredangle_1^{ta})^2 + (\measuredangle_2^{ta})^2 - (\measuredangle_1^{ta})^2 \cdot (\measuredangle_2^{ta})^2}, \\ \sqrt{(\measuredangle_1^{tb})^2 + (\measuredangle_2^{tb})^2 - (\measuredangle_1^{tb})^2 \cdot (\measuredangle_2^{tb})^2} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{(\sigma_1^{ia})^2 + (\sigma_2^{ia})^2 - (\sigma_1^{ia})^2 \cdot (\sigma_2^{ia})^2}, \\ \sqrt{(\sigma_1^{ib})^2 + (\sigma_2^{ib})^2 - (\sigma_1^{ib})^2 \cdot (\sigma_2^{ib})^2} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} (\varrho_1^{fa})^2 + (\varrho_2^{fa})^2 - (\varrho_1^{fa})^2 \cdot (\varrho_2^{fa})^2 \\ -(\varrho_1^{fa})^2 \cdot (\measuredangle_2^{ta})^2 - (\measuredangle_1^{ta})^2 \cdot (\varrho_2^{fa})^2 \end{array}}, \\ \sqrt{\begin{array}{c} (\varrho_1^{fb})^2 + (\varrho_2^{fb})^2 - (\varrho_1^{fb})^2 \cdot (\varrho_2^{fb})^2 \\ -(\varrho_1^{fb})^2 \cdot (\measuredangle_2^{tb})^2 - (\measuredangle_1^{tb})^2 \cdot (\varrho_2^{fb})^2 \end{array}} \end{array}\right] \end{array}\right]$$

$$(2)\ \daleth_1 \barwedge \daleth_2 = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\begin{array}{c}(\measuredangle_1^{ta})^2+(\measuredangle_2^{ta})^2-(\measuredangle_1^{ta})^2\cdot(\measuredangle_2^{ta})^2 \\ -(\measuredangle_1^{ta})^2\cdot(\varrho_2^{fa})^2-(\varrho_1^{fa})^2\cdot(\measuredangle_2^{ta})^2\end{array}}, \\ \sqrt{\begin{array}{c}(\measuredangle_1^{tb})^2+(\measuredangle_2^{tb})^2-(\measuredangle_1^{tb})^2\cdot(\measuredangle_2^{tb})^2 \\ -(\measuredangle_1^{tb})^2\cdot(\varrho_2^{fb})^2-(\varrho_1^{fb})^2\cdot(\measuredangle_2^{tb})^2\end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{(\sigma_1^{ia})^2+(\sigma_2^{ia})^2-(\sigma_1^{ia})^2\cdot(\sigma_2^{ia})^2}, \\ \sqrt{(\sigma_1^{ib})^2+(\sigma_2^{ib})^2-(\sigma_1^{ib})^2\cdot(\sigma_2^{ib})^2} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{(\varrho_1^{fa})^2+(\varrho_2^{fa})^2-(\varrho_1^{fa})^2\cdot(\varrho_2^{fa})^2}, \\ \sqrt{(\varrho_1^{fb})^2+(\varrho_2^{fb})^2-(\varrho_1^{fb})^2\cdot(\varrho_2^{fb})^2} \end{array}\right] \end{array}\right]$$

$$(3)\ \zeta\cdot\daleth_1 = \left[\begin{array}{c} \left[\sqrt{1-\left(1-(\measuredangle_1^{ta})^2\right)^{\zeta}}, \sqrt{1-\left(1-(\measuredangle_1^{tb})^2\right)^{\zeta}}\right] \\ \left[\sqrt{1-\left(1-(\sigma_1^{ia})^2\right)^{\zeta}}, \sqrt{1-\left(1-(\sigma_1^{ib})^2\right)^{\zeta}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1-(\measuredangle_1^{ta})^2\right)^{\zeta}-\left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\zeta}}, \\ \sqrt{\left(1-(\measuredangle_1^{tb})^2\right)^{\zeta}-\left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\zeta}} \end{array}\right] \end{array}\right]$$

$$(4)\ \daleth_1^{\zeta} = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\left(1-(\varrho_1^{fa})^2\right)^{\zeta}-\left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\zeta}}, \\ \sqrt{\left(1-(\varrho_1^{fb})^2\right)^{\zeta}-\left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\zeta}} \end{array}\right] \\ \left[\sqrt{1-\left(1-(\sigma_1^{ia})^2\right)^{\zeta}}, \sqrt{1-\left(1-(\sigma_1^{ib})^2\right)^{\zeta}},\right] \\ \left[\sqrt{1-\left(1-(\varrho_1^{fa})^2\right)^{\zeta}}, \sqrt{1-\left(1-(\varrho_1^{fb})^2\right)^{\zeta}}\right] \end{array}\right]$$

2.1. *IPNIWAO*

Definition 2.2. Let $\daleth_\partial = \langle(\measuredangle_\partial^{ta}, \measuredangle_\partial^{tb}), (\sigma_\partial^{ia}, \sigma_\partial^{ib}), (\varrho_\partial^{fa}, \varrho_\partial^{fb})\rangle$ be the IPNNs, $\partial = 1, 2, ..., \propto$, $\backepsilon_\partial$ be the weight of $\daleth_\partial$ and $\backepsilon_\partial \geq 0, \Diamond_{\partial=1}^{\propto} \backepsilon_\partial = 1$. Then the IPNIWAO $(\daleth_1, \daleth_2, ..., \daleth_\propto) = \Diamond_{\partial=1}^{\propto} \backepsilon_\partial\ \daleth_\partial$.

Theorem 2.3. *Let $\daleth_\partial = \langle(\measuredangle_\partial^{ta}, \measuredangle_\partial^{tb}), (\sigma_\partial^{ia}, \sigma_\partial^{ib}), (\varrho_\partial^{fa}, \varrho_\partial^{fb})\rangle$ be the IPNNs, $\partial = 1, 2, ..., \propto$.*

$$Then, IPNIWAO(\daleth_1, \daleth_2, ..., \daleth_\propto) = \left[\begin{array}{c} \left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{ta})^2\right)^{\backepsilon_\partial}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{tb})^2\right)^{\backepsilon_\partial}}\right] \\ \left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-(\sigma_\partial^{ia})^2\right)^{\backepsilon_\partial}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-(\sigma_\partial^{ib})^2\right)^{\backepsilon_\partial}}\right] \\ \left[\begin{array}{c} \sqrt{\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{ta})^2\right)^{\backepsilon_\partial}-\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{ta}+\varrho_\partial^{fa})^2\right)^{\backepsilon_\partial}}, \\ \sqrt{\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{tb})^2\right)^{\backepsilon_\partial}-\Box_{\partial=1}^{\propto}\left(1-(\measuredangle_\partial^{tb}+\varrho_\partial^{fb})^2\right)^{\backepsilon_\partial}} \end{array}\right] \end{array}\right].$$

Proof. If $\partial = 2$, IPNIWAO$(\dashv_1, \dashv_2) = \backepsilon_1 \dashv_1 \veebar \backepsilon_2 \dashv_2$,

where,

$$\backepsilon_1 \dashv_1 = \left[\begin{array}{c} \left[\sqrt{1-\left(1-(\measuredangle_1^{ta})^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-(\measuredangle_1^{tb})^2\right)^{\backepsilon_1}}\right] \\ \left[\sqrt{1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1-(\measuredangle_1^{ta})^2\right)^{\backepsilon_1}-\left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\backepsilon_1}}, \\ \sqrt{\left(1-(\measuredangle_1^{tb})^2\right)^{\backepsilon_1}-\left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\backepsilon_1}} \end{array}\right] \end{array}\right]$$

and

$$\backepsilon_2 \dashv_2 = \left[\begin{array}{c} \left[\sqrt{1-\left(1-(\measuredangle_2^{ta})^2\right)^{\backepsilon_2}}, \sqrt{1-\left(1-(\measuredangle_2^{tb})^2\right)^{\backepsilon_2}}\right] \\ \left[\sqrt{1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}}, \sqrt{1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1-(\measuredangle_2^{ta})^2\right)^{\backepsilon_2}-\left(1-(\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^{\backepsilon_2}}, \\ \sqrt{\left(1-(\measuredangle_2^{tb})^2\right)^{\backepsilon_2}-\left(1-(\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^{\backepsilon_2}} \end{array}\right] \end{array}\right]$$

We get

$$\backepsilon_1 \dashv_1 \veebar \backepsilon_2 \dashv_2 = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-(\measuredangle_1^{ta})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-(\measuredangle_2^{ta})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\measuredangle_1^{ta})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-(\measuredangle_2^{ta})^2\right)^{\backepsilon_2}\right), \end{array}} \\ \sqrt{\begin{array}{c} \left(1-\left(1-(\measuredangle_1^{tb})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-(\measuredangle_2^{tb})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\measuredangle_1^{tb})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-(\measuredangle_2^{tb})^2\right)^{\backepsilon_2}\right), \end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}\right), \end{array}} \\ \sqrt{\begin{array}{c} \left(1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}\right), \end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-\varrho_1^{fa})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-\varrho_1^{fa})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \\ -\left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\backepsilon_1}\cdot\left(1-(\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^{\backepsilon_2} \end{array}} \\ \sqrt{\begin{array}{c} \left(1-\left(1-\varrho_1^{fb})^2\right)^{\backepsilon_1}\right)+\left(1-\left(1-\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-\varrho_1^{fb})^2\right)^{\backepsilon_1}\right)\cdot\left(1-\left(1-\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \\ -\left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\backepsilon_1}\cdot\left(1-(\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^{\backepsilon_2} \end{array}} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c}\left[\sqrt{1-\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{2}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}}\right] \\ \left[\begin{array}{c}\sqrt{\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}-\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}-\square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}\end{array}\right]\end{array}\right].$$

Using induction $\partial \geq 3$,IPNIWAO$(\daleth_1, \daleth_2, ..., \daleth_\propto)$

$$= \left[\begin{array}{c}\left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}}\right] \\ \left[\begin{array}{c}\sqrt{\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}\end{array}\right]\end{array}\right].$$

If $\partial = \propto +1$, then $IPNIWAO(\daleth_1, \daleth_2, ..., \daleth_\propto, \daleth_{\propto+1})$

$$= \left[\begin{array}{c}\left[\begin{array}{c}\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}\cdot\Big(1-(\measuredangle_{\propto+1}^{ta})^2\Big)^{\ni\propto+1}}, \\ \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}\cdot\Big(1-(\measuredangle_{\propto+1}^{tb})^2\Big)^{\ni\propto+1}}\end{array}\right] \\ \left[\begin{array}{c}\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}\cdot\Big(1-(\sigma_{\propto+1}^{ia})^2\Big)^{\ni\propto+1}}, \\ \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}\cdot\Big(1-(\sigma_{\propto+1}^{ib})^2\Big)^{\ni\propto+1}}\end{array}\right] \\ \left[\begin{array}{c}\sqrt{\begin{array}{c}\Big(\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\Big)\cdot \\ \Big(\Big((\measuredangle_{\propto+1}^{ta})^2\Big)^{\ni\propto+1}-(\measuredangle_{\propto+1}^{ta}+\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}\Big)\end{array}} \\ \sqrt{\begin{array}{c}\Big(\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\Big)\cdot \\ \Big(\Big((\measuredangle_{\propto+1}^{tb})^2\Big)^{\ni\propto+1}-(\measuredangle_{\propto+1}^{tb}+\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}\Big)\end{array}}\end{array}\right]\end{array}\right]$$

$$= \left[\begin{array}{c}\left[\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}},\right] \\ \left[\begin{array}{c}\sqrt{\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{ta})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}} \\ \sqrt{\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{tb})^2\Big)^{\ni\partial}-\square_{\partial=1}^{\propto+1}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}\end{array}\right]\end{array}\right]$$

Theorem 2.4. *If* $\daleth_\partial = \left\langle(\measuredangle_{\partial}^{ta}, \measuredangle_{\partial}^{tb}), (\sigma_{\partial}^{ia}, \sigma_{\partial}^{ib}), (\varrho_{\partial}^{fa}, \varrho_{\partial}^{fb})\right\rangle$ *be the IPNNs and* $\daleth_\partial = \daleth$ *and* $\measuredangle^{tb} \cdot \varrho^{fb} = 0$,*then the* $IPNIWAO(\daleth_1, \daleth_2, ..., \daleth_\propto) = \daleth$,$\partial = 1, 2, ..., \propto$.

Proof. Note that, $\left\langle (\measuredangle_\partial^{ta}, \measuredangle_\partial^{tb}), (\sigma_\partial^{ia}, \sigma_\partial^{ib}), (\varrho_\partial^{fa}, \varrho_\partial^{fb}) \right\rangle = \left\langle (\measuredangle^{ta}, \measuredangle^{tb}), (\sigma^{ia}, \sigma^{ib}), (\varrho^{fa}, \varrho^{fb}) \right\rangle$, $\partial = 1, 2, ..., \propto$ and $\Diamond_{\partial=1}^{\propto} \ni_\partial = 1$. We get, $IPNIWAO(\dashv_1, \dashv_2, ..., \dashv_\propto)$

$$= \begin{bmatrix} \left[\sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{ta})^2\right)^{\ni_\partial}}, \sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{tb})^2\right)^{\ni_\partial}}\right] \\ \left[\sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\sigma^{ia})^2\right)^{\ni_\partial}}, \sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\sigma^{ib})^2\right)^{\ni_\partial}}\right] \\ \left[\begin{array}{c} \sqrt{\Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{ta})^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{ta} + \varrho^{fa})^2\right)^{\ni_\partial}}, \\ \sqrt{\Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{tb})^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle^{tb} + \varrho^{fb})^2\right)^{\ni_\partial}} \end{array}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\sqrt{1 - \left(1 - (\measuredangle^{ta})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}}, \sqrt{1 - \left(1 - (\measuredangle^{tb})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}},\right] \\ \left[\sqrt{1 - \left(1 - (\sigma^{ia})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}}, \sqrt{1 - \left(1 - (\sigma^{ib})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1 - (\measuredangle^{ta})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial} - \left(1 - (\measuredangle^{ta} + \varrho^{fa})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}}, \\ \sqrt{\left(1 - (\measuredangle^{tb})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial} - \left(1 - (\measuredangle^{tb} + \varrho^{fb})^2\right)^{\Diamond_{\partial=1}^{\propto} \ni_\partial}} \end{array}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\sqrt{1 - \left(1 - (\measuredangle^{ta})^2\right)}, \sqrt{1 - \left(1 - (\measuredangle^{tb})^2\right)}\right] \\ \left[\sqrt{1 - \left(1 - (\sigma^{ia})^2\right)}, \sqrt{1 - \left(1 - (\sigma^{ib})^2\right)}\right] \\ \left[\begin{array}{c} \sqrt{\left(1 - (\measuredangle^{ta})^2\right) - \left(1 - (\measuredangle^{ta} + \varrho^{fa})^2\right)}, \\ \sqrt{\left(1 - (\measuredangle^{tb})^2\right) - \left(1 - (\measuredangle^{tb} + \varrho^{fb})^2\right)} \end{array}\right] \end{bmatrix}$$

$$= \left((\measuredangle^{ta}, \measuredangle^{tb}), (\sigma^{ia}, \sigma^{ib}), (\varrho^{fa}, \varrho^{fb})\right) = \dashv$$

2.2. *Interaction weighted geometric(IPNIWGO)*

Definition 2.5. Let $\dashv_\partial = \left\langle (\measuredangle_\partial^{ta}, \measuredangle_\partial^{tb}), (\sigma_\partial^{ia}, \sigma_\partial^{ib}), (\varrho_\partial^{fa}, \varrho_\partial^{fb}) \right\rangle$ be the IPNNs, $\ni_\partial$ be the weight of $\dashv_\partial$. Then the IPNIWGO $(\dashv_1, \dashv_2, ..., \dashv_\propto) = \Box_{\partial=1}^{\propto} \dashv_\partial^{\ni_\partial}$.

Theorem 2.6. *If* $\dashv_\partial = \left\langle (\measuredangle_\partial^{ta}, \measuredangle_\partial^{tb}), (\sigma_\partial^{ia}, \sigma_\partial^{ib}), (\varrho_\partial^{fa}, \varrho_\partial^{fb}) \right\rangle$ *be the IPNNs. Then,*

$$IPNIWGO(\dashv_1, \dashv_2, ..., \dashv_\propto) = \begin{bmatrix} \left[\begin{array}{c} \sqrt{\Box_{\partial=1}^{\propto}\left(1 - (\varrho_\partial^{fa})^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle_\partial^{ta} + \varrho_\partial^{fa})^2\right)^{\ni_\partial}}, \\ \sqrt{\Box_{\partial=1}^{\propto}\left(1 - (\varrho_\partial^{fb})^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1 - (\measuredangle_\partial^{tb} + \varrho_\partial^{fb})^2\right)^{\ni_\partial}} \end{array}\right] \\ \left[\sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\sigma_\partial^{ia})^2\right)^{\ni_\partial}}, \sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\sigma_\partial^{ib})^2\right)^{\ni_\partial}}\right] \\ \left[\sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\varrho_\partial^{fa})^2\right)^{\ni_\partial}}, \sqrt{1 - \Box_{\partial=1}^{\propto}\left(1 - (\varrho_\partial^{fb})^2\right)^{\ni_\partial}}\right] \end{bmatrix}$$

Proof. If $\partial = 2$,then IPNIWGO$(\daleth_1, \daleth_2) = \daleth_1^{\backepsilon_1} \overline{\wedge} \daleth_2^{\backepsilon_2}$,

where,

$$\daleth_1^{\backepsilon_1} = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1} - \left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\backepsilon_1}}, \\ \sqrt{\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1} - \left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\backepsilon_1}} \end{array}\right] \\ \left[\sqrt{1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}}\right] \\ \left[\sqrt{1-\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1}}\right] \end{array}\right]$$

$$\daleth_2^{\backepsilon_2} = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2} - \left(1-(\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^{\backepsilon_2}}, \\ \sqrt{\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2} - \left(1-(\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^{\backepsilon_2}} \end{array}\right] \\ \left[\sqrt{1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}}, \sqrt{1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}}\right] \\ \left[\sqrt{1-\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2}}, \sqrt{1-\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2}}\right] \end{array}\right]$$

We get,

$$\daleth_1^{\backepsilon_1} \overline{\wedge} \daleth_2^{\backepsilon_2} = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \\ -\left(\left(1-(\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^{\backepsilon_1} \cdot \left(1-(\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \end{array}} \\ \sqrt{\begin{array}{c} \left(1-\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \\ -\left(\left(1-(\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^{\backepsilon_1} \cdot \left(1-(\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\sigma_1^{ia})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\sigma_2^{ia})^2\right)^{\backepsilon_2}\right) \end{array}} \\ \sqrt{\begin{array}{c} \left(1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\sigma_1^{ib})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\sigma_2^{ib})^2\right)^{\backepsilon_2}\right) \end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} \left(1-\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\varrho_1^{fa})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\varrho_2^{fa})^2\right)^{\backepsilon_2}\right) \end{array}}, \\ \sqrt{\begin{array}{c} \left(1-\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1}\right) + \left(1-\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \\ -\left(1-\left(1-(\varrho_1^{fb})^2\right)^{\backepsilon_1}\right) \cdot \left(1-\left(1-(\varrho_2^{fb})^2\right)^{\backepsilon_2}\right) \end{array}} \end{array}\right] \end{array}\right]$$

$$\text{Hence, IPNIWGO}(\Finv_1, \Finv_2) = \left[\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\square_{\partial=1}^{2}\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial} - \square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \\
\sqrt{\square_{\partial=1}^{2}\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial} - \square_{\partial=1}^{2}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}},
\end{array}\right] \\
\left[\sqrt{1-\square_{\partial=1}^{2}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}}\right] \\
\left[\sqrt{1-\square_{\partial=1}^{2}\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}\right]
\end{array}\right]$$

$$\text{IPNIWGO}(\Finv_1, \Finv_2, ..., \Finv_{\propto}) = \left[\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\square_{\partial=1}^{\propto}\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial} - \square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \\
\sqrt{\square_{\partial=1}^{\propto}\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial} - \square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}
\end{array}\right] \\
\left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}}\right] \\
\left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}}\right]
\end{array}\right]$$

If $\partial = \propto +1$,then IPNIWGO$(\Finv_1, ..., \Finv_{\propto}, \Finv_{\propto+1})$

$$= \left[\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\cdot\Big(1-(\measuredangle_{\propto+1}^{ta}+\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}
\end{array}} \\
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\cdot\Big(1-(\measuredangle_{\propto+1}^{tb}+\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}
\end{array}}
\end{array}\right] \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\sigma_{\propto+1}^{ia})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\sigma_{\partial}^{ia})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\sigma_{\propto+1}^{ia})^2\Big)^{\ni\propto+1}\Big)
\end{array}}, \\
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\sigma_{\propto+1}^{ib})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\sigma_{\partial}^{ib})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\sigma_{\propto+1}^{ib})^2\Big)^{\ni\propto+1}\Big)
\end{array}}
\end{array}\right] \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fa})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\varrho_{\propto+1}^{fa})^2\Big)^{\ni\propto+1}\Big)
\end{array}}, \\
\sqrt{\begin{array}{c}
\Diamond_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\Big) + \Big(1-\Big(1-(\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}\Big) \\
-\square_{\partial=1}^{\propto}\Big(1-\Big(1-(\varrho_{\partial}^{fb})^2\Big)^{\ni\partial}\Big)\cdot\Big(1-\Big(1-(\varrho_{\propto+1}^{fb})^2\Big)^{\ni\propto+1}\Big)
\end{array}}
\end{array}\right]
\end{array}\right]$$

$$
= \begin{bmatrix} \begin{bmatrix} \sqrt{\begin{matrix} \left(\square_{\partial=1}^{\propto}\left(1-(\varrho_{\partial}^{fa})^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto}\left(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^{\ni\partial}\right)\cdot \\ \left(\left((\varrho_{\propto+1}^{fa})^2\right)^{\ni\propto+1} - (\measuredangle_{\propto+1}^{ta}+\varrho_{\propto+1}^{fa})^2\right)^{\ni\propto+1}\Big) \end{matrix}}, \\ \sqrt{\begin{matrix} \left(\square_{\partial=1}^{\propto}\left(1-(\varrho_{\partial}^{fb})^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto}\left(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^{\ni\partial}\right)\cdot \\ \left(\left((\varrho_{\propto+1}^{fb})^2\right)^{\ni\propto+1} - (\measuredangle_{\propto+1}^{tb}+\varrho_{\propto+1}^{fb})^2\right)^{\ni\propto+1}\Big) \end{matrix}}, \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\square_{\partial=1}^{\propto}\left(1-(\sigma_{\partial}^{ia})^2\right)^{\ni\partial}\cdot\left(1-(\sigma_{\propto+1}^{ia})^2\right)^{\ni\propto+1}}, \\ \sqrt{1-\square_{\partial=1}^{\propto}\left(1-(\sigma_{\partial}^{ib})^2\right)^{\ni\partial}\cdot\left(1-(\sigma_{\propto+1}^{ib})^2\right)^{\ni\propto+1}} \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\square_{\partial=1}^{\propto}\left(1-(\varrho_{\partial}^{fa})^2\right)^{\ni\partial}\cdot\left(1-(\varrho_{\propto+1}^{fa})^2\right)^{\ni\propto+1}}, \\ \sqrt{1-\square_{\partial=1}^{\propto}\left(1-(\varrho_{\partial}^{fb})^2\right)^{\ni\partial}\cdot\left(1-(\varrho_{\propto+1}^{fb})^2\right)^{\ni\propto+1}} \end{bmatrix} \end{bmatrix}
$$

$$
= \begin{bmatrix} \begin{bmatrix} \sqrt{\square_{\partial=1}^{\propto+1}\left(1-(\varrho_{\partial}^{fa})^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto+1}\left(1-(\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto+1}\left(1-(\varrho_{\partial}^{fb})^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto+1}\left(1-(\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^{\ni\partial}} \end{bmatrix} \\ \left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-(\sigma_{\partial}^{ia})^2\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-(\sigma_{\partial}^{ib})^2\right)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-(\varrho_{\partial}^{fa})^2\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-(\varrho_{\partial}^{fb})^2\right)^{\ni\partial}}\right] \end{bmatrix}
$$

Corollary 2.7. *Let* $\Finv_{\partial} = \left\langle(\measuredangle_{\partial}^{ta},\measuredangle_{\partial}^{tb}),(\sigma_{\partial}^{ia},\sigma_{\partial}^{ib}),(\varrho_{\partial}^{fa},\varrho_{\partial}^{fb})\right\rangle$ *be the IPNNs and all are equal and* $\measuredangle^{tb}\cdot\varrho^{fb}=0$. *Then IPNIWGO*$(\Finv_1,\Finv_2,...,\Finv_{\propto})=\Finv$.

2.3. *generalized IPNIWAO (GIPNIWAO)*

Definition 2.8. Let $\Finv_{\partial} = \left\langle(\measuredangle_{\partial}^{ta},\measuredangle_{\partial}^{tb}),(\sigma_{\partial}^{ia},\sigma_{\partial}^{ib}),(\varrho_{\partial}^{fa},\varrho_{\partial}^{fb})\right\rangle$ be the IPNNs, $\ni_{\partial}$ be a weight of $\Finv_{\partial}$. Then, the GIPNIWAO $(\Finv_1,\Finv_2,...,\Finv_{\propto}) = \left(\Diamond_{\partial=1}^{\propto} \ni_{\partial} \Finv_{\partial}^2\right)^{1/2}$.

Theorem 2.9. *Let* $\Finv_{\partial} = \left\langle\langle(\measuredangle_{\partial}^{ta},\measuredangle_{\partial}^{tb}),(\sigma_{\partial}^{ia},\sigma_{\partial}^{ib}),(\varrho_{\partial}^{fa},\varrho_{\partial}^{fb})\right\rangle$ *be the IPNNs. Then GIPNIWAO* $(\Finv_1,\Finv_2,...,\Finv_{\propto})=$

$$
\begin{bmatrix} \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\ni\partial}}\right] \\ \begin{bmatrix} \sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\ni\partial} - \square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni\partial}} \end{bmatrix} \end{bmatrix}.
$$

Proof. First, we have to prove that

$$\Diamond_{\partial=1}^{\propto} \backepsilon_{\partial} \daleth_{\partial}^{2} = \left[\begin{array}{c} \left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\backepsilon_{\partial}}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\backepsilon_{\partial}}}\right] \\ \left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\backepsilon_{\partial}}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\backepsilon_{\partial}}}\right] \\ \left[\begin{array}{c} \sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\backepsilon_{\partial}} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^2\right)^{\backepsilon_{\partial}}}, \\ \sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\backepsilon_{\partial}} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^2\right)^{\backepsilon_{\partial}}} \end{array}\right] \end{array}\right].$$

If $\partial = 2$,then

$$\backepsilon_{1} \daleth_{1}^{2} = \left[\begin{array}{c} \left[\sqrt{1-\left(1-\left((\measuredangle_{1}^{ta})^2\right)^2\right)^{\backepsilon_{1}}}, \sqrt{1-\left(1-\left((\measuredangle_{1}^{tb})^2\right)^2\right)^{\backepsilon_{1}}},\right] \\ \left[\sqrt{1-\left(1-\left((\sigma_{1}^{ia})^2\right)^2\right)^{\backepsilon_{1}}}, \sqrt{1-\left(1-\left((\sigma_{1}^{ib})^2\right)^2\right)^{\backepsilon_{1}}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1-\left((\measuredangle_{1}^{ta})^2\right)^2\right)^{\backepsilon_{1}} - \left(1-\left((\measuredangle_{1}^{ta}+\varrho_{1}^{fa})^2\right)^2\right)^{\backepsilon_{1}}}, \\ \sqrt{\left(1-\left((\measuredangle_{1}^{tb})^2\right)^2\right)^{\backepsilon_{1}} - \left(1-\left((\measuredangle_{1}^{tb}+\varrho_{1}^{fb})^2\right)^2\right)^{\backepsilon_{1}}} \end{array}\right] \end{array}\right]$$

and

$$\backepsilon_{2} \daleth_{2}^{2} = \left[\begin{array}{c} \left[\sqrt{1-\left(1-\left((\measuredangle_{2}^{ta})^2\right)^2\right)^{\backepsilon_{1}}}, \sqrt{1-\left(1-\left((\measuredangle_{2}^{tb})^2\right)^2\right)^{\backepsilon_{1}}},\right] \\ \left[\sqrt{1-\left(1-\left((\sigma_{2}^{ia})^2\right)^2\right)^{\backepsilon_{1}}}, \sqrt{1-\left(1-\left((\sigma_{2}^{ib})^2\right)^2\right)^{\backepsilon_{1}}}\right] \\ \left[\begin{array}{c} \sqrt{\left(1-\left((\measuredangle_{2}^{ta})^2\right)^2\right)^{\backepsilon_{1}} - \left(1-\left((\measuredangle_{2}^{ta}+\varrho_{2}^{fa})^2\right)^2\right)^{\backepsilon_{1}}}, \\ \sqrt{\left(1-\left((\measuredangle_{2}^{tb})^2\right)^2\right)^{\backepsilon_{1}} - \left(1-\left((\measuredangle_{2}^{tb}+\varrho_{2}^{fb})^2\right)^2\right)^{\backepsilon_{1}}} \end{array}\right] \end{array}\right].$$

We get, $\ni_1 \dashv_1 \veebar \ni_2 \dashv_2 =$

$$
\left[\begin{array}{l}
\left[\begin{array}{l}
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\measuredangle_2^{ta})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\measuredangle_2^{ta})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}} \\
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\measuredangle_2^{tb})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\measuredangle_2^{tb})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}}
\end{array}\right] \\
\left[\begin{array}{l}
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_2^{ia})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_2^{ia})^2\right)^2\right)^{\ni_1}}\right)^2,\end{array}} \\
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_2^{ib})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_2^{ib})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}}
\end{array}\right] \\
\left[\begin{array}{l}
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\ni_1}}\right)^2, \\ \qquad -\left[\begin{array}{l}\left(\sqrt{\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^2\right)^{\ni_1}}\right)^2\cdot \\ \left(\sqrt{\left(1-\left((\measuredangle_2^{ta})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}\right]\end{array}} \\
\sqrt{\begin{array}{l}\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\ni_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\ni_1}}\right)^2, \\ -\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\ni_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\ni_1}}\right)^2, \\ \qquad -\left[\begin{array}{l}\left(\sqrt{\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^2\right)^{\ni_1}}\right)^2\cdot \\ \left(\sqrt{\left(1-\left((\measuredangle_2^{tb})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}\right]\end{array}}
\end{array}\right]
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\left[\sqrt{1-\Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\backepsilon\partial}}\right] \\
\left[\sqrt{1-\Box_{\partial=1}^{2}\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\Box_{\partial=1}^{2}\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon\partial}},\right] \\
\left[\begin{array}{c}
\sqrt{\Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\backepsilon\partial} - \Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^2\right)^{\backepsilon\partial}}, \\
\sqrt{\Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\backepsilon\partial} - \Box_{\partial=1}^{2}\left(1-\left((\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^2\right)^{\backepsilon\partial}}
\end{array}\right]
\end{array}\right]
$$

In general,

$$
= \left[\begin{array}{c}
\left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\backepsilon\partial}},\right] \\
\left[\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon\partial}}\right] \\
\left[\begin{array}{c}
\sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{ta})^2\right)^2\right)^{\backepsilon\partial} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^2\right)^{\backepsilon\partial}}, \\
\sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{tb})^2\right)^2\right)^{\backepsilon\partial} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^2\right)^{\backepsilon\partial}}
\end{array}\right]
\end{array}\right].
$$

If $\partial = \propto +1$,then $\Diamond_{\partial=1}^{\propto} \backepsilon_\partial \Finv_\partial^2 + \backepsilon_{\propto+1} \Finv_{\propto+1}^2 = \Diamond_{\partial=1}^{\propto+1} \backepsilon_\partial \Finv_\partial^2$.

Now,$\Diamond_{\partial=1}^{\propto} \backepsilon_\partial \Finv_\partial^2 + \backepsilon_{\propto+1} \Finv_{\propto+1}^2 = \backepsilon_1 \Finv_1^2 \veebar \backepsilon_2 \Finv_2^2 \veebar \ldots \veebar \backepsilon_\propto \Finv_\propto^2 \veebar \backepsilon_{\propto+1} \Finv_{\propto+1}^2$

$$
= \left[\begin{array}{c}
\left[\sqrt{\begin{array}{c}\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\ni_\partial}}\right)^2+\left(\sqrt{1-\left(1-\left((\measuredangle_{\propto+1}^{ta})^2\right)^2\right)^{\ni_1}}\right)^2,\\ -\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\measuredangle_{\propto+1}^{ta})^2\right)^2\right)^{\ni_1}}\right)^{2}\end{array}}, \sqrt{\begin{array}{c}\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\ni_\partial}}\right)^2+\left(\sqrt{1-\left(1-\left((\measuredangle_{\propto+1}^{tb})^2\right)^2\right)^{\ni_1}}\right)^2,\\ -\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\measuredangle_{\propto+1}^{tb})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}}\right]\\
\left[\sqrt{\begin{array}{c}\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\ni_\partial}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ia})^2\right)^2\right)^{\ni_1}}\right)^2,\\ -\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ia})^2\right)^2\right)^{\ni_1}}\right)^{2}\end{array}}, \sqrt{\begin{array}{c}\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\ni_\partial}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ib})^2\right)^2\right)^{\ni_1}}\right)^2,\\ -\left(\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ib})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}}\right]\\
-\left[\begin{array}{c}\left(\sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta})^2\right)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\\ \left(\sqrt{\left(1-\left((\measuredangle_{\propto+1}^{ta})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_{\propto+1}^{ta}+\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^2,\\ \left(\sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb})^2\right)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2\cdot\\ \left(\sqrt{\left(1-\left((\measuredangle_{\propto+1}^{tb})^2\right)^2\right)^{\ni_1}-\left(1-\left((\measuredangle_{\propto+1}^{tb}+\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2\end{array}\right]
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{ta})^2\right)^2\right)^{\ni_\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{tb})^2\right)^2\right)^{\ni_\partial}}\right]\\
\left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\sigma_{1}^{ia})^2\right)^2\right)^{\ni_\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\sigma_{1}^{ib})^2\right)^2\right)^{\ni_\partial}}\right]\\
\left[\begin{array}{c}\sqrt{\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{ta})^2\right)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{ta}+\varrho_{1}^{fa})^2\right)^2\right)^{\ni_\partial}},\\ \sqrt{\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{tb})^2\right)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_{1}^{tb}+\varrho_{1}^{fb})^2\right)^2\right)^{\ni_\partial}}\end{array}\right]
\end{array}\right]
$$

and $\lozenge_{\partial=1}^{\propto+1}\Big(\quad \ni_\partial \quad \daleth_\partial^2 \Big) =$

$$\left[\begin{array}{c} \left[\left(\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{ta})^2\Big)^2\Big)^{\ni_\partial}}\right),\left(\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{tb})^2\Big)^2\Big)^{\ni_\partial}}\right)\right] \\ \left[\left(\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\sigma_\partial^{ia})^2\Big)^2\Big)^{\ni_\partial}}\right),\left(\sqrt{1-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\sigma_\partial^{ib})^2\Big)^2\Big)^{\ni_\partial}}\right)\right] \\ \left[\begin{array}{c}\left(\sqrt{\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{ta})^2\Big)^2\Big)^{\ni_\partial}-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{ta}+\varrho_\partial^{fa})^2\Big)^2\Big)^{\ni_\partial}}\right), \\ \left(\sqrt{\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{tb})^2\Big)^2\Big)^{\ni_\partial}-\square_{\partial=1}^{\propto+1}\Big(1-\Big((\measuredangle_\partial^{tb}+\varrho_\partial^{fb})^2\Big)^2\Big)^{\ni_\partial}}\right)\end{array}\right] \end{array}\right].$$

Corollary 2.10. *Let* $\daleth_\partial = \big\langle(\measuredangle_\partial^{ta},\measuredangle_\partial^{tb}),(\sigma_\partial^{ia},\sigma_\partial^{ib}),(\varrho_\partial^{fa},\varrho_\partial^{fb})\big\rangle$ *be the IPNNs and all are equal. Then GIPNIWAO* $(\daleth_1,\daleth_2,...,\daleth_\propto)=\daleth$.

2.4. *Generalized IPNIWGO (GIPNIWGO)*

Definition 2.11. Let $\daleth_\partial = \big\langle(\measuredangle_\partial^{ta},\measuredangle_\partial^{tb}),(\sigma_\partial^{ia},\sigma_\partial^{ib}),(\varrho_\partial^{fa},\varrho_\partial^{fb})\big\rangle$ be the IPNNs and $\ni_\partial$ be the weight of $\daleth_\partial$,where $\partial=1,2,...,\propto$. Then,the GIPNIWGO$(\daleth_1,\daleth_2,...,\daleth_\propto)=\frac{1}{\zeta}\Big(\square_{\partial=1}^{\propto}(\zeta\daleth_\partial)^{\ni_\partial}\Big)$.

Theorem 2.12. *Let* $\daleth_\partial = \big\langle\big\langle(\measuredangle_\partial^{ta},\measuredangle_\partial^{tb}),(\sigma_\partial^{ia},\sigma_\partial^{ib}),(\varrho_\partial^{fa},\varrho_\partial^{fb})\big\rangle$ *be the collection of IPNNs. Then the GIPNIWGO* $(\daleth_1,\daleth_2,...,\daleth_\propto)=$

$$\left[\begin{array}{c}\left[\begin{array}{c}\sqrt{\square_{\partial=1}^{\propto}\left(1-\Big((\varrho_\partial^{fa})^2\Big)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto}\left(1-\Big((\measuredangle_\partial^{ta}+\varrho_\partial^{fa})^2\Big)^2\right)^{\ni_\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto}\left(1-\Big((\varrho_\partial^{fb})^2\Big)^2\right)^{\ni_\partial}-\square_{\partial=1}^{\propto}\left(1-\Big((\measuredangle_\partial^{tb}+\varrho_\partial^{fb})^2\Big)^2\right)^{\ni_\partial}}\end{array}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\Big((\sigma_\partial^{ia})^2\Big)^2\right)^{\ni_\partial}},\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\Big((\sigma_\partial^{ib})^2\Big)^2\right)^{\ni_\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\Big((\varrho_\partial^{fa})^2\Big)^2\right)^{\ni_\partial}},\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\Big((\varrho_\partial^{fb})^2\Big)^2\right)^{\ni_\partial}}\right]\end{array}\right]$$

Proof. Using the induction method,

$$\square_{\partial=1}^{\propto}(\zeta\daleth_\partial)^{\ni_\partial}=\left[\begin{array}{c}\left[\begin{array}{c}\sqrt{\square_{\partial=1}^{\propto}\Big(1-\Big((\varrho_\partial^{fa})^2\Big)^2\Big)^{\ni_\partial}-\square_{\partial=1}^{\propto}\Big(1-\Big((\measuredangle_\partial^{ta}+\varrho_\partial^{fa})^2\Big)^2\Big)^{\ni_\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto}\Big(1-\Big((\varrho_\partial^{fb})^2\Big)^2\Big)^{\ni_\partial}-\square_{\partial=1}^{\propto}\Big(1-\Big((\measuredangle_\partial^{tb}+\varrho_\partial^{fb})^2\Big)^2\Big)^{\ni_\partial}}\end{array}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-\Big((\sigma_\partial^{ia})^2\Big)^2\Big)^{\ni_\partial}},\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-\Big((\sigma_\partial^{ib})^2\Big)^2\Big)^{\ni_\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-\Big((\varrho_\partial^{fa})^2\Big)^2\Big)^{\ni_\partial}},\sqrt{1-\square_{\partial=1}^{\propto}\Big(1-\Big((\varrho_\partial^{fb})^2\Big)^2\Big)^{\ni_\partial}}\right]\end{array}\right]$$

If $\partial = 2$,then

$$(\zeta\daleth_1)^{\backepsilon_1} = \begin{bmatrix} \begin{bmatrix} \sqrt{\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1} - \left(1-\left((\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}, \\ \sqrt{\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1} - \left(1-\left((\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}} \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon_1}} \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}} \end{bmatrix} \end{bmatrix}$$

and

$$(\zeta\daleth_2)^{\backepsilon_2} = \begin{bmatrix} \begin{bmatrix} \sqrt{\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1} - \left(1-\left((\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}, \\ \sqrt{\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1} - \left(1-\left((\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}} \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\left(1-\left((\sigma_2^{ia})^2\right)^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-\left((\sigma_2^{ib})^2\right)^2\right)^{\backepsilon_1}} \end{bmatrix} \\ \begin{bmatrix} \sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}, \sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}, \end{bmatrix} \end{bmatrix}$$

We get, $(\zeta\daleth_1)^{\backepsilon_1}\,\bar{\wedge}\,(\zeta\daleth_2)^{\backepsilon_2}$

$$
= \left[\begin{array}{l}
\left[\begin{array}{l}
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left[\begin{array}{l}
\left(\sqrt{\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}-\left(1-\left((\measuredangle_1^{ta}+\measuredangle_1^{ta})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot \\
\left(\sqrt{\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}-\left(1-\left((\measuredangle_2^{ta}+\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}\right]
\end{array}}, \\
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left[\begin{array}{l}
\left(\sqrt{\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}-\left(1-\left((\measuredangle_1^{tb}+\measuredangle_1^{tb})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot \\
\left(\sqrt{\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}-\left(1-\left((\measuredangle_2^{tb}+\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}\right]
\end{array}}
\end{array}\right] \\
\left[\begin{array}{l}
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_2^{ia})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_2^{ia})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}}, \\
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\sigma_2^{ib})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\sigma_2^{ib})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}}
\end{array}\right] \\
\left[\begin{array}{l}
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fa})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}}, \\
\sqrt{\begin{array}{l}
\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2+\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2 \\
-\left(\sqrt{1-\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2\cdot\left(\sqrt{1-\left(1-\left((\varrho_2^{fb})^2\right)^2\right)^{\backepsilon_1}}\right)^2
\end{array}}
\end{array}\right]
\end{array}\right]
$$

$$= \left[\begin{array}{l} \left[\begin{array}{l} \sqrt{\square_{\partial=1}^{2}\left(1-\left((\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}-\square_{\partial=1}^{2}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{2}\left(1-\left((\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}-\square_{\partial=1}^{2}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}}, \end{array}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{2}\left(1-\left((\sigma_{\partial}^{ia})^{2}\right)^{2}\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\left(1-\left((\sigma_{\partial}^{ib})^{2}\right)^{2}\right)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{2}\left(1-\left((\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{2}\left(1-\left((\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}}\right] \end{array}\right]$$

If $\partial = \propto$,then

$$= \left[\begin{array}{l} \left[\begin{array}{l} \sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}-\square_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}} \end{array}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^{2}\right)^{2}\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^{2}\right)^{2}\right)^{\ni\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^{2}\right)^{2}\right)^{\ni\partial}}, \sqrt{1-\square_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^{2}\right)^{2}\right)^{\ni\partial}}\right] \end{array}\right]$$

If $\partial = \propto +1$,then $\square_{\partial=1}^{\propto}(\zeta\dashv_{\partial})^{\ni\partial}\cdot(\zeta\dashv_{\propto+1})^{\ni\propto+1}=\square_{\partial=1}^{\propto+1}(\zeta\dashv_{\partial})^{\ni\partial}$.

Now,$\square_{\partial=1}^{\propto}(\zeta\dashv_{\partial})^{\ni\partial}\cdot(\zeta\dashv_{\propto+1})^{\ni\propto+1}=(\zeta\dashv_{1})^{\ni 1}\,\overline{\wedge}\,(\zeta\dashv_{2})^{\ni 2}\,\overline{\wedge}\,...\,\overline{\wedge}\,(\zeta\dashv_{\propto})^{\ni\propto}\,\overline{\wedge}\,(\zeta\dashv_{\propto+1})^{\ni\propto+1}$

$$
= \left[\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left[\begin{array}{c}
\left(\sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{ta}+\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \\
\left(\sqrt{\left(1-\left((\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1} - \left(1-\left((\measuredangle_{\propto+1}^{ta}+\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^2
\end{array}\right]
\end{array}}, \\
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left[\begin{array}{c}
\left(\sqrt{\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial} - \Box_{\partial=1}^{\propto}\left(1-\left((\measuredangle_{\partial}^{tb}+\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \\
\left(\sqrt{\left(1-\left((\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1} - \left(1-\left((\measuredangle_{\propto+1}^{tb}+\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2
\end{array}\right]
\end{array}}
\end{array}\right] \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ia})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ia})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ia})^2\right)^2\right)^{\ni_1}}\right)^{2},
\end{array}} \\
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ib})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\sigma_{\partial}^{ib})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\sigma_{\propto+1}^{ib})^2\right)^2\right)^{\ni_1}}\right)^2
\end{array}}
\end{array}\right] \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fa})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fa})^2\right)^2\right)^{\ni_1}}\right)^{2},
\end{array}} \\
\sqrt{\begin{array}{c}
\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2 + \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2 \\
-\left(\sqrt{1-\Box_{\partial=1}^{\propto}\left(1-\left((\varrho_{\partial}^{fb})^2\right)^2\right)^{\ni_\partial}}\right)^2 \cdot \left(\sqrt{1-\left(1-\left((\varrho_{\propto+1}^{fb})^2\right)^2\right)^{\ni_1}}\right)^2
\end{array}}
\end{array}\right]
\end{array}\right]
$$

$$= \left[\begin{array}{l} \left[\begin{array}{l} \sqrt{\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon\partial} - \square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_1^{ta}+\varrho_1^{fa})^2\right)^2\right)^{\backepsilon\partial}}, \\ \sqrt{\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon\partial} - \square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_1^{tb}+\varrho_1^{fb})^2\right)^2\right)^{\backepsilon\partial}} \end{array}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\sigma_1^{ia})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\sigma_1^{ib})^2\right)^2\right)^{\backepsilon\partial}}\right] \\ \left[\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_1^{fa})^2\right)^2\right)^{\backepsilon\partial}}, \sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_1^{fb})^2\right)^2\right)^{\backepsilon\partial}}\right] \end{array}\right]$$

Hence

$$\frac{1}{\zeta}\left(\square_{\partial=1}^{\propto+1}(\zeta\dashv_\partial)^{\backepsilon\partial}\right) =$$

$$\left[\begin{array}{c} \left(\sqrt{\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_\partial^{fa})^2\right)^2\right)^{\backepsilon\partial} - \square_{\partial=1}^{\propto+1}\left(1-\left((\measuredangle_\partial^{ta}+\varrho_\partial^{fa})^2\right)^2\right)^{\backepsilon\partial}}\right), \\ \left(\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\sigma_\partial^{ia})^2\right)^2\right)^{\backepsilon\partial}}\right)\left(\sqrt{1-\square_{\partial=1}^{\propto+1}\left(1-\left((\varrho_\partial^{fa})^2\right)^2\right)^{\backepsilon\partial}}\right) \end{array}\right]$$

Corollary 2.13. *Let $\dashv_\partial = \left\langle(\measuredangle_\partial^{ta},\measuredangle_\partial^{tb}),(\sigma_\partial^{ia},\sigma_\partial^{ib}),(\varrho_\partial^{fa},\varrho_\partial^{fb})\right\rangle$ be the collection of IPNNs and all are equal. Then the GIPNIWGO $(\dashv_1,\dashv_2,...,\dashv_\propto) = \dashv$.*

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