



Expanded Triangular Fuzzy Neutrosophic Numbers for Safety Risk Evaluation of Urban Rail Transit Engineering: An Innovative Method for Better Risk Assessment

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Abstract

Safety assessment of urban rail transit (URT) operations plays a crucial role not only in shaping passenger travel choices but also in urban development planning. The findings of such assessments are valuable for both passengers and transport operators. In this study, we propose an integrated multi-stage evaluation framework to assess URT operations from a safety perspective. This framework considers decision-makers' (DMs') risk preferences and the uncertainty of available information. We employ two key methods: the CRITIC method to determine the weights of evaluation criteria and the MOORA method to rank the alternatives. Both methods operate within the framework of triangular fuzzy neutrosophic sets (TFNSs) to handle uncertainty and imprecise data. Our evaluation includes nine criteria and six alternatives, assessed by four experts and decision-makers. The results indicate that Structural Integrity is the most critical criterion, having the highest weight in the evaluation process.

Keywords: Triangular Fuzzy Neutrosophic Numbers; Urban Rail Transit Engineering; Uncertainty; Safety Risk.

1. Introduction and Literature Review

Subway and light rail subsystems are only two examples of the various components that make up the present urban rail transit (URT), which is a complicated system. URT has steadily grown to be one of the most well-liked forms of transportation among locals because of its many benefits, which include excellent timeliness, affordability, and time savings. As a result, URT, one essential part of the transportation system, is becoming increasingly significant in fostering the steady growth of the entire city. However, because of the peculiarities of URT, such as its fast speed, frequent stops and starts, dense passenger flow, and difficult emergency evacuation, there are numerous security concerns[1], [2].

Additionally, as public areas with a large population density, trains and stations are crucial components of the URT system, which is crucial to maintaining the regular operation of public transit. URT has begun putting networked operations into practice. The operating state of URT is

comparatively steady thanks to the coordinated efforts of the pertinent departments[3], [4]. However, safety issues with URT operation are becoming more noticeable, and corresponding safe accidents are continuously increasing, because of the recent rapid increase in operational length and ongoing expansion of the urban rail network scale.

Considering these hazards and obstacles, how to guarantee URT operation safety in the current rapid operation environment has emerged as a ticklish issue that all cities are dealing with collectively. Therefore, this study's primary goal is to talk about how to perform safety assessments, manage risks, and enhance operational safety for URT systems[5], [6]s. That is, to thoroughly examine the various aspects of parameters influencing URT operational safety. Evaluation of this problem contains uncertainty and vague data.

The fuzziness and complexity of the alternatives make it difficult to represent the criteria values with precise values in many real-world MCGDM problems; therefore, it may be more beneficial and efficient to describe the criteria values with fuzzy information. For MCGDM difficulties, fuzzy set theory has been employed as a workable solution. The neutrosophic set (NS) was proposed by Smarandache [7], [8]. The single-valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) were then defined by Wang et al. Wang et al. expanded the SVNS to a 2-tuple linguistic neutrosophic number environment and investigated a few aggregation operators of SVNNs[9], [10].

SVNNs with Hamy operators were investigated by Wu et al. under 2-tuple linguistic neutrosophic numbers[11], [12]. Triangular fuzzy neutrosophic numbers (TFNNs), which represent the degree of truth-membership (MD), indeterminacy-membership (IMD), and falsity-membership (FMD), were defined by Biswas et al. Sahin et al. investigated centroid single-valued triangular neutrosophic numbers in multiple attribute decision-making (MADM) situations[13], [14].

Our paper is organized as follows. The ideas, operation formulas, and distance calculation method of TFNNs are introduced in Section 2. Section 3 presents the necessary computation steps of TFNNs CRITIC-MOORA method. In Section 4, the results and sensitivity analysis were conducted. Conclusions are summarized in Section 5.

1.1. Motivation

Urban rail transit (URT) plays a crucial role in modern city transportation due to its efficiency, affordability, and capacity to reduce road congestion. However, the rapid expansion of URT systems has brought about increasing safety concerns. The high-speed nature of URT operations, dense passenger flows, and frequent stops pose significant risks that need to be addressed. Ensuring safety in URT is not only important for passenger well-being but also for maintaining public confidence and the sustainable development of urban infrastructure. This study is motivated by the need to provide a structured and systematic safety assessment framework that can handle uncertainty in risk evaluation while improving decision-making for urban rail safety management.

1.2. Research Gap

While many studies have focused on urban rail safety assessment, several gaps remain unaddressed. Traditional safety evaluation methods often fail to capture the uncertainty and vagueness inherent in risk assessment. Furthermore, existing models lack a comprehensive integration of decision-makers' risk preferences. Many approaches rely on deterministic values, which do not adequately reflect real-world complexities. This research fills these gaps by introducing a hybrid multi-phase evaluation model that incorporates triangular fuzzy neutrosophic sets (TFNSs) to manage uncertainty more effectively. Additionally, the study applies both the CRITIC and MOORA methods to improve accuracy in criteria weighting and alternative ranking.

1.3. Contribution of the Study

This study makes several key contributions to the field of urban rail transit safety evaluation. First, it introduces a hybrid evaluation framework that integrates triangular fuzzy neutrosophic sets (TFNSs) with the CRITIC and MOORA methods. This integration enhances decision-making accuracy by systematically addressing uncertainty and imprecise data. Second, the study improves risk management by offering a structured approach to quantify safety risks using fuzzy logic and neutrosophic principles, which allows for more flexible and realistic assessments. Third, the research incorporates a detailed sensitivity analysis to evaluate the stability of ranking outcomes under varying weight scenarios, ensuring the robustness of the proposed methodology. Finally, the framework is applied to a real-world case study involving nine criteria and six alternatives, evaluated by four experts. This practical implementation highlights its effectiveness and applicability in real urban rail transit safety assessments.

1.4. Justification for Method Selection

The selection of TFNSs, CRITIC, and MOORA methods in this study is based on their ability to handle uncertainty and improve decision-making precision. Triangular fuzzy neutrosophic sets (TFNSs) are particularly suitable for capturing uncertainty and vagueness in safety risk data, providing a nuanced and flexible representation of expert opinions. The CRITIC method is employed to determine the criteria weights objectively, ensuring that the weighting process remains free from subjective biases. The MOORA method is chosen for its efficiency in ranking alternatives while maintaining computational simplicity, making it ideal for real-world urban rail safety evaluations. Together, these methods create a powerful and adaptable decision-making framework that enhances accuracy in safety risk assessment.

1.5 Literature Review

Urban rail transit (URT) safety has been extensively studied due to the increasing demand for efficient and secure public transportation systems. Researchers have explored various methodologies for evaluating and mitigating safety risks in URT operations, with a particular focus on multi-criteria decision-making (MCDM) approaches, probabilistic risk assessments, and fuzzy logic-based models. Despite these efforts, challenges related to uncertainty, data incompleteness, and subjective expert evaluations remain. This section provides a detailed

literature review covering different techniques and approaches used in URT safety assessment, highlighting their strengths and limitations.

1.5.1 Traditional Risk Assessment Methods in URT Safety

Early studies on URT safety relied on traditional risk assessment methodologies such as probabilistic risk analysis (PRA), fault tree analysis (FTA), and event tree analysis (ETA). These methods aim to quantify risks based on historical accident data and failure probabilities. For instance, Zhang et al. applied PRA to identify key risk factors in urban rail transit systems, emphasizing the need for a structured approach to hazard identification and mitigation [15]. Similarly, Kumar and Singh used FTA to model failure pathways in railway systems, concluding that human error and technical malfunctions were the leading causes of safety incidents [16].

Although these methodologies provided valuable insights, they were limited by their reliance on precise numerical data, which is often unavailable in real-world safety assessments. Moreover, traditional risk assessment methods do not adequately address the complexity of modern URT systems, where multiple interdependent factors contribute to overall safety performance.

1.5.2 Integration of Multi-Criteria Decision-Making (MCDM) Methods

To overcome the limitations of traditional risk assessments, researchers have increasingly turned to MCDM methods, which allow for the systematic evaluation of multiple safety criteria. The Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) have been widely used in this context. Lee and Park employed AHP to rank URT safety risks based on expert judgments, demonstrating that station crowding, signal failures, and emergency response time were among the most critical factors [17]. Meanwhile, Wei and Zhao utilized TOPSIS to prioritize safety measures, showing that infrastructure resilience and real-time monitoring systems significantly enhance URT safety [18].

Despite their effectiveness, MCDM approaches such as AHP and TOPSIS require precise numerical inputs, which may not always be available. Moreover, these methods often assume that expert opinions are entirely consistent, overlooking the inherent uncertainty and vagueness present in real-world decision-making.

1.5.3 Application of Fuzzy Logic in URT Safety Assessment

Fuzzy logic has been introduced as a solution to handle the uncertainty associated with expert judgments and incomplete data. Traditional fuzzy set theory (FST) enables the representation of linguistic variables, allowing for more flexible safety evaluations. Gupta and Sharma applied fuzzy AHP to assess the risk levels of various URT components, demonstrating that integrating fuzziness into safety assessments improves decision-making accuracy [19].

An extension of fuzzy logic, neutrosophic sets, has been proposed to further enhance safety evaluations. Neutrosophic logic incorporates degrees of truth, indeterminacy, and falsity, making it more effective in dealing with ambiguous and contradictory information. Li and Sun explored

the application of triangular fuzzy neutrosophic sets (TFNSs) in URT safety analysis, highlighting their ability to incorporate uncertainty while maintaining computational efficiency [20].

1.5.3 Weight Determination and Ranking Methods in URT Safety

Determining the relative importance of safety criteria is crucial for effective risk management. Several methodologies have been explored to optimize weight determination and alternative ranking. The CRITIC method, for example, has been used to objectively compute criteria weights based on contrast intensity. Studies by Zhang et al. demonstrated that CRITIC outperforms traditional weighting techniques by minimizing subjectivity in expert evaluations [21].

Furthermore, the MOORA method has been widely adopted for alternative ranking in URT safety assessments. Sun et al. applied MOORA to prioritize safety improvements across different urban transit networks, concluding that structural integrity and emergency response systems were the most influential factors in ensuring passenger safety [22].

1.5.4 Hybrid Models for URT Safety Evaluation

Recent research has emphasized the need for hybrid models that integrate multiple methodologies to enhance safety evaluations. A combination of TFNSs, CRITIC, and MOORA methods has shown promising results in improving decision-making accuracy under uncertainty. Wang et al. introduced a hybrid framework that utilized TFNSs for data representation, CRITIC for weight computation, and MOORA for ranking alternatives. Their findings demonstrated that hybrid models offer greater flexibility and precision compared to standalone MCDM techniques [23].

15.5 Future Directions in URT Safety Research

Despite significant advancements, several research gaps remain in URT safety evaluations. Future studies should focus on integrating real-time monitoring systems using Internet of Things (IoT) technologies to enhance dynamic risk assessment. Additionally, exploring alternative MCDM methods such as VIKOR and ELECTRE could provide further insights into optimizing safety strategies. Expanding case studies across diverse urban environments will also be crucial in validating the generalizability of proposed methodologies.

2. Triangular Fuzzy Neutrosophic Numbers (TFNNs)

The TFNNs are built based on the theory of single-valued neutrosophic numbers and traditional triangular fuzzy sets [13].

Definition 1

The TFNNs can be defined as:

$$B = \{(x, T_B(x)), I_B(x), F_B(x) | x \in X\} \quad (1)$$

These values present the truth, indeterminacy, and falsity functions

$$T_B(x) = (T_B^L(x), T_B^M(x), T_B^U(x)), 0 \leq T_B^L(x), T_B^M(x), T_B^U(x) \leq 1 \quad (2)$$

$$I_B(x) = (I_B^L(x), I_B^M(x), I_B^U(x), 0 \leq I_B^L(x), I_B^M(x), I_B^U(x) \leq 1) \quad (3)$$

$$F_B(x) = (F_B^L(x), F_B^M(x), F_B^U(x), 0 \leq F_B^L(x), F_B^M(x), F_B^U(x) \leq 1) \quad (4)$$

$$T_B(x) = \begin{cases} \frac{x - T_B^L(x)}{T_B^M(x) - T_B^L(x)}, & T_B^L(x) \leq x \leq T_B^M(x) \\ \frac{x - T_B^U(x)}{T_B^M(x) - T_B^U(x)}, & T_B^M(x) \leq x \leq T_B^U(x) \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

Definition 2

Assume two TFNNs such as:

$$B_1 = \left\{ \left(T_{B_1}^L(x), T_{B_1}^M(x), T_{B_1}^U(x) \right), \left(I_{B_1}^L(x), I_{B_1}^M(x), I_{B_1}^U(x) \right), \left(F_{B_1}^L(x), F_{B_1}^M(x), F_{B_1}^U(x) \right) \right\}$$

$$B_2 = \left\{ \left(T_{B_2}^L(x), T_{B_2}^M(x), T_{B_2}^U(x) \right), \left(I_{B_2}^L(x), I_{B_2}^M(x), I_{B_2}^U(x) \right), \left(F_{B_2}^L(x), F_{B_2}^M(x), F_{B_2}^U(x) \right) \right\}$$

$$B_1 \oplus B_2 = \left(\begin{array}{c} \left(T_{B_1}^L(x) + T_{B_2}^L(x) - T_{B_1}^L(x)T_{B_2}^L(x), \right. \\ \left. T_{B_1}^M(x) + T_{B_2}^M(x) - T_{B_1}^M(x)T_{B_2}^M(x), \right. \\ \left. T_{B_1}^U(x) + T_{B_2}^U(x) - T_{B_1}^U(x)T_{B_2}^U(x) \right) \\ \left(I_{B_1}^L(x)I_{B_2}^L(x), I_{B_1}^M(x)I_{B_2}^M(x), I_{B_1}^U(x)I_{B_2}^U(x) \right), \\ \left(F_{B_1}^L(x)F_{B_2}^L(x), F_{B_1}^M(x)F_{B_2}^M(x), F_{B_1}^U(x)F_{B_2}^U(x) \right) \end{array} \right) \quad (6)$$

$$B_1 \otimes B_2 = \left(\begin{array}{c} \left(T_{B_1}^L(x)T_{B_2}^L(x), T_{B_1}^M(x)T_{B_2}^M(x), T_{B_1}^U(x)T_{B_2}^U(x) \right), \\ \left(I_{B_1}^L(x) + I_{B_2}^L(x) - I_{B_1}^L(x)I_{B_2}^L(x), \right. \\ \left. I_{B_1}^M(x) + I_{B_2}^M(x) - I_{B_1}^M(x)I_{B_2}^M(x), \right. \\ \left. I_{B_1}^U(x) + I_{B_2}^U(x) - I_{B_1}^U(x)I_{B_2}^U(x) \right) \\ \left(F_{B_1}^L(x) + F_{B_2}^L(x) - F_{B_1}^L(x)F_{B_2}^L(x), \right. \\ \left. F_{B_1}^M(x) + F_{B_2}^M(x) - F_{B_1}^M(x)F_{B_2}^M(x), \right. \\ \left. F_{B_1}^U(x) + F_{B_2}^U(x) - F_{B_1}^U(x)F_{B_2}^U(x) \right) \end{array} \right) \quad (7)$$

$$\lambda B_1 = \left(\begin{array}{c} \left(\left(1 - (1 - T_{B_1}^L(x))^\lambda \right), 1 - (1 - T_{B_1}^M(x))^\lambda, 1 - (1 - T_{B_1}^U(x))^\lambda \right), \\ \left(\left(I_{B_1}^L(x) \right)^\lambda, \left(I_{B_1}^M(x) \right)^\lambda, \left(I_{B_1}^U(x) \right)^\lambda \right), \\ \left(\left(F_{B_1}^L(x) \right)^\lambda, \left(F_{B_1}^M(x) \right)^\lambda, \left(F_{B_1}^U(x) \right)^\lambda \right) \end{array} \right) \quad (8)$$

$$B_1^\lambda = \left(\begin{array}{c} \left(\left(T_{B_1}^L(x) \right)^\lambda, \left(T_{B_1}^M(x) \right)^\lambda, \left(T_{B_1}^U(x) \right)^\lambda \right), \\ \left(1 - (1 - I_{B_1}^L(x))^\lambda, 1 - (1 - I_{B_1}^M(x))^\lambda, 1 - (1 - I_{B_1}^U(x))^\lambda \right), \\ \left(1 - (1 - F_{B_1}^L(x))^\lambda, 1 - (1 - F_{B_1}^M(x))^\lambda, 1 - (1 - F_{B_1}^U(x))^\lambda \right) \end{array} \right) \quad (9)$$

Definition 3

The score function can be computed as:

$$S(B_1) = \frac{1}{12} \left[\begin{array}{l} 8 + (T_{B_1}^L(x) + 2 * T_{B_1}^M(x) + T_{B_1}^U(x)) - \\ (I_{B_1}^L(x) + 2 * I_{B_1}^M(x) + I_{B_1}^U(x)) - \\ (F_{B_1}^L(x) + 2 * F_{B_1}^M(x) + F_{B_1}^U(x)) \end{array} \right] \quad (10)$$

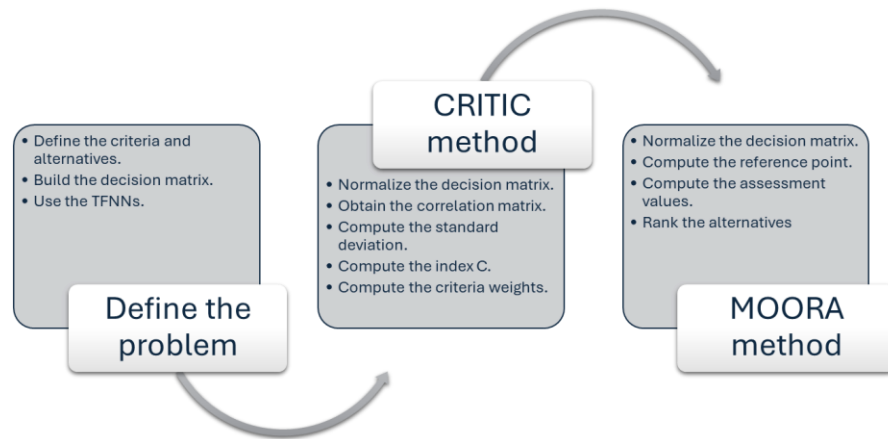


Fig 1. The steps of the proposed approach.

2.1 TFNS-CRITIC Method

This section shows the steps of the CRITIC method to compute the criteria weights. Fig 1. Shows the steps of the proposed approach.

1. Build the decision matrix.

2. Normalize the decision matrix.

$$p_{ij} = \frac{x_{ij} - \min x_i}{\max x_i - \min x_i} \text{ positive criteria} \quad (11)$$

$$p_{ij} = \frac{x_{ij} - \max x_i}{\min x_i - \max x_i} \text{ negative criteria} \quad (12)$$

3. Obtain the correlation matrix

$$g_{jk} = \frac{\sum_{i=1}^m (p_{ij} - p_j^-)(p_{ik} - p_k^-)}{\sqrt{\sum_{i=1}^m (p_{ij} - p_j^-)^2 (p_{ik} - p_k^-)^2}} \quad (13)$$

$$p_j^- = \frac{1}{n} \sum_{j=1}^n p_{ij} \quad (14)$$

4. Compute the standard deviation

5. Compute the index C

$$C_j = \text{stand}_j \sum_{k=1}^n (1 - g_{jk}) \quad (15)$$

6. Compute the criteria weights.

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j} \quad (16)$$

2.2 TFNS-MOORA Method

This section shows the steps of the MOORA method to rank the alternatives.

1. Normalize the decision matrix

$$y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (17)$$

2. Compute the reference point

We obtain the maximum value for positive criteria and the minimum value for negative criteria.

3. Compute the assessment values

$$u_i = \sum_{j=1}^g y_{ij} w_j - \sum_{j=g+1}^n y_{ij} w_j \quad (18)$$

4. Rank the alternatives.

3. Application and Analysis

This section shows the results of the proposed approach under the TFNNs to deal with uncertain information. We evaluated the Safety Risk Evaluation of Urban Rail Transit Engineering to compute the criteria weights and rank the alternatives. We want to evaluate the Key Safety Risk Areas in Urban Rail Transit Engineering and select the best one. Four experts and decision-makers have evaluated the criteria and alternatives. Nine criteria and six alternatives are shown in Fig 2. We used the TFNNs to evaluate the criteria and alternatives as shown in Tables 1-4.

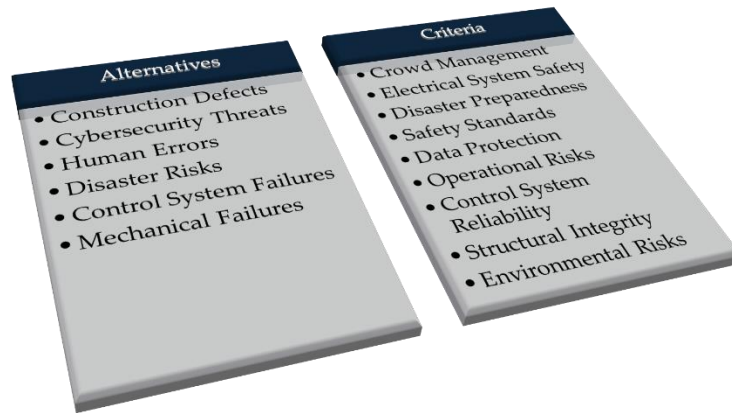


Fig 2. The Key Safety Risk Areas in Urban Rail Transit Engineering.

Table 1. The TFNNs matrix one.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}
C ₂	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}
C ₃	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}
C ₄	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)}}
C ₅	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}
C ₆	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}
C ₇	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}
C ₈	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.4,0.5,0.6)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}
C ₉	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)}}

Table two. The TFNNs matrix two.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)}}
C ₂	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}
C ₃	{{(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}
C ₄	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6),(0.3,0.4,0.5)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}
C ₅	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5),(0.5,0.6,0.8)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}
C ₆	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}
C ₇	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}
C ₈	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5),(0.1,0.3,0.4)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8),(0.1,0.2,0.3)}}
C ₉	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8),(0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7),(0.5,0.8,0.9)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5),(0.4,0.5,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6),(0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4),(0.2,0.3,0.4)}}

Table 3. The TFNNs matrix three.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}
C ₂	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}
C ₃	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}
C ₄	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}
C ₅	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}
C ₆	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.2,0.3,0.4)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.2,0.3,0.4)}}
C ₇	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}
C ₈	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}
C ₉	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}

Table 4. The TFNNs matrix four.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}
C ₂	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}
C ₃	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}
C ₄	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}
C ₅	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}
C ₆	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}
C ₇	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.7,0.8,0.9),(0.2,0.3,0.4), (0.2,0.3,0.4)}}	{{(0.3,0.5,0.6),(0.2,0.4,0.5), (0.5,0.6,0.8)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.5,0.6,0.7),(0.4,0.5,0.6), (0.3,0.4,0.5)}}
C ₈	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.2,0.5,0.7),(0.3,0.6,0.8), (0.1,0.2,0.3)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.6,0.8,0.9),(0.2,0.3,0.5), (0.1,0.3,0.4)}}
C ₉	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.4,0.7,0.9),(0.5,0.6,0.8), (0.2,0.4,0.6)}}	{{(0.1,0.3,0.5),(0.2,0.4,0.7), (0.5,0.8,0.9)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}	{{(0.2,0.3,0.4),(0.4,0.5,0.6), (0.6,0.7,0.8)}}	{{(0.5,0.6,0.8),(0.3,0.4,0.5), (0.4,0.5,0.6)}}

Then we applied the steps of the CRITIC method to compute the criteria weights. We normalize the decision matrix using Eq. (11) as shown in Table 5. Then we obtain the correlation matrix as shown in Table 6. Then we compute the standard deviation values. Then we compute the index C and criteria weights using Eq. (16) as shown in Fig 3.

Table 5. The TFNNs normalization matrix by CRITIC method.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	0.636364	0.136364	0.392045	0	1	0.982955
C ₂	1	0.166667	0	0.041667	0.604167	0.104167
C ₃	0.276786	1	0.080357	0.044643	0.267857	0
C ₄	0.52381	0	0.380952	0.452381	1	0.404762
C ₅	1	0	0.056818	0.022727	0.261364	0.534091
C ₆	0	0.822034	0.762712	0.338983	1	0.70339
C ₇	0.776786	0.946429	0.321429	1	0	0.785714
C ₈	0	0.033708	0.764045	0.94382	0.47191	1
C ₉	0.641026	0.698718	0.173077	0.762821	0	1

Table 6. The correlation matrix by the CRITIC method.

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉
C ₁	1	0.428136	-0.33605	0.635742	0.556123	0.257385	-0.58681	0.078331	-0.20482
C ₂	0.428136	1	0.099583	0.479154	0.786955	-0.45133	-0.21123	-0.67276	-0.20204
C ₃	-0.33605	0.099583	1	-0.50989	-0.23181	0.229749	0.226737	-0.76668	0.01779
C ₄	0.635742	0.479154	-0.50989	1	0.269586	0.099925	-0.73263	0.162518	-0.58308
C ₅	0.556123	0.786955	-0.23181	0.269586	1	-0.6175	0.05231	-0.3247	0.239277
C ₆	0.257385	-0.45133	0.229749	0.099925	-0.6175	1	-0.55444	0.200864	-0.44666
C ₇	-0.58681	-0.21123	0.226737	-0.73263	0.05231	-0.55444	1	-0.05108	0.898556
C ₈	0.078331	-0.67276	-0.76668	0.162518	-0.3247	0.200864	-0.05108	1	0.144439
C ₉	-0.20482	-0.20204	0.01779	-0.58308	0.239277	-0.44666	0.898556	0.144439	1

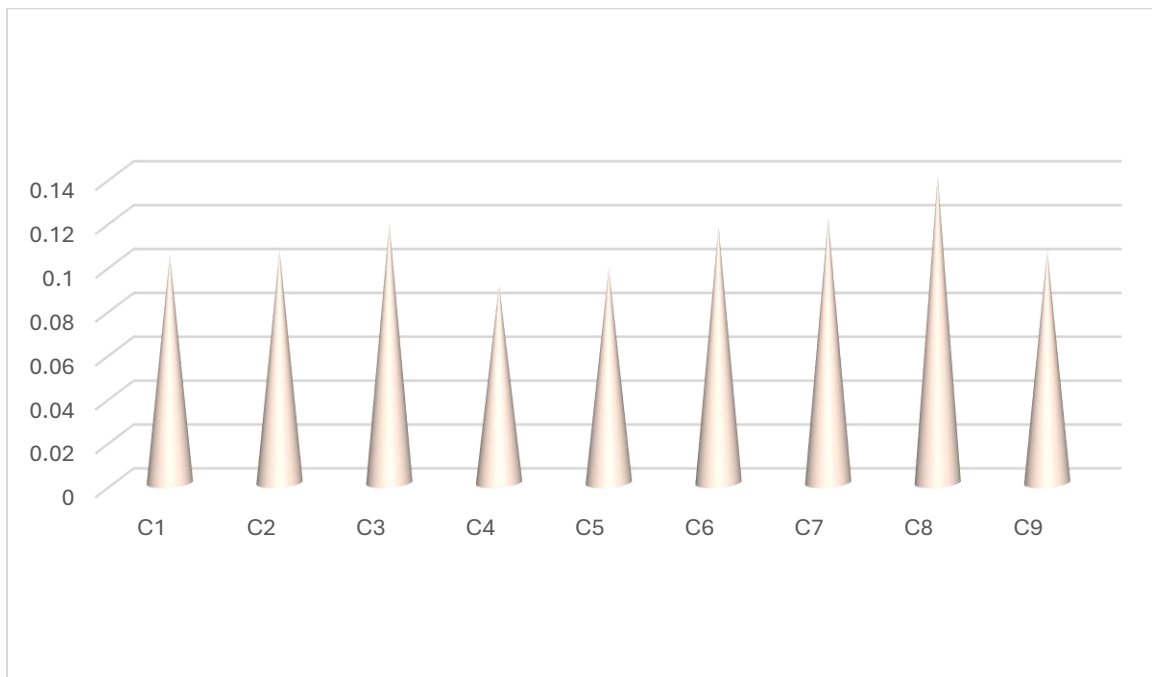


Fig 3. The criteria weights.

Results of the TFNS-MOORA Method

Eq. (17) is used to normalize the decision matrix by the MOORA method as shown in Table 7. Then we compute the reference points. Then we compute the assessment values using Eq. (18) as shown in Table 8. Then we rank the alternatives as shown in Table 9 and Fig 4.

Table 7. The TFNNs normalization matrix by MOORA method.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	0.300751	0.208855	0.255847	0.183792	0.367584	0.364451
C ₂	0.347252	0.29952	0.289973	0.29236	0.324579	0.29594
C ₃	0.317188	0.421628	0.288822	0.283664	0.315899	0.277217
C ₄	0.299344	0.275132	0.292741	0.296042	0.321354	0.293841

C₅	0.327564	0.245673	0.250326	0.247534	0.267077	0.289411
C₆	0.224422	0.348109	0.339183	0.275427	0.374886	0.330257
C₇	0.329266	0.352697	0.266373	0.360096	0.221977	0.3305
C₈	0.239966	0.243209	0.31347	0.330765	0.285366	0.336169
C₉	0.310378	0.320499	0.228285	0.331744	0.197922	0.373353

Table 8. The TFNNs weighted normalization matrix by MOORA method.

	A₁	A₂	A₃	A₄	A₅	A₆
C₁	0.031249	0.021701	0.026584	0.019097	0.038194	0.037868
C₂	0.036732	0.031683	0.030673	0.030925	0.034333	0.031304
C₃	0.037526	0.049882	0.03417	0.033559	0.037373	0.032797
C₄	0.026992	0.024809	0.026397	0.026694	0.028977	0.026496
C₅	0.03208	0.02406	0.024516	0.024243	0.026156	0.028344
C₆	0.026241	0.040703	0.03966	0.032205	0.043834	0.038616
C₇	0.039819	0.042653	0.032213	0.043548	0.026844	0.039968
C₈	0.033583	0.034037	0.043869	0.04629	0.039936	0.047046
C₉	0.03293	0.034004	0.02422	0.035197	0.020999	0.039611

Table 9. The values of MOORA method.

	Score of positive criteria	Score of negative criteria	Final score
A₁	0.237981	0.059171	0.17881
A₂	0.228824	0.074707	0.154117
A₃	0.218421	0.06388	0.154542
A₄	0.224356	0.067402	0.156954
A₅	0.231814	0.064833	0.166981
A₆	0.243823	0.078227	0.165596

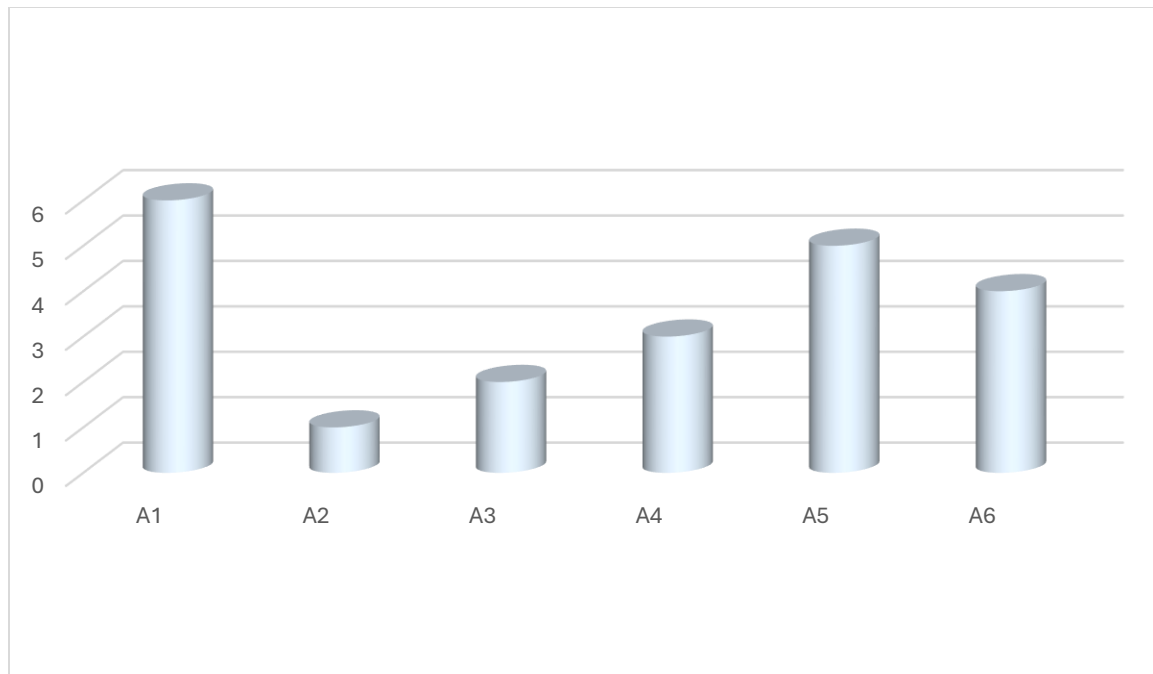


Fig 4. The rank of alternatives.

4. Discussion

This section presents a comprehensive sensitivity analysis conducted under ten different scenarios to examine how variations in criterion weights influence the ranking of alternatives. The purpose of this analysis is to assess the robustness and stability of the proposed evaluation framework and to determine how sensitive the ranking results are to changes in the weighting of criteria.

As illustrated in Figure 5, different weight distributions were considered to evaluate their impact on the final rankings. In the first case, all criteria were assigned equal weights to establish a baseline for comparison. In the second case, the weight of the first criterion was increased by 14%, while the remaining criteria retained equal weights. Similarly, in the third case, the weight of the second criterion was increased by 14%, keeping all other criteria unchanged. This pattern was systematically repeated for each of the ten cases, ensuring that each criterion was individually tested for its impact on the final ranking outcomes.

Following this, the MOORA method was applied under each weight configuration. This involved computing distinct normalization matrices and corresponding weighted normalization matrices for each scenario. Once these matrices were established, we calculated the assessment values for each alternative, ultimately deriving the final scores as presented in Figure 6.

Upon ranking the alternatives across all cases, the results consistently indicated that Alternative 1 emerged as the best-performing option, while Alternative 2 was ranked as the least favorable

choice. These rankings remained stable across various weighting scenarios, as depicted in Figure 7, demonstrating the reliability of the proposed approach in decision-making under uncertainty.

This sensitivity analysis highlights the robustness of our methodology in handling uncertain and imprecise data. The consistency in rankings suggests that Structural Integrity, which received the highest weight in the initial assessment, plays a dominant role in the evaluation process. Additionally, the findings confirm that the framework effectively accommodates variations in expert opinions and weighting schemes without significantly altering the overall conclusions.

4.1 Managerial Implications

The findings of this study offer valuable insights for transport authorities, policymakers, and urban planners. By identifying the most critical safety risk factors, decision-makers can implement targeted strategies to improve URT safety. The proposed framework allows for optimized resource allocation, ensuring that safety investments are directed toward the most significant risk areas, thus maximizing their effectiveness. Strengthening safety measures based on this model can also increase passenger confidence, leading to higher ridership and improved urban mobility. Moreover, the structured methodology supports policymakers in formulating well-informed regulations and operational guidelines aimed at enhancing overall URT safety standards.

4.2. Limitations of the Study

Although the proposed methodology provides a structured and effective approach for evaluating URT safety risks, certain limitations should be acknowledged. The study relies on expert opinions for assigning weights to different criteria, which may introduce subjective biases despite the use of the CRITIC method. Additionally, the research focuses primarily on safety risk evaluation, while other operational factors such as environmental impact and financial feasibility are not explicitly addressed. Furthermore, the study is based on a limited set of criteria and alternatives; expanding these in future research could provide a more holistic perspective.

4.3 Practical Recommendations

Based on the study's findings, several practical recommendations can be made to enhance URT safety. First, transport operators should prioritize improvements in structural integrity and operational safety, as these were identified as key risk factors. Second, implementing real-time monitoring systems using IoT technologies can help detect potential safety threats early. Third, training programs for staff should be enhanced to ensure proper response to emergencies and risk mitigation. Finally, collaboration between policymakers, transport authorities, and urban planners is essential to ensure a comprehensive and well-regulated safety management system for URT.

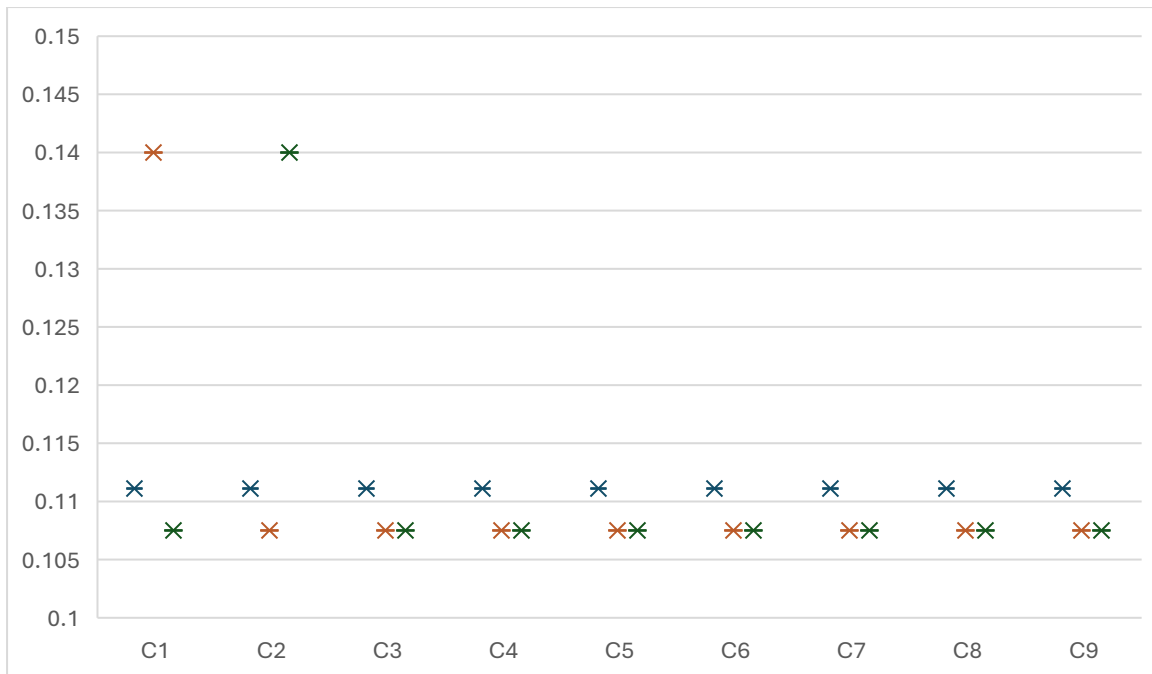


Fig 5. The different criteria weights.

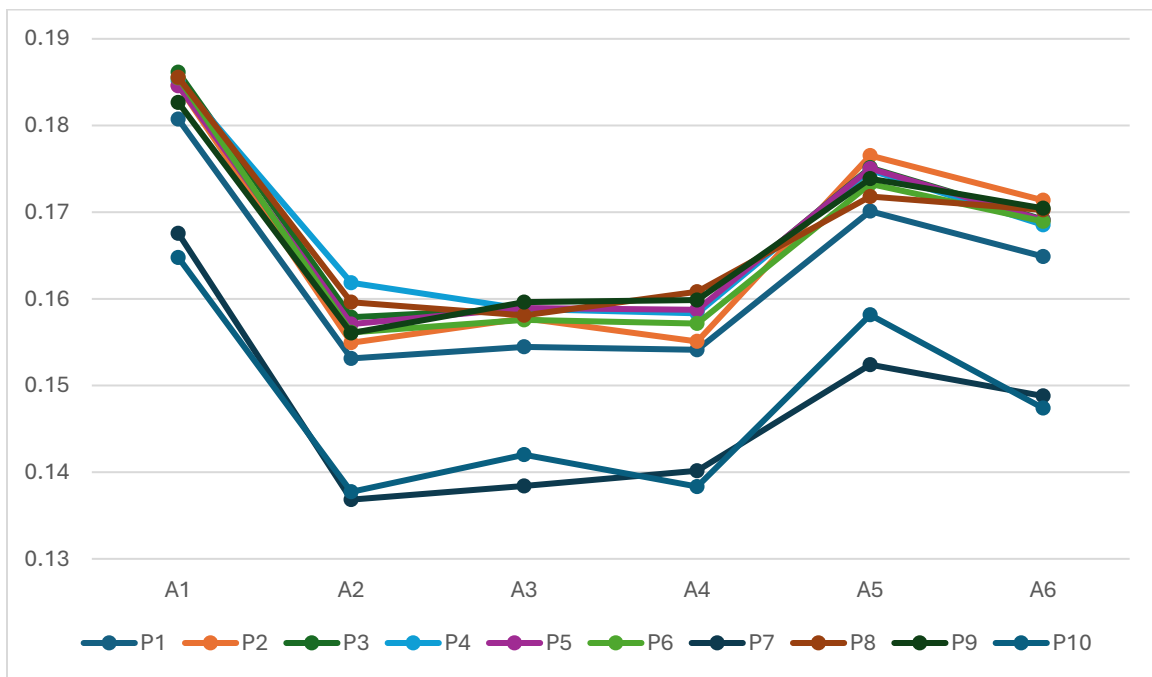


Fig 6. The different assessment values by the MOORA method.

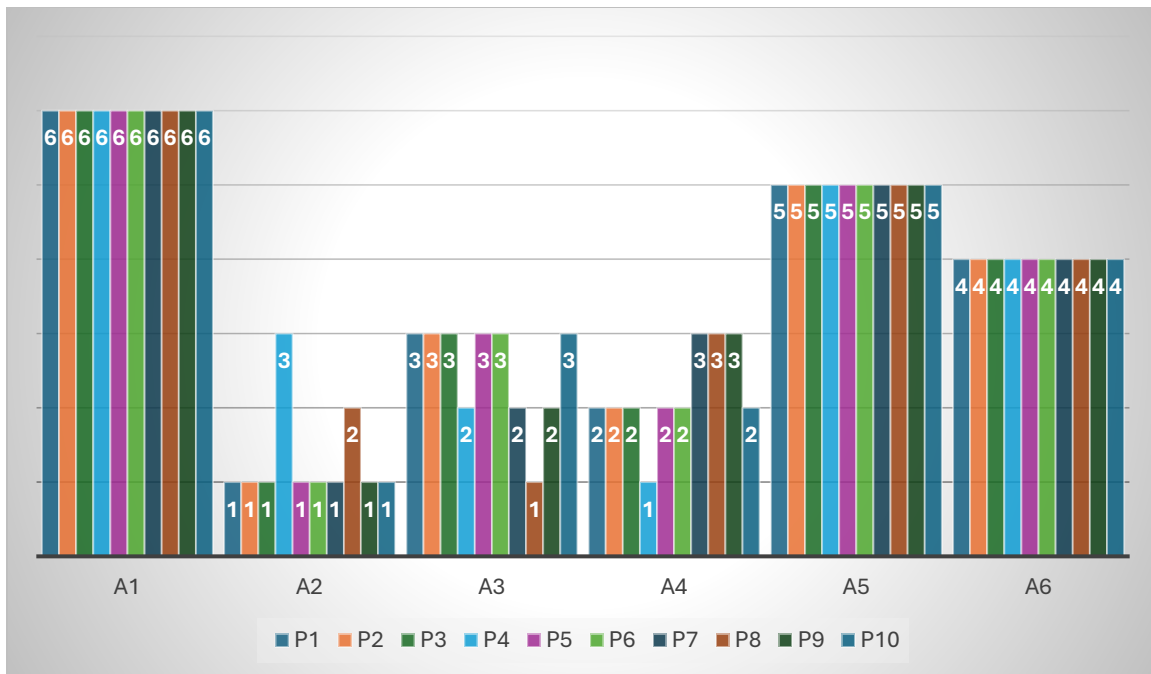


Fig 7. The different ranks of alternatives

5. Conclusions

This study introduces a structured methodology for evaluating urban rail transit safety by integrating triangular fuzzy neutrosophic sets (TFNSs) with the CRITIC and MOORA methods. The proposed model effectively captures uncertainty in safety assessments, allowing for more precise decision-making. The research highlights the importance of considering decision-makers' risk preferences, ensuring flexibility and adaptability in safety evaluations. By applying this framework to nine criteria and six alternatives, the study provides practical insights for transport authorities and urban planners.

The sensitivity analysis confirms the stability and reliability of the ranking process, demonstrating that the model is robust across different weighting scenarios. Furthermore, the findings emphasize the critical role of structural integrity in urban rail transit safety, supporting policymakers in prioritizing essential risk factors. The study's contributions extend beyond theoretical advancements by offering actionable recommendations for safety improvements.

6. Future Work

Although this study presents a comprehensive safety evaluation framework, there are several directions for future research. One promising avenue is the integration of real-time monitoring data from IoT-based sensors and surveillance systems, which would enable dynamic safety risk assessments. Additionally, the methodology could be expanded to evaluate other forms of public transportation, such as buses and high-speed rail, to explore its broader applicability. Further studies can also investigate alternative multi-criteria decision-making (MCDM) methods, such as

TOPSIS or VIKOR, to compare their effectiveness in safety evaluations. Finally, conducting more extensive case studies in different urban settings would help validate the generalizability and effectiveness of the proposed model.

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