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Quality Evaluation of Immersive Virtual Reality Interactive Art Design under Cultural Heritage Digitization under Interval Valued Neutrosophic Numbers

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Abstract: The integration of disciplines has become a crucial issue that the entire society must deal with due to the economy's and society's rapid development. Particularly for teaching art and design, the new features of quickly iterative technology and the developing theory of the digital age presented new difficulties. Simultaneously, as computer hardware continues to advance, new simulation technologies are continually emerging, opening new avenues for creative design work. As a methodical endeavor, teaching art design should incorporate cutting-edge instructional strategies with scientific theories. This study proposes a multi-criteria decision-making (MCDM) approach to evaluate the immersive virtual reality interactive art design under cultural heritage digitization. This study uses two MCDM methods such as the CRITIC method to compute the criteria weights and the ARAS method to rank the alternatives. Two MCDMs are used under the interval-valued neutrosophic sets (IVNSs) to deal with vague and uncertain information. We collect ten criteria and five alternatives to be evaluated by four experts. The sensitivity analysis is conducted to show the stability of the rank under different cases.

Keywords: Interval Valued Neutrosophic Numbers; Virtual Reality; Cultural Heritage Digitization.

1. Introduction

Since science and technology have advanced so quickly in recent years, the virtual reality sector has grown steadily. Advanced technologies like artificial intelligence, 5G, IoT (the Internet of Things), and big data are continuously integrating into virtual reality. The industrial use of virtual reality is facilitated by science and technology, which also creates new businesses and their ecosystems, like, Digital Twins, and other innovative technological ideas[1], [2]. Richer mixed virtual reality applications are emerging because of the widespread use of 5G and the expansion of numerous virtualization technologies into other industries[3], [4]. One area that needs new technology like these to make virtual reality more intelligent is education.

In recent years, virtual reality technology has gained popularity in the network technology sector. Other names for it include "virtual environment," "spiritual environment technology," and "cyberspace." Numerous technological domains are involved in virtual reality technology, which is a synthesis of numerous technologies[5], [6]. It is under the broad category of simulation technology, which also includes network technology, computer perception technology, multimedia technology, and graphics technology.

The technology behind virtual reality consists of four parts. The first is the virtual scene, a dynamic 3D image created by a computer[7], [8]. It has a very lifelike effect that creates an immersive experience. The second component is perception, which encompasses all the senses that people ought to possess, such as vision, touch, movement, and so forth.

Virtual reality can produce a sensation of reality thanks to these perceptual simulations. The third section consists of interactive features such as handle operation, hand-and-foot position movement, and angle of view movement. Information about the user's behavior will be obtained by the computer, which will then process and react to it before providing the user with virtual reality feedback. Currently, it mostly uses the handle and helmet to interact with the virtual environment. The handle controls how hands interact with virtual things, while the head theft creates the three-dimensional display effect[9], [10]. There will be more gadgets in the future that can mimic different interactive functions as virtual reality technology advances. As a result, virtual reality technology aligns with the idea of contemporary educational advancement.

This study evaluates Immersive Virtual Reality Interactive Art Design under Cultural Heritage Digitization. This problem is a multi-criteria decision-making (MCDM) issue. In the framework of the MCDM issue, management science and decision analysis have made significant contributions. The MCDM approaches offer appropriate models for supporting intricate and interrelated decision issues with significant tactics, concepts, and outcomes in contrast to other types of decision-making methods. A collection of multiple criterion approaches that evaluate and choose options based on several factors directly characterize it[11], [12]. It discusses a methodical quantitative strategy that attempts to assist DMs in making logical and effective judgments by taking significant goals and criteria into account.

Therefore, setting up models that can assist DMs in dealing with unclear situations is the goal of MCDM researchers. The fuzzy set (FS) approach is the basic idea behind overcoming uncertainty in decision-making processes[13], [14]. In several applications, classical fuzzy decision-making models have received the attention they deserve. In these circumstances, interval numbers, intuitionistic models, rough sets (RS), and FS are frequently used[15], [16]. Neutrosophic set (NS) notions are the most modern method in this category.

When addressing real-time issues, NSs consider elemental uncertainties and inconsistencies in addition to the membership and non-membership degrees of elements[17], [18]. NSs represent the incompleteness and inconsistency of an element to set, which makes them more relevant for solving complicated problems than FSs and intuitionistic FSs (IFSs).

1.1 Questions of this Study

In essence, this essay aims to respond to the following questions:

A. Determining the key factors for choosing an immersive Virtual Reality Interactive Art Design type in the face of uncertainty.
B. Examining whether the factors taken into consideration are appropriate for immersive Virtual Reality Interactive Art Design issues.
C. Examining the level of uncertainty surrounding these standards.
D. Developing strategies to deal with ambiguity and inaccurate information.
E. Examining which kind of uncertainty model would be best for enabling experts to compare and make decisions when assessing criteria and alternatives.

F. Confirming that the best-performing alternative may be found by applying the suggested neutrosophic model.
G. When addressing real-time issues, NSs consider elemental uncertainties and inconsistencies in addition to the membership and non-membership degrees of elements. NSs convey an element's inconsistency and incompleteness.

1.2 Contributions of this study

The following goals and contemporary issues that this paper aims to address can be identified based on the questions raised above:

1) This study presents an integrated approach to find, assess, and choose the best types of immersive Virtual Reality Interactive Art Design.
2) Assisting DMs in obtaining dependable solutions using the suggested neutrosophic MCDM model while taking information imprecision, inconsistency, and indeterminacy into consideration.
3) Use the interval-valued neutrosophic numbers to evaluate the criteria and alternatives.
4) Two MCDM methods are used in this study such as the CRITIC method to compute the criteria weights and the ARAS method to rank the alternatives.
5) The sensitivity analysis is conducted to show the stability of the results.

1.3 Organization of this study

The study is organized as follows: Section 2 presents the necessary mathematical formulations for the suggested methods. This section shows the steps of the CRITIC method to compute the criteria weights and the ARAS method to rank the alternatives. Deduced results, case study implementation, and performance sensitivity analysis are shown in section 3. Section 4 shows the conclusions.

2. IVN-CRITIC-ARAS Approach

In this section, an integrated CRITIC-ARAS method under the neutrosophic environment to rank the different types of Immersive Virtual Reality Interactive Art Design. This approach has two main stages. In the first stage, the criteria weights are computed using the IVN-CRITIC method. In the second stage, the rank of alternatives is computed using the IVN-ARAS approach. Figure 1 shows the two main stages. Types of Immersive Virtual Reality Interactive Art Design are determined such as $(A_1, \dots, A_m)$ *and n criteria* $(C_1, \dots, C_2)$.

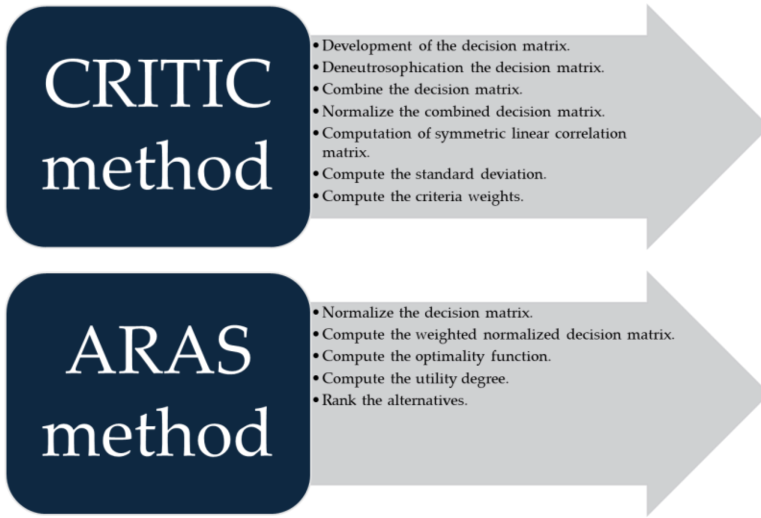

Fig 1. Methodological procedure.

2.1 Neutrosophic Approach

In this section, some definitions of the IVNs are described[19].

Definition 1.

The neutrosophic sets have three membership functions such as Truth. $T_A(x)$, Indeterminacy $I_A(x)$, and Falsity $F_A(x)$.

$$0- \leq \sup T_A(x) + \sup T_A(x) + \sup T_A(x) \leq 3+ \quad (1)$$

Definition 2.

Let two intervals valued neutrosophic numbers (IVNNs) with operations such as:

$$\pi A = \begin{pmatrix} \left[1-\left(1-T_A^L(x)\right)^{\pi}, 1-\left(1-T_A^U(x)\right)^{\pi}\right], \\ \left[\left(I_A^L(x)\right)^{\pi}, \left(I_A^U(x)\right)^{\pi}\right], \\ \left[\left(F_A^L(x)\right)^{\pi}, \left(F_A^U(x)\right)^{\pi}\right] \end{pmatrix} \quad (2)$$

$$A^{\pi} = \begin{pmatrix} \left[\left(T_A^L(x)\right)^{\pi}, \left(T_A^U(x)\right)^{\pi}\right], \\ \left[1-\left(1-I_A^L(x)\right)^{\pi}, 1-\left(1-I_A^U(x)\right)^{\pi}\right], \\ \left[1-\left(1-F_A^L(x)\right)^{\pi}, 1-\left(1-F_A^U(x)\right)^{\pi}\right] \end{pmatrix} \quad (3)$$

$$A + B = \begin{pmatrix} [T_A^L(x) + T_B^L(x) - T_A^L(x)T_B^L(x), T_A^U(x) + T_B^U(x) - T_A^U(x)T_B^U(x)], \\ [I_A^L(x)I_B^L(x), I_A^U(x)I_B^U(x)], \\ [F_A^L(x)F_B^L(x), F_A^U(x)F_B^U(x)] \end{pmatrix} \quad (4)$$

$$AB = \begin{pmatrix} [T_A^L(x)T_B^L(x), T_A^U(x)T_B^U(x)], \\ [I_A^L(x) + I_B^L(x) - I_A^L(x)I_B^L(x), I_A^U(x) + I_B^U(x) - I_A^U(x)I_B^U(x)], \\ [F_A^L(x) + F_B^L(x) - F_A^L(x)F_B^L(x), F_A^U(x) + F_B^U(x) - F_A^U(x)T_B^U(x)] \end{pmatrix} \quad (5)$$

$$A - B = \begin{pmatrix} [T_A^L(x) - T_B^U(x), T_A^U(x) - T_B^L(x)], \\ \left[\max\left(I_A^L(x), I_B^L(x)\right), \max\left(I_A^U(x), I_B^U(x)\right)\right], \\ [F_A^L(x) - F_B^U(x), F_A^U(x) - F_B^L(x)] \end{pmatrix} \quad (6)$$

$$\frac{A}{\pi} = \begin{pmatrix} \left[\min\left(\frac{T_A^L(x)}{\pi}, 1\right), \min\left(\frac{T_A^U(x)}{\pi}, 1\right)\right], \\ \left[\min\left(\frac{I_A^L(x)}{\pi}, 1\right), \min\left(\frac{I_A^U(x)}{\pi}, 1\right)\right] \\ \left[\min\left(\frac{F_A^L(x)}{\pi}, 1\right), \min\left(\frac{F_A^U(x)}{\pi}, 1\right)\right] \end{pmatrix} \quad (7)$$

$$\frac{A}{B} = \begin{pmatrix} \begin{bmatrix} \min\left\{\frac{T_A^L(x)}{T_B^L(x)}, \frac{T_A^L(x)}{T_B^U(x)}, \frac{T_A^U(x)}{T_B^L(x)}, \frac{T_A^U(x)}{T_B^U(x)}\right\}, \\ \max\left\{\frac{T_A^L(x)}{T_B^L(x)}, \frac{T_A^L(x)}{T_B^U(x)}, \frac{T_A^U(x)}{T_B^L(x)}, \frac{T_A^U(x)}{T_B^U(x)}\right\} \end{bmatrix}, \\ \begin{bmatrix} \min\left\{\frac{I_A^L(x)}{I_B^L(x)}, \frac{I_A^L(x)}{I_B^U(x)}, \frac{I_A^U(x)}{I_B^L(x)}, \frac{I_A^U(x)}{I_B^U(x)}\right\}, \\ \max\left\{\frac{I_A^L(x)}{I_B^L(x)}, \frac{I_A^L(x)}{I_B^U(x)}, \frac{I_A^U(x)}{I_B^L(x)}, \frac{I_A^U(x)}{I_B^U(x)}\right\} \end{bmatrix}, \\ \begin{bmatrix} \min\left\{\frac{F_A^L(x)}{F_B^L(x)}, \frac{F_A^L(x)}{F_B^U(x)}, \frac{F_A^U(x)}{F_B^L(x)}, \frac{F_A^U(x)}{F_B^U(x)}\right\}, \\ \max\left\{\frac{F_A^L(x)}{F_B^L(x)}, \frac{F_A^L(x)}{F_B^U(x)}, \frac{F_A^U(x)}{F_B^L(x)}, \frac{F_A^U(x)}{F_B^U(x)}\right\} \end{bmatrix} \end{pmatrix} \quad (8)$$

$$A^{-1} = \begin{pmatrix} \left[\left(T_A^L(x)\right)^{-1}, \left(T_A^U(x)\right)^{-1}\right], \\ \left[\left(I_A^L(x)\right)^{-1}, \left(I_A^U(x)\right)^{-1}\right], \\ \left[\left(F_A^L(x)\right)^{-1}, \left(F_A^U(x)\right)^{-1}\right] \end{pmatrix} \quad (9)$$

Definition 3.

We can compute the de-neutrosophication function such as:

$$S(A) = \begin{pmatrix} \frac{T_A^L(x)+T_A^U(x)}{2} + \\ \left(1 - \frac{\left(I_A^L(x)+I_A^U(x)\right)}{2}\right)\left(I_A^U(x)\right) - \\ \left(\frac{\left(F_A^L(x)+F_A^U(x)\right)}{2}\right)\left(1 - F_A^U(x)\right) \end{pmatrix} \quad (10)$$

2.2 CRITIC Method

This method is used to compute the criteria weights.

1. Development of the decision matrix such as:

$$y_{ij} = \begin{pmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{m1} & \cdots & y_{mn} \end{pmatrix}; i = 1, \dots, m; j = 1, \dots, n \quad (11)$$

2. Deneutrosophication of the decision matrix

3. Combine the decision matrix.

The decision matrix is combined using the average method.

4. Normalize the combined decision matrix.

$$u_{ij} = \frac{y_{ij} - \min y_{ij}}{\max y_{ij} - \min y_{ij}} \text{ for positive criteria} \quad (12)$$

$$u_{ij} = \frac{\min y_{ij} - y_{ij}}{\max y_{ij} - \min y_{ij}} \text{ for cost criteria} \quad (13)$$

5. Computation of symmetric linear correlation matrix.

We compute the correlation values between the criteria. Then we build the correlation matrix between the main criteria. Then we subtract one from the correlation matrix.

6. Compute the standard deviation.

7. Then we multiply the standard deviation by the correlation matrix by subtracting one.

$$C_j = Stand * \sum_{j=1}^{n} 1 - m_{ij} \quad (14)$$

8. Compute the criteria weights.

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j} \quad (15)$$

2.3 IVN-ARAS Method

This part shows the steps of the ARAS method under neutrosophic sets to rank the alternatives.

9. Normalize the decision matrix.

$$r_{ij} = \frac{y_{ij}}{\sum_{i=0}^{m} y_{ij}} \quad (16)$$

10. Compute the weighted normalized decision matrix.

$$T_{ij} = w_j * r_{ij} \quad (17)$$

11. Compute the optimality function.

$$O_i = \sum_{j=1}^{n} T_{ij} \quad (18)$$

12. Compute the utility degree

$$D_i = \frac{O_i}{K} \quad (19)$$

Where k is the optimality value of O_i

13. Rank the alternatives.

3. Illustrative Example and Results

This section shows the results and discussion.

3.1 Case Study

This study uses the MCDM approach to rank the Immersive Virtual Reality Interactive Art Design under Cultural Heritage Digitization. This study uses types of Immersive Virtual Reality Interactive Art Design to select the best one based on a set of criteria. We collected ten criteria and five alternatives as shown in Table 1. Four experts have evaluated the criteria and alternatives based on their experience. Table 2 shows the details of four experts.

Table 1. The criteria and alternatives.

Criteria	Alternatives
Technical Performance	Interactive AR/VR Installations
Narrative and Storytelling	Educational VR Experiences
Sustainability and Preservation	Virtual Museum Exhibits
User Experience (UX) and Interface Design	Immersive Storytelling and Roleplay
Accessibility and Inclusivity	
Immersion and Presence	
Historical and Cultural Accuracy	
Visual and Aesthetic Quality	
Interactivity and User Engagement	
Emotional and Educational Impact	VR Heritage Site Reconstructions

Table 2. Sets of experts.

Expert	Gender	Work experience	Job position
1	Male	18 years of experience	Virtual Reality Interactive
2	Male	20 years of experience	Virtual Reality Interactive
3	Male	23 years of experience	Virtual Reality Interactive
4	Female	15 years of experience	Virtual Reality Interactive

3.2 Model Implementation, Results Analysis and Discussion

After the determination of the criteria and alternatives, the next step is to obtain the data and information. The data and information are obtained using four experts. Four experts evaluated the criteria and alternatives using the terms of IVNSs as shown in Table 3. Table 4 shows the IVNNs for the criteria and alternatives. Then we apply the score function to obtain one matrix. Then we combined the decision matrix into one matrix.

Table 3. IVN values.

A	([0.1,0.2],[0.1,0.2],[0.8,0.9])
B	([0.2,0.3],[0.3,0.4],[0.7,0.8])
C	([0.3,0.4],[0.4,0.5],[0.6,0.7])
D	([0.4,0.5],[0.5,0.6],[0.5,0.6])
E	([0.5,0.5],[0.6,0.7],[0.4,0.5])
F	([0.5,0.6],[0.5,0.6],[0.4,0.5])
G	([0.6,0.7],[0.4,0.5],[0.3,0.4])
H	([0.7,0.8],[0.2,0.3],[0.2,0.3])
I	([0.8,0.9],[0.1,0.2],[0.1,0.2])

Table 4. The decision matrix.

E_1	A_1	A_2	A_3	A_4	A_5
C_1	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_2	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_3	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])
C_4	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.2,0.3],[0.3,0.4],[0.7,0.8])
C_5	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.1,0.2],[0.1,0.2],[0.8,0.9])
C_6	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.8,0.9],[0.1,0.2],[0.1,0.2])
C_7	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.7,0.8],[0.2,0.3],[0.2,0.3])
C_8	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.6,0.7],[0.4,0.5],[0.3,0.4])
C_9	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.5,0.6],[0.5,0.6],[0.4,0.5])
C_{10}	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
E_2	A_1	A_2	A_3	A_4	A_5
C_1	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_2	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_3	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])
C_4	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_5	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.1,0.2],[0.1,0.2],[0.8,0.9])
C_6	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_7	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.7,0.8],[0.2,0.3],[0.2,0.3])
C_8	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.6,0.7],[0.4,0.5],[0.3,0.4])
C_9	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.5,0.6],[0.5,0.6],[0.4,0.5])
C_{10}	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
E_3	A_1	A_2	A_3	A_4	A_5
C_1	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_2	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_3	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_4	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.2,0.3],[0.3,0.4],[0.7,0.8])
C_5	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_6	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_7	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.7,0.8],[0.2,0.3],[0.2,0.3])
C_8	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.6,0.7],[0.4,0.5],[0.3,0.4])
C_9	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.5,0.6],[0.5,0.6],[0.4,0.5])
C_{10}	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
E_4	A_1	A_2	A_3	A_4	A_5
C_1	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.5,0.5],[0.6,0.7],[0.4,0.5])
C_2	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.4,0.5],[0.5,0.6],[0.5,0.6])
C_3	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])
C_4	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.2,0.3],[0.3,0.4],[0.7,0.8])
C_5	([0.5,0.5],[0.6,0.7],[0.4,0.5])	([0.3,0.4],[0.4,0.5],[0.6,0.7])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])
C_6	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.8,0.9],[0.1,0.2],[0.1,0.2])
C_7	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.7,0.8],[0.2,0.3],[0.2,0.3])
C_8	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.6,0.7],[0.4,0.5],[0.3,0.4])
C_9	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.6,0.7],[0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.2,0.3],[0.2,0.3])	([0.8,0.9],[0.1,0.2],[0.1,0.2])	([0.5,0.6],[0.5,0.6],[0.4,0.5])
C_{10}	([0.1,0.2],[0.1,0.2],[0.8,0.9])	([0.2,0.3],[0.3,0.4],[0.7,0.8])	([0.5,0.6],[0.5,0.6],[0.4,0.5])	([0.4,0.5],[0.5,0.6],[0.5,0.6])	([0.5,0.5],[0.6,0.7],[0.4,0.5])

3.3 Determination of Criteria Weights Using IVN-CRITIC Method

The IVN-CRITIC method is used to compute the criteria weights. We normalize the decision matrix as shown in Table 5. Then we Compute the correlation between criteria as shown in table 6. The correlation matrix shows a high correlation between criteria and a low correlation between criteria. There are positive and negative correlations between the criteria. Then we subtract one from the correlation matrix. Then we obtain the sum of each row and compute the criteria weights as shown in Figure 2.

Table 5. The normalization matrix by the IVN-CRITIC method.

	A_1	A_2	A_3	A_4	A_5
C_1	0.108434	1	0.783133	0.361446	0
C_2	0	1	0.799392	0.860182	0.416413
C_3	0.613546	0	1	0.705179	0.294821
C_4	0.473373	0.970414	1	0.686391	0
C_5	0.689441	0	0.956522	1	0.304348

C_6	0.380282	0.408451	0	0.105634	1
C_7	0.893836	0.363014	0.304795	0	1
C_8	0.833948	0.697417	0.538745	0	1
C_9	1	0	0.311475	0.688525	0
C_{10}	0	0.28169	1	0.650704	0.630986

Table 6. The correlation matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	1	0.788017	-0.11021	0.90724	-0.19444	-0.54649	-0.62788	-0.26338	-0.39699	0.219911
C_2	0.788017	1	-0.12628	0.649938	-0.0907	-0.39517	-0.80691	-0.55304	-0.5455	0.524795
C_3	-0.11021	-0.12628	1	0.217932	0.951961	-0.6463	-0.29369	-0.43935	0.539292	0.5164
C_4	0.90724	0.649938	0.217932	1	0.172479	-0.83905	-0.75425	-0.49434	0.007436	0.182784
C_5	-0.19444	-0.0907	0.951961	0.172479	1	-0.66517	-0.41213	-0.63174	0.655427	0.434529
C_6	-0.54649	-0.39517	-0.6463	-0.83905	-0.66517	1	0.789216	0.736426	-0.44621	-0.26256
C_7	-0.62788	-0.80691	-0.29369	-0.75425	-0.41213	0.789216	1	0.912958	-0.04846	-0.42611
C_8	-0.26338	-0.55304	-0.43935	-0.49434	-0.63174	0.736426	0.912958	1	-0.33636	-0.35406
C_9	-0.39699	-0.5455	0.539292	0.007436	0.655427	-0.44621	-0.04846	-0.33636	1	-0.39525
C_{10}	0.219911	0.524795	0.5164	0.182784	0.434529	-0.26256	-0.42611	-0.35406	-0.39525	1

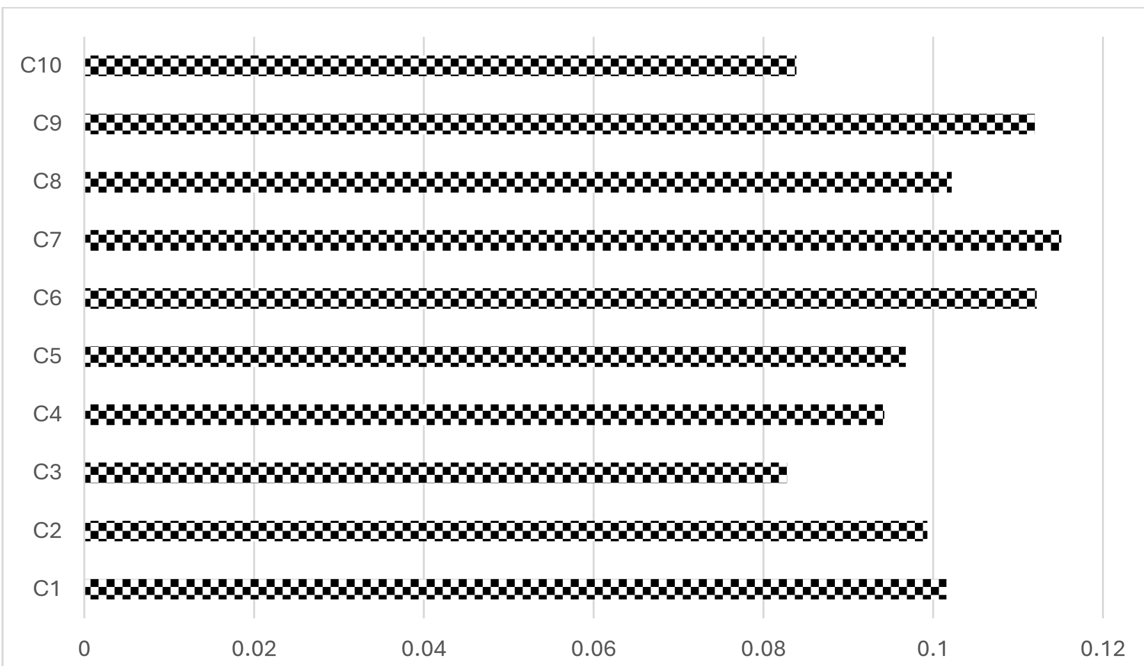

Fig. 2. Criteria weights.

3.4 Rank the alternatives Using the IVN-ARAS Method

The IVN-ARAS method is used to rank the alternatives. We start with the combined decision matrix between the criteria and alternatives. Eq. (16) is used to compute the normalized decision matrix as shown in Table 7. Then we compute the weighted normalized decision matrix using Eq. (17) as shown in Table 8.

Then we compute the optimal function using Eq. (18). Then we compute the utility degree using Eq. (19). Then we rank the alternatives as shown in Table 9.

Table 7. The normalization matrix by the IVN-ARAS method.

	A_1	A_2	A_3	A_4	A_5
C_1	0.176854	0.237164	0.222494	0.193969	0.169519
C_2	0.113021	0.254405	0.226042	0.234637	0.171895
C_3	0.210983	0.136802	0.257707	0.222062	0.172447
C_4	0.187882	0.227337	0.229685	0.204791	0.150305
C_5	0.208	0.1525	0.2295	0.233	0.177
C_6	0.200084	0.201765	0.177385	0.183691	0.237074
C_7	0.244775	0.182476	0.175643	0.139871	0.257235
C_8	0.225503	0.209671	0.191271	0.128798	0.244758
C_9	0.251046	0.165969	0.192469	0.224547	0.165969
C_{10}	0.101622	0.155676	0.293514	0.226486	0.222703

Table 8. The weighted normalized decision matrix.

	A_1	A_2	A_3	A_4	A_5
C_1	0.017961	0.024086	0.022596	0.019699	0.017216
C_2	0.011225	0.025267	0.02245	0.023304	0.017072
C_3	0.017464	0.011323	0.021331	0.018381	0.014274
C_4	0.017706	0.021425	0.021646	0.0193	0.014165
C_5	0.020127	0.014757	0.022208	0.022547	0.017128
C_6	0.02245	0.022638	0.019903	0.02061	0.0266
C_7	0.028174	0.021003	0.020217	0.0161	0.029608
C_8	0.02304	0.021423	0.019543	0.01316	0.025008
C_9	0.028111	0.018585	0.021552	0.025144	0.018585
C_{10}	0.008525	0.013059	0.024622	0.019	0.018682

Table 9. The rank of alternatives.

Alternatives	O_i	D_i	Rank
A_1	0.194784	0.901493	2
A_2	0.193567	0.895859	1
A_3	0.216068	1	5
A_4	0.197243	0.912875	3
A_5	0.198338	0.917942	4

4. Discussion, Validation and Sensitivity Analysis

This section shows the discussion of the results. The results show the criterion Historical and Cultural Accuracy has the highest weights and the Sustainability and Preservation criterion has the lowest weights. Then we applied the ARAS method to rank the alternatives. The results show the Virtual Museum Exhibits has the highest rank and the Educational VR Experiences has the lowest rank.

Then we applied the sensitivity analysis results to show different ranks of alternatives. We conducted 11 cases in the weight criteria. In the first case, we put all criteria with the same weights. In the second case, we increase the first criterion by 25% and other criteria have the same weights. In the third case, we increase the sconed criterion by 25% and other criteria have the same weights. Figure 3 shows the different cases in criteria weights.

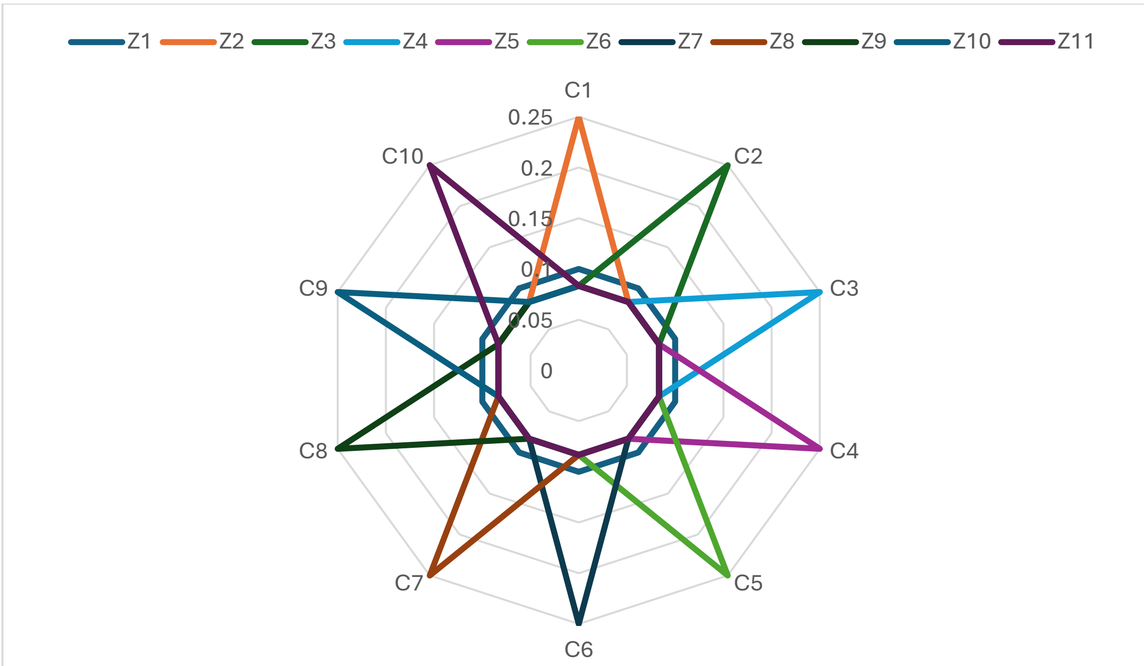

Fig. 3. Different criteria weights.

Then we rank the alternatives under these cases to show the stability of the results. The results show that alternative 3 is the best alternative in all cases. We show that cases 1,2,3,7,11 obtained alternative 1 is the worst alternative. Case 5 shows that alternative 5 is the worst alternative. The cases 4,6,10 show that alternative 2 is the worst. Cases 8 and 9 show that alternative 4 is the worst. We show the rank of alternative is stable under different cases. Table 10 shows the rank of alternatives.

Table 10. The rank of alternatives under different cases.

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}
A_1	1	1	1	3	2	3	1	3	3	3	1
A_2	2	4	3	1	3	1	2	2	2	1	2
A_3	5	5	5	5	5	5	5	5	5	5	5
A_4	4	3	4	4	4	4	3	1	1	4	4
A_5	3	2	2	2	1	2	4	4	4	2	3

5. Conclusions

This study assesses the relationship between the criteria to establish criteria weights using IVN-CRITIC. Then, using the IVN-ARAS approach, alternatives are evaluated and ranked concerning the established criteria. According to the study's specialists and research team, the suggested decision-making model would enable policymakers to reconsider comparable procedures and make sensible choices in real-world scenarios. The suggested model helps the organization create a knowledge-based window. The model's results can be used and compared to determine the strengths and weaknesses of various MCDM issues. The results show that the Historical and Cultural Accuracy criterion has the highest weights and Sustainability and Preservation have the lowest weights. Then the rank of alternatives is computed. The Virtual Museum Exhibits have the highest rank, and the Educational VR Experiences have the lowest rank.

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