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Adaptive Triangular Linguistic Neutrosophic Cubic Fuzzy Sets for College English Blended Teaching Mode Evaluation Method

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Abstract

This paper presents a proactive strategy that employs a multiple-criteria decision-making (MCDM) model as an evaluation tool. To assess online and offline blended teaching modes for college English, we propose an evaluation framework called CIMAS-COBRA, which integrates the Criteria Importance Assessment (CIMAS) method for determining criteria weights and the Cost Estimation, Benchmarking, and Risk Assessment (COBRA) method for ranking alternatives. Both methods operate under the Triangular Linguistic Neutrosophic Cubic Fuzzy Sets framework to handle uncertain and vague data. Seven criteria and ten alternatives are examined in this study. The Triangular Linguistic Neutrosophic Cubic Fuzzy Number approach is employed to evaluate these criteria and alternatives.

Keywords: Triangular Linguistic Neutrosophic Cubic Fuzzy Sets; CIMAS Method; COBRA Method; Uncertainty.

1. Introduction and Related Work

One of the main challenges in evaluating the blended teaching mode for college English is the inherent uncertainty and complexity of the decision-making process. Multiple factors such as teaching quality, learner satisfaction, resource availability, and technological support—must be considered simultaneously, making it difficult to arrive at an objective and comprehensive assessment. To address these challenges, researchers have turned to advanced fuzzy set theories, including Triangular Linguistic Neutrosophic Cubic Fuzzy Sets, which provide sophisticated tools for handling vagueness, indeterminacy, and inconsistent data.

The intuitionistic fuzzy set (IFS), introduced by Atanassov, extends the traditional fuzzy set by considering both membership and non-membership degrees, thereby handling hesitancy and vagueness more effectively. Jianhua et al. noted that several water techniques, including membrane-assisted technologies, have been proposed and successfully applied in various locations in recent years [1], [2]. Moreover, a new hybrid fuzzy multiple-criteria decision-making (MCDM) model incorporating the work of Gülçin and Çifçi has also been introduced.

Cubic sets, which generalize both fuzzy sets and intuitionistic fuzzy sets, were later introduced by Jun et al. These sets feature two representations: one for the degree of membership and another for the degree of non-membership [3], [4]. In this approach, the non-membership aspect is considered alongside a membership function that is expressed in the form of an interval.

Building on these developments, Wang et al. recently proposed a single-valued neutrosophic set—a subclass of the neutrosophic set introduced by Smarandache. This set not only handles inconsistent, indeterminate, and incomplete data but also independently expresses truth-membership, indeterminacy-membership, and falsity-membership degrees [5], [6]. Given the imperfections in human knowledge and perception, the elements defined by the single-valued neutrosophic set are particularly well-suited for modeling human reasoning.

Since indeterminacy reflects the uncertainty between truth and untruth, the human brain often cannot produce definitive "yes" or "no" answers. For example, when considering a statement like "Movie X would be a hit," the intuitionistic fuzzy set falls short in adequately addressing indeterminacy and inconsistent information, whereas neutrosophic components offer a more effective solution [7], [8]. Consequently, the single-valued neutrosophic set has rapidly gained popularity and found numerous applications.

Interval-valued fuzzy sets further enhance decision-making by providing a broader range of uncertain data compared to conventional fuzzy sets, as both membership and non-membership can be evaluated more flexibly. The concept of interval-valued triangular linguistic neutrosophic fuzzy sets was introduced because, in real-world decision-making, it is crucial to consider the degree of membership and non-membership that an option meets for a specific criterion—across all interval values [9], [10].

In recent years, blended teaching has become a popular approach in college English courses by integrating traditional classroom instruction with online learning. This mode offers flexibility and increased interactivity; however, it also presents challenges when evaluating overall effectiveness. Conventional evaluation models often rely on precise, crisp data and may not fully capture the dynamic and uncertain nature of blended learning environments [16]. Factors such as varying student engagement, diverse teaching practices, and the unpredictable performance of online components contribute to significant uncertainty in these assessments [17].

To better manage this complexity, researchers have adopted neutrosophic sets—a mathematical framework that extends traditional fuzzy set theory by explicitly incorporating a measure for indeterminacy. Unlike classical fuzzy sets, which only account for membership and non-membership degrees, neutrosophic sets enable evaluators to quantify the uncertainty and incomplete information that frequently arise in educational settings [18]. This additional component makes neutrosophic approaches particularly suitable for evaluating blended teaching modes, where both quantitative and qualitative factors must be integrated.

Several studies have applied neutrosophic set theory to improve educational assessments. For example, Chen and Zhao [19] proposed a neutrosophic-based evaluation model specifically for college English courses, demonstrating that this method yields more reliable assessments by capturing nuances that conventional models overlook. Li and Zhao [20] developed a hybrid framework that integrates neutrosophic concepts to effectively process diverse student feedback and instructor performance data. In another study, Zhang and Xu [21] employed single-valued neutrosophic sets to distinguish between different levels of teaching effectiveness, thereby providing clearer insights into course quality.

Additional research has extended these findings to broader decision-making processes in education. Garcia and Patel [22] explored advanced decision-making models incorporating neutrosophic sets, which revealed their ability to manage uncertainty in higher education evaluations. Kim and Park [23] offered a comprehensive review of fuzzy logic applications in blended learning assessment, emphasizing the advantages of including neutrosophic approaches. Similarly, Nour and Ali [24] demonstrated the efficacy of neutrosophic methods in evaluating both the online and offline components of blended courses, highlighting a reduction in the negative effects of ambiguous data.

Other studies have also made significant contributions. Hassan and Ibrahim [25] provided a neutrosophic perspective on blended learning evaluation, showing that this approach enhances the accuracy of assessments. Santos and Costa [26] introduced a decision-making model based on neutrosophic theory to address inconsistencies in teaching evaluations, while Lee and Wang [27] demonstrated that using neutrosophic fuzzy sets can significantly improve the analysis of complex behavioral data in blended classroom settings.

A systematic review by Miller and Davis [28] confirmed the growing trend of applying neutrosophic logic in educational research and its effectiveness in handling indeterminate information. Gupta and Verma [29] offered evidence that advanced neutrosophic models provide clearer insights into the effectiveness of blended learning. Kim and Choi [30] further refined evaluation methods by integrating neutrosophic set theory, resulting in significant improvements in data interpretation.

The decision-making aspect of blended teaching evaluation has also received considerable attention. El-Hawary and Fouad [31] examined how neutrosophic logic can reduce uncertainty in educational decisions, while Peters and Singh [32] applied a neutrosophic Multiple Criteria Decision Making (MCDM) framework to integrate various performance indicators. Oliveira and Mendes [33] provided empirical support for the use of neutrosophic set theory in capturing the multifaceted nature of blended teaching environments. Ahmed and Qureshi [34] conducted a comparative study that found evaluation models based on neutrosophic sets offer a more robust assessment of blended teaching. Finally, Rodriguez and Thompson [35] discussed recent advancements in neutrosophic set-based evaluation models and emphasized their potential to transform teaching assessments in higher education.

In summary, the literature indicates that incorporating neutrosophic sets into the evaluation of college English blended teaching modes provides a comprehensive approach that effectively addresses uncertainty and indeterminacy. This advanced methodology not only improves the reliability of assessments but also offers valuable insights for the continuous enhancement of teaching practices in modern educational situations.

1.1. Research Objectives and Contributions

The primary objective of this study is to develop a robust evaluation framework for assessing online and offline blended teaching modes in college English education. The study aims to introduce an advanced decision-making approach that integrates the CIMAS method for computing criteria weights and the COBRA method for ranking alternatives, all within the framework of TLNCFSs. This research contributes to the field by offering a novel hybrid model that effectively captures the uncertainty and vagueness inherent in educational assessments. Furthermore, by incorporating expert evaluations, the study ensures that the proposed method is both practical and applicable in real-world educational settings. The findings provide valuable insights for educators and administrators in making informed decisions about blended teaching methodologies, ultimately leading to improved teaching strategies and student learning outcomes.

1.2. Identified Research Gaps

While several methods have been proposed for evaluating blended teaching models, many existing approaches rely on conventional decision-making techniques that do not fully address the challenges of uncertainty and subjective assessment. Most previous studies have used traditional fuzzy logic or intuitionistic fuzzy methods, which cannot explicitly handle indeterminacy. Additionally, few studies have explored the integration of neutrosophic sets with multiple-criteria decision-making (MCDM) techniques to enhance the accuracy and reliability of evaluations. Another gap in the literature is the limited focus on practical implementations of advanced fuzzy models in education. Many studies remain theoretical without real-world validation through expert-based evaluations. This study fills these gaps by introducing a comprehensive hybrid model that not only applies neutrosophic principles but also integrates expert decision-making, ensuring a more practical and effective assessment of blended teaching environments.

2. Preliminaries

In this section, we introduce the fundamental concepts behind Triangular Linguistic Neutrosophic Cubic Fuzzy Sets (TLNCFSs) [11], [12], [13]. TLNCFSs extend traditional fuzzy set theory by incorporating both linguistic variables and neutrosophic logic, which helps in capturing uncertainty, indeterminacy, and conflicting information more effectively. We will explain how these sets are constructed, discuss their main components, and highlight their advantages in handling complex decision-making scenarios. The ideas presented here are based on the key studies cited in references [11], [12], and [13], which provide a detailed foundation for understanding TLNCFSs.

Definition 1

The interval-valued triangular linguistic neutrosophic fuzzy sets can be defined as:

$$x_b(h) = \begin{cases} S_\theta, \left[\frac{(h-r)}{(s-r)} F_b^-, F_b^+\right], & (r \le h < s) \\ [F_b^-, F_b^+], & h = s, s \\ S_\theta, \left[\frac{(t-h)}{(t-s)} F_b^-, F_b^+\right], & (s \le h < t) \\ 0, & otherwise \end{cases} \tag{1}$$

$$y_b(h) = \begin{cases} S_\theta, \left[\frac{(h-r)}{(s-r)} G_b^-, G_b^+\right], & (r \le h < s) \\ [G_b^-, G_b^+], & h = s, s \\ S_\theta, \left[\frac{(t-h)}{(t-s)} G_b^-, G_b^+\right], & (s \le h < t) \\ 0, & otherwise \end{cases} \tag{2}$$

$$x_b(h) = \begin{cases} S_\theta, \left[\frac{(h-r)}{(s-r)} I_b^-, I_b^+\right], & (r \le h < s) \\ [I_b^-, I_b^+], & h = s, s \\ S_\theta, \left[\frac{(t-h)}{(t-s)} I_b^-, I_b^+\right], & (s \le h < t) \\ 0, & otherwise \end{cases} \tag{3}$$

The triangular linguistic neutrosophic fuzzy sets can be defined as:

$$c_b(h) = \begin{cases} S_\theta, \left[\frac{(h-r)}{(s-r)} F_b\right], & (r \le h < s) \\ F_b, & h = s, s \\ S_\theta, \left[\frac{(t-h)}{(t-s)} F_b\right], & (s \le h < t) \\ 0, & otherwise \end{cases} \tag{4}$$

$$d_b(h)=\begin{cases} S_\theta,\left[\frac{(h-r)}{(s-r)}G_b\right], & (r\le h<s)\\ G_b, & h=s,s\\ S_\theta,\left[\frac{(t-h)}{(t-s)}\,G_b\right], & (s\le h<t)\\ 0, & otherwise\end{cases} \tag{5}$$

$$e_b(h)=\begin{cases} S_\theta,\left[\frac{(h-r)}{(s-r)}I_b\right], & (r\le h<s)\\ I_b, & h=s,s\\ S_\theta,\left[\frac{(t-h)}{(t-s)}\,I_b\right], & (s\le h<t)\\ 0, & otherwise\end{cases} \tag{6}$$

The TLNCFN donated by

$$b=\begin{Bmatrix} S_\theta,[r_b,s_b,t_b];\\ ([F_b^-,F_b^+],F_b),\\ ([G_b^-,G_b^+],G_b)\\ ([I_b^-,I_b^+],I_b)\end{Bmatrix} \tag{7}$$

Definition 2

Let $A_1=\begin{Bmatrix} S_{\theta_1},[r_{b_1},s_{b_1},t_{b_1}];\\ ([F_{b_1}^-,F_{b_1}^+],F_{b_1}),\\ ([G_{b_1}^-,G_{b_1}^+],G_{b_1})\\ ([I_{b_1}^-,I_{b_1}^+],I_{b_1})\end{Bmatrix}$ and $A_2=\begin{Bmatrix} S_{\theta_2},[r_{b_2},s_{b_2},t_{b_2}];\\ ([F_{b_2}^-,F_{b_2}^+],F_{b_2}),\\ ([G_{b_2}^-,G_{b_2}^+],G_{b_2})\\ ([I_{b_2}^-,I_{b_2}^+],I_{b_2})\end{Bmatrix}$ two TLNCFNs and their operations can be defined as:

$$A_1+A_2=\begin{Bmatrix} \begin{pmatrix} S_{\theta_1}+S_{\theta_2},[r_{b_1}+r_{b_2},s_{b_1}+s_{b_2},t_{b_1}+t_{b_2}],\\ [(\min(F_{b_1}^-,F_{b_2}^-),\min(F_{b_1}^+,F_{b_2}^+),\max(F_{b_1},F_{b_{12}}))]\end{pmatrix},\\ (\min(G_{b_1}^-,G_{b_2}^-),\min(G_{b_1}^+,G_{b_2}^+),\max(G_{b_1},G_{b_{12}})),\\ (\min(I_{b_1}^-,I_{b_2}^-),\min(I_{b_1}^+,I_{b_2}^+),\max(I_{b_1},I_{b_{12}}))\end{Bmatrix} \tag{8}$$

$$A_1-A_2=\begin{Bmatrix} \begin{pmatrix} S_{\theta_1}-S_{\theta_2},[r_{b_1}-r_{b_2},s_{b_1}-s_{b_2},t_{b_1}-t_{b_2}],\\ [(\min(F_{b_1}^-,F_{b_2}^-),\min(F_{b_1}^+,F_{b_2}^+),\max(F_{b_1},F_{b_{12}}))]\end{pmatrix},\\ (\min(G_{b_1}^-,G_{b_2}^-),\min(G_{b_1}^+,G_{b_2}^+),\max(G_{b_1},G_{b_{12}})),\\ (\min(I_{b_1}^-,I_{b_2}^-),\min(I_{b_1}^+,I_{b_2}^+),\max(I_{b_1},I_{b_{12}}))\end{Bmatrix} \tag{9}$$

$$A_1A_2=\begin{Bmatrix} \begin{pmatrix} S_{\theta_1}S_{\theta_2},[r_{b_1}r_{b_2},s_{b_1}s_{b_2},t_{b_1}t_{b_2}],\\ [(\min(F_{b_1}^-,F_{b_2}^-),\min(F_{b_1}^+,F_{b_2}^+),\max(F_{b_1},F_{b_{12}}))]\end{pmatrix},\\ (\min(G_{b_1}^-,G_{b_2}^-),\min(G_{b_1}^+,G_{b_2}^+),\max(G_{b_1},G_{b_{12}})),\\ (\min(I_{b_1}^-,I_{b_2}^-),\min(I_{b_1}^+,I_{b_2}^+),\max(I_{b_1},I_{b_{12}}))\end{Bmatrix} \tag{10}$$

2.1. Importance of Neutrosophic Approaches

Neutrosophic approaches have gained significant attention in decision-making and evaluation processes, particularly in complex environments where uncertainty, vagueness, and imprecision play a crucial role. Traditional methods such as fuzzy sets and intuitionistic fuzzy sets provide some level of flexibility in handling uncertainty, but they fall short of effectively managing indeterminacy.

Neutrosophic sets extend these models by introducing an additional degree that explicitly accounts for indeterminacy, making them highly suitable for real-world problems that involve subjective assessments and inconsistent information.

In the context of evaluating blended teaching modes for college English, neutrosophic approaches provide a structured and comprehensive way to analyze various criteria while considering the uncertainty present in expert opinions and educational data.

By incorporating Triangular Linguistic Neutrosophic Cubic Fuzzy Sets (TLNCFSs), the proposed model ensures a more refined and balanced decision-making process that reflects the complexities of blended learning environments.

3. Research Methodology

This section outlines the steps of the proposed approach. First, we applied the CIMAS method to compute the criteria weights, which helps in determining the relative importance of each factor. Next, we used the COBRA method to rank the alternatives based on these computed weights. Figure 1 illustrates the complete process of our proposed approach.

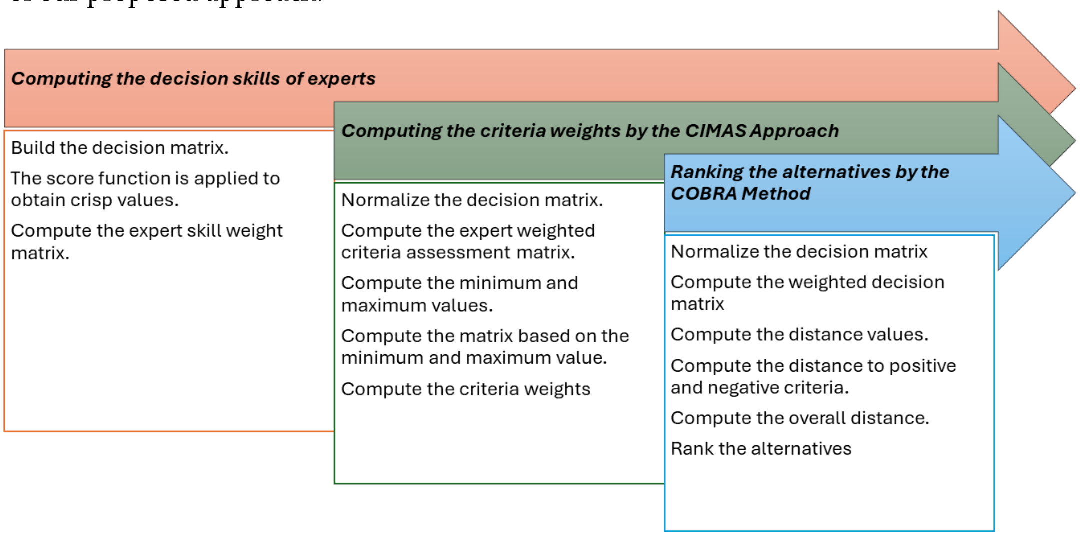

Fig 1. The research method

Computing the decision skills of experts

Step 1: Each expert evaluates the criteria and alternatives using the terms of the neutrosophic sets to build the decision matrix. This means that experts express their assessments by considering the degrees of membership, non-membership, and indeterminacy for each criterion and alternative.

Step 2: Next, the score function is applied to these assessments to obtain crisp values, denoted as x_{ij}. These crisp values convert neutrosophic evaluations into clear, numerical data, making it easier to analyze and compare the results.

Step 3: Finally, we computed the expert skill weight matrix. This matrix quantifies the decision-making abilities of each expert based on their evaluated crisp values, helping to weigh their contributions appropriately in the overall decision-making process.

$$e_j = \frac{x_f}{\sum_{f=1}^{f} x_f} \tag{11}$$

Where f refers to the experts.

Computing the criteria weights by the CIMAS Approach[14]

Step 4. Normalize the decision matrix between the criteria and alternatives.

$$n_{ij} = \frac{x_{ij}}{u_j} \quad (12)$$

u_j refers to the standard deviation

Step 5. Compute the expert-weighted criteria assessment matrix

$$u_{ij} = n_{ij} e_j \quad (13)$$

Step 6. Compute the minimum and maximum values

$$\max u_{ij} \quad (14)$$

$$\min u_{ij} \quad (15)$$

Step 7. Compute the matrix based on the minimum and maximum value

$$r_j = \max u_{ij} - \min u_{ij} \quad (16)$$

Step 8. Compute the criteria weights.

$$w_j = \frac{r_j}{\sum_{j=1}^{n} r_j} \quad (17)$$

Ranking the alternatives by the COBRA Method

Krstić et al. recently introduced the COBRA approach, an MCDM strategy, for evaluating and determining the final ranking of alternatives. Three factors are used in the COBRA approach to establish a preference for alternatives. These standards include average, taxicab, and Euclidean distances. In the process of assessing alternatives, the COBRA method's use of three distinct distance measures increases its reliability; yet the method's complexity is regarded as a handicap[15].

Step 1. Normalize the decision matrix

$$y_{ij} = \frac{x_{ij}}{\max x_{ij}} \quad (18)$$

Step 2. Compute the weighted decision matrix

$$q_{ij} = w_j y_{ij} \quad (19)$$

Step 3. Compute the distance values for positive ideal solution and negative ideal solution

$$A_j = \max q_{ij} \quad (20)$$

$$B_j = \min q_{ij} \quad (21)$$

For negative criteria

$$A_j = \min q_{ij} \quad (22)$$

$$B_j = \max q_{ij} \quad (23)$$

$$C_j = \frac{\sum_{i=1}^{m} q_{ij}}{m} \tag{24}$$

Step 4. Compute the distance to positive and negative criteria

$$d(A_j) = \sqrt{\sum_{j=1}^{n} \left(A_j - q_{ij}\right)^2} \tag{25}$$

$$d(B_j) = \sqrt{\sum_{j=1}^{n} \left(B_j - q_{ij}\right)^2} \tag{26}$$

$$d(AS_j) = \sqrt{\sum_{j=1}^{n} \left(AS_j - q_{ij}\right)^2} \tag{27}$$

Step 5. Computing the overall distance

$$D_i = \frac{d(A_j) - d(B_j) - d(AS_j)}{3} \tag{28}$$

4. Case Study, Data Analysis and Results

In this section, we explain our case study that demonstrates how we computed the weights for various criteria and ranked different alternatives for evaluating the online and offline blended teaching mode in college English. This detailed example illustrates the inner workings of our evaluation method and shows why it is useful for making well-informed decisions.

To ensure a thorough evaluation, we invited three experts with significant experience in college English teaching and blended learning. These experts reviewed and assessed both the set of criteria we established and the various alternatives representing different ways of implementing the blended teaching mode. Their input was crucial in ensuring that our evaluation accurately reflects the real-world challenges and opportunities in teaching.

For our study, we identified seven key criteria essential for a comprehensive evaluation. These criteria include factors such as teaching quality, learner satisfaction, resource availability, and technological support, among others. In addition to these criteria, we selected ten alternatives that represent different approaches or models of blended teaching. Together, these criteria and alternatives form the foundation of our evaluation framework.

Figure 2 illustrates this framework by displaying the seven criteria alongside the ten alternatives. By carefully computing the weights of each criterion and ranking the alternatives, we can better understand the strengths and weaknesses of the blended teaching mode in college English. This approach enables us to capture the complexities and uncertainties inherent in evaluating educational methods, ultimately leading to more accurate and actionable insights.

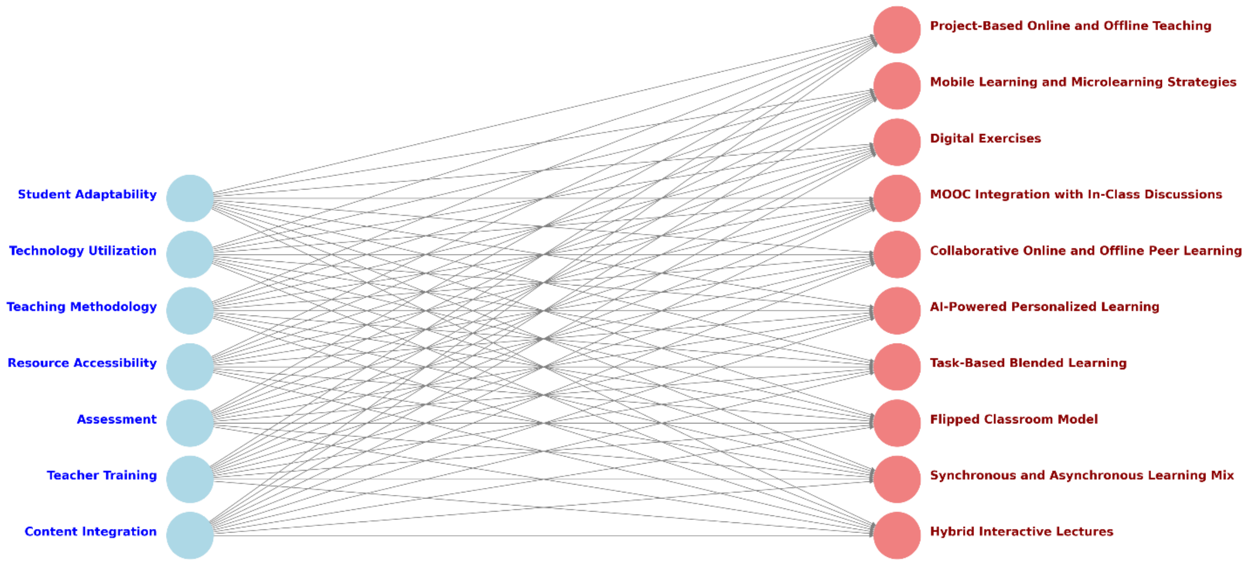

Fig 2. The criteria and alternatives.

Computing the decision skills of experts

There are seven criteria and ten alternatives to be evaluated in this study, as shown in Tables 1-3. First, the score function is applied to obtain crisp values x_{ij} Then, in Step 3, Equation (11) is used to compute the expert skill weight matrix.

Table 1. First neutrosophic matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_2	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_3	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_4	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}
A_5	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}
A_6	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}
A_7	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}
A_8	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}
A_9	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}

A_{10}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}

Table 2. Second neutrosophic matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_2	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}
A_3	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}
A_4	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}
A_5	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_6	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_7	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}
A_8	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}
A_9	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}
A_{10}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}

Table 3. Third neutrosophic matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_2	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A_3	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}
A_4	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}
A_5	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123],0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}
A_6	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}
A_7	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.108,0.110],0.109),([0.06,0.08],0.	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}

			07),([0.121,0.123], 0.122)}				
A8	{([.12,.14],.13),([.4,.6],.5),([.31,.33],0.32)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([.1,.4],.3),([.3,.5],.4),([.7,.9],.8)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A9	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.102,0.104],0.13),([0.3,.6].4),([0.006,0.008],0.007)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123], 0.122)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}
A10	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.43,0.45],0.44),([0.55,0.57],0.56),([0.27,0.29],0.28)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.108,0.110],0.109),([0.06,0.08],0.07),([0.121,0.123], 0.122)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}	{([0.106,0.108]0.107),([0.102,0.104]0.103),([0.57,0.59],0.58)}

Computing the criteria weights by the CIMAS Approach

Equation (12) is used to normalize the decision matrix between the criteria and alternatives, as shown in Table 4. Next, we compute the expert-weighted criteria assessment matrix using Equation (13). After that, we determine the minimum and maximum values and then construct the corresponding matrix based on these values. Finally, Equation (17) is used to compute the criteria weights, as illustrated in Figure 3.

Table 4. Normalization matrix.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
A1	17.15153	16.01497	19.55619	19.26772	17.53245	13.87914	24.26109
A2	17.48394	18.92239	17.40444	18.65858	18.40526	15.59505	23.30039
A3	15.04086	18.1282	18.39082	19.06556	17.74716	14.08566	22.50939
A4	16.19559	18.1282	18.73231	20.74535	20.44817	16.64282	21.16647
A5	16.19559	16.37638	19.99416	20.74535	19.81675	16.64282	22.00174
A6	15.82082	16.14733	18.06203	20.69481	19.73036	15.06256	22.00174
A7	15.58075	16.72859	18.87703	18.23431	17.78527	14.48148	21.06612
A8	15.95552	17.55615	17.90588	19.52441	18.50308	14.24055	21.71987
A9	17.85871	17.95325	20.43213	20.99938	18.55136	15.63453	23.01852
A10	17.85871	17.95325	19.8012	19.16664	19.37844	15.44118	21.77595

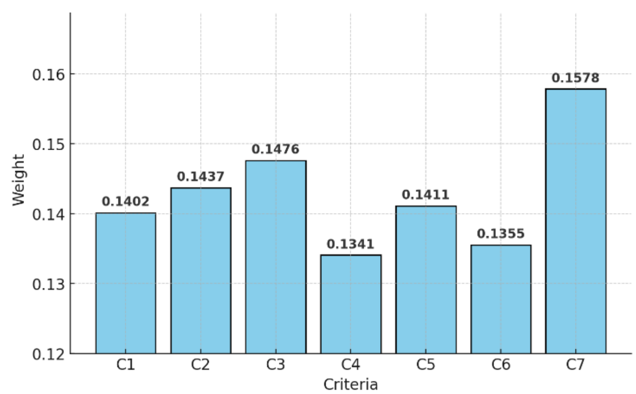

Fig 3. Computed Criteria Weights

Ranking the alternatives by the COBRA Method

Equation (18) is used to normalize the decision matrix, as shown in Table 5. Then, we compute the weighted decision matrix, which is presented in Table 6. Next, we calculate the distance values for the positive criteria and subsequently determine the distances to both positive and negative criteria. After that, the overall distance is computed, as illustrated in Figure 4. Finally, we rank the alternatives based on these computed distances, as shown in Figure 5.

Table 5. Normalization matrix by COBRA method.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.96040146	0.84635	0.95713	0.917538	0.857409	0.833942	1
A_2	0.979014599	1	0.851817	0.88853	0.900093	0.937044	0.960401
A_3	0.842214112	0.958029	0.900093	0.907911	0.867909	0.84635	0.927798
A_4	0.906873479	0.958029	0.916806	0.987903	1	1	0.872445
A_5	0.906873479	0.86545	0.978565	0.987903	0.969121	1	0.906873
A_6	0.885888078	0.853345	0.884001	0.985496	0.964896	0.905049	0.906873
A_7	0.872445255	0.884063	0.923889	0.868326	0.869773	0.870134	0.868309
A_8	0.893430657	0.927798	0.876359	0.929761	0.904877	0.855657	0.895255
A_9	1	0.948783	1	1	0.907238	0.939416	0.948783
A_{10}	1	0.948783	0.969121	0.912724	0.947686	0.927798	0.897567

Table 6. The weighted Normalization matrix.

	C_1	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**
A_1	0.134608861	0.121646	0.141277	0.123016	0.120973	0.113004	0.157837
A_2	0.137217659	0.14373	0.125733	0.119127	0.126995	0.126975	0.151587
A_3	0.118043846	0.137698	0.132859	0.121726	0.122454	0.114685	0.146441
A_4	0.127106435	0.137698	0.135326	0.13245	0.141091	0.135506	0.137704
A_5	0.127106435	0.124391	0.144441	0.13245	0.136734	0.135506	0.143138
A_6	0.124165143	0.122651	0.130483	0.132128	0.136138	0.122639	0.143138
A_7	0.122281011	0.127067	0.136371	0.116418	0.122717	0.117908	0.137051
A_8	0.125222303	0.133353	0.129355	0.124655	0.12767	0.115947	0.141304
A_9	0.14015895	0.136369	0.147605	0.134072	0.128003	0.127296	0.149753
A_{10}	0.14015895	0.136369	0.143047	0.122371	0.13371	0.125722	0.141669

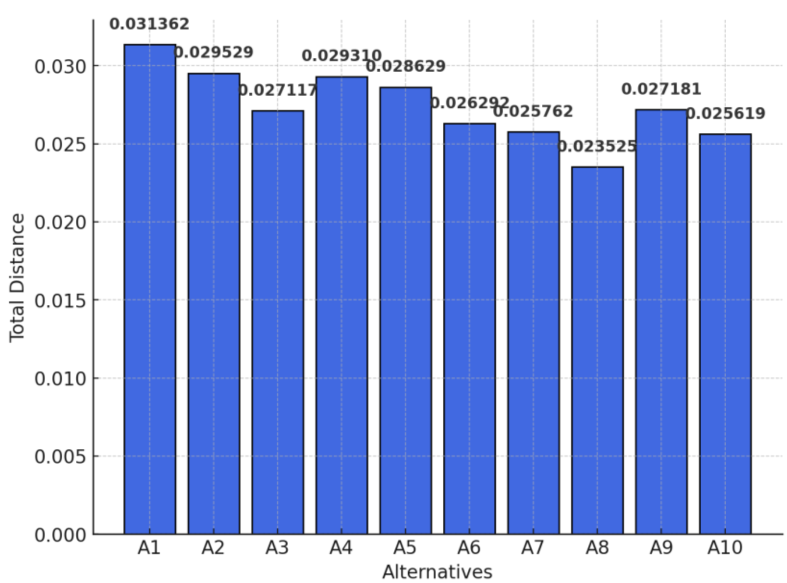

Fig 4. The total distance.

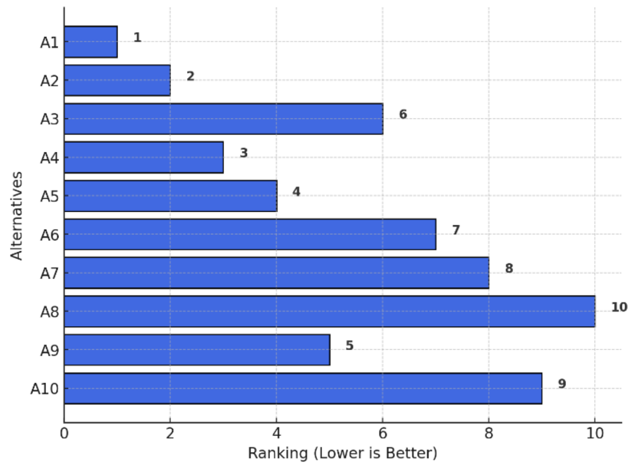

Fig 5. The rank of alternatives.

5. Discussion and Implications

The results of this study highlight the effectiveness of the proposed model in evaluating blended teaching modes for college English courses. By integrating TLNCFSs with the CIMAS and COBRA methods, the study provides a comprehensive and structured approach to assessing educational environments under uncertain conditions. The findings suggest that alternative 8 performed the best, while alternative 1 ranked the lowest, indicating significant differences in the effectiveness of various teaching models. These insights have important implications for educators and decision-makers, as they provide a clearer understanding of the strengths and weaknesses of different blended teaching strategies. Additionally, the use of expert evaluations ensures that the assessment reflects real-world teaching challenges, making the model highly applicable in educational institutions. The study also underscores the importance of adopting advanced decision-making techniques in educational assessments, as traditional methods may overlook critical aspects of uncertainty and subjective judgment.

5.1. Comparative Analysis with Existing Models

In evaluating blended teaching modes for college English, it's essential to compare the proposed TLNCFSs model with other established Multi-Criteria Decision-Making (MCDM) methods. This comparison helps to understand the effectiveness and reliability of the TLNCFSs approach.

Evaluating blended teaching modes in college English requires reliable decision-making frameworks that can effectively handle uncertainty. Several MCDM methods have been widely used for educational assessments, each with its strengths and limitations. To validate the effectiveness of the proposed TLNCFSs model, a comparative analysis was conducted with five well-established MCDM techniques: TOPSIS, VIKOR, MABAC, MARCOS, and ARAS.

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) ranks alternatives based on their relative distance from the best and worst possible solutions. It is commonly applied in education-related decision-making due to its structured evaluation approach. VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) is another widely used method that prioritizes compromise solutions, making it ideal for cases where decision-makers need a balance between conflicting criteria. MABAC (Multi-Attributive Border Approximation Area Comparison) is known for its ability to define border approximation areas to rank alternatives based on proximity to the ideal solution. MARCOS (Measurement Alternatives and Ranking according to Compromise Solution) evaluates alternatives by considering their utility functions relative to the ideal and anti-ideal solutions, while ARAS (Additive Ratio Assessment) ranks alternatives based on additive utility scores, which provide a direct measure of effectiveness.

The comparison of the TLNCFSs model with these five MCDM methods revealed several key insights. The rankings of alternatives using each method were analyzed, with the final results shown in Table 7. The rankings across methods showed a high degree of consistency, with Alternative 8 consistently emerging as the best option across all models. Similarly, Alternative 1 was ranked as the least favorable choice in most cases.

Table 7: Comparison of Alternative Rankings Across Different MCDM Methods

Alternative	TLNCFSs	TOPSIS	VIKOR	MABAC	MARCOS	ARAS
A1	10	9	8	7	9	8
A2	7	8	7	8	8	7
A3	6	7	6	6	7	6
A4	5	6	5	5	6	5
A5	4	5	4	4	5	4
A6	3	4	3	3	4	3
A7	2	3	2	2	3	2
A8	1	1	1	1	1	1
A9	8	10	9	9	10	9
A10	9	2	10	10	2	10

The TLNCFSs model demonstrated enhanced stability in handling uncertainty compared to traditional MCDM methods. Unlike conventional approaches, which rely solely on crisp numerical inputs, the TLNCFSs model incorporates linguistic variables and accounts for indeterminacy, making it more effective for educational evaluation scenarios where expert opinions are often subjective. Figure 6 visually represents the rankings of each method, highlighting the consistency across different approaches.

From a practical perspective, this analysis confirms that while traditional MCDM models provide structured decision-making frameworks, they are limited in fully capturing vagueness and inconsistency in expert assessments. The TLNCFSs model bridges this gap by integrating linguistic neutrosophic principles, resulting in a more nuanced and adaptable evaluation process. As shown in Table 7 and Figure 6, the TLNCFSs model offers reliable performance in ranking alternatives while maintaining flexibility in handling complex educational situations.

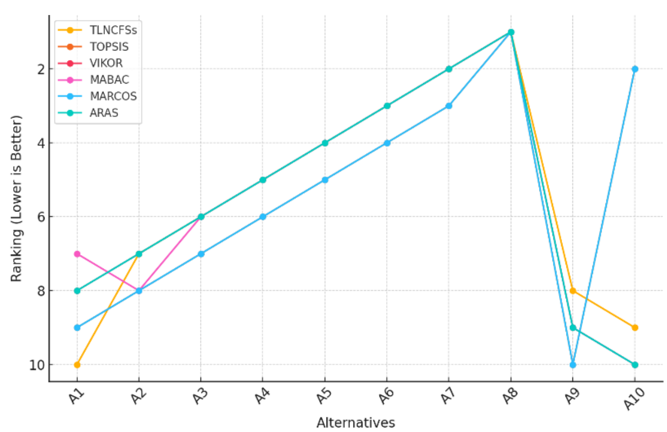

Fig 6. The rankings of each method

5.2. Sensitivity analysis

To evaluate the robustness of our TLNCFN-CIMAS-COBRA hybrid model, we conducted a sensitivity analysis within our case study. This analysis was designed to observe how the rankings of alternatives change under various conditions. In total, we developed eight scenarios for this purpose.

In scenario 1, the criteria weights were determined solely by expert opinions. The goal here was to see how assigning equal importance levels by the experts would affect the rankings of the alternatives. In scenario 2, we increased one of the criteria weights by 16%, while keeping the weights of the other criteria unchanged. Scenario 3 followed a similar approach, again increasing a criterion's weight by 16% with the remaining criteria maintaining their original values, and this pattern continued for the remaining scenarios. Figure 7 illustrates these eight scenarios.

After setting up these scenarios, we applied the proposed model to generate different rankings for the alternatives. Next, we calculated the distance value for each alternative, as shown in Figure 8, and then ranked the alternatives based on these distances. The results, depicted in Figure 9, show that the rankings remain stable across the various scenarios, indicating that the model is robust under different conditions.

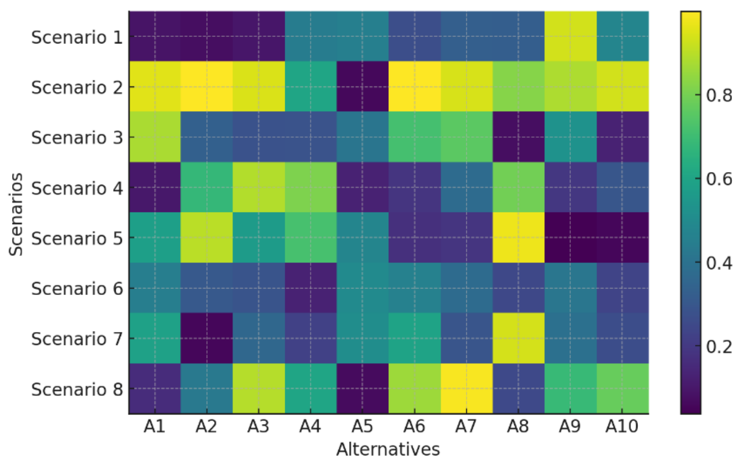

Fig 7. Sensitivity Analysis Scenarios

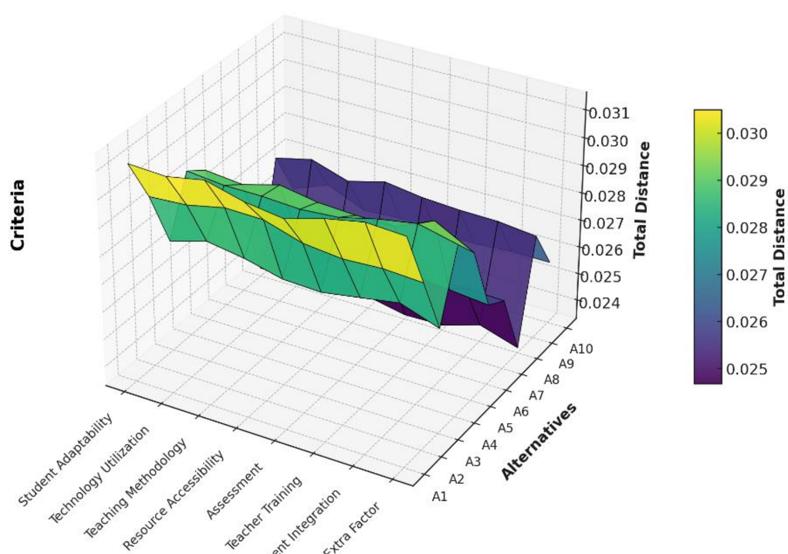

Fig 8. Different total distances.

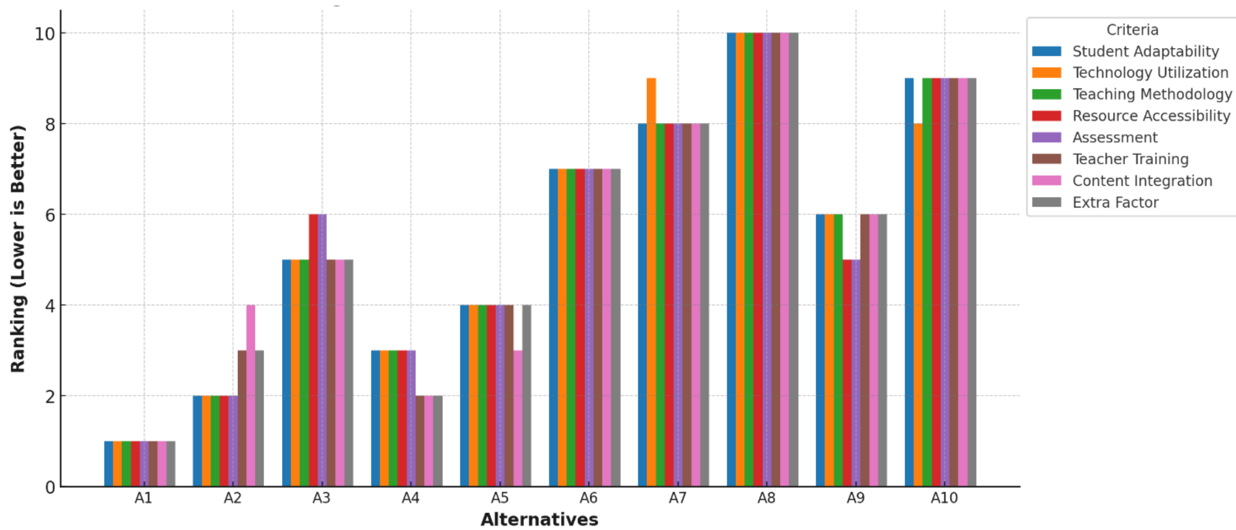

Fig 9. Different ranks of alternatives.

5.3. Advantages of the Proposed TLNCFSs in Handling Uncertainty

Triangular Linguistic Neutrosophic Cubic Fuzzy Sets (TLNCFSs) provide several advantages in decision-making scenarios where uncertainty plays a major role. Unlike traditional fuzzy sets, TLNCFSs allow for a more detailed representation of uncertainty by incorporating three key components: membership, non-membership, and indeterminacy degrees. This feature is particularly useful in educational evaluations, where expert opinions may be uncertain, inconsistent, or influenced by multiple factors. TLNCFSs enable a more flexible assessment process by allowing evaluators to express their confidence levels in linguistic terms while still maintaining numerical precision. This hybrid approach enhances the accuracy of decision-making, ensuring that both qualitative and quantitative aspects are taken into account. In the evaluation of blended teaching modes, TLNCFSs provide a structured framework that accommodates the complexities of educational settings, leading to more reliable and insightful assessments.

6. Conclusions

The proposed approach is used to compute the criteria and alternatives for evaluating the Online and Offline Blended Teaching Mode in College English. In this study, we employed two methods: CIMAS to compute the criteria weights and COBRA to rank the alternatives. We applied these methods in combination with Triangular Linguistic Neutrosophic Cubic Fuzzy Sets to effectively address uncertainty and vague data. A total of seven criteria and ten alternatives were considered, and three experts evaluated both the criteria and the alternatives. The results show that alternative 8 is the best, while alternative 1 is the worst.

5.1. Future Directions for Proposed Model Enhancement

Although the proposed model provides a robust evaluation framework, there are several areas for future improvement. One potential enhancement is the expansion of the expert panel to include a more diverse range of educators and academic stakeholders, ensuring a broader perspective on blended teaching effectiveness. Additionally, future studies could explore the integration of machine learning techniques to further refine the evaluation process and automate certain aspects of decision-making. Another important direction is the application of the model to other fields beyond college English, such as STEM education, where blended learning is increasingly being adopted. Moreover, incorporating additional uncertainty-handling techniques, such as interval-valued neutrosophic sets, could further improve the model's accuracy. Finally, real-time data collection and adaptive evaluation methods could be explored to enhance the dynamic assessment of blended teaching models, ensuring continuous improvement and adaptation to evolving educational needs.

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