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Application of the Neutrosophic Fuzzy Sets in Profession Determination

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Abstract: We present several operations, such as algebraic sum, algebraic product, and arithmetic mean, over the neutrosophic sets extended with moment in time and sketch numbers. Through mathematical illustrations, we explore various properties and apply this concept to professional decision-making. Existing neutrosophic set (NS) and fuzzy set theories lack the capability to dynamically account for time and evolving circumstances. Many approaches fail to provide satisfactory solutions for handling evolving decision criteria or dynamic uncertainties. While operations like algebraic sum, algebraic product, and arithmetic mean exist for neutrosophic sets, their adaptation and extension to dynamic and temporal dimensions (moment in time and sketch numbers) remain underexplored. Additionally, there is a lack of studies applying neutrosophic set extensions in critical domains such as professional decision-making, which involves dynamic and complex evaluation processes. Many existing models do not adequately demonstrate practical applicability with robust mathematical illustrations. The introduction of Interval-Valued Temporal Neutrosophic Fuzzy Sets (IVTNFS), incorporating advanced operations and additional dimensions, provides a stronger framework for addressing evolving uncertainties in decision-making.

Keywords: Intuitionistic-FS (IFS), Temporal-IFS (TIFS), Neutrosophic Set (NS), Interval Valued-IFS (IVIFS) and IV-Temporal NFS (IVTNFS).

1. Introduction

L.A. Zadeh [40] introduced fuzzy set (FS) structure to scientifically express uncertainty in the year 1965. This abstract concept aimed to address the challenge of assigning a degree of membership to every component of a specified set. This, inside turn, laid the foundation for FS theory. Zadeh established the notion of a vague set, which generalizes the membership function of a set, allowing the degree of membership of an component to be more flexible than just "true" or "false." A vague set is mathematically constructed by assigning each element in the universe a value that represents its degree of membership in the fuzzy set. This value corresponds to the extent to which the element aligns with the conception represent through the vague set. Membership values are often expressed as real numbers within the closed interval [0, 1].

The word fuzzy sense was introduced by Zadeh [40] in 1962. His work progressed further in 1965 with the formalization of this mathematical concept, now known as Fuzzy Set Theory. Later, Krassimir T. Atanassov [2] introduced IFS, which extend fuzzy sets by incorporating together a membership degree with a non-membership degree. Since their introduction, intuitionistic fuzzy sets have garnered significant attention from researchers, leading to their application in various domains such as model discovery.

A wide-ranging IFSs - GIFS was proposed by Samanta & Mondal [16] beneath the condition to facilitate the sum of the membership and non-membership degrees does not exceed one. Additionally, Gargov and K.T. Atanassov [5] developed interval-valued IFSs, second-type IFSs, and temporal IFSs. Further extensions include rough IFSs proposed by Rizvi et al. [25] and IF soft sets introduced by Majiet et al. [14]. In 2015, R. Srinivasan and K. Rajesh [18] developed an extension of IVIFS, studying its fundamental operations and operators. They further introduced various distance measures over IVIFSST in 2017 and the second type of temporal IFS [19], applying the concept to real-life situations in 2019. Later, K. Rajesh [20] introduced Certain Level Operators over TIFS in 2022.

There are two types of time: instantaneous time and interval time. An instant represents a single moment, whereas an interval denotes the duration among two points in occasion. A chronological representation can be based on one or together of these types. If the granularity of the moment aspect is adequately refined, or else if start and end points of a closed interval are equal, then a time instant can be considered a time interval. While most existing research on time modeling focuses on defining distinct and precise temporal information, in reality, temporal information is often uncertain and ambiguous.

Smarandache [29] established the notion of Neutrosophic Sets (NS) and its properties in 1999. Later, in 2005, he proposed Neutrosophic Sets with Intervals [28], where each element's uncertain membership status is represented using three components: truth, indeterminacy, and falsity. In 2023, Smarandache [30] introduce the idea of Excellent Anxious Soft Sets and Fuzzy Expansion of Excellent Anxious Soft Sets, along with developments in Revolutionary Topologies, New Types of Topologies, and Neutrosophic Topologies [36]. In 2024, he further introduced Appurtenance & Insertion Equations to construct neutrosophic operations essential in neutrosophic statistics [27].

In 2021, Deli, Uluçay, and Polat [9] introduced N-Valued Neutrosophic Trapezoidal Numbers with similarity measures and applications in MCDM problems. Riad K with Al-Hamido [24] explored Neutrosophic Algebraic Structure II in 2024, following Smarandache's [27] introduction of Neutrosophic

Two-Fold Algebra that same year. In 2024, K. Rajesh [21, 22] and M.C.J. Anand et al. introduce the idea of IVNFSs, incorporating both temporal and interval aspects along with various operations. In 2025, Khalid and Saeed [12] developed an approach for hybridizing N-Subalgebra with quantified neutrosophic sets using G-Algebra. Ram and Singh [23] introduced Neutrosophic Automata and Its Algebraic Properties in 2025. These concepts have since been applied in diverse fields, including preference structures, relational databases, consistency theory, expert systems, and more.

Traditional neutrosophic set extensions that rely solely on membership and non-membership degrees often fail to adequately capture dynamic uncertainty. However, the IVTNFS plays a more significant role by incorporating time-based extensions into neutrosophic sets, yielding more accurate results. This advancement is a key motivation for further developing neutrosophic fuzzy sets.

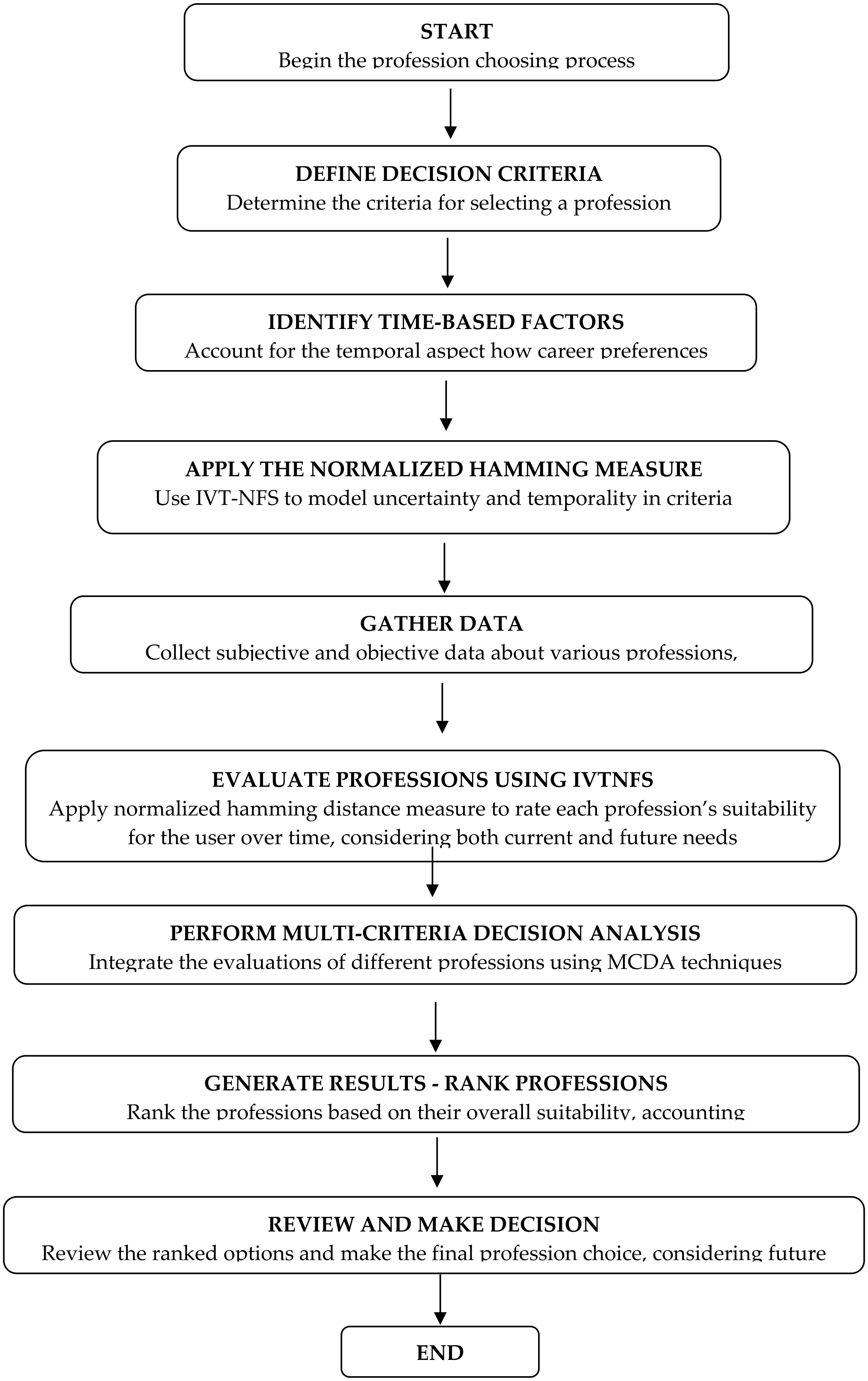
Motivation of this work:

Many real-world problems (e.g., career determination, image processing, and pattern recognition) involve uncertainty, vagueness, and incomplete information. Existing models struggle to handle these complexities effectively. Traditional fuzzy sets and neutrosophic sets are valuable but insufficient for scenarios that demand a dynamic representation of uncertainty over time or additional factors. Decision-making often requires incorporating the element of time, where preferences and conditions evolve. The proposed moment-based extension of neutrosophic sets adds temporal relevance and adaptability, making it a more realistic and practical tool. The extended framework has significant potential in solving real-life problems such as professional career selection, image processing, and decision-making, which are critical and impactful fields.

Novelties of the work:

This work introduces a novel thought of intervals with the temporal neutrosophic fuzzy sets IVTNFS, which incorporates a temporal dimension to better represent dynamic changes in real-world scenarios. The development and application of algebraic sum, algebraic product, and arithmetic mean operations over IVTNFS provide innovative tools for handling complex decision-making scenarios. Mathematical demonstrations and examples are provided to validate the efficiency of the proposed approach in real-life contexts, showcasing its practicality. The proposed framework is applied to career determination, demonstrating its potential to handle uncertainty and dynamic factors in significant life decisions. This can be further extended to areas like image processing and pattern recognition. By leveraging the temporal aspect and advanced mathematical operations, this work offers a refined methodology for achieving precise and reliable decisions in complex and evolving environments.

This research paper is going to organize while tag along: in the first segment we present a few essential definitions which determination be use in the 2nd section. In the third segment, we introduced a few operations like, addition, multiplication, arithmetic mean operation and geometric mean operation over IVTNFS and also studied several properties of IVTNFS finally we conclude this research.


START
Begin the profession choosing process
DEFINE DECISION CRITERIA
Determine the criteria for selecting a profession
IDENTIFY TIME-BASED FACTORS
Account for the temporal aspect how career preferences
APPLY THE NORMALIZED HAMMING MEASURE
Use IVT-NFS to model uncertainty and temporality in criteria
GATHER DATA
Collect subjective and objective data about various professions,
EVALUATE PROFESSIONS USING IVTNFS
Apply normalized hamming distance measure to rate each profession's suitability for the user over time, considering both current and future needs
PERFORM MULTI-CRITERIA DECISION ANALYSIS
Integrate the evaluations of different professions using MCDA techniques
GENERATE RESULTS - RANK PROFESSIONS
Rank the professions based on their overall suitability, accounting
REVIEW AND MAKE DECISION
Review the ranked options and make the final profession choice, considering future
END

2. Preliminary

Definition: 2.1 [2] Allow us consider K exist located in a filled sets. An Intuitionistic Fuzzy Sets (IFS) Y in K which is precise by K.T. Atanassov as the subsequent formula.

$$Y = \{< e, \sigma_Y(e), \varphi_Y(e) > | \ e \in K\}$$

Wherever the affiliation function denoted as σ_Y map from K to [0, 1] and the non-affiliation function denoted as φ_Y map from K to [0, 1] respectively with $0 \leq \sigma_Y(e) + \varphi_Y(e) \leq 1$ and for each components $e \in$ K.

Definition: 2.2 [5] Allow us consider K exist located in a filled sets. An intuitionistic fuzzy sets with time moments (TIFS) Y in K which is defined by K.T. Atanassov as the subsequent formula.

$$Y = \{< (e, s), \sigma_Y(e, s), \varphi_Y(e, s) > |(e, s) \in K \times L\}$$

Wherever the affiliation function denoted as σ_Y map from K to [0, 1] and the non-affiliation function denoted as φ_Y map from K to [0, 1] respectively with the condition $0 \leq \sigma_Y(e, s) + \varphi_Y(e, s) \leq 1$ and for each components $e \in$ K.

Definition: 2.3 [4] Let us consider K be located in a non-unfilled set. A fuzzy set with non membership values and intervals (name as IVIFS) Y in K which is defined by K.T. Atanassov and Gargov as the subsequent formula.

$$Y = \left\{< (e, s), \alpha_Y(e, s), \beta_Y(e, s) > |(e, s) \in K \times L\right\}$$

Everywhere $\alpha_Y: K \rightarrow [0, 1], \beta_Y: K \rightarrow [0, 1]$. The interludes $\alpha_Y(e, s)$ as well as $\beta_Y(e, s)$ symbolise the gradation of affiliation and gradation of non-affiliation of the component K to a set R, everywhere $\alpha_Y(e, s) = [\alpha_{YL}(e, s), \ \alpha_{YU}(e, s)]$ and $\beta_Y(e, s) = [\beta_{YL}(e, s), \beta_{YU}(e, s)]$ using the circumstance that $\alpha_{YU}(e, s) + \beta_{YU}(e, s) \leq 1$ for all $e \in K$.

Definition: 2.4 [33] Let us consider K be located in a non-unfilled set. A Neutrosophic Sets (NS) Y voguish K is pigeon-holed by an accuracy, indeterminacy & inaccuracy affiliation utility which exist, correspondingly represented as T_Y, I_Y as well F_Y in addition it is designated such as the succeeding formula

$$Y = \{< e, T_Y(e), I_Y(e), F_Y(e) >| e \in K\}$$

The values of $T_Y(e), I_Y(e)$ & $F_Y(e)$ be presentusual or unusual subcategories of $]0^-, 1^+[$, .i.e., $T_Y(e) : K \rightarrow]0^-, 1^+[, I_Y(e) : K \rightarrow]0^-, 1^+[$ as well as $F_Y(e): K \rightarrow]0^-, 1^+[$ certainly not constraint is realistic on the totality of $T_Y(e), I_Y(e)$ & $F_Y(e)$, with

$$0^- \leq T_Y(e) + I_Y(e) + F_Y(e) \leq 3^+$$

Where $T_Y(e), I_Y(e)$ also $F_Y(e)$ is termed as neutrosophic integers.

Definition: 2.5 [34] Let us consider K be located in a non-unfilled set. A Neutrosophic Fuzzy Sets (NFS) Y voguish K is pigeon-holed by an accuracy, indeterminacy & inaccuracy affiliation utility which exist, correspondingly represented as M_Y, T_Y, I_Y as well F_Y in addition it is designated such as the succeeding formula

$$Y = \{< e, M_Y(e), T_Y(e), I_Y(e), F_Y(e) > | e \in K\}$$

The values of $M_Y(e), T_Y(e), I_Y(e)$ & $F_Y(e)$ be present usual or unusual sub categories of $]0^-, 1^+[$ i.e., $M_Y(e) : K \to]\, 0^-, 1^+[, T_Y(e) : K \to]\, 0^-, 1^+[, I_Y(e) : K \to]0^-, 1^+[$ as well as $F_Y(e) : K \to]\, 0^-, 1^+[$ certainly not constraint is realistic on the totality of $M_Y(e), T_Y(e), I_Y(e)$ &
$F_Y(e)$, therefore

$$0^- \leq M_Y(e) + T_Y(e) + I_Y(e) + F_Y(e) \leq 3^+$$

Where $M_Y(e), T_Y(e), I_Y(e)$ also $F_Y(e)$ is termed as neutrosophic integers.

Definition: 2.6 [42] Let us consider K be located in a non-unfilled set. A Neutrosophic Sets in intervals (IVNS) Y voguish K is pigeon-holed by an accuracy, indeterminacy &inaccuracy affiliation utility which exist, correspondingly represented as T_Y, I_Y as well F_Y in addition it is designated such as the succeeding formula

$$\mathrm{Y} = \{< e, [\mathrm{T_{YL}(e), T_{YU}(e)}], [\mathrm{I_{YL}(e), I_{YU}(e)}], [\mathrm{F_{YL}(e), F_{YU}(e)}] > | \mathrm{e} \in \mathrm{K}\}$$

Everywhere the utilities $T_Y(e), I_Y(e)$ & $F_Y(e) : K \to]0^-, 1^+[$ with the condition
$0^- \leq \mathrm{T_{YU}(e)} + \mathrm{I_{YU}(e)} + \mathrm{F_{YU}(e)} \leq 3^+$.

Definition: 2.7 [24] Let us consider K be located in a non-unfilled set. A Temporal Neutrosophic Sets in intervals (IVTNFS) Y voguish K is pigeon-holed by an accuracy, indeterminacy & inaccuracy affiliation utility which exist, correspondingly represented as $\mathrm{T_Y, I_Y}$ as well $\mathrm{F_Y}$ in addition it is designated such as the succeeding formula

$$\mathrm{Y} = \{< (\mathrm{e,s}), [\mathrm{M_{YL}(e,s), M_{YU}(e,s)}], [\mathrm{T_{YL}(e,s), T_{YU}(e,s)}], [\mathrm{I_{YL}(e,s), I_{YU}(e,s)}], [\mathrm{F_{YL}(e,s), F_{YU}(e,s)}] > | e \in K \;\&\; s \in L\}$$

Everywhere the utilities $T_Y(e), I_Y(e)$ & $F_Y(e) : K \to]0^-, 1^+[$ with the time moments and satisfy the following condition

$$0^- \leq \mathrm{M_{YU}(e,s)} + \mathrm{T_{YU}(e,s)} + \mathrm{I_{YU}(e,s)} + \mathrm{F_{YU}(e,s)} \leq 3^+.$$

Definition: 2.8 [24] Let us consider Y be any IVTNFSs on K, then the accompaniment of Y is designated as $\bar{Y}$ further it is well-defined partakes the succeeding formula

$$\bar{Y} = \{< (\mathrm{e,s}), [\mathrm{M_{YL}(e,s), M_{YU}(e,s)}], [\mathrm{F_{YL}(e,s), F_{YU}(e,s)}], [1 - \mathrm{I_{YU}(e,s)}, 1 - \mathrm{I_{YU}(e,s)}], [\mathrm{T_{YL}(e,s), T_{YU}(e,s)}] > | e \in K \;\&\; s \in L\}$$

3. Various operations over the branch of Neutrosophic fuzzy sets with intervals

We introduce some new operations over IVTNFS through arithmetical example and in addition establish a number of his property.

Definition: 3.1 Allow M, N are the wing of Neutrosophic Fuzzy Sets then, we define algebraic sum in the following form

$$M + N = \{\langle (e,s),((R_{ML}(e,s) + R_{NL}(e,s) - \big(R_{ML}(e,s).R_{NL}(e,s)\big),$$
$$(R_{MU}(e,s) + R_{NU}(e,s) - \big(R_{MU}(e,s).R_{NU}(e,s)\big)),$$
$$\big\{((T_{ML}(e,s) + T_{NL}(e,s) - \big(T_{ML}(e,s).T_{NL}(e,s)\big),$$
$$((T_{MU}(e,s) + T_{NU}(e,s) - \big(T_{MU}(e,s).T_{NU}(e,s)\big)),$$
$$((I_{ML}(e,s) + I_{NL}(e,s) - \big(I_{ML}(e,s).I_{NL}(e,s)\big),$$
$$(I_{MU}(e,s) + I_{NU}(e,s) - \big(I_{MU}(e,s),I_{NU}(e,s)\big)),$$
$$((F_{ML}(e,s) + F_{NL}(e,s) - \big(F_{ML}(e,s).F_{NL}(e,s)\big),$$
$$(F_{MU}(e,s) + F_{NU}(e,s) - \big(F_{MU}(e,s).F_{NU}(e,s)\big))\big\} > |(e,s) \in K \times L\}$$

Definition: 3.2 Let M, N are in the expansion of Neutrosophic Fuzzy Sets then, the algebraic product is describe as the succeeding formula

$$M \cdot N = \{\langle (e,s),(\, R_{ML}(e,s).R_{NL}(e,s), R_{MU}(e,s).R_{NU}(e,s)),$$
$$\{(T_{ML}(e,s).T_{NL}(e,s), T_{MU}(e,s).T_{NU}(e,s)), (I_{ML}(e,s).I_{NL}(e,s),$$
$$I_{MU}(e,s).I_{NU}(e,s)), (F_{ML}(e,s) + F_{NL}(e,s) - F_{ML}(e,s).F_{NL}(e,s),$$
$$F_{MU}(e,s) + F_{NU}(e,s) - F_{MU}(e,s).F_{NU}(e,s)\}\rangle |(e,s) \in K \times L\}$$

Definition: 3.3 Let M, N are in the extension of Neutrosophic Fuzzy Sets then, the arithmetic mean operation is describe as the succeeding formula

$$M \;@\; N = \Bigg\{\langle (e,s), \left[\frac{R_{ML}(e,s) + R_{NL}(e,s)}{2}, \frac{R_{MU}(e,s) + R_{NU}(e,s)}{2}\right]$$
$$\left[\frac{T_{ML}(e,s) + T_{NL}(e,s)}{2}, \frac{T_{MU}(e,s) + T_{NU}(e,s)}{2}\right],$$
$$\left[\frac{I_{ML}(e,s) + I_{NL}(e,s)}{2}, \frac{I_{MU}(e,s) + I_{NU}(e,s)}{2}\right],$$
$$\left[\frac{F_{ML}(e,s) + F_{NL}(e,s)}{2}, \frac{F_{MU}(e,s) + F_{NU}(e,s)}{2}\right]\rangle \,|(e,s) \in K \times L\}$$

Definition: 3.4 Let us consider M, N are in the extension of Neutrosophic Fuzzy Sets then, the geometric mean operation is define as the following form

$$M \,\$\, N = \Big\{\langle (e,s), \left[\sqrt{R_{ML}(e,s) \cdot R_{NL}(e,s)}, \sqrt{R_{MU}(e,s) \cdot R_{NU}(e,s)}\right],$$
$$\Big\{\left[\sqrt{T_{ML}(e,s) \cdot T_{NL}(e,s)}, \sqrt{T_{MU}(e,s) \cdot T_{NU}(e,s)}\right],$$
$$\left[\sqrt{I_{ML}(e,s) \cdot I_{NL}(e,s)}, \sqrt{I_{MU}(e,s) \cdot I_{NU}(e,s)}\right],$$

$$\left[\sqrt{F_{ML}(e,s)\cdot F_{NL}(e,s)},\sqrt{F_{MU}(e,s)\cdot F_{NU}(e,s)}\right]\}\rangle|(e,s)\in K\times L\}$$

Example: 3.5 Let us consider an IVTNFSs M and N over K given by

$$M=\{<a,[0.2,0.4],[0.6,0.3],[0.3,0.5],[0.2.0.3]>,<b,[0.6,0.8],[0.5,0.6],[0.1,0.4],[0.4,0.7]>\}$$

And

$$N=\{<a,[0.3,0.5],[0.4,0.2],[0.4,0.5],[0.2.0.6]>,<b,[0.3,0.4],[0.3,0.6],[0.1,0.8],[0.5,0.4]>\}$$

Then,

$$M+N=\{\langle a,[0.44,0.7],[0.76,0.44],[0.58,0.75],[0.36,0.72]\rangle, \langle b,[0.72,0.88],[0.65,0.84],[0.19,0.88],[0.7,0.82]\rangle\}$$

$$M\cdot N=\{\langle a,[0.06,0.2],[0.24,0.06],[0.12,0.25],[0.36,0.72]\rangle, \langle b,[0.18,0.32],[0.15,0.36],[0.01,0.32],[0.7,0.82]\rangle\}$$

$$M@N=\{\langle a,[0.25,0.45],[0.5,0.25],[0.35,0.5],[0.2,0.45]\rangle,\langle b,[0.45,0.6],[0.4,0.6],[0.1,0.6],[0.45,0.55]\rangle\}$$

$$M\$N$$
$$=\{\langle a,[0.24,0.44],[0.49,0.24],[0.34,0.5],[0.2,0.42]\rangle,\langle b,[0.42,0.57],[0.39,0.6],[0.1,0.57],[0.45,0.53]\rangle\}$$

Therefore M and N satisfied the interval valued temporal neutrosophic fuzzy sets condition

Algorithm: 3.6

STEP 1: $Y=\{<e,\alpha_Y(e)>|\ e\in K\}$ The affiliation function lies between (0 - 1), then we move next step. (Then the function R is fuzzy sets.)

STEP 2: $Y=\left\{<e,\alpha_Y(e),\beta_Y(e)>|\ e\in K\right\}$ The affiliation & non-affiliation function lies between (0 - 1) & $0\le\alpha_Y(e)+\beta_Y(e)\le 1$ Then, we move next step. (Then the function E is Intuitionistic fuzzy sets.)

STEP 3: $Y=\left\{<e,\alpha_Y(e),\beta_Y(e)>|\ e\in K\right\}$ the addition of upper limit of affiliation and non-affiliation values is 1 then, we move the next step. (Then the function E is said to be IVIFS.)

STEP 4: $Y=\left\{<(e,s),\alpha_Y(e,s),\beta_Y(e,s)>|\ (e,s)\in K\times L\right\}$ & the addition of affiliation value, Truth, Indeterminacy and falsity value is lies between zero to one, then the function E is said to be IVTIFS. We move the next step.

STEP 5: $Y=\{<(e,s),[M_{YL}(e,s),M_{YU}(e,s)],[T_{YL}(e,s),T_{YU}(e,s)],[I_{YL}(e,s),I_{YU}(e,s)],$ $[F_{YL}(e,s),F_{YU}(e,s)]>|e\in K\ \&\ s\in L\}$ with the sum of all upper bound values of affiliation, Truth, Indeterminacy and falsity is lies between zero to one, then the function R is called IVTNFS.

Theorem 3.7 Allow M and N be a IVTNFSs on K. Then and there, we partake the resulting

(i) $\boldsymbol{M @ N = N @ M}$,

(ii) $\boldsymbol{M \$ N = N \$ M}$.

Proof:

For M, N ∈ IVTNFS, then the arithmetic and geometric mean procedure are defined as

$$M \,@\, N = \Big\{\langle (e,s), \left[\frac{R_{ML}(e,s)+R_{NL}(e,s)}{2}, \frac{R_{MU}(e,s)+R_{NU}(e,s)}{2}\right]$$
$$\left[\frac{T_{ML}(e,s)+T_{NL}(e,s)}{2}, \frac{T_{MU}(e,s)+T_{NU}(e,s)}{2}\right],$$
$$\left[\frac{I_{ML}(e,s)+I_{NL}(e,s)}{2}, \frac{I_{MU}(e,s)+I_{NU}(e,s)}{2}\right],$$
$$\left[\frac{F_{ML}(e,s)+F_{NL}(e,s)}{2}, \frac{F_{MU}(e,s)+F_{NU}(e,s)}{2}\right]\rangle \,|(e,s) \in K \times L\}$$

$$M \,\$\, N = \Big\{\langle (e,s), \left[\sqrt{R_{ML}(e,s)\cdot R_{NL}(e,s)}, \sqrt{R_{MU}(e,s)\cdot R_{NU}(e,s)}\right],$$
$$\Big\{\left[\sqrt{T_{ML}(e,s)\cdot T_{NL}(e,s)}, \sqrt{T_{MU}(e,s)\cdot T_{NU}(e,s)}\right],$$
$$\left[\sqrt{I_{ML}(e,s)\cdot I_{NL}(e,s)}, \sqrt{I_{MU}(e,s)\cdot I_{NU}(e,s)}\right],$$
$$\left[\sqrt{F_{ML}(e,s)\cdot F_{NL}(e,s)}, \sqrt{F_{MU}(e,s)\cdot F_{NU}(e,s)}\right]\Big\}\rangle |(e,s) \in K \times L\}$$

(i) $M \,@\, N \;= M \,@\, N$

$$M \,@\, N = \Big\{\langle (e,s), \left[\frac{R_{ML}(e,s)+R_{NL}(e,s)}{2}, \frac{R_{MU}(e,s)+R_{NU}(e,s)}{2}\right]$$
$$\left[\frac{T_{ML}(e,s)+T_{NL}(e,s)}{2}, \frac{T_{MU}(e,s)+T_{NU}(e,s)}{2}\right],$$
$$\left[\frac{I_{ML}(e,s)+I_{NL}(e,s)}{2}, \frac{I_{MU}(e,s)+I_{NU}(e,s)}{2}\right],$$
$$\left[\frac{F_{ML}(e,s)+F_{NL}(e,s)}{2}, \frac{F_{MU}(e,s)+F_{NU}(e,s)}{2}\right]\rangle \,|(e,s) \in K \times L\}$$

$$= \Big\{\langle (e,s), \left[\frac{R_{NL}(e,s)+R_{ML}(e,s)}{2}, \frac{R_{NU}(e,s)+R_{MU}(e,s)}{2}\right]$$
$$\left[\frac{T_{NL}(e,s)+T_{ML}(e,s)}{2}, \frac{T_{NU}(e,s)+T_{MU}(e,s)}{2}\right],$$
$$\left[\frac{I_{NL}(e,s)+I_{ML}(e,s)}{2}, \frac{I_{NU}(e,s)+I_{MU}(e,s)}{2}\right],$$
$$\left[\frac{F_{NL}(e,s)+F_{ML}(e,s)}{2}, \frac{F_{NU}(e,s)+F_{MU}(e,s)}{2}\right]\rangle \,|(e,s) \in K \times L\}$$

$$= N \,@\, M$$

which is prooved the part (i) of the theorem.

(ii) $M \,\$\, N \;= M \,\$ N$

$$M \,\$\, N = \Big\{\langle (e,s), \left[\sqrt{R_{ML}(e,s)\cdot R_{NL}(e,s)}, \sqrt{R_{MU}(e,s)\cdot R_{NU}(e,s)}\right],$$
$$\Big\{\left[\sqrt{T_{ML}(e,s)\cdot T_{NL}(e,s)}, \sqrt{T_{MU}(e,s)\cdot T_{NU}(e,s)}\right],$$

$$\left[\sqrt{I_{ML}(e,s)\cdot I_{NL}(e,s)},\sqrt{I_{MU}(e,s)\cdot I_{NU}(e,s)}\right],$$
$$\left[\sqrt{F_{ML}(e,s)\cdot F_{NL}(e,s)},\sqrt{F_{MU}(e,s)\cdot F_{NU}(e,s)}\right]\Big\}\rangle|(e,s)\in K\times L\}$$

$$=\Big\{\langle(e,s),\left[\sqrt{R_{NL}(e,s)\cdot R_{ML}(e,s)},\sqrt{R_{NU}(e,s)\cdot R_{MU}(e,s)}\right],$$
$$\Big\{\left[\sqrt{T_{NL}(e,s)\cdot T_{ML}(e,s)},\sqrt{T_{NU}(e,s)\cdot T_{MU}(e,s)}\right],$$
$$\left[\sqrt{I_{NL}(e,s)\cdot I_{ML}(e,s)},\sqrt{I_{NU}(e,s)\cdot I_{MU}(e,s)}\right],$$
$$\left[\sqrt{F_{NL}(e,s)\cdot F_{ML}(e,s)},\sqrt{F_{NU}(e,s)\cdot F_{MU}(e,s)}\right]\Big\}\rangle|(e,s)\in K\times L\}$$

$= N\ \$\ M$

Which is complete the evidence of $M\ @\ N\ = N\ @\ M$ and $M\ \$\ N\ = N\ \$\ M$.

Theorem: 3.8 Let us choose M also N be a IVTNFSs on K. Then, we partake the resulting

- **(i)** $(\mathbf{M^c@N^c})^{\mathbf{c}} = \mathbf{M\ @\ N}$,
- **(ii)** $(\mathbf{M^c\$N^c})^{\mathbf{c}} \neq \mathbf{M\ \$\ N}$.

Proof: Consider

$$M=\{<(e,s),[M_{ML}(e,s),M_{MU}(e,s)],[T_{ML}(e,s),T_{MU}(e,s)],$$
$$[I_{ML}(e,s),I_{MU}(e,s)],[F_{ML}(e,s),F_{MU}(e,s)]>|e\in K\ \&\ s\in L\}$$

And

$$N=\{<(e,s),[M_{NL}(e,s),M_{NU}(e,s)],[T_{NL}(e,s),T_{NU}(e,s)],$$
$$[I_{NL}(e,s),I_{NU}(e,s)],[F_{NL}(e,s),F_{NU}(e,s)]>|e\in K\ \&\ s\in L\}$$

Then, the complement of M, N and arithmetic operation as follows

$$M^c=\{<(e,s),[M_{ML}(e,s),M_{MU}(e,s)],[F_{ML}(e,s),F_{MU}(e,s)],$$
$$[1-I_{MU}(e,s),1-I_{ML}(e,s)],[T_{ML}(e,s),T_{MU}(e,s)]>|e\in K\ \&\ s\in L\}$$
$$N^c=\{<(e,s),[M_{NL}(e,s),M_{NU}(e,s)],[F_{NL}(e,s),F_{NU}(e,s)],$$
$$[1-I_{NL}(e,s),1-I_{NU}(e,s)],[T_{NL}(e,s),T_{NU}(e,s)]>|e\in K\ \&\ s\in L\}$$

Now,

$$M^c@N^c\ =\Big\{\langle(e,s),\left[\frac{M_{ML}(e,s)+M_{NL}(e,s)}{2},\frac{M_{MU}(e,s)+M_{NU}(e,s)}{2}\right]$$
$$\left[\frac{F_{ML}(e,s)+F_{NL}(e,s)}{2},\frac{F_{MU}(e,s)+F_{NU}(e,s)}{2}\right],$$
$$\left[\frac{1-I_{MU}(e,s)+1-I_{NU}(e,s)}{2},\frac{1-I_{ML}(e,s)+1-I_{NL}(e,s)}{2}\right],$$
$$\left[\frac{T_{ML}(e,s)+T_{NL}(e,s)}{2},\frac{T_{MU}(e,s)+T_{NU}(e,s)}{2}\right]\rangle\ |(e,s)\in K\times L\}$$

$$M^c@N^c\ =\Big\{\langle(e,s),\left[\frac{M_{ML}(e,s)+M_{NL}(e,s)}{2},\frac{M_{MU}(e,s)+M_{NU}(e,s)}{2}\right]$$

$$\left[\frac{F_{ML}(e,s)+F_{NL}(e,s)}{2},\frac{F_{MU}(e,s)+F_{NU}(e,s)}{2}\right],$$

$$\left[\frac{I_{ML}(e,s)+I_{NL}(e,s)}{2},\frac{I_{MU}(e,s)+I_{NU}(e,s)}{2}\right],$$

$$\left[\frac{T_{ML}(e,s)+T_{NL}(e,s)}{2},\frac{T_{MU}(e,s)+T_{NU}(e,s)}{2}\right]\rangle\,|(e,s)\in K\times L\}$$

$$(M^c@N^c)^c=\left\{\langle(e,s),\left[\frac{R_{NL}(e,s)+R_{ML}(e,s)}{2},\frac{R_{NU}(e,s)+R_{MU}(e,s)}{2}\right]\right.$$

$$\left[\frac{F_{NL}(e,s)+F_{ML}(e,s)}{2},\frac{F_{NU}(e,s)+F_{MU}(e,s)}{2}\right],$$

$$\left[\frac{1-I_{MU}(e,s)+1-I_{NU}(e,s)}{2},\frac{1-I_{ML}(e,s)+1-I_{NL}(e,s)}{2}\right],$$

$$\left[\frac{T_{NL}(e,s)+T_{ML}(e,s)}{2},\frac{T_{NU}(e,s)+T_{MU}(e,s)}{2}\right]\rangle\,|(e,s)\in K\times L\}$$

$$=\left\{\langle(e,s),\left[\frac{R_{NL}(e,s)+R_{ML}(e,s)}{2},\frac{R_{NU}(e,s)+R_{MU}(e,s)}{2}\right]\right.$$

$$\left[\frac{T_{NL}(e,s)+T_{ML}(e,s)}{2},\frac{T_{NU}(e,s)+T_{MU}(e,s)}{2}\right],$$

$$\left[\frac{I_{NL}(e,s)+I_{ML}(e,s)}{2},\frac{I_{NU}(e,s)+I_{MU}(e,s)}{2}\right],$$

$$\left[\frac{F_{NL}(e,s)+F_{ML}(e,s)}{2},\frac{F_{NU}(e,s)+F_{MU}(e,s)}{2}\right]\rangle\,|(e,s)\in K\times L\}$$

$$=M\ @\ N$$

Therefore, $(M^c@N^c)^c=M\ @\ N$.

Easy to verify the second part of the proof $(M^c\$N^c)^c\neq M\ \$\ N$.

4. Application of the Neutrosophic Fuzzy Sets in Profession determination

Profession choice is one of the most important challenges students face in making decisions about their future. It plays a crucial role in shaping their professional paths within the field of education. When students select a course, they initiate their academic journey and work towards finding a suitable career. Upon entering university, students take on greater responsibility and often prioritize choosing the right course over selecting a school. Their career and course decisions are influenced by various factors, such as academic interests and career aspirations, leading to diverse experiences and knowledge. In such cases, uncertainty arises, making decision-making more complex. To address this uncertainty, neutrosophic fuzzy set theory can be effectively utilized to analyze and identify problems in students' decision-making processes.

From the sample collection, we provide the shortest academic and professional usage of the $d_{NH}(M, N)$ measure of IVTNFS. The Student set is S = {First Student, Second Student, Third Student, Fourth Student}, the Occupation set is C = {Data Analysis, Data Science, Software developer & ML Engineer} and the subject set is Sub = {Physics, Electronics, Mathematics & Computer Science}, which is a group of work-related courses. Table 1 shows the requirements for majors and other courses taught according to IVTNFS values. Each performance is defined by four numbers, R_M, T_M, I_M and F_M, which are the membership existence, true, uncertain and false membership utility.

	Physics	**Electronics**	**Mathematics**	**Computer science**
Data Analysis	[0.3,0.3],[0.5,0.4], [0.6,0.3], [0.5,0.7]	[0.4,0.5],[0.3,0.4], [0.7,0.3], [0.2,0.3]	[0.5, 0.5], [0.3, 0.5], [0.6, 0.7], [0.4, 0.3]	[0.4, 0.6],[0.3,0.2], [0.5, 0.4], [0.5,0.6]
Data Science	[0.4,0.6],[0.5,0.4], [0.3, 0.5],[0.3,0.7]	[0.3,0.4],[0.4,0.6], [0.5,0.5], [0.9,0.5]	[0.4, 0.4], [0.4, 0.5], [0.6, 0.5], [0.5, 0.6]	[0.7, 0.9],[0.4,0.3], [0.5, 0.3], [0.4,0.4]
Software developer	[0.3,0.6],[0.2,0.4], [0.6,0.3], [0.5,0.8]	[0.5,0.4],[0.6,0.3], [0.4, 0.3],[0.7,0.5]	[0.5, 0.6], [0.4, 0.5], [0.3, 0.4], [0.3, 0.4]	[0.4,0.6], [0.7,0.5], [0.4,0.5], [0.4,0.2]
ML Engineer	[0.6,0.5],[0.6,0.3], [0.5,0.4], [0.7,0.9]	[0.6,0.3],[0.2,0.7], [0.5,0.3], [0.4,0.5]	[0.2, 0.3], [0.5, 0.6], [0.5, 0.4], [0.4, 0.7]	[0.3, 0.5],[0.4,0.7], [0.6,0.6], [0.3,0.2]

Table 1: Professions vs Subjects

The results obtained by the students after several tests are shown in Table 2

	Physics	**Electronics**	**Mathematics**	**Computer science**
First Student	[0.7,0.3],[0.5,0.4], [0.5,0.5], [0.3,0.5]	[0.6,0.7],[0.2,0.3], [0.6,0.5], [0.4,0.7]	[0.5, 0.4], [0.6, 0.3], [0.3, 0.5], [0.4, 0.8]	[0.5,0.4], [0.6,0.3], [0.7,0.3], [0.4,0.5]
Second Student	[0.5,0.4],[0.3,0.6], [0.7,0.4], [0.4,0.6]	[0.3,0.4],[0.5,0.3], [0.5,0.3], [0.5,0.3]	[0.5, 0.3], [0.3, 0.3], [0.4, 0.3], [0.5, 0.1]	[0.6,0.3], [0.2,0.7], [0.3,0.3], [0.4,0.5]
Third Student	[0.1,0.3],[0.6,0.3], [0.5,0.3], [0.4,0.6]	[0.5,0.4],[0.7,0.8], [0.7,0.2], [0.4,0.9]	[0.2, 0.7], [0.8, 0.6], [0.3, 0.5], [0.1, 0.3]	[0.2,0.6], [0.5,0.4], [0.6,0.5], [0.6,0.6]
Fourth Student	[0.6,0.5],[0.6,0.3], [0.5,0.4], [0.7,0.9]	[0.7,0.4],[0.5,0.6], [0.5,0.4], [0.3,0.6]	[0.6, 0.5], [0.6, 0.3], [0.4, 0.4], [0.6, 0.6]	[0.3,0.4], [0.4,0.9], [0.3,0.5], [0.6,0.5]

Table 2: Students vs Subjects

Result: Using Hamming normalized distance to calculate with the study, we get the following message with the distance for all students and all subjects.

IVTNFS	Data Analysis	Data Science	Software Developer	ML Engineer
First Student	0.1688	**0.1625**	0.1906	0.1750
Second Student	0.1594	0.1750	**0.1563**	0.2000
Third Student	**0.1781**	0.2344	0.1875	0.1969
Fourth Student	0.1719	0.1844	0.1688	**0.1594**

Table 3: shortest distance among Students vs Professions

In Table 3, we conclude that the shortest distance provides accurate location determination among students and professions by using the normalized Hamming distance measure.

Discussion: The first student chooses the Data Science Department, the second student chooses the Software Development Department, the third student chooses the Data Analytics Department, and the fourth student chooses the Machine Learning Engineer Department. We conclude that the use of normalized Hamming distance measure in IVTNFS act as a significant role in obtaining shortest distance. Suppose the distance between the student and the job is the shortest, the student chooses that job.

5. Conclusion

In this paper, we introduced few operations over the extended type of neutrosophic sets, namely IVTNFS, and studied few properties with numerical examples. Further we applied this IVTNFS concept in the determination of their Professions.

Future Work: We will study the concepts of similarity measure and also study the properties of various distance measures among the intervals temporal neutrosophic fuzzy sets with members. Further we can apply the above mentioned concepts in the real-life application of IVTNFS, such as pattern recognition, supplier selection, and image processing, and so on.

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