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Solving a Global-Mixed Integer Signomial Geometric Fractional Programming Problem

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Abstract. This article addresses mixed integer fractional signomial geometric programming (MIFSGP) problems, which have been widely used in industrial design. In this paper, first, we convert fractional signomial programming into a nonfractional problem so that it maintains its geometric structure. Then, convex relaxation is used to reach a mixed integer global solution. Although, in many cases, we obtain a better objective function value with this process, designers may still be dissatisfied with the rupture between the original objective function value and the relaxed value. Therefore, we apply a spatial branch and bound algorithm to decrease that distance to an acceptable extent and maintain the global solution. Finally, a real design problem is considered to evaluate the efficiency and accuracy of the proposed technique.

Keywords: geometric programming, fractional programming, mixed integer programming, non-convex functions, spatial branch and bound algorithm.

1. **Introduction**

Fractional geometric programming (FGP) is applied to solve a class of geometric programming problems to minimize the fractional objective function under definite constraints. A few methods have been utilized in recent decades to convert a fractional signomial objective function into a nonfractional signomial objective function to reach the optimal solution via common mathematical programming techniques. In a mixed integer fractional signomial geometric programming problem (MIFSGP), the objective function is a quotient of two signomial functions subject to certain constraints with integer and continuous decision variables; this approach is an essential part of geometric programming problems in the wide scope of engineering design, management and finance. For instance, Ray and Saini (2001) [10], Arora (1989) [3], Tsai (2005) [12], and Shirinnejad et al [11] addressed a few methods to solve real (FGP) problems in design engineering.

In a real nonfractional signomial case, the number of iterations of the solver is also significantly reduced, and an integer solution is obtained by reformulating the problem, changing negative power variables to positive power variables and applying our proposed approach. In this work, first, we define a new variable for the fractional objective function. This technique formulates an MIFGP problem by adding new constraints to a nonfractional mixed integer geometric programming (MIGP) problem.

Since the formulated problem still contains some concave terms in the objective functions or constraints and is sometimes more than before, this feature still results in local solutions to the problem. In addition, the existence of integer variables generally makes the feasible region non-convex. Therefore, applying convex relaxation in most cases will yield the lowest possible value for the objective function of the original (MIFGP) problem. Therefore, by using a spatial branch and bound algorithm (SBB), we find a feasible solution for the MIGP problems and the tightest global lower bound to the local original lower bound for the objective function simultaneously.

2. Mixed integer signomial geometric fractional programming (MISGFP) problems

A signomial function consists of a sum of positive or negative terms that are products of power functions, i.e.,

$$p(x) = f(x_1, x_2, \ldots, x_n) = \sum_{t=1}^{T} \sigma_t c_t \prod_{j=1}^{n} x_j^{\alpha_{tj}}, \tag{1}$$

Where x is a vector that contains positive variables of real or integer types. In each signomial function, T and n represent the number of terms and variables, respectively. C_t is the absolute value of the coefficients, and σ_t is the sign of the coefficient (+1 or −1). If $\sigma_t = +1$ in all terms of a signomial function, the function is called a posynomial. A (MISGFP) programming problem is defined in its typical form as follows:

Obtain $x = (x_1, x_2, x_3, \ldots, x_n)^T$ to

$$\begin{aligned} &\text{Minimize} \qquad f(x) = \frac{p(x)}{q(x)}, \qquad q(x) > 0, \\ &\text{Subject to} \qquad g_i(x) \le \xi_i, \qquad \xi_i = \pm 1, \quad i = 1, 2, \ldots, m. \end{aligned} \tag{2}$$

where $p(x)$, $q(x)$ and $g_i(x)$ are signomial functions and

$$x = (x_1, x_2, x_3, \ldots, x_n)^T \in \boldsymbol{X},$$

$\boldsymbol{X}$ is a vector of real or integer positive variables x,

$$0 \le \underline{x}_j \le x_j \le \bar{x}_j, \qquad j = 1, 2, \ldots, n.$$

It is assumed that the mentioned problem is feasible and has an optimal solution.

3. Strategy of reformulation

This article presents a convenient technique for converting a non-convex problem (MISGFP) into a convex nonfractional mixed integer signomial geometric programming problem (MISGP). Consider the following signomial geometric fractional programming problem:

$$\begin{aligned} &\text{Minimize} \qquad f(x) = \frac{p(x)}{q(x)} = \frac{\sum_{t=1}^{T_0} \sigma_{0t} c_{0t} \prod_{j=1}^{n} \ x_j^{\alpha_{0tj}}}{\sum_{t=1}^{T_0'} \sigma_{0t}' c_{0t}' \prod_{j=1}^{n} \ x_j^{\alpha_{0tj}'}}, \\ &\text{Subject to} \qquad g_i(x) \le \xi_i, \qquad \xi_i = \mp 1, \qquad i = 1, 2, \ldots, m. \end{aligned} \tag{3}$$

where

$$g_i(x) = \sum_{t=1}^{T_i} \sigma_{it} c_{it} \prod_{j=1}^{n} x_j^{\alpha_{it}},$$

$q(x) > 0,$

for $x \in \boldsymbol{X}$ and for all $x = (x_1, x_2, x_3, \ldots, x_n)^T \in \boldsymbol{X} \subseteq \mathbb{R}, \qquad 0 \le \underline{x}_j \le x_j \le \bar{x}_j,$

$j = 1, 2, \ldots, n,$

$T_0 =$ Number of terms in $p(x)$,

$T_0' =$ Number of terms in $q(x)$,

$c_{0t}, c_{0t}' \in \mathbb{R}^+,$

let $q(x) = x_{n+1},\ x_{n+1} > 0.$

We have:

$$f(x) = x_{n+1}^{-1} * p(x),$$

To find the lower and upper bounds of x_{n+1}, we should solve the following two sub problems:

$$\begin{aligned} L_{n+1} &:= \min q(x), \\ \text{Subject to} \quad & g_i(x) \le \xi_i, \\ & \xi = \mp 1, \qquad i = 1, 2, \dots, m, \quad \text{for } x \in \boldsymbol{X}. \end{aligned}$$

and

$$\begin{aligned} U_{n+1} &:= \max q(x), \\ \text{Subject to} \quad & g_i(x) \le \xi_i, \\ & \xi = \mp 1, \qquad i = 1, 2, \dots, m, \quad \text{for } x \in \boldsymbol{X}. \end{aligned}$$

Therefore, problem (3) leads to the following signomial geometric programming problem, which contains a new equality constraint, $x_{n+1}^{-1} * q(x) = 1$.

$$\text{Minimize} \quad x_{n+1}^{-1} * p(x) \tag{4}$$

$$\text{Subject to} \quad x_{n+1}^{-1} * q(x) = 1, \tag{5}$$

$$\begin{aligned} & g_i(x) \le \xi_i, \\ & \xi_i = \mp 1, \qquad i = 1, 2, \dots m, \\ & L_{n+1} \le x_{n+1} \le U_{n+1}, \\ & x > 0, \end{aligned}$$

We can replace equation (4) with two inequalities as follows:

$$x_{n+1}^{-1} * q(x) \le 1, \tag{6}$$

$$-x_{n+1}^{-1} * q(x) \le -1. \tag{7}$$

Or equivalent of(7)

$$x_{n+1} * q^{-1}(x) \le 1. \tag{8}$$

Therefore, the original MISGFP problem is reformulated to:

$$\begin{aligned} \text{Minimize} \quad & x_{n+1}^{-1} * p(x), \\ \text{Subject to} \quad & x_{n+1}^{-1} * q(x) \le 1, \\ & x_{n+1} * q^{-1}(x) \le 1, \\ & g_i(x) \le \xi_i, \\ & \xi_i = \mp 1, \qquad i = 1, 2, \dots, m, \\ & L_{n+1} \le x_{n+1} \le U_{n+1}, \\ & x > 0. \end{aligned} \tag{9}$$

Where

$$p\left(x\right)=\sum_{t=1}^{T_0}\sigma_{0t}c_{0t}\prod_{j=1}^{n}x_j^{\alpha_{0tj}},$$
$$q\left(x\right)=\sum_{t=1}^{T_0'}\sigma_{0t}'c_{0t}'\prod_{j=1}^{n}x_j^{\alpha_{0tj}'},$$
$$g_i(x)=\sum_{t=1}^{T_i}\sigma_{it}c_{it}\prod_{j=1}^{n}x_j^{\alpha_{itj}},$$
$$q(x)>0, \qquad \forall\, x=\left(x_1,x_2,\ldots,x_n\right)^T\in \boldsymbol{X}\subseteq\mathbb{R}, \qquad 0\le \underline{x}_j\le x_j\le \bar{x}_j, \qquad j=1,2,\ldots,n.$$

Proposition 3.1. *If* $0<q\left(x^*\right)=x_{n+1}^*$*, where* x^* *is the optimal solution of* (4)*,* x_n^* *is the optimal solution of* (3)*,*

$$\forall x\neq x^*, \qquad \left(x_{n+1}^*\right)^{-1}*p\left(x^*\right)\le\left(x_{n+1}^*\right)^{-1}*p(x). \tag{10}$$

Proof. Suppose $x^*=(x_1^*,x_2^*,\ldots x_n^*)$ is an optimal solution of (4); consequently,

$$\left(x_{n+1}^*\right)^{-1}*q\left(x^*\right)=1,$$
$$g_i(x^*)\le\xi_i,$$
$$\xi=\mp1, \qquad i=1,2,\ldots,m.$$

Thus, x^*, is satisfied in the constraint of (3). Additionally, by replacing

$$q\left(x\right)=x_{n+1},$$

and

$$q\left(x^*\right)=x_{n+1}^*,$$

in the objective function (4), we have:

$$\begin{gathered} x_{n+1}^{*-1}*\sum_{t=1}^{T_0}\sigma_{0t}c_{0t}\prod_{j=1}^{n}x_j^{*\alpha_{0tj}}\le x_{n+1}^{-1}*\sum_{t=1}^{T_0}\sigma_{0t}c_{0t}\prod_{j=1}^{n}x_j^{\alpha_{0t}} \\ \frac{\sum_{t=1}^{T_0}\sigma_{0t}c_{0t}\prod_{j=1}^{n}x_j^{*\alpha_{0tj}}}{\sum_{t=1}^{T_0'}\sigma_{0t}'c_{0t}'\prod_{j=1}^{n}x_j^{*\alpha_{0tj}'}}\leqslant\frac{\sum_{t=1}^{T_0}\sigma_{0t}c_{0t}\prod_{j=1}^{n}x_j^{\alpha_{0tj}}}{\sum_{t=1}^{T_0'}\sigma_{0t}'c_{0t}'\prod_{j=1}^{n}x_j^{\alpha_{0tj}'}}. \end{gathered} \tag{11}$$

Hence, x^* is an optimal solution of (3). □

4. Convexification strategies

Converting to convex relaxation is an efficient tool for obtaining a global solution in non-convex (MIGP) problems. It is applied to expand the feasible set of (4) and achieve a lower bound on the optimal solution of the (MIGP) problems. Since the nonlinear functions in the constraints and objective of (4) are signomial functions, each signomial function is convex if all the terms are convex [7]. The requirements of convexification for every signomial term are provided with the following theorem. See [8].

Theorem 4.1 (Maranas and Floudas (1995)). *A positive signomial term*

$$f(x) = c\prod_{i=1}^{n} x_i^{\alpha_i},$$

is convex if one of the following requirements holds:

(1) $\alpha_i \leq 0, \qquad (i = 1, \ldots, n).$

(2) $\exists\, k \neq i, \qquad \alpha_k > 0, \qquad \alpha_i \leq 0, \qquad \sum_{i=1}^{n} \alpha_i \geq 1.$

A negative signomial term

$$f(x) = c\prod_{i=1}^{n} x_i^{\alpha_i} (c < 0),$$

is convex if $\alpha_i > 0$ *(for* $i = 1, \ldots, n$*) and*

$$\sum_{i}^{n} \alpha_i \leq 1.$$

Theorem 4.1 states that it is possible to convexify every positive or negative signomial term by using power transformations; therefore, this fact is applied in this paper to convexify non-convex signomial functions.

After utilizing power transformations to convexify a signomial term, the power transformation functions need to be approximated by piecewise linear functions. This study used a standard piecewise linear approximation, special order set type 2 (SOS2) (Beale and Tomlin) [5].

5. Bound assessment algorithm

The spatial branch-and-bound algorithm is one of the best-known techniques for obtaining a precise or at least an ε-approximate solution to a non-convex mixed integer geometric programming problem. The lower bound of the objective function is computed for the entire feasible region. The SBB algorithm works in a common application of tight lower bounds estimated through convex relaxation. To improve the quality of our method used to determine the lower bounds of the MIFGP (2), we applied (using AMPL [6]), a streamlined “partial SBB” algorithm [1, 9], to find ε-approximate solutions for every small positive ε. This algorithm works recursively by dividing the search area along the coordinate direction that contributes most to the difference between the lower and upper bounds on the optimal objective function value determined in every subproblem. For a nonlinear minimization problem, the lower bound is usually computed by constructing and solving a convex relaxation. The upper bound can simply be a local minimum found by an NLP solver.

At each branching step, the most reassuring node is registered, and the others are discarded. Since our purpose is to solve an MIGFP problem with integer variables, branching may be necessary for both continuous and integer decision variables.

6. **Proposed Algorithm: The partial depth-first SBB algorithm**

Input P as a non-convex MIGP problem.

Devote count while converting the non-convex problem to a convex one.

(1) Allocate o→ count and

Find x^*, through an objective function with the value S^*, which can solve P in local form.

(2) A convex relaxation R is used for the primal problem P.

(3) Find the minimum value of S to obtain the optimum solution S^* using relaxation R.

(4) Choose branching point $\grave{x}_i$ for the ith variable.

(5) We add a new constraint as follows to define P_0 as P

$$x_i^L \leq x_i \leq \grave{x}_i.$$

(6) P_1 is defined as P by adding the constraint $\grave{x}_i \leq x_i \leq x_i^U$.

(7) P_k is used as a convex relaxation for R_k, $k \in \{0, 1\}$.

(8) Similarly, we used $\grave{x}_k$ as the optimum of P_k by the S_k value, $k \in \{0, 1\}$.

(9) Let $l = (S_0, S_1)$, (the best lower bound to S^*).

(10) For $S_l > S^*$, the algorithm is stopped since the node cannot be further improved.

(11) For $S_l < S^*$, devote $S_l \to S$ and $\grave{x}_l \to \grave{x}$.

(12) In the case of the feasibility of x^* in P_l, solve local P_l to find x_i^* with S_i^*.

(13) ?For $S_i^* < S^*$, let $S_i^* \to S^*$ and $x_i^* \to x^*$, to improve the non-convex problem solution.

(14) The global optimum is achieved if $|S^* - S| < \varepsilon$. Here ε is an acceptable error.

(15) Dedicates $P_l \to P$ and $\grave{x}_l \to \grave{x}$ to update the ith branching variable and branching point$\grave{x}_i$.

(16) Finally, the count is increased, and the algorithm is repeated.

7. **Numerical example**

Example 7.1. [2] This case pertains to a design problem of a journal bearing. Its structure is an inverse problem, where the eccentricity and attitude angle are achieved for a certain load and speed. The volume of steel, the thickness of the intermediate layer and nickel barrier, and the dimensions of the plated overlay of the journal bearing are assumed to be unknown. Hence, some of these parameters of the model are known and estimated by engineers. Suppose that x_1 is the radial clearance, x_2 is the fluid force, x_3 is the diameter, x_4 the rotation speed and x_5 is the length-to-diameter ratio. The variables of these parameters are concentrated between some specified lower and upper bounds.

The following mathematical programming formulation depicts the above mentioned problem as a signomial geometric programming problem:

$$\begin{aligned} \text{Minimize} \quad & 0.5x_1^2x_2x_4x_5 + 1.1x_1^{-1}x_2^{-1}x_3^{-1}, \\ \text{Subject to} \quad & 8.4x_1x_2^{-1}x_3^{-1}x_4^{-1}x_5 \le 4.2, \\ & 0.5x_2x_3 + x_1 + x_4^{-1}x_5^{-1} + 1.6x_3x_4 \le 1. \end{aligned}$$

The above problem is a concave signomial programming problem. Upon solving the problem, the local objective value was found to be $F = 3.561$. The objective function of the above problem can be easily converted into a fractional function as follows:

$$\frac{0.5x_1^3x_2^2x_3x_4x_5 + 1.1}{x_1x_2x_3},$$

Therefore, the original problem is modified by defining x_6 as a new variable where $x_6 = x_1x_2x_3$,

$$\begin{aligned} \text{Minimize} \quad & 0.5x_1^3x_2^2x_3x_4x_5x_6^{-1} + 1.1x_6^{-1}, \\ \text{Subject to} \quad & x_1x_2x_3x_6^{-1} \le 1, \\ & x_1^{-1}x_2^{-1}x_3^{-1}x_6 \le 1, \\ & 8.4x_1x_2^{-1}x_3^{-1}x_4^{-1}x_5 \le 4.2, \\ & 0.5x_2x_3 + x_1 + x_4^{-1}x_5^{-1} + 1.6x_3x_4 \le 1. \end{aligned}$$

The mentioned problem has seven non-convex terms for obtaining a global solution. Several power transformations and piecewise linear approximations are applied to underestimate convexified terms. Therefore, the non-convex primary programming problem is now converted to the folowing convex programming problem:

$$\begin{aligned} \text{Minimize} \quad & 0.5Z_1^6 y_2^{-2} y_3^{-1} y_4^{-1} y_5^{-1} x_6^{-1} + 1.1 x_6^{-1}, \\ & y_1^{-1} y_2^{-1} y_3^{-1} x_6^{-1} \leq 1, \\ & 8.4 k_1^5 x_2^{-1} x_3^{-1} x_4^{-1} y_5^{-1} \leq 4.2, \\ & 0.5 y_2^{-1} z_3^2 + k_1^5 x_4^{-1} x_5^{-1} + 1.6 z_3^2 y_4^{-1} \leq 1, \\ & x_1^{-1} x_2^{-1} x_3^{-1} y_6^{-1} \leq 1. \end{aligned}$$

where

$$\begin{aligned} & z_1 = x_1^{0.5}, Z_1 = L\left(x_1^{0.5}\right) \\ & k_1 = x_1^{0.2},\ K_1 = L\left(x_1^{0.2}\right) \\ & z_3 = x_3^{0.5},\ Z_3 = L\left(x_3^{0.5}\right) \\ & y_i = x_i^{-1}, \qquad Y_i = L\left(x_i^{-1}\right), \qquad\qquad i = 1,2,3,4,5,6, \\ & x_2 \in Z, \qquad x_1, x_2, x_3, x_4, x_5, x_6 \in \mathbb{R} \\ & 0.1 \leqslant x_1 \leqslant 0.5, \\ & 9 \leqslant x_2 \leqslant 17, \\ & 0.01 \leqslant x_3 \leqslant 0.21, \\ & 0.1 \leqslant x_4 \leqslant 1.7, \\ & 0.1 \leqslant x_5 \leqslant 1.8, \\ & 0.1 \leqslant x_6 \leqslant 0.5. \end{aligned}$$

$L\left(x_1^{0.5}\right), L(x_1^{0.2}), L\left(x_3^{0.5}\right), L\left(x_i^{-1}\right)$ are the piecewise linearized expressions of the non-convex terms $x_1^{0.5}, x_1^{0.2}, x_3^{0.5}$ and convex terms x_i^{-1} for $i = 1, 2, \ldots, 6$, respectively. Since $x_2 \in Z$, we choose integer break points to have an integer solution directly. By solving this program with LINGO 18.0 software, where the tolerable error was specified as 0.001, the following solution is obtained. A comparison of the original non-convex problem and the convex problem is shown in Table 1. Although the optimum value of the convex problem is much better than that of the non-convex problem and the number of iterations is much less than that of the original non-convex problem, this inequality may not be desirable for decision makers.

Table 1. The optimal conciliation obtained solutions.

problem	F	x_1	x_2	x_3	x_4	x_5	iteration
Non-convex(P)	$F^* = 3.55257$	0.3317	9.7297	0.1199	0.8687	1.5275	130
Convex(R)	$\hat{F} = 2.82846$	0.5	9	0.1275	0.9	0.9	43

Therefore, the difference between the objectives of two problems is:

$$\left|F^* - \hat{F}\right| = 0.72511.$$

Suppose the maximum difference between objectives F^* and $\hat{F}$ is accepted for $\varepsilon = 10^{-2}$. This means that we should find a tighter relaxation when implementing the proposed Algorithm.

Applying the suggested Algorithm, the following steps are implemented to reach the minimum value of $\varepsilon \leq 10^{-2}$. Therefore, the global solution of the convex problem is obtained with an acceptable difference from that of the primary non-convex problem. The remaining steps are presented in Table 2 as follows:

Hint: The branching variable, objective value and punching node are abbreviated as B.V., O.V., and P.N., respectively.

TABLE 2. Table 2 shows the global solution obtained after 16 iterations with an acceptable tolerance of $\varepsilon = 0.0086$ as follows.

steps	B.V	V.v in R_0	O. $V\grave{F}_0$	V. V in R_1	O. V$\hat{F}_1$	$\left\|\grave{F}_0 - F_0^*\right\|$	$\left\|\grave{F}_1 - F_0^*\right\|$	pruned node
1	x_5	0.85	1.97	1.5	$2.85b$	$b1.58257$	$b0.7$	R_0
2	x_5	1.5	2.875	1.52	2.895	0.67757	0.65757	R_0
3	x_2	8	2.895	10	2.82125	0.67757	0.73132	R_1
4	x_2	8	2.895	9	2.921	0.67757	0.63157	R_1
5	x_3	0.120	3.752271	0.2	2.41922	0.1997	1.1335	R_0
6	x_3	0.11	3.41	0.12	3.75227	0.14257	0.1997	R_0
7	x_3	0.105	3.85211	0.11	3.41	0.29954	0.14257	R_0
8	x_3	0.108	3.81242	0.110	3.41	0.25985	0.14257	R_0
9	x_3	0.109	3.43	0.110	3.41	0.12257	0.14257	R_1
10	x_3	0.1085	3.85	0.109	3.43	0.24743	0.12257	R_0
11	x_3	0.1088	3.44	0.109	3.43	0.11257	0.12257	R_1
12	x_4	0.7	3.44372	0.85	3.6887	0.10885	0.13613	R_1
13	x_4	0.76	3.4801	0.85	3.6887	0.07247	0.13613	R_1
14	x_4	0.756	3.54006	0.8	3.8799	0.01251	0.32733	R_1
15	x_1	0.45	3.5422	0.5	3.54006	0.01037	0.0125	R_1
16	x_1	0.43	3.54398	0.45	3.5422	0.0086	0.01037	R_0

Table 3. Comparison of the results obtained from the initial problem and the relaxed problem.

Problem	**O.V**	x_1	x_2	x_3	x_4	x_5
Original (P)	$F^* = 3.55257$	0.3317	9.729	0.1199	0.8687	1.5275
Relaxed (R)	$\hat{F} = 3.54398$	0.43	9	0.1088	0.706	1.52

The results of the present algorithm indicate the effectiveness of the spatial branch and bound technique. According to Table 3, we conclude that with the help of the proposed algorithm, we reach a global optimum, which is very close to the solution of the original problem and is sometimes even an integer solution to the problem of interest.

8. Conclusion

In this paper, a mixed-integer fractional geometric programming problem (MIFGP) as an NP-hard problem is discussed. To reach a global solution, we first converted the non-convex geometric programming problem to a convex problem to underestimate the original non-convex problem. We used the spatial branch and bound technique to increase the consistency and efficiency of the relaxation strategies. The results showed that implementing this algorithm on the convexified signomial programming problem can make the lower bound of the relaxed problem much closer to the lower bound of its non-convex programming problem.

Conflict of interest: the authors are hereby declare that there is no any conflict of interest regarding the present article.

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