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Edge connectivity of a neutrosophic graph

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Abstract

Neutosophic graphs are an extension of fuzzy and intuitionistic fuzzy graphs by including the uncertainty, vagueness, and indeterminacy that are normal in the real world. This paper looks into the edge connectivity of a neutrosophic graph, which is a basic parameter that shows how strong and fault-tolerant networks are that are modelled by these graphs. Edge connectivity, which is the smallest number of edges that need to be taken away from a graph to make it trivial or disconnected, is a key concept in figuring out how strong and resilient networks are in many situations. This paper also addresses computational challenges related to determining edge connectivity in neutrosophic graphs. We develop efficient algorithms that minimize computational overhead and ensure accuracy in identifying the critical edge sets. We analyze the performance of these algorithms through both theoretical complexity assessments and empirical evaluations on benchmark datasets. Some of the most important things that the study found were critical edges that, when removed, have a big effect on how connected the graph is and how indeterminacy affects the strength of networks. The research underscores the importance of incorporating neutrosophic parameters into graph connectivity studies to better model and analyse systems characterized by uncertainty and partial knowledge.

Keywords: Cut-edge; Edge connectivity; Graph algorithms; Neutrosophic graph

1. Introduction

Zadeh [1] introduced the theory of fuzzy sets, revolutionizing the handling of uncertainty and imprecision in mathematical models. Fuzzy set theory added vagueness to classical set theory by giving membership grades from 0 to 1. This enabled its application in various domains, including decision-making [2, 3]. Over the years, researchers have built on this base to create fuzzy graphs (FGs), which model relationships when there is uncertainty. FGs have been used in communication networks, social systems, and optimization problems [4, 5]. An important generalization of fuzzy sets, known as intuitionistic fuzzy sets (IFSs), was introduced by Atanassov [6]. IFSs incorporate an additional parameter, the degree of non-membership, to address uncertainty with greater precision. This addition made it possible for intuitionistic fuzzy graphs (IFGs), which have been studied a lot for their ability to model systems with two levels of uncertainty, like social networks, decision support systems, and cell phone networks [7, 8, 9]. Researchers have extensively studied the connectivity properties of fuzzy and intuitionistic fuzzy graphs. They have focused on measures like cut vertices, bridges, and strong paths, which are important for understanding how stable and resilient networks are.

In spite of these improvements, both FGs and IFGs don't deal with systems that are inherently uncertain, like relationships that can't be clearly put into membership or nonmembership categories. To get around this problem, Smarandache came up with neutrosophic sets [10], a structure with three parts: truth-membership (TT), indeterminacy-membership (I), and falsity-membership (F). Then Neutrosophic graphs (NGs) take this idea and apply it to graph theory, giving us a better way to model networks that are uncertain, vague, and uncertain all at the same time. Then Raut et al. wrote a lot of papers about neutrosophic theory, Fermatean neutrosophic theory, and how they could be used [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Connectivity is a fundamental parameter for any graph structure, as it reflects the robustness and reliability of the network it represents. The edge connectivity of a graph, defined as the minimum number of edges whose removal disconnects the graph, plays a critical role in evaluating network stability. A lot of research has been done on edge connectivity for crisp, fuzzy, and intuitionistic fuzzy graphs, but not as much on how it applies to neutrosophic graphs. The fact that NGs contain uncertainty adds to their complexity, calling for new ways to measure and analyze connectivity.

In this paper, we look into how edges connect in neutrosophic graphs and come up with new definitions and methods that work with this type of graph. We include the truth, indeterminacy, and falsity parameters of edges in the analysis. This gives us a more complete picture of how robust a network is. Section-2 provides the necessary preliminaries, including fundamental concepts of neutrosophic graphs. Section-3 presents the mathematical formulation of neutrosophic edge removal and the neutrosophic edge measure. Section-4 presents a theorem, which presents neutrosophic edge connectivity is the classical edge connectivity of the graph. Section-5 presents an algorithm to compute edge connectivity in a neutrosophic graph. Section-6 presents an example, and section-7 defines results and Finally, Section-8 concludes the study with key findings and outlines potential directions for future research. This study adds to the basic ideas of neutrosophic graph theory and gives us useful tools for looking at how strong complex systems are when there is uncertainty, vagueness, and indeterminacy.

2. Preliminaries

2.1 Neutrosophic Graph

A neutrosophic graph $G = (V, E, \tau, \iota, \phi)$ consists of:

- V: Set of vertices.
- E: Set of edges.
- $\tau: E \to [0,1]$ represent Truth membership function.
- $\iota: E \to [0,1]$ represent Indeterminacy membership function.
- $\phi: E \to [0,1]$ represent Falsity membership function.

Each edge $e \in E$ is represented as $e = (u, v, \tau, \iota, \phi)$, where $\tau, \iota \, and \, \phi$ satisfy $0 \leq \tau_e + \iota_e + \phi_e \leq 1$.

2.2 Edge Connectivity

For a graph G:

- Edge connectivity $\lambda(G)$: Minimum number of edges that need to be removed to disconnect G.

- For neutrosophic graphs, edge connectivity considers the neutrosophic weights (τ, ι, ϕ)

3. Mathematical Formulation

3.1 Neutrosophic Edge Removal

Let $E' \subseteq E$ be a subset of edges. Removing E' results in a subgraph $G' = (V, E \setminus E')$. The neutrosophic impact of removing E' is defined as:

$d(G, G') = \sum_{e \in E'} ((1 - \tau_e) + \iota_e + \phi_e)$.

3.2 Edge Connectivity Measure

The neutrosophic edge connectivity $\lambda_{\mathrm{N}}(\mathrm{G})$ is:

- $\lambda_N(G) = min(\lambda_N(G), d(G, G'))$is disconnected $d(G, G') = \sum_{e \in E'} ((1 - \tau_e) + \iota_e + \phi_e)$.

4. Theorem:

For any neutrosophic graph $G = (V, E, \tau, \iota, \phi)$ the neutrosophic edge connectivity $\lambda_N(G)$ satisfies:$\lambda_N(G) \geq \lambda(\mathrm{G})$, where $\lambda(G)$, is the classical edge connectivity of the graph.

Proof:
From Definitions of Classical Edge Connectivity λ(G), The minimum number of edges that need to be removed to disconnect the graph G.

Neutrosophic Edge Connectivity $\lambda_N(G)$ Defined as:

$$\lambda_N(G) = min \sum_{e \in E'} \big((1 - \tau_e) + \iota_e + \phi_e\big)$$

where (τ, ι, ϕ) are the truth, indeterminacy, and falsity membership values of the edges in E'.

Steps-1: Classical Case as a Special Case:

- If we assign $\tau_e = 1, \iota_e = 0$ and $\phi_e = 0$ for all $e \in E$, the neutrosophic graph G reduces to a classical graph.
- Under this assignment: $\sum_{e\in E'}((1-\tau_e)+\iota_e+\phi_e) = \sum_{e\in E'}\big((1-1)+0+0\big) = \mathrm{E}'$

  Hence, $\lambda_N(G) = \lambda(\mathrm{G})$ in the classical case.

Steps-2: General Neutrosophic Graph:

For a neutrosophic graph, edge weights include τ_e, ι_e, ϕ_e The neutrosophic impact of removing an edge e is given by: $w(e) = 1 - \tau_e + \iota_e + \phi_e$.. This weight $w(e) \geq 0$ since $0 \leq \tau_e + \iota_e + \phi_e \leq 1$ and $\tau_e + \iota_e + \phi_e \leq 1$.

Steps-3 : Minimum Edge Set to Disconnect G:

- Let E' be the edge set with the minimum neutrosophic impact that disconnects G.
- The neutrosophic impact is:

$$d(G, G') = \sum w(e) = \sum_{e\in E'} ((1 - \tau_e + \iota_e + \phi_e)$$

- Since $w(e) \geq 1 - \tau_e$, the neutrosophic impact satisfies: $d(G, G') \geq \mid E' \mid$, where $\mid E' \mid$ is the number of edges in E'.

Steps-4: Relation to Classical Edge Connectivity:

The classical edge connectivity $\lambda(G)$ corresponds to the minimum $\mid E' \mid$ for which G' becomes disconnected.

- Hence, the neutrosophic edge connectivity, which considers $w(e) \geq 1$ for all edges in the classical sense, satisfies: $\lambda_N(G) \geq \lambda(G)$.

The neutrosophic edge connectivity $\lambda_N(G)$ is always greater than or equal to the classical edge connectivity $\lambda(G)$ because neutrosophic weights assign additional contributions from indeterminacy and falsity measures, making $\lambda_N(G)$at least as large as $\lambda(G)$.

5. Algorithm: Compute Edge Connectivity in a Neutrosophic Graph

Input:

- A neutrosophic graph $G = (V, E, \tau, \iota, \phi)$

  Where, V is Set of vertices and E is Set of edges with neutrosophic weights (τ, ι, ϕ).

Output:

- The neutrosophic edge connectivity $\lambda_N(G)$, the minimum neutrosophic impact required to disconnect the graph.

Steps:

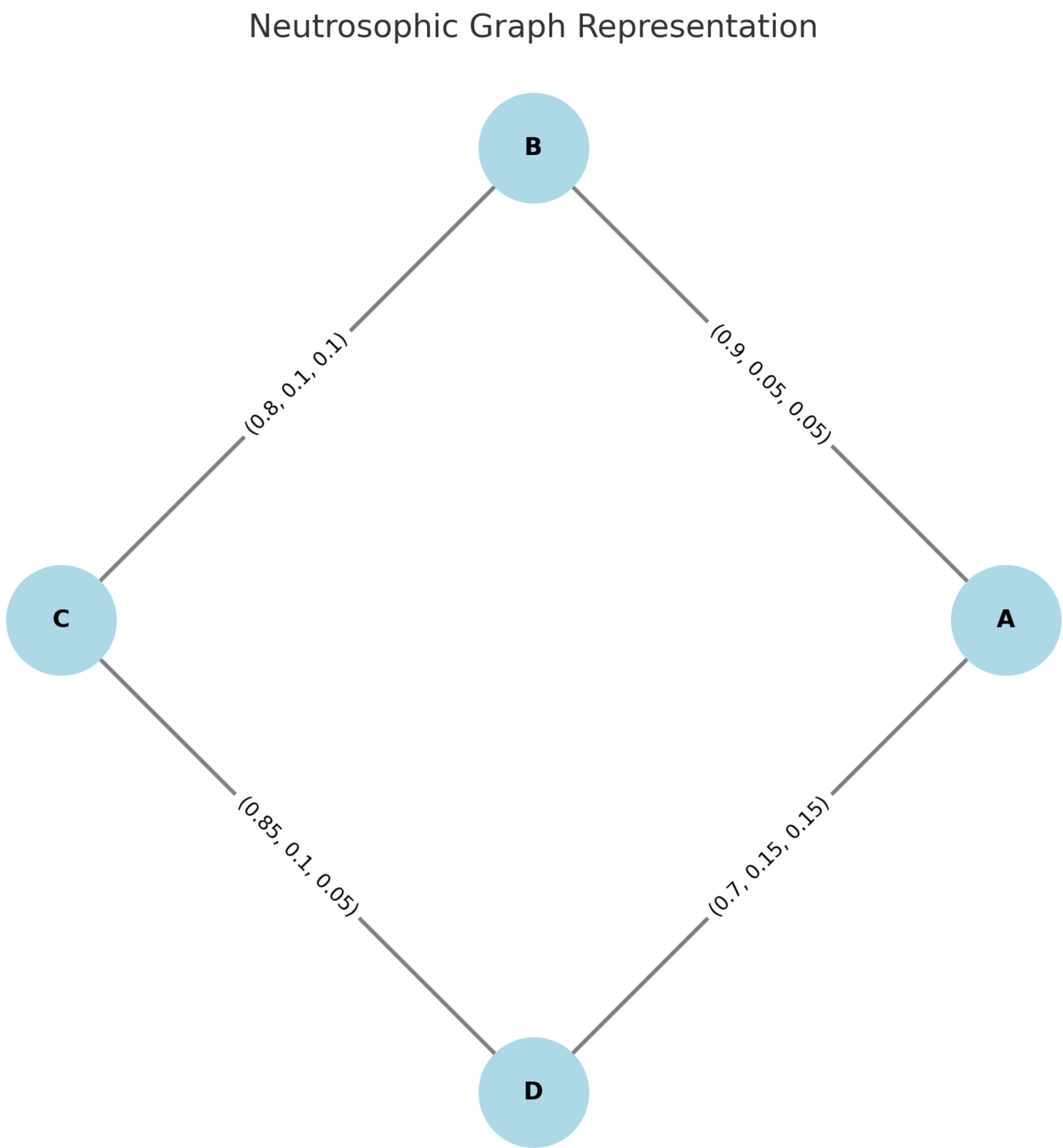
2. Initialization:
   - Set $\lambda_N(G) \rightarrow \infty$.
3. Identify Edge Subsets:
   - List all subsets of edges $E' \subseteq E$.
4. Iterate Over Subsets:
   - For each E', perform the following:

- Remove edges in E' from G, forming a subgraph $G' = (V, E \setminus E')$.
- Check if G' is disconnected (using graph traversal techniques like DFS or BFS).

5. Calculate Neutrosophic Impact:
   - For each subset E' that disconnects G, compute the neutrosophic impact: $d(G, G') = \sum_{e \in E'}((1 - \tau_e) + \iota_e + \phi_e)$.
6. Update Edge Connectivity:
   - Update $\lambda_N(G) = min(\lambda_N(G), d(G, G'))$
7. Return Result:
   - Return the computed $\lambda_N(G)$.

6. Example: Edge Connectivity Computation

Here is the graphical representation of the neutrosophic graph from the provided example. Each edge is labelled with its corresponding neutrosophic weights (τ, ι, ϕ).



Graph Description:

- Vertices: $V = \{A, B, C, D\}$.
- Edges: $E = \{e_1, e_2, e_3, e_4\}$
  - $e_1 = (\mathrm{A}, \mathrm{B}) = (0.9, 0.05, 0.05)$
  - $e_2 = (\mathrm{B}, \mathrm{C}) = (0.8, 0.1, 0.1)$
  - $e_3 = (\mathrm{C}, \mathrm{D}) = (0.85, 0.1, 0.05)$

- $e_4 = (\mathrm{D}, \mathrm{A}) = 0.7, 0.15, 0.15)$

Steps:

1. Edge Subsets:
   - Identify subsets of edges that disconnect the graph:
     - $\{e_1, e_3\}$: Disconnects A and C from B and D.
     - $\{e_2, e_4\}$: Disconnects B and D from A and C.
2. Compute Neutrosophic Impact:
   - For$\{e_1, e_3\}$

   $d(G, G') = (1 - 0.9 + 0.05 + 0.05) + (1 - 0.85 + 0.1 + 0.05) = 0.2 + 0.3 = 0.5$

   - For$\{e_2, e_4\}$:

   $d(G, G') = (1 - 0.8 + 0.1 + 0.1) + (1 - 0.7 + 0.15 + 0.15) = 0.4 + 0.6 = 1.0$
3. Minimum Neutrosophic Impact:

$$\lambda_N(G) = 0.5$$

7. Result

The neutrosophic edge connectivity

$$\lambda_N(G) = 0.5$$

8. Conclusion

In this paper, we advance our theoretical understanding of edge connectivity in neutrosophic graphs and provide practical tools for its application in diverse fields. There are suggestions for future research, such as looking into how vertex connectivity changes in neutrosophic graphs and how it might be used in new areas like artificial intelligence, cybersecurity, and complex system analysis. This work lays the groundwork for further exploration of neutrosophic graph theory and its implications for network science and decision-making under uncertainty.

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